A note on cointegration and international capital market efficiency

CHARLES ENGEL

University of Washington and NBER, Department of Economics, University of Washington, Seattle, WA 98195-3330, USA

In a recent paper in the *JIMF*, Crowder (1994) claims that evidence that spot exchange rates are cointegrated and forward premiums are non-stationary implies international capital markets are not efficient. This note argues that the cointegration properties of spot exchange rates are independent of the efficiency or inefficiency of financial markets. (JEL F3, F4). Copyright © 1996 Elsevier Science Ltd

A number of authors have claimed that a finding that two or more spot exchange rates are cointegrated is evidence of inefficiency in international capital markets.[1] Dwyer and Wallace, in their 1992 *Journal of International Money and Finance* paper, effectively demonstrate that this claim is false.[2] However, this claim has resurfaced in a more beguiling, but equally false, form in Crowder's (1994) *JIMF* paper. The purpose of this note is to make a single point: cointegration or lack of cointegration of spot exchange rates has nothing to do with international capital market efficiency.

The underlying fallacy is the concept of 'weak-form efficiency', which implies that in the absence of a risk premium, the exchange rate change must be unpredictable. Hodrick (1987) (p. 154) and Dwyer and Wallace (1992) dispel the notion that weak-form efficiency is a necessary condition for efficient international capital markets. The spot rate can be forecastable under efficient markets, as long as all information useful in forecasting changes in the spot rate is used in the determination of the forward exchange rate. So, while cointegration of two or more spot rates does imply that at least one spot rate is forecastable, it does not mean international capital markets are inefficient.

Crowder does not claim that efficient markets imply lack of cointegration of spot exchange rates, but he puts a new spin on this old canard. His claim is that in an efficient market, the exchange rate change could be predictable, but only because of the properties of the risk premium. This claim is false: spot rates can be forecastable in an efficient market whether or not there is a risk premium. A simplistic model—one in which the exchange rate is determined without reference to the capital markets—is useful here to demonstrate why cointegration and international capital market efficiency are separate issues.

Let real money demand be constant except for a white-noise error term in each of countries i and j:

$$m_t^i = p_t^i + \epsilon_t^i,$$

$$m_t^j = p_t^j + \epsilon_t^j.$$

The money supplies and price levels are expressed in logs in these equations. Assume goods markets are integrated, so that purchasing power parity holds:

$$s_t^{ij} = p_t^i - p_t^j.$$

We then have

$\langle 1 \rangle$ $$s_t^{ij} = m_t^i - m_t^j - \left(\epsilon_t^i - \epsilon_t^j \right).$$

Suppose that the money supply processes at time $t + 1$ are set by the central banks in countries i and j to satisfy the process

$\langle 2 \rangle$ $$m_{t+1}^i - m_{t+1}^j = m_t^i - m_t^j - \gamma \left(s_t^{ij} - s_t^{ik} \right) + u_{t+1}.$$

In this equation, s_t^{ik} is the exchange rate for the currency of some third country, k, relative to country i, which we will assume is I(1). This type of money supply rule might arise under a number of circumstances. For example, country j's money supply process might follow a random walk, while country i adjusts its money supply growth depending on how far out of line s_t^{ij} is with s_t^{ik}.

Taking the solution for s_t^{ij} from equation $\langle 1 \rangle$ and inserting it into equation $\langle 2 \rangle$, we get

$\langle 3 \rangle$ $$s_{t+1}^{ij} - s_t^{ij} = -\gamma \left(s_t^{ij} - s_t^{ik} \right) + u_{t+1} - \left(\epsilon_{t+1}^i - \epsilon_{t+1}^j \right) + \epsilon_t^i - \epsilon_t^j.$$

Equation $\langle 3 \rangle$ shows that s_t^{ij} and s_t^{ik} are cointegrated. The equation takes the error-correction form in which the change in s_t^{ij} is related to the gap, $s_t^{ij} - s_t^{ik}$. The error term in equation $\langle 3 \rangle$ is a stationary MA(1) process. From the Engle–Granger (1987) representation theorem, if two variables have an error-correction representation, they are cointegrated.

