Long Swings in the Dollar: Are They in the Data and Do Markets Know It?

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The value of the dollar appears to move in one direction for long periods of time. We develop a new statistical model of exchange rate dynamics as a sequence of stochastic, segmented time trends. We reject the null hypothesis that exchange rates follow a random walk in favor of our model of long swings. Our model also generates better forecasts than a random walk. The specification is a natural framework for assessing the importance of the "peso problem" for the dollar. We nonetheless reject uncovered interest parity. (JEL 431)

Why did the dollar rise so dramatically in the early 1980s only to fall precipitously afterward? Explanations have focused on such factors as the effects of U.S. monetary and fiscal policy on real interest rates (Jeffrey Frankel, 1988, and Martin Feldstein, 1986), lower capital taxes (Olivier Blanchard and Lawrence Summers, 1984), or a "safe haven" effect (Michael Dooley and Peter Isard, 1985).

Important features of the dollar's movements are difficult to reconcile with these explanations under the dominant models of exchange rate determination. Figure 1 plots the number of U.S. dollars required to obtain a German mark, French franc, or British pound over the period 1973:III–1988:I.[1] One is tempted to share Feldstein's (1988, p. 21) summary of these data: "the dollar has experienced three big swings." The first of these is marked by a sustained rise of foreign currencies against the dollar; between the beginning of 1977 and the end of 1979, the mark gained 33 percent against the dollar, the franc gained 21 percent, and the pound 26 percent. This was followed by a five-year surge in the dollar, at the end of which these three European currencies fell 60–90 percent (logarithmically) against the dollar. Early in 1985, foreign currencies once more began to rise, gaining 50–70 percent against the dollar by the end of 1987.

The apparent long swings in the exchange rate pose important challenges for existing theory. In Rudiger Dornbusch's (1976) model, a monetary or fiscal policy change that drives up real interest rates results in a one-time upward jump in the value of the dollar. The dollar is then supposed to depreciate steadily, so as to equate expected returns across countries. Yet as Dornbusch himself noted in 1983,

> The [overshooting] model for the real interest rate does well in explaining that a rise in U.S. interest rates should lead to an appreciation of the real exchange rate. But it fails when it predicts that the real exchange rate should also be depreciating. That has not in fact been happening, and a theory is needed that will explain why the dollar —real or nominal—is both high *and* stuck. [p. 83]

Indeed, the picture seems to have been even worse than Dornbusch painted it—the dollar was high *and rising* for three years prior and two years subsequent to Dornbusch's remarks. Accounting for the gradual, sustained fall in the dollar beginning in 1985 in a way that is consistent with

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[1]The data are normalized so that 1973:III = 1.0 for all three currencies.

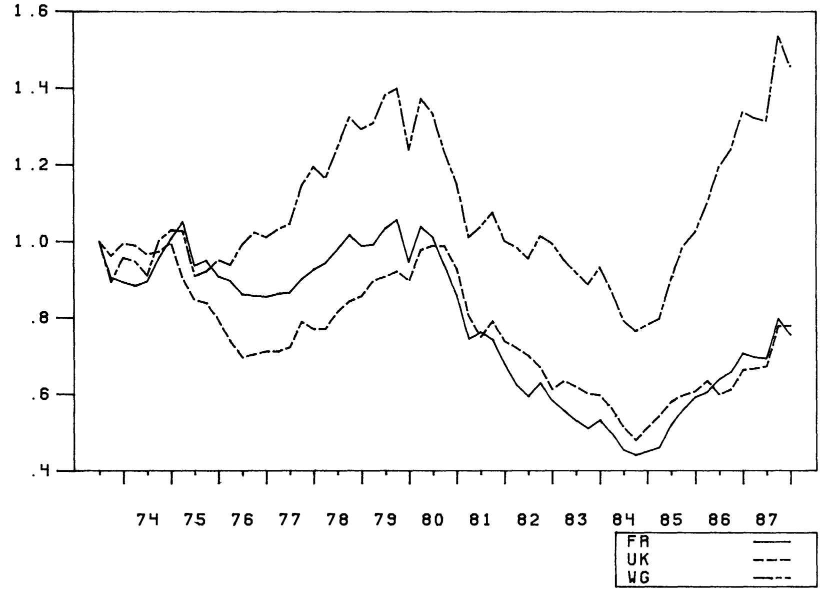

FIGURE 1

the explanation given for its rise is equally problematic.[2]

A further aspect of the apparent long swings that causes difficulty for theories of the exchange rate is that during the period of the strengthening dollar, forward exchange rates (in dollars per unit of foreign currency) were consistently above the spot exchange rate. If the forward rate reflects expectations of future spot rates, then the market appeared to believe over a long period of time that a depreciation of the dollar was imminent. Yet the dollar continued its climb upward until the end of 1984. It could be argued that this forward rate behavior represents an example of William Krasker's (1980) "peso problem." If investors perceived a small probability of a large depreciation, then we might see a forward rate in excess of the current spot rate for a sustained period of time.

For these reasons, it seems useful to formalize the concept of long swings in the exchange rates. What does one mean by long swings, and what magnitudes are plausibly associated with the population parameters? Are long swings a systematic part of the process that generated the data in Figure 1, or a pattern imposed by the eye on the directionless drift of a random walk? If long swings are an accurate description of population dynamics, what sorts of expectations on the part of foreign exchange speculators are consistent with this process? Addressing these questions can provide us with a systematic basis for discussing the issues raised above.

