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## Expectations, Monetary Policy, and the Misalignment of Traded Goods Prices

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Between the last day of March 2002, and the last day of December 2004, the price of a barrel of crude oil rose from \$26.31 to \$43.45, a 65.1 percent increase. This represents a 55.1 percent real price increase, relative to the U.S. consumer price index (CPI). Over the same period, the price of a barrel of oil rose from €30.18 to €32.09, a 6.3 percent increase. Relative to the French CPI, this was a 0.7 percent increase, and relative to the German CPI, a 2.5 percent increase. Apparently, the United States experienced a major oil price increase, but Europe did not.

How can this be? Mechanically, these figures indicate that there was a very large depreciation of the real CPI dollar exchange rate relative to France and Germany. In fact, in nominal terms, the dollar depreciated 55.3 percent over this period, while inflation in the United States was only slightly higher than in France and Germany.

It would be absurd to attempt to explain these movements in a purely real, neoclassical model. Such an explanation would require a massive increase in the excess supply of goods that are weighted heavily in the U.S. consumption basket, relative to the excess supply of goods weighted heavily in the French and German baskets.

However, the movement in the nominal exchange rate does not seem as difficult to comprehend. Asset prices are known to be volatile. While there are difficulties reconciling the volatility of nominal exchange rates with the predictions of rational expectations present value models, these asset pricing models might be able to account for big swings in nominal exchange rates by appealing to changes in expectations about future inflation or money growth, for example.<sup>1</sup>

If there are some nominal goods prices that are sticky in euros, and some that are sticky in dollars, these nominal exchange rate swings will lead to relative price changes that do not reflect efficient responses to relative scarcity. This is the point made by Devereux and Engel (2006, here-

inafter referred to as DE06)—when some goods prices are sticky, there is a conflict between the exchange rate's role as an asset price, and its role as a determinant of relative goods prices. As an asset price, the exchange rate might efficiently respond to news about future conditions that affect the value of one currency relative to the other. But these factors that determine the nominal exchange rate are unlikely to be the same factors that would yield efficient relative goods price changes. Goods prices should be determined by supply and demand, and not by expectations of future monetary conditions.

It is important to emphasize here that we are not referring to a situation in which foreign exchange is mispriced. In recent years, policymakers have examined the problems for monetary policy when asset prices contain bubbles or otherwise appear not to be priced rationally. Although such mispricing of exchange rates might lead to problems similar to the ones we examine here, we specifically only consider the case in which the nominal exchange rate is determined by a no-bubbles rational expectations equilibrium.

Devereux and Engel (2006) recommend that monetary policy be used to eliminate the effects of news of future fundamentals on exchange rates. Since exchange rates and other asset prices are primarily determined in the short run by reaction to news of the future rather than shocks to the current fundamentals, DE06 suggest that an optimal monetary policy would largely eliminate unanticipated exchange rate movements.

Our purpose in this paper is to enrich the model of DE06. Earlier work, which has not considered the effects of news on exchange rates, has emphasized the importance of the price-setting assumption for monetary policy rules in the open economy. Producers in each country set the price of their output in advance, but there are two different models of pricing behavior. If the price is set in the producer's currency and sold at the same price in the foreign country, the model assumes producer-currency pricing (PCP). If producers set a price in the home country for sale to home consumers, and a price in the foreign currency for sale to foreign households, we say there is local-currency pricing (LCP). The literature has emphasized that when pricing decisions are closer to the LCP configuration, optimal monetary policy stabilizes nominal exchange rates.<sup>2</sup>

Here, we build a model very similar to that in Devereux and Engel (2003), in which prices are set one period in advance. However, we show that optimal (cooperative) monetary policy rules should expunge the effects of news of future fundamentals on exchange rates, irrespective of

the assumption of price setting. We shall refer to this as stabilizing the exchange rate, because the optimal policy makes the conditional variance of exchange rates small. Exchange rate shocks should occur only when there are shocks to current fundamentals.

It is certainly true that stabilizing the nominal exchange rate is a more important consideration for more open economies. But some (for example, Kollmann 2004) have claimed that there is essentially no gain from stabilizing the exchange rate of the currency of two countries that trade very little. So, since U.S.-European trade represents a small fraction of their respective gross domestic products (GDPs), monetary policy can safely ignore the euro/dollar fluctuations according to this point of view.

But our view is that the volume of U.S.-European trade is a poor yardstick to gauge the impact of the exchange rate on the economies of the United States and Europe. A change in that exchange rate leads potentially to misalignments in the prices of all traded goods. The fraction of goods whose prices are set or strongly influenced by international markets is much larger than the import/GDP ratio. Oil prices are one example. The drop in the value of the dollar in 2002 to 2004 meant that Europe was insulated from the oil price shock that hit the United States. Or put another way, if the dollar had not depreciated, the price of oil in Europe would have risen also. A smaller dollar price increase would have equilibrated the oil market, reducing the oil shock for the United States.<sup>3</sup>

We do not attempt here to build and calibrate a realistic full macro model. We explore a series of toy models in this paper as a prelude to further work. We believe that this approach helps to isolate the important features of open economies that might influence monetary policy decisions.

As in DE06, it is helpful to conclude this introductory section with a discussion of how our arguments in this paper differ from the existing literature. First, the existing monetary policy literature does not explicitly consider the effect of news in open-economy models, except as agents learn from changes in current macro fundamentals. Second, optimal policy in this setting is not achieved simply by eliminating inflation. The sticky price distortion requires eliminating the effects of news on exchange rate changes, even if inflation is controlled. Third, the argument for stabilizing exchange rates is different than arguments put forth for stabilizing other asset prices. The only distortion in our stylized model is from sticky prices. We deliberately have eliminated all asset

market distortions to make our point clear. The reason that exchange rates need to be controlled even when inflation is driven to zero, even though the only distortion comes from sticky prices, is that prices are set in each currency so an exchange rate change leads to a potentially undesirable relative price movement.

The rest of the paper is organized as follows. The next section develops the basic model used throughout the paper. Section 3.2 derives the flexible price solution of the model and establishes the principle that efficient relative prices should depend only on contemporaneous fundamentals. Section 3.3 analyzes the solution under sticky prices, both with PCP and LCP pricing, and obtains the optimal monetary policy rules in each case, assuming money supply as an instrument of monetary policy. Section 3.4 shows the implications of the model for relative prices as well as some other extensions. Section 3.5 illustrates that the main results carry over to the more realistic case of interest rate targeting for monetary policy. Some conclusions then follow.

### 3.1 Model

There are two symmetric countries, each with a continuum of households normalized in size to equal one. In each country, a continuum of monopolists produces goods that are considered imperfect substitutes by consumer-households. Goods are produced using labor and a commodity. Each country is endowed with equal amounts of the nondurable commodity each period, and it is sold in a competitive market to producers in each country. Households consume output of both countries, but their preferences exhibit home bias—they weight goods produced in their own country more heavily than imported goods. We assume there is a complete market for state-contingent bonds.