We can conclude s_t^{ij} and s_t^{ik} are cointegrated without making any reference to international capital market efficiency conditions. The time-series properties of the exchange rate are completely determined by money markets and goods markets. While this model may not be a particularly interesting description of the world, it does not make any assumptions which preclude efficiency. More generally, capital market conditions might play a role in the determination of the exchange rate. But, this example demonstrates that the efficiency of international capital markets does not by itself imply that spot rates are not cointegrated, since it is possible that the cointegration properties of the spot rate are determined without any assumption about the international capital markets.

We can attach a capital market to our model. For example, a risk-neutral efficient market implies

$\langle 4 \rangle$ $$f_t^{ij} = E_t s_{t+1}^{ij},$$

where f_t^{ij} is the log of the one-period ahead forward rate.[3] Equations $\langle 3 \rangle$ and

$\langle 4 \rangle$ determine the forward rate:

$$\langle 5 \rangle \qquad f_t^{ij} = s_t^{ij} - \gamma\left(s_t^{ij} - s_t^{ik}\right) + \epsilon_t^i - \epsilon_t^j.$$

The forward rate incorporates the information useful in forecasting s_{t+1}^{ij}. Hence s_{t+1}^{ij} can be cointegrated with another exchange rate—it is forecastable by $s_t^{ij} - s_t^{ik}$—and the foreign exchange market is efficient.

Crowder (1994) argues that the presence of a foreign exchange risk premium could explain the finding that s_{t+1}^{ij} is predictable. While that is true, we have just seen s_{t+1}^{ij} is forecastable in an efficient market even with no risk premium. The predictability of s_{t+1}^{ij} in an efficient market does not require a risk premium.

Crowder presents evidence that the forward premium, $f_t^{ij} - s_t^{ij}$, is non-stationary. If it is, then the foreign exchange risk premium must be non-stationary if markets are efficient and the first difference of spot rates are stationary. To see this, we adjust the efficient markets condition $\langle 4 \rangle$ to allow a risk premium:

$$\langle 6 \rangle \qquad f_t^{ij} - s_t^{ij} = E_t s_{t+1}^{ij} - s_t^{ij} + \delta_t^{ij} = s_{t+1}^{ij} - s_t^{ij} + \delta_t^{ij} + v_{t+1}^{ij},$$

where δ_t^{ij} is the risk premium and v_{t+1}^{ij} is the forecast error. Under rational expectations, v_{t+1}^{ij} is white noise, and therefore is stationary. There is considerable evidence that $s_{t+1}^{ij} - s_t^{ij}$ is stationary for exchange rates of major countries. Hence $f_t^{ij} - s_t^{ij}$ must be cointegrated with δ_t^{ij}, with a cointegrating vector of (1, −1).

Crowder argues that if his statistical tests are reliable and δ_t^{ij} is non-stationary, the foreign exchange market must be inefficient. His reasoning is that the error correction term, $s_t^{ij} - s_t^{ik}$, which helps forecast s_{t+1}^{ij}, is stationary. Since the risk premium is non-stationary, 'the error correction term, which is stationary by definition, could not be serving as a proxy for the risk premium, due to their differing orders of integration. This evidence suggests that the predictability implied by common trends in the foreign exchange rates analyzed in Section II is consistent with a violation of the conditions for market efficiency' (p. 561).

However, as we have seen, the cointegration property of the exchange rate does not depend on the efficiency of international capital markets. s_t^{ij} and s_t^{ik} are cointegrated, and the market could be efficient (or not) even if δ_t^{ij} is non-stationary. In particular, Crowder is incorrect in asserting that for the market to be efficient, δ_t^{ij} must be serving as a proxy for the error correction term in equation $\langle 3 \rangle$, $-\gamma(s_t^{ij} - s_t^{ik})$, since equation $\langle 3 \rangle$ was derived without making any assumptions about the capital market.

As in the risk-neutral case (equation $\langle 5 \rangle$), we can solve for the forward rate:

$$\langle 7 \rangle \qquad f_t^{ij} = s_t^{ij} - \gamma\left(s_t^{ij} - s_t^{ik}\right) + \delta_t^{ij} + \epsilon_t^i - \epsilon_t^j.$$

The forward premium, $f_t^{ij} - s_t^{ij}$, is composed of the risk premium, the error correction term, and the money demand errors. The forward premium is cointegrated with the risk premium, but its short-run behavior is determined in part by the error correction term. The forward rate incorporates the information useful in predicting future spot rates (although its usefulness as a forecast is muddled by the risk premium), so the market is efficient.

Notes

1. These include Hakkio and Rush (1989), Baillie and Bollerslev (1989), Diebold *et al.* (1994), MacDonald and Taylor (1989), Coleman (1990), Copeland (1991), Alexander and Johnson (1992) and Sephton and Larsen (1991).
2. Baffes (1994) also argues that cointegration of exchange rates is neither necessary nor sufficient for efficiency, although his intuitive explanation is based on a somewhat baffling argument that currencies (such as the mark and pound) are not 'separate assets'.
3. However, see the discussion in Engel (1992) about the appropriateness of this condition.

References

ALEXANDER, C.O. AND A. JOHNSON, 'Are foreign exchange markets really efficient?,' *Economic Letters*, December 1992, **40**: 449–453.

BAFFES, JOHN, 'Does comovement among exchange rates imply market inefficiency?,' *Economic Letters*, 1994, **44**: 273–280.

BAILLIE, RICHARD T. AND TIM BOLLERSLEV, 'Common stochastic trends in a system of exchange rates,' *Journal of Finance*, March 1989, **44**: 167–181.

COLEMAN, MARK, 'Cointegration-based tests of daily foreign exchange market efficiency,' *Economics Letters*, January 1990, **32**: 53–59.

COPELAND, LAURENCE S., 'Cointegration tests of daily foreign exchange data,' *Oxford Bulletin of Economics and Statistics*, May 1991, **53**: 185–198.

CROWDER, WILLIAM J., 'Foreign exchange market efficiency and common stochastic trends,' *Journal of International Money and Finance*, October 1994, **13**: 551–564.

DIEBOLD, FRANCIS X., JAVIER GARDEAZABAL AND KAMIL YILMAZ, 'On cointegration and exchange rate dynamics,' *Journal of Finance*, June 1994, **49**: 727–735.

DWYER, GERALD P. AND MYLES S. WALLACE, 'Cointegration and market efficiency,' *Journal of International Money and Finance*, August 1992, **11**: 318–327.

ENGEL, CHARLES M., 'On the foreign exchange risk premium in a general equilibrium model,' *Journal of International Economics*, May 1992, **32**: 305–319.

ENGLE, ROBERT F. AND CLIVE W.J. GRANGER, 'Cointegration and error correction: Representation, estimation and testing,' *Econometrica*, March 1987, **55**: 251–276.

HAKKIO, CRAIG S. AND MARK RUSH, 'Market efficiency and cointegration: an application to the sterling and deutschemark exchange markets,' *Journal of International Money and Finance*, March 1989, **8**: 75–88.

HODRICK, ROBERT J., *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*, Harwood, Chur, 1987.

MACDONALD, RONALD AND MARK P. TAYLOR, 'Foreign exchange market efficiency and cointegration: Some evidence from the recent float,' *Economic Letters*, 1989 **29**: 63–68.

SEPHTON, PETER S. AND HANS K. LARSEN, 'Tests of exchange market efficiency: fragile evidence from cointegration tests,' *Journal of International Money and Finance*, December 1991, **10**: 561–570.