#### 3.1.1 Households

Households in each country maximize expected discounted utility over an infinite horizon. The preferences of the representative household in the home country are given by:

$$Y = E_0 \sum_{t=0}^{\infty} \beta^t v_t, \quad 0 < \beta < 1,$$

where

$$v_t = \frac{1}{1-\rho} C_t^{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M_t}{P_t} \right)^{1-\varepsilon} - \eta L_t, \quad \rho > 0, \varepsilon > 0.$$

In this expression,  $C_t$  represents aggregate consumption, which is a Cobb-Douglas function of home- and foreign-produced consumption goods:

$$C_t = (C_{Ht})^{1-(\gamma/2)} (C_{Ft})^{\gamma/2}, \quad \gamma \leq 1.$$

Preferences exhibit home bias when  $\gamma < 1$ , so that home consumers put more weight  $[1 - (\gamma / 2)]$  on consumption of the home good aggregate,  $C_{Ht}$ . Foreign households have symmetric, but not identical preferences. They put weight  $1 - (\gamma / 2)$  on consumption of the foreign good aggregate, so that

$$C_t^* = (C_{Ht}^*)^{\gamma/2} (C_{Ft}^*)^{1-(\gamma/2)}.$$

(Throughout the paper, a \* signifies the foreign country values.) In turn,  $C_{Ht}$  and  $C_{Ft}$  are Center for Economic Studies (CES) aggregates that equally weight the continuum of goods produced in each country, with elasticity of substitution equal to  $\lambda$ .

The exact price index is given (up to a constant of proportionality) by:

$$P_t = (P_{Ht})^{1-(\gamma/2)} (P_{Ft})^{\gamma/2}.$$

Variables  $P_{Ht}$  and  $P_{Ft}$  are CES price indexes defined over the continuum of goods produced in the home country and the foreign country, respectively. These prices are expressed in nominal terms in the domestic currency.

We assume real balances appear in the utility function. We follow the blueprint of Devereux and Engel (2003), who in turn follow Obstfeld and Rogoff (1995, 2000, 2002), but generalize the model to include home bias in preferences and production that uses a commodity as an input. Monetary policy is expressed as a money supply rule. This is not a realistic description of how monetary policy is set by most central banks today, but this paper is not trying to deliver a policy rule that can be taken off the shelf and used by policymakers. The main ideas we are trying to convey hold whether the central bank controls interest rates directly, or controls them indirectly through the money supply. The last section of this paper shows how the results carry through when monetary policy is set by an interest rate rule (which is the case considered in DE06.) However, for pedagogical reasons, it is clearer to present the model initially using money supply rules.

Maximization is done subject to a standard budget constraint. Without loss of generality, home households can be assumed to own home firms. Households receive income from profits, wages, rents earned from their commodity endowment, and the payoffs from their state con-

tingent claims. They carry over money balances from the previous period, and receive lump-sum monetary transfers from the government. They use their resources to buy consumption goods, to acquire money balances to hold in the current period, and to acquire state contingent bonds (which can be held in negative quantities) that pay off in the next period.

Derivations in this model are quite straightforward, so in order to save space we will present only the log-linearized first-order conditions of the households (where the linearization is done around the nonstochastic steady state). Lowercase letters represent the logs of the corresponding uppercase letters.

The tradeoffs between leisure and consumption for home and foreign households, respectively, are given by:

$$w_t = p_t + \rho c_t \quad (1)$$

$$w_t^* = p_t^* + \rho c_t^*. \quad (2)$$

The log of the nominal wage in the home country is  $w_t$ .

The money demand functions are derived from the Euler equation for money balances, in the home and foreign countries:

$$m_t - p_t = \frac{\rho}{\varepsilon} c_t - \frac{1}{i\varepsilon} (E_t p_{t+1} + \rho E_t c_{t+1} - p_t - \rho c_t) \quad (3)$$

$$m_t^* - p_t^* = \frac{\rho}{\varepsilon} c_t^* - \frac{1}{i\varepsilon} (E_t p_{t+1}^* + \rho E_t c_{t+1}^* - p_t^* - \rho c_t^*). \quad (4)$$

Here,  $i = (1 + \pi)/\beta - 1$  is the steady-state nominal interest rate, where  $\pi$  is the steady-state inflation rate, which will be the same in both countries under the following assumptions.

With Cobb-Douglas preferences, the ratio of expenditure on home and foreign aggregates is a constant in each country:

$$p_{Ft} + c_{Ft} = p_{Ht} + c_{Ht} \quad (5)$$

$$p_{Ft}^* + c_{Ft}^* = p_{Ht}^* + c_{Ht}^*. \quad (6)$$

The consumption aggregate in each country can be defined:

$$c_t = \left(1 - \frac{\gamma}{2}\right) c_{Ht} + \frac{\gamma}{2} c_{Ft} \quad (7)$$

$$c_t^* = \frac{\gamma}{2} c_{Ht}^* + \left(1 - \frac{\gamma}{2}\right) c_{Ft}^*. \quad (8)$$

The price indexes are:

$$p_t = \left(1 - \frac{\gamma}{2}\right) p_{Ht} + \frac{\gamma}{2} p_{Ft} \quad (9)$$

$$p_t^* = \frac{\gamma}{2} p_{Ht}^* + \left(1 - \frac{\gamma}{2}\right) p_{Ft}^*. \quad (10)$$

Through trade in state-contingent bonds, households in each country equate the marginal utility of an additional dollar of consumption across all states at each time. These first-order conditions can be summarized by:

$$p_t + \rho c_t = s_t + p_t^* + \rho c_t^*, \quad (11)$$

where  $s_t$  is the log of the home currency price of foreign currency.

### 3.1.2 Firms

Output is produced using labor and a commodity in each country. In addition, there are country-specific productivity shocks.

As we have noted, in each country there is a continuum of monopolistic firms that each produces a final consumption good.

The production functions for representative firm  $i$  in each country are CES, with elasticity of substitution of  $\theta$ :

$$Y_{Ht}(i) = \Psi_t \{ \alpha^{1/\theta} [L_t(i)]^{(\theta-1)/\theta} + (1 - \alpha)^{1/\theta} [X_t(i)]^{(\theta-1)/\theta} \}^{\theta/(\theta-1)}.$$

$$Y_{Ft}^*(i) = \Psi_t^* \{ \alpha^{1/\theta} [L_t^*(i)]^{(\theta-1)/\theta} + (1 - \alpha)^{1/\theta} [X_t^*(i)]^{(\theta-1)/\theta} \}^{\theta/(\theta-1)}.$$

Labor input in the home country for firm  $i$  is  $L_t(i)$ ,  $X_t(i)$  is the input of the commodity for firm  $i$ , and  $\Psi_t$  is the home productivity level, common to all home firms.

The log-linearized production functions (dropping the index  $i$ ) are given by:

$$y_{Ht} = \psi_t + \alpha \ell_t + (1 - \alpha) x_t \quad (12)$$

$$y_{Ft}^* = \psi_t^* + \alpha \ell_t^* + (1 - \alpha) x_t^*. \quad (13)$$

The log of the home productivity shock can be decomposed into two components:

$$\psi_t = u_t + v_{t-1}.$$

The notion is that agents receive news about productivity in advance. We express that by saying that the  $v_{t-1}$  component of time  $t$  productivity is observed one period in advance, at time  $t - 1$ . Similarly,

$$\psi_t^* = u_t^* + v_{t-1}^*.$$

We will consider two different models for the evolution of shocks: one in which all components are purely independent and identically distributed (i.i.d.) so all shocks are temporary; and another in which all components are pure random walks, so that all shocks are permanent.

Factor demands are given by:

$$x_t - \ell_t = -\theta(p_{xt} - w_t) \quad (14)$$

$$x_t^* - \ell_t^* = -\theta(p_{xt}^* - w_t^*). \quad (15)$$

The nominal price of the commodity in the home country is  $P_{xt}$ .

When goods prices are flexible, each firm sets its goods price as a markup over unit cost. We shall assume, however, that the government subsidizes output of each firm so that in fact it produces at an efficient level, and sets price equal to unit cost. This assumption will guarantee us that the flexible-price equilibrium is also efficient.<sup>4</sup> The price equations are given by:

$$p_{Ht} = \alpha w_t + (1 - \alpha)p_{xt} - \psi_t \quad (16)$$

$$p_{Ft}^* = \alpha w_t^* + (1 - \alpha)p_{xt}^* - \psi_t^*. \quad (17)$$

Each of the unique consumption goods could, in principle, be priced differently in the home and foreign countries, because we will assume that firms can costlessly segment the markets. However, since the demand function for each good is the same among home and foreign households, the desired price set by each firm for the home and foreign market is identical. So we have that the law of one price holds for each good:

$$p_{Ht} = s_t + p_{Ht}^* \quad (18)$$

$$p_{Ft} = s_t + p_{Ft}^*. \quad (19)$$

While all of the other equations presented so far hold whether prices are flexible or set in advance, we emphasize that equations (16) and (17) pertain only to the flexible-price version of the model. Equations (18) and (19) will hold in the PCP version of the sticky-price model, but not in the LCP version.

### 3.1.3 Equilibrium

The commodity is freely traded on world markets, and so it has a single world price:

$$p_{xt} = s_t + p_{xt}^*. \quad (20)$$

The world endowment of the commodity is  $\bar{x}_t$ , which is a random variable. We will assume that no information about  $\bar{x}_t$  is known until time  $t$ , at which time it is fully known.

The log-linearized commodity market resource constraint is:

$$\bar{x}_t = \frac{1}{2}x_t + \frac{1}{2}x_t^*. \quad (21)$$

The weights are  $1/2$ , on home and foreign log commodity demand because of the symmetry of the home and foreign country in the non-stochastic steady state.

The log-linearized market clearing conditions for home and foreign goods can be summarized as:

$$y_{Ht} = \left(1 - \frac{\gamma}{2}\right)c_{Ht} + \frac{\gamma}{2}c_{Ht}^* \quad (22)$$

$$y_{Ft}^* = \frac{\gamma}{2}c_{Ft} + \left(1 - \frac{\gamma}{2}\right)c_{Ft}^*. \quad (23)$$

When prices are flexible, equations (1) through (23) determine the twenty-three variables:  $w_t, w_t^*, y_{Ht}, y_{Ft}^*, \ell_t, x_t, \ell_t^*, x_t^*, c_t, c_{Ht}, c_{Ft}, c_t^*, c_{Ht}^*, c_{Ft}^*, p_t, p_{Ht}, p_{Ft}, p_{xt}, p_t^*, p_{Ht}^*, p_{Ft}^*, p_{xt}^*$ , and  $s_t$ . In the PCP model, we have that  $p_{Ht}$  and  $p_{Ft}^*$  are set one period ahead, so that equations (16) and (17) do not hold. The remaining twenty-one equations determine the remaining twenty-one variables. In the LCP model,  $p_{Ht}, p_{Ft}, p_{Ht}^*$ , and  $p_{Ft}^*$  are set in advance. Equations (16) to (19) do not hold in the LCP model, and the remaining nineteen equations determine the remaining nineteen variables.

## 3.2 Flexible Price Solution

The solution to the model under the assumption of flexible prices provides a benchmark. We have noted that, with the optimal production subsidies to monopolists in place, the equilibrium under flexible prices is efficient. We will see that in some cases analyzed below, even with

sticky prices, the efficient allocation is obtainable with appropriate monetary policy.

For any variable  $z_t$  and its foreign counterpart,  $z_t^*$ , we will define

$$z_t^k \equiv z_t - z_t^*,$$

$$\bar{z}_t = \frac{z_t + z_t^*}{2},$$

the relative and world values of these variables.<sup>5</sup>

Devereux and Engel (2006) emphasize that the theorem derived by Barro and King (1984) applies to this model: even though expectations are forward looking, because preferences are time separable, there are no durable goods (except the moneys), and markets are complete, the (real) prices and quantities depend only on the current period values of the exogenous variables— $\psi_t$ ,  $\psi_t^*$ , and  $\bar{x}_t$ —and not on the expected future values. The solutions do not depend on any assumptions about the stochastic processes for these exogenous variables.

This is a key insight: relative prices are determined by relative scarcity of goods or factors, not by expectations about the future. Expectations can play a role in determining money prices (and demand for money), because money is a durable asset. But under flexible prices, money is neutral in this model, so expectations of the future play no role in the determination of equilibrium relative prices or real allocations.

Here we present the solutions for aggregate consumption and output in each country, and the solutions for relative prices:

$$c_t = \frac{1-\gamma}{2\rho} \psi_t^k + \frac{1-\alpha(1-\theta)}{1-\alpha(1-\theta\rho)} \bar{\psi}_t + \frac{1-\alpha}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (24)$$

$$c_t^* = \frac{\gamma-1}{2\rho} \psi_t^k + \frac{1-\alpha(1-\theta)}{1-\alpha(1-\theta\rho)} \bar{\psi}_t + \frac{1-\alpha}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (25)$$

$$y_{Ht} = \psi_t - \frac{(\rho-1)(1-\gamma)^2}{2\rho} \psi_t^k + \frac{\alpha\theta(1-\rho)}{1-\alpha(1-\theta\rho)} \bar{\psi}_t + \frac{1-\alpha}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (26)$$

$$y_{Ft}^* = \psi_t^* + \frac{(\rho-1)(1-\gamma)^2}{2\rho} \psi_t^k + \frac{\alpha\theta(1-\rho)}{1-\alpha(1-\theta\rho)} \bar{\psi}_t + \frac{1-\alpha}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (27)$$

$$p_{Ft} - p_{Ht} = p_{Ft}^* - p_{Ht}^* = \psi_t^k \quad (28)$$

$$w_t - p_{Ht} = \frac{1}{2} \psi_t^k + \frac{\rho(1-\alpha(1-\theta))}{1-\alpha(1-\theta\rho)} \bar{\psi}_t + \frac{\rho(1-\alpha)}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (29)$$

$$p_{Ft} - p_{Ht} = \frac{1}{2} \psi_t^k + \frac{1-\alpha(1-\theta)}{1-\alpha(1-\theta\rho)} \bar{\psi}_t - \frac{\rho\alpha}{1-\alpha(1-\theta\rho)} \bar{x}_t \quad (30)$$

Consumption and prices depend only on the current level of productivity and supply of the commodity. Note also, the log of the real exchange rate is proportional to the terms of trade:

$$q_t = s_t + p_t^* - p_t = (1-\gamma)\psi_t^k.$$

We could use equations (3) and (4) along with the solutions for consumption in (24) and (25), in conjunction with some assumptions about the money supply process to solve for nominal price levels. Because equations (3) and (4) are forward looking, the nominal price levels depend upon expectations of future consumption, and hence future productivity levels, as well as expectations about money supplies. However, the nominal price levels have no influence on real prices or allocations under flexible prices.

### 3.3 Sticky Prices Solution and Monetary Policy

We begin this section by making some assumptions about the money supply process. We assume that money supplies in each country are determined by:

$$m_t = m_{t-1} + \mu_t + \delta_{t-1} \quad (31)$$

$$m_t^* = m_{t-1}^* + \mu_t^* + \delta_{t-1}^* \quad (32)$$

Monetary policy rules are designed to respond to unanticipated shocks, so  $E_{t-1}(\mu_t) = E_{t-1}(\mu_t^*) = 0$ , and  $E_{t-2}(\delta_{t-1}) = E_{t-2}(\delta_{t-1}^*) = 0$  will hold. Here  $\mu_t(\mu_t^*)$  is an addition to the time  $t$  information set, while  $\delta_{t-1}(\delta_{t-1}^*)$  is an addition to the time  $t-1$  information set. Note that this assumption means that conditionally (on time  $t$  information) expected money growth will vary over time, although the unconditional expectation of money growth is zero. This monetary rule is designed so that the  $\mu_t$  component reacts to the components of productivity,  $u_t$  and  $u_t^*$ , that are news in period  $t$ , while the  $\delta_{t-1}$  component reacts to the components of this period's productivity,  $v_t$  or  $v_t^*$ , that were learned in period  $t-1$ . In other words, the monetary authorities commit in period  $t$  to change the money supply between  $t+1$  and  $t$  by an amount that is expected to equal  $\delta_t$ .

We introduce some new notation. For any variable  $z_{t+j}$ ,  $j \geq 0$  define

$$\hat{E}_t z_{t+j} = E_t z_{t+j} - E_{t-1} z_{t+j}.$$

In other words,  $\hat{E}_t z_{t+j}$  is the news received at time  $t$  about variable  $z_{t+j}$ . When  $j = 0$ , we will write simply  $\hat{z}_t = z_t - E_{t-1} z_t$  for innovations in  $z_t$ .<sup>6</sup> Under this notation,  $\hat{\psi}_t = u_t$ , and  $\hat{E}_t \psi_{t+1} = v_t$ , for example.

We will assume that nominal prices are set one period in advance. Firms set prices to maximize the value of the firm. In the first case, we will consider PCP firms. Home firms set prices in home currency and foreign firms set prices in foreign currency. In that case, equations (16) and (17) are replaced by:

$$p_{Ht} = E_{t-1}[\alpha w_t + (1 - \alpha)p_{xt} - \psi_t] \quad (33)$$

$$p_{Ft}^* = E_{t-1}[\alpha w_t^* + (1 - \alpha)p_{xt}^* - \psi_t^*]. \quad (34)$$

We will also consider LCP firms. These firms set a price one period in advance in domestic currency for sale to households in the home country, and a price in foreign currency for sale to foreign households. Equations (33) and (34) determine  $p_{Ht}$  and  $p_{Ft}^*$ . In addition, for LCP firms, equations (18) and (19) are replaced by:

$$p_{Ht} = E_{t-1} s_t + p_{Ht}^* \quad (35)$$

$$p_{Ft} = E_{t-1} s_t + p_{Ft}^*. \quad (36)$$

In this section, we assume that the shocks— $u_t$ ,  $u_t^*$ ,  $v_t$ ,  $v_t^*$ , and  $\bar{x}_t$ —are purely transitory (mean zero, i.i.d.)

Because prices are expected (at time  $t$ ) to be at their flexible price equilibrium levels at time  $t + 1$  (compare equations (33) through (34) to (16) through (17), and additionally for the LCP case, (35) to (36) to (18) to (19)), consumption in period  $t + 1$  is expected to be at its efficient level. From equation (26) we have:

$$\hat{E}_t c_{t+1} = \frac{1 - \gamma}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \quad (37)$$

$$\hat{E}_t c_{t+1}^* = \frac{\gamma - 1}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \quad (38)$$

It is useful to note that  $\hat{E}_t m_{t+k} = \hat{E}_t m_{t+1}$ , and  $\hat{E}_t c_{t+k} = 0$ ,  $k \geq 2$ . It follows from pushing equation (3) ahead two periods that  $\hat{E}_t p_{t+2} = \hat{E}_t m_{t+2} = \hat{E}_t m_{t+1}$ . Then pushing equation (3) ahead one period, we have

$$\hat{E}_t m_{t+1} - \hat{E}_t p_{t+1} = \frac{\rho}{\varepsilon} \hat{E}_t c_{t+1} - \frac{1}{i\varepsilon} (\hat{E}_t p_{t+2} + \rho \hat{E}_t c_{t+2} - \hat{E}_t p_{t+1} - \rho \hat{E}_t c_{t+1}).$$

We can solve this out to write:

$$\hat{E}_t p_{t+1} = \hat{E}_t m_{t+1} - \frac{\rho(1 + i)}{1 + i\varepsilon} \hat{E}_t c_{t+1}. \quad (39)$$

### 3.3.1 PCP

Recall  $p_t = (1 - \gamma/2)p_{Ht} + (\gamma/2)(s_t + p_{Ft}^*)$ , so under PCP we have  $\hat{p}_t = (\gamma/2)\hat{s}_t$ . Also note  $\hat{E}_t m_{t+1} = \hat{m}_t + \delta_t$ . Taking innovations in equation (3), we get

$$\hat{m}_t - \hat{p}_t = \frac{\rho}{\varepsilon} \hat{c}_t - \frac{1}{i\varepsilon} (E_t \hat{p}_{t+1} + \rho E_t \hat{c}_{t+1} - \hat{p}_t - \rho \hat{c}_t).$$

Using these relationships along with equation (39) to get:

$$\hat{c}_t = \phi \left( \hat{m}_t - \frac{\gamma}{2} \hat{s}_t \right) + \frac{1}{\rho(1 + i)} \delta_t + \frac{i(\varepsilon - 1)}{(1 + i)(1 + i\varepsilon)} \hat{E}_t c_{t+1} \quad (40)$$

where  $\phi \equiv (1 + i\varepsilon)/[\rho(1 + i)]$ .

The analogous expression for innovations in foreign consumption is given by

$$\hat{c}_t^* = \phi (\hat{m}_t^* + \frac{\gamma}{2} \hat{s}_t) + \frac{1}{\rho(1 + i)} \delta_t^* + \frac{i(\varepsilon - 1)}{(1 + i)(1 + i\varepsilon)} \hat{E}_t c_{t+1}^*. \quad (41)$$

From the risk-sharing condition (11) we have

$$(1 - \gamma)\hat{s}_t = \rho(\hat{c}_t - \hat{c}_t^*).$$

Taking the difference between equations (40) and (41), and using the difference between equations (37) and (38), we can derive:

$$\begin{aligned} \hat{s}_t &= \frac{1 + i\varepsilon}{1 + i[1 - \gamma(1 - \varepsilon)]} \hat{m}_t^R + \frac{1}{1 + i[1 - \gamma(1 - \varepsilon)]} \delta_t^R \\ &\quad + \frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)\{1 + i[1 - \gamma(1 - \varepsilon)]\}} \hat{E}_t c_{t+1}^R \\ &= \frac{1 + i\varepsilon}{1 + i[1 - \gamma(1 - \varepsilon)]} m_t^R + \frac{1}{1 + i[1 - \gamma(1 - \varepsilon)]} \delta_t^R \\ &\quad + \frac{(1 - \gamma)i(\varepsilon - 1)}{(1 + i\varepsilon)\{1 + i[1 - \gamma(1 - \varepsilon)]\}} v_t^R. \end{aligned} \quad (42)$$

The last term shows the influence of expected future fundamentals on the nominal exchange rate. Substituting (42) and (37) into equation (41), we can also see the influence of expected future fundamentals on current consumption.

The exchange rate is the sum of revisions to current fundamentals and expectations of future fundamentals. Current fundamentals are unanticipated movements in relative money growth across the home and foreign country. Expectations of future productivity shocks, however, affect the exchange rate currently. This is explained as follows. When  $v_t > v_t^*$ , there is a shock to future home productivity that exceeds that to future foreign productivity. If in addition  $\gamma < 1$ , this must increase anticipated consumption at home more than in the foreign country, since home residents' consumption is more sensitive to home productivity in the presence of home bias in preferences. From (3), holding the current monetary innovation constant, a rise in expected future home relative consumption will increase the home nominal interest rate, relative to the foreign nominal interest rate, when  $\varepsilon > 1$ . This will reduce demand for money at home relative to the foreign country, and as a result there is an unanticipated home currency depreciation. Finally, future fundamentals also incorporate future changes in the relative money supplies,  $\delta_t - \delta_t^*$ , which can be forecasted based on announcements of future relative technology growth rates.

Note that the key feature of this mechanism is that the exchange rate responds to future fundamentals. That is, the time  $t + 1$  productivity shock becomes known at time  $t$ , and generates news, which leads the current exchange rate to move, and the resulting changes in the expected future money supply have a similar effect.

Using equations (5)–(11), we can derive:

$$\hat{y}_{Ht} - \hat{y}_{Ft}^* = \frac{(1 - \gamma)^2 + \rho\gamma(2 - \gamma)}{\rho} \hat{s}_t. \quad (43)$$

A future productivity boom in the home country or anticipated future money growth in the home country (in both cases, relative to the foreign counterpart) will lead to a home depreciation ( $\varepsilon > 1$ ). From the risk-sharing condition, home consumption rises relative to foreign consumption when  $s$  rises. This is associated with a boom in domestic output relative to foreign.

It is well known that in this setting, when no shocks are anticipated, a monetary rule can be established that replicates the flexible price alloca-

tions.<sup>7</sup> Suppose  $m_t - m_{t-1} = \mu_t$  and  $\psi_t = u_t$ . Then with a bit of work, equations (40) through (42) can be solved to give:

$$c_t = \phi \left\{ \frac{2(1 - \gamma)(1 + i) + \gamma(1 + i\varepsilon)}{2 + 2i[1 - \gamma(1 - \varepsilon)]} \mu_t + \frac{\gamma(1 + i\varepsilon)}{2 + 2i[1 - \gamma(1 - \varepsilon)]} \mu_t^* \right\} \quad (44)$$

$$c_t^* = \phi \left\{ \frac{\gamma(1 + i\varepsilon)}{2 + 2i[1 - \gamma(1 - \varepsilon)]} \mu_t + \frac{2(1 - \gamma)(1 + i) + \gamma(1 + i\varepsilon)}{2 + 2i[1 - \gamma(1 - \varepsilon)]} \mu_t^* \right\}. \quad (45)$$

These two equations give  $c_t$  and  $c_t^*$  as linear functions of  $\mu_t$  and  $\mu_t^*$ . The optimal solutions for  $c_t$  and  $c_t^*$ , given in equations (24) and (25), have home and foreign consumption solved as functions of  $u_t$ ,  $u_t^*$ , and  $x_t$ . So, monetary policy can replicate the optimal consumption levels using a policy that equates the right-hand sides of equations (44) and (45) with the right-hand sides of equations (24) and (25), respectively, and then solves for  $\mu_t$  and  $\mu_t^*$  as linear functions of  $u_t$ ,  $u_t^*$ , and  $x_t$ . It is tedious but straightforward to confirm that the policies that solve these equations will also yield solutions for relative prices under PCP that replicate the flexible price solutions given in equations (28) through (30). We shall not elaborate on this aspect of the optimal monetary policy further.

What is of interest, here, however, is the distortion caused by the effect of anticipated future disturbances—anticipated future productivity and money growth—on current output, consumption, and price levels. Can we design a policy that eliminates these effects as well, and delivers the flexible price allocation? What are the properties of such a policy?

In the PCP model, the sticky price distortion manifests itself through terms of trade movements that do not replicate the optimal reaction of the terms of trade to current productivity levels, as given in equation (28). If there were no anticipated shocks, the optimal exchange rate policy would deliver an exchange rate that mimicked the terms of trade. That is, with  $p_{Ft}^*$  and  $P_{Ht}$  fixed one period in advance, innovations in the terms of trade,  $s_t + p_{Ft}^* - P_{Ht}$ , are just determined by innovations in the nominal exchange rate. Common wisdom (supported by the empirical work of DE06) is that short-run exchange rate movements are largely driven by news about the future. In the context of this model, this means that the variances of  $v_t$ ,  $v_t^*$ ,  $\delta_t$ , and  $\delta_t^*$  are much larger than the variances of  $\mu_t$ ,  $\mu_t^*$ ,  $u_t$ ,  $u_t^*$ , and  $x_t$ . Therefore, the policies that target news about the future are far more important in delivering desirable terms of trade movements and real exchange rate movements than the policies that target current fundamentals.

PROPOSITION 1. *The optimal monetary policy sets*

$$\delta_t = -\frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \hat{E}_t c_{t+1} = -\frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \left[ \frac{1 - \gamma}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \right]$$

$$\delta_t^* = -\frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \hat{E}_t c_{t+1}^* = -\frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \left[ \frac{\gamma - 1}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \right].$$

*This policy eliminates the effects of news on nominal exchange rates.*

PROOF. It follows directly from equations (40) and (41) that these policies eliminate the influence of news on current consumption. Inspection of equation (42) shows that it also eliminates the effects of news on exchange rates.

In practical terms, a policy that eliminates the effects of news on exchange rates will substantially stabilize exchange rates (making the conditional variance of exchange rates very small). Notice if the home country follows the policy set out in the Proposition for  $\delta_t$ , the foreign country could set its policy to drive the effect of news on exchange rates to zero. That is, from (42), the foreign country could eliminate the impact of news on the exchange rate by setting

$$\delta_t^* = \delta_t + \frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \hat{E}_t c_{t+1}^R.$$

With the home country setting  $\delta_t$  as given in the Proposition, this policy for the foreign country would be identical to the optimal rule for the foreign country given in the Proposition. Ignoring the effects of contemporary shocks to the fundamentals on exchange rates, the optimal cooperative policy could be implemented by having the home country follow the policy that delivers its individually optimal level of consumption, and having the foreign country target shocks to exchange rates.

### 3.3.2 LCP

Under LCP we have  $\hat{p}_t = 0$ . Following the steps used to derive equation (40), but with this expression for the innovation in the consumer price level, we find under LCP:

$$\hat{c}_t = \phi \hat{m}_t + \frac{1}{\rho(1 + i)} \delta_t + \frac{i(\varepsilon - 1)}{(1 + i)(1 + i\varepsilon)} \hat{E}_t c_{t+1}. \quad (46)$$

$$\hat{c}_t^* = \phi \hat{m}_t^* + \frac{1}{\rho(1 + i)} \delta_t^* + \frac{i(\varepsilon - 1)}{(1 + i)(1 + i\varepsilon)} \hat{E}_t c_{t+1}^*. \quad (47)$$

From the risk-sharing condition (11) we have under LCP

$$\hat{s}_t = \rho(\hat{c}_t - \hat{c}_t^*).$$

As with the derivation of equation (42), take the difference between equations (46) and (47), and use the difference between equations (37) and (38), to derive:

$$\hat{s}_t = \frac{1 + i\varepsilon}{1 + i} \hat{m}_t^R + \frac{1}{1 + i} \delta_t^R + \frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)(1 + i)} \hat{E}_t c_{t+1}^R$$

$$= \frac{1 + i\varepsilon}{1 + i} \hat{m}_t^R + \frac{1}{1 + i} \delta_t^R + \frac{(1 - \gamma)i(\varepsilon - 1)}{(1 + i\varepsilon)(1 + i)} v_t^R. \quad (47)$$

Qualitatively, news has a similar impact under local-currency pricing as it does under producer-currency pricing. We also now have:

$$\hat{y}_{tH} - y_{tH}^* = \frac{(1 - \gamma)}{\rho} \hat{s}_t. \quad (49)$$

But we can make an even stronger statement about optimal policy:

PROPOSITION 2. *The optimal rules for  $\delta_t$  and  $\delta_t^*$  are the same as under PCP. They also eliminate the effects of news on exchange rates, as under PCP.*

PROOF. This comes from inspection of (46) through (48).

Devereux and Engel (2003, 2007), Monacelli (2005), Sutherland (2005), and Corsetti and Pesenti (2005) have emphasized that under LCP, monetary policy's response to shocks to the current fundamentals cannot replicate the flexible-price equilibrium. That result holds in this model as well. But if exchange rates are primarily driven by news, the optimal response to news will involve stabilizing the exchange rate. In contrast to what previous studies have emphasized, in response to news, the optimal policy is identical under PCP and LCP.

Standard Optimal Currency Area (OCA) reasoning suggests that it is efficient to allow the exchange rate to respond to country-specific productivity shocks. We find, in the absence of a monetary response, that indeed the exchange rate will respond to announcements of country-specific productivity shocks. The direction of movement depends on the size of  $\varepsilon$ . For  $\varepsilon > 1$ , the exchange rate will depreciate in response to an

announced future home productivity expansion. It is tempting to interpret this movement along efficiency (or OCA) lines—the future home productivity expansion should cause a home-country terms of trade deterioration. Hence, the response of agents forecasting this in financial markets leads to an immediate nominal exchange rate depreciation.

But the problem with this reasoning is that the immediate response of the current nominal exchange rate causes a change in the current real exchange rate (by different degrees in the PCP and LCP environments), because current nominal prices cannot respond to the announced future shock. In the absence of a *current* (as opposed to future) productivity shock, however, there is no efficiency reason for the real exchange rate to move at all. In fact, movements in the real exchange rate are associated with welfare losses since they push consumption and employment away from their efficient levels.

Thus, in a sticky price environment, when the exchange rate responds to news, there is no guarantee that it will do so in an efficient manner. Indeed, in our model, the optimal monetary rule should prevent the exchange rate from responding to news about future fundamentals at all. The critical requirement is that there not be any *unanticipated* movements in the exchange rate. That is, the time  $t$  exchange rate will be known in time  $t - 1$ .

Of course the model is quite stylized, since we have assumed that all prices can adjust before the news takes effect. But this is not necessarily unrealistic. At an anecdotal level, we see the exchange rate responding to all types of potential events (e.g., effects of Social Security changes that may affect the budget deficit in five or more years' time) that may occur much further in the future than would be relevant for business cycle frequencies. These exchange rate movements are not necessarily desirable, because we have to recognize that the response to future shocks may not be consistent with the currently desired structure of relative prices.

### 3.4 International Goods Prices and Extensions

From equations (1) through (2), (7) through (8), and (11) through (15), we can write in all models of price setting:

$$p_{xt} - p_t = \rho c_t + \frac{1}{\alpha\theta} (\bar{c}_t - \bar{\psi}_t - \bar{x}_t) \quad (50)$$

$$p_{xt}^* - p_t^* = \rho c_t^* + \frac{1}{\alpha\theta} (\bar{c}_t - \bar{\psi}_t - \bar{x}_t). \quad (51)$$

Holding consumption levels and current productivity levels constant, a decline in the world supply of the commodity raises the relative CPI price of the commodity in both countries.

Solving out for consumption, in a flexible price world, we find:

$$p_{xt} - p_t = \frac{1 - \gamma}{2} \psi_t^R + \frac{1 - \alpha\rho(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{\psi}_t - \frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t \quad (52)$$

$$p_{xt}^* - p_t^* = \frac{\gamma - 1}{2} \psi_t^R + \frac{1 - \alpha\rho(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{\psi}_t - \frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t. \quad (53)$$

Optimally, relative prices should reflect only current productivity levels and the current supply of the commodity.

Under sticky prices, relative prices do not respond optimally to productivity levels or scarcity of the commodity. If there were no news about the future, the relative prices would still not be optimal under nominal price stickiness, unless monetary policy can deliver the correct levels of consumption in equations (50) and (51) that allow the economy to replicate the relative price solutions given in (52) through (53). We have noted that optimal monetary policy can achieve this allocation under PCP but not under LCP.

As we emphasized in the introduction, we are concerned with the large swings in  $P_{xt} - p_t$  relative to  $p_{xt}^* - p_t^*$ —a difference that does not seem by changes in relative current productivity levels in final goods. This difference is equal to the real exchange rate, and the swings may be driven by the expectations of future productivity or monetary growth that influence the nominal exchange rate.

The following example illustrates the issue. Suppose the commodity supply follows an i.i.d. process, and there is an unexpected decline in the supply of commodities, so  $\bar{x}_t$  is negative. Assume there are no innovations in the current productivity levels:  $u_t = u_t^* = 0$ . From (52) and (53), under flexible prices we have:

$$\hat{p}_{xt} - \hat{p}_t = - \frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t \quad \hat{p}_{xt}^* - \hat{p}_t^* = - \frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t.$$

Under sticky prices, the actual innovations in relative prices in each country depend on expectations and monetary policy rules. Suppose, for example under PCP, that  $\mu_t$  and  $\mu_t^*$  are set optimally to respond to current commodity supply disturbances, but the rules for  $\delta_t$  and  $\delta_t^*$  deviate from the optimal rules set forth in proposition 1. Specifically, define the deviations from optimal policy:

$$\xi_t \equiv \delta_t + \frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \hat{E}_t c_{t+1}, \quad \xi_t^* \equiv \delta_t^* + \frac{\rho i(\varepsilon - 1)}{(1 + i\varepsilon)} \hat{E}_t c_{t+1}^*.$$

Then, we can write:

$$\begin{aligned} \hat{p}_{x,t} - \hat{p}_t &= -\frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t + \frac{1}{1 + i} \xi_t \\ &\quad - \frac{\gamma(1 + i\varepsilon)}{2(1 + i)\{1 + i[1 - \gamma(1 - \varepsilon)]\}} \xi_t^R + \frac{1}{\alpha\theta\rho(1 + i)} \bar{\xi}_t \\ \hat{p}_{x,t} - \hat{p}_t^* &= -\frac{\alpha\rho}{1 - \alpha(1 - \theta\rho)} \bar{x}_t + \frac{1}{1 + i} \xi_t^* \\ &\quad + \frac{\gamma(1 + i\varepsilon)}{2(1 + i)\{1 + i[1 - \gamma(1 - \varepsilon)]\}} \xi_t^{*R} + \frac{1}{\alpha\theta\rho(1 + i)} \bar{\xi}_t^*. \end{aligned}$$

Under this policy, the relative prices respond correctly (by construction) to the supply shock, but their value also depends on anticipated future productivity and money growth. The difference between the two relative prices is given by:

$$\hat{p}_{x,t} - \hat{p}_t - (\hat{p}_t^* - \hat{p}_t) = (1 - \gamma)\hat{s}_t = \frac{1 - \gamma}{1 + i[1 - \gamma(1 - \varepsilon)]} \xi_t^R,$$

and thus will be influenced by anticipated future shocks, unless monetary policy is set optimally as in proposition 1.

Examination of these equations show how a commodity supply shock,  $\bar{x}_t$ , could have different effects on relative prices in the two countries if optimal monetary policies are not followed. Depending on the policy errors,  $\xi_t$  and  $\xi_t^*$ , we could see a situation where the entire price effect of a decline in commodity supply is felt in the home country—a positive  $\hat{p}_{x,t} - \hat{p}_t$ —with no change in the foreign country ( $\hat{p}_{x,t}^* - \hat{p}_t^* = 0$ ). That is precisely the situation that could lead to a situation in which the United States experienced the large increase in the relative price of oil in 2002 to 2004, while the relative price increase in Europe was minimal.

### 3.4.1 Permanent Shocks

We have assumed so far that productivity shocks are transitory, but nothing depends on that assumption. Here we turn briefly to the case of permanent productivity shocks.

The derivations of equations (1) through (38) are unaffected by this as-

sumption. However, now we have  $\hat{E}_t c_{t+k} = \hat{E}_t c_{t+1}$  and (as before)  $\hat{E}_t m_{t+k} = \hat{E}_t m_{t+1}$ ,  $k \geq 2$ . It follows from equation (3) pushed ahead one period that

$$\hat{E}_t p_{t+1} = E_t \hat{m}_{t+1} - \frac{\rho}{\varepsilon} \hat{E}_{t+1} \hat{c}_{t+1}.$$

It is straightforward to verify under the PCP assumption that:

$$\hat{c}_t = \phi \left( i\hat{m}_t - \frac{\gamma}{2} \hat{s}_t \right) + \frac{1}{\rho(1 + i)} \delta_t + \frac{\varepsilon - 1}{\varepsilon(1 + i)} \hat{E}_t c_{t+1}$$

$$\hat{c}_t^* = \phi \left( i\hat{m}_t^* + \frac{\gamma}{2} \hat{s}_t \right) + \frac{1}{\rho(1 + i)} \delta_t^* + \frac{\varepsilon - 1}{\varepsilon(1 + i)} \hat{E}_t c_{t+1}^*$$

$$\begin{aligned} \hat{s}_t &= \frac{1 + i\varepsilon}{1 + i[1 - \gamma(1 - \varepsilon)]} \hat{m}_t^R + \frac{1}{1 + i[1 - \gamma(1 - \varepsilon)]} \delta_t^R \\ &\quad + \frac{\rho(\varepsilon - 1)}{\varepsilon[1 + i[1 - \gamma(1 - \varepsilon)]]} \hat{E}_t c_{t+1}^R. \end{aligned}$$

Proposition 1 still holds, however, with the optimal policies modified to:

$$\delta_t = -\frac{\rho(\varepsilon - 1)}{\varepsilon} \hat{E}_t c_{t+1} = -\frac{\rho(\varepsilon - 1)}{\varepsilon} \left[ \frac{1 - \gamma}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \right]$$

$$\delta_t^* = -\frac{\rho(\varepsilon - 1)}{\varepsilon} \hat{E}_t c_{t+1}^* = -\frac{\rho(\varepsilon - 1)}{\varepsilon} \left[ \frac{\gamma - 1}{2\rho} v_t^R + \frac{1 - \alpha(1 - \theta)}{1 - \alpha(1 - \theta\rho)} \bar{v}_t \right].$$

Under LCP, we have:

$$\hat{c}_t = \phi i\hat{m}_t + \frac{1}{\rho(1 + i)} \delta_t + \frac{\varepsilon - 1}{\varepsilon(1 + i)} \hat{E}_t c_{t+1}$$

$$\hat{c}_t^* = \phi i\hat{m}_t^* + \frac{1}{\rho(1 + i)} \delta_t^* + \frac{\varepsilon - 1}{\varepsilon(1 + i)} \hat{E}_t c_{t+1}^*$$

$$\hat{s}_t = \frac{1 + i\varepsilon}{1 + i} \hat{m}_t^R + \frac{1}{1 + i} \delta_t^R + \frac{\rho(\varepsilon - 1)}{\varepsilon(1 + i)} \hat{E}_t c_{t+1}^R.$$

Proposition 2 still holds as stated.

### 3.5 Interest Rate Rules

Devereux and Engel (2006) examine a model similar to the one presented previously. However, instead of one-period ahead pricing, DE06

assume prices are set according to a modified Calvo rule.<sup>8</sup> That leads to asynchronized price setting—firms with identical costs and facing identical demand curves will have different market prices because they have adjusted prices at different dates. Devereux and Engel (2006) consider the optimal interest rate rule in this setting. The production structure is a simplified version of the model here, because in DE06 only labor is used as an input, and the optimal interest rate rule is derived only when prices are set by PCP and productivity shocks are transitory.

The standard result in this literature is that with PCP firms, policy-makers have an incentive to eliminate the price distortion by driving inflation to zero, and the resulting allocation is efficient.<sup>9</sup> Distortions are eliminated if policy eliminates changes in  $p_{Ht}$  and  $p_{Ft}^*$ . Indeed, if there were no news about the future, policies that drive producer-price inflation to zero in each country allow the economy to achieve the first-best allocation. The nominal exchange rate would adjust endogenously to achieve the optimal terms of trade.

However, in the set up of DE06, when news affects exchange rates, then simply targeting inflation is not sufficient. Monetary policy must act in a way to eliminate the effects of anticipated future changes on current allocations. Such a policy, DE06 show, implies stabilizing the exchange rate response to news.

We have examined optimal money supply rules, but here we shall discuss interest rate rules, and show that our central conclusions are not altered.<sup>10</sup> We shall follow DE06, and consider only the case of PCP with transitory productivity shocks. It is straightforward to go through the entire taxonomy of other cases. We will also follow DE06 and focus on the effects of anticipated future productivity shocks, and accordingly set  $u_t = u_t^* = 0$ , so that all productivity changes are foreseen one period in advance. Analogously, we will hold the supply of the commodity constant. As such, we are now essentially considering a special case of DE06, one where the fraction of firms that adjust prices each period is one.<sup>11</sup> Obviously the general results of DE06 go through in this case, but it is worthwhile to draw the link explicitly.

Suppose the home and foreign interest rate rules are given by:

$$i_t = \sigma \pi_t + \delta_t \quad (54)$$

$$i_t^* = \sigma \pi_t^* + \delta_t^*, \quad (55)$$

where  $\pi_t \equiv p_{Ht} - p_{H,t-1}$  and  $\pi_t^* \equiv p_{Ft}^* - p_{F,t-1}^*$ . We want to consider the form of the “placeholder” variables  $\delta_t$  and  $\delta_t^*$  that achieve optimal policy.

In DE06, part of an optimal rule requires setting  $\sigma$  to be very large, to eliminate the distortion caused by asynchronous price setting in each country. With one period ahead price setting, all price setting is synchronized. But we note here that the Taylor principle holds even in this context: we need  $\sigma > 1$  for price-level determinacy.

The first-order conditions for the households’ optimization problems in the home and foreign country yield, respectively:

$$\dot{i}_t = \rho(E_t c_{t+1} - c_t) + E_t p_{t+1} - p_t \quad (56)$$

$$\dot{i}_t^* = \rho(E_t c_{t+1}^* - c_t^*) + E_t p_{t+1}^* - p_t^*. \quad (57)$$

The optimal allocations are then given by equations (24) through (30), recognizing that we are assuming  $\psi_t = v_{t-1}$  and  $\psi_t^* = v_{t-1}^*$ .

Under PCP, prices are set according to (33) and (34). If we use these equations in conjunction with (54) to (57), the risk-sharing condition (11), and the definitions of the price indexes (9) and (10), the model can be solved.

First, suppose that the central banks target only inflation, so that  $\delta_t = \delta_t^* = 0$ . Use the notation for the terms of trade,  $\tau_t \equiv s_t + p_{Ft}^* - p_{Ht}$ . Under PCP, innovations in the terms of trade are equivalent to innovations in the nominal exchange rate.

Equating the right-hand sides of (54) and (56), and taking expectations at time  $t-1$ , we get a solution for  $\pi_t$ :

$$\sigma \pi_t = -\rho E_{t-1} c_t - \frac{\gamma}{2} E_{t-1} \tau_t = \frac{-1}{2} v_{t-1}^R - \frac{\rho[1 - \alpha(1 - \theta)]}{1 - \alpha(1 - \theta\rho)} \bar{v}_{t-1}, \quad (58)$$

where the second equality follows from equations (24) and (28). The equation for the foreign interest rate is symmetric, so that

$$\sigma(\pi_t - \pi_t^*) = -v_{t-1}^R.$$

We have  $\dot{i}_t - \dot{i}_t^* = E_t \tau_{t+1} - \tau_t + E_t(\pi_{t+1} - \pi_{t+1}^*)$ , since interest parity holds to a first-order approximation. Using  $E_t \tau_{t+1} = v_t - v_t^*$ , we get:

$$\dot{\tau}_t = -(\dot{i}_t - \dot{i}_t^*) + v_t - v_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*). \quad (59)$$

Combining this with (58) and its foreign counterpart, we find

$$\dot{\tau}_t = v_{t-1} - v_{t-1}^* + \frac{\sigma - 1}{\sigma} (v_t - v_t^*). \quad (60)$$

Recall that the optimal terms of trade are given by  $\tau_t = v_{t-1} - v_{t-1}^*$ . We see in (60) that no matter what the degree of inflation targeting, the terms of

trade respond optimally to the current productivity level. But for all admissible values of  $\sigma$  (since we must have  $\sigma > 1$ ), expected future productivity levels also influence the terms of trade.

It is apparent from inspection of equation (59) that the general form of the interest rate rules (54) and (55) that will eliminate the effects of anticipated shocks on the terms of trade require:

$$\delta_t - \delta_t^* = \frac{\sigma - 1}{\sigma}(v_t - v_t^*).$$

Under this type of rule, we arrive at  $\tau_t = v_{t-1} - v_{t-1}^*$ . Such a policy eliminates innovations in exchange rates:

$$\hat{s}_t = \hat{\tau}_t = 0.$$

We have so far discussed only the properties of the relative interest rate rules, and their implications for relative prices and exchange rates. Equating the right-hand sides of (54) and (56), we have:

$$\sigma\pi_t + \delta_t = \rho(E_t c_{t+1} - c_t) + E_t p_{t+1} - p_t.$$

To find the optimal value of  $\delta_t$ , we note that at the optimum,  $c_t = E_{t-1} c_t$  and  $\tau_t = E_{t-1} \tau_t$ . Then, using (58), we get:

$$\begin{aligned} \delta_t &= \rho E_t c_{t+1} + E_t \tau_{t+1} + E_t \pi_{t+1} = (1 - \sigma) E_t \pi_{t+1} \\ &= \frac{-(1 - \sigma)}{2} v_t^R - \frac{\rho(1 - \sigma)[1 - \alpha(1 - \theta)]}{1 - \alpha(1 - \theta\rho)} \bar{v}_t. \end{aligned}$$

By symmetry, the optimal foreign monetary policy sets:

$$\delta_t^* = \frac{1 - \sigma}{2} v_t^R - \frac{\rho(1 - \sigma)[1 - \alpha(1 - \theta)]}{1 - \alpha(1 - \theta\rho)} \bar{v}_t.$$

The message is unchanged from the model with money supply rules: Monetary policy should target anticipated future shocks in such a way as to eliminate unanticipated changes in nominal exchange rates.

### 3.6 Conclusions

An optimally designed monetary policy must react differentially to changes in fundamentals that are anticipated and changes that are unanticipated. In practice, of course, such a policy is not practical to imple-

ment. Our point here, however, is to stress that under the optimal policy, unanticipated changes to exchange rates are largely eliminated. There should only be shocks to exchange rates when the current fundamentals change unexpectedly. But it is widely recognized that most exchange rate changes are in response to news about the future, not in response to the current levels of productivity or monetary policy. However we might in practice implement the optimal policy, a gauge of its success is that the effects of news on exchange rates is eliminated.

We should emphasize that we are not attempting here to develop a new insight about the deep properties of monetary policymaking in sticky-price models. We have deliberately built a series of models in which the only distortion is nominal price stickiness, and the optimal policy is to replicate the flexible price equilibrium. The contribution is a practical one. By and large, the flexible price equilibrium will not be one in which anticipated future shocks, which determine the relative prices of two currencies, should determine relative goods prices. Optimal policy should attempt to eliminate unanticipated shocks to exchange rates.

We note, however, that our model also has several unrealistic features. We have no durable goods—either capital, storable inputs, or durable consumption goods. Our economy has complete and unrestricted capital markets. Our future work aims to assess realistic policy rules not only in economies that have these realistic features, but also in economies in which agents receive signals about future fundamentals.

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### Notes

1. Engel and West (2004) and Engel, Mark, and West (2007) have recently argued that present-value models of exchange rates can account for a large fraction of the volatility of exchange rates, and potentially all of the volatility if we could measure such unobserved fundamentals as money demand errors or monetary policy shocks.
2. See, for example, Smets and Wouters (2002), Devereux and Engel (2003, 2007), Sutherland (2005), Corsetti and Pesenti (2005), Monacelli (2005), and Duarte and Obstfeld (2007).

3. Although it is frequently stated that oil is priced in dollars, there is no real significance to that statement since crude oil is priced on a spot market. It is not a sticky price in any currency.
4. This statement is not quite true, because a fully efficient allocation would require that interest rates be set by the Fisherian rule. But, as is standard in the literature, we will assume that the weight on real balances in the utility function,  $\chi$ , is vanishingly small, so that the utility from real balances is insignificant.
5. Note that we have defined  $\bar{y}_t^w$  to be the world endowment of the commodity, but by equation (21) it still fits our definition of a world variable.
6. Since all variables with a  $t$  subscript are known at time  $t$ ,  $\hat{\pi}_t$  means the same thing as  $\hat{\pi}_t^e$ . We introduce the  $\pi_t$  because it is used a lot and is less cumbersome.
7. See, for example, Obstfeld and Rogoff (2000, 2002), Devereux and Engel (2003), and Corsetti and Pesenti (2005).
8. The rule is modified because it is assumed that even when firms are allowed to change prices, the price change is not implemented for one period.
9. For example, see Clarida, Gali, and Gertler (2001, 2002), Kollman (2002), Benigno and Lippi (2003, 2006), and Gali and Monacelli (2005).
10. If we do not alter the model, and leave real balances in the utility function (with a very small weight), then when an interest rate rule is followed we assume the central banks adjust the money supplies endogenously so that money market equilibrium attains the desired interest rate.
11. One small difference is that our production function does not have output proportional to labor input, since we still assume a (now constant) input of the commodity.

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