The model we investigate is a special case of that introduced in James Hamilton (1989a).[3] The basic idea is to decompose a

[2]Changes in nominal price differentials between countries were small over this period compared to changes in nominal exchange rates. Thus the real and nominal exchange rates exhibit essentially the same patterns (see Michael Mussa, 1986, on this point). This fact poses additional challenges for theory. In our empirical analysis we focus on the dynamics of nominal exchange rates rather than real exchange rates. When we extend the process to a bivariate system including the nominal interest differential, this permits us to obtain a clean parameterization for testing uncovered interest parity without having to commit ourselves to a model of price level expectation.

[3]Graciela Kaminsky (1988) has also fit Hamilton's model to exchange rate data. She uses monthly data on the pound, whereas we investigate quarterly data on

nonstationary time-series into a sequence of stochastic, segmented time trends. Specifically, we model any given quarter's change in the exchange rate as deriving from one of two regimes, which could correspond to episodes of a rising or falling exchange rate, respectively. The regime at any given date is presumed to be the outcome of a Markov chain whose realizations are unobserved by the econometrician. The task facing the econometrician is to characterize the two regimes and the law that governs the transition between them. These parameter estimates can then be used to infer which regime the process was in at any historical date and provide forecasts for future values of the series.

Our maximum likelihood estimates correspond closely to the visual impressions of Figure 1. In regime 1 the mark is rising 4 percent per quarter against the dollar, the franc 3.3 percent, and the pound 2.6 percent. Regime 2 is associated with quarterly declines in the foreign currencies of -1.2 percent, -2.7 percent, and -3.8 percent, respectively. A given regime is likely to persist for several years, and the econometrically imputed historical change points are close to those the eye is tempted to draw directly from Figure 1.

We perform both Wald tests and likelihood ratio tests that compare the null hypothesis that exchange rates follow a martingale with the segmented trends alternative. In every test but one (the likelihood ratio tests for German data), we reject the martingale hypothesis. The segmented trends model reduces the within sample mean forecast error by 9–14 percent at horizons from two quarters to a year for all three currencies, relative to a random walk specification. Comparable improvements characterize the post-sample forecasts at horizons from one to two quarters. We conclude that long swings in the exchange rate may well be a real feature of the data-generating process.

In exploring the second question posed by our paper—whether markets perceive these swings—we investigate the hypothesis of uncovered interest parity. This hypothesis holds that the nominal interest differential between two countries forecasts future exchange rate changes. This is essentially equivalent to the claim that the three-month forward exchange rate is a rational forecast of the future spot exchange rate. We find no evidence to support this hypothesis in the data. We conclude that either (a) investors did not know the population parameters of the long swings model that generated the historical data, as our rational-expectations calculations assume, or (b) uncovered interest parity does not hold. Big differences in the volatility of exchange rates between the two regimes make it possible that (b) is due to risk aversion on the part of foreign exchange speculators.

The plan of the paper is as follows. Section I sets out the basic model we use to formalize the long swings hypothesis. Section II characterizes our estimation procedure. Empirical results are presented in Section III, while Section IV analyzes the hypothesis of uncovered interest parity. Conclusions are offered in Section V.

I. A Model of Stochastic Segmented Trends

Our model postulates the existence of an unobserved variable (denoted s_t) that takes on the value one or two. This variable characterizes the "state" or "regime" that the process was in at date t. When $s_t = 1$, the observed change in the exchange rate y_t is presumed to have been drawn from a $N(\mu_1, \sigma_1^2)$ distribution, whereas when $s_t = 2$, y_t is distributed $N(\mu_2, \sigma_2^2)$; thus when $s_t = 1$, the trend in the exchange rate is μ_1, whereas when $s_t = 2$, the trend is μ_2.

We further postulate a Markov chain for the evolution of the unobserved state vari-

the pound, mark, and franc. She does not explore the hypothesis of uncovered interest parity and does not devote as much detail to analysis of the random walk hypothesis. She extends Hamilton's model to a case where information besides realized exchange rates is available on the state of the process. Stephan Schulmeister (1987) has also documented the "long swings" phenomenon on the basis of nonparametric statistical analysis.

able:

$$(1)\quad \begin{aligned} p(s_t=1|s_{t-1}=1) &= p_{11} \\ p(s_t=2|s_{t-1}=1) &= 1-p_{11} \\ p(s_t=1|s_{t-1}=2) &= 1-p_{22} \\ p(s_t=2|s_{t-1}=2) &= p_{22}. \end{aligned}$$

The process for s_t is presumed to depend on past realizations of y and s only through s_{t-1}.

Note the variety of behavior that the model allows; in particular, we do not impose that exchange rates are described by long swings. For example, there can be asymmetry in the persistence of the two regimes—upward moves could be short but sharp (μ_1 large and positive, p_{11} small), whereas downward moves could be gradual and drawn out (μ_2 negative and small in absolute value, p_{22} large). Alternatively, the exchange rate change this period could be completely independent of the state that prevailed last period, as in a random walk, if $p_{11}=1-p_{22}$. A third possibility is the long swings hypothesis, which we represent as the claim that μ_1 and μ_2 are opposite in sign and that values for both p_{11} and p_{22} are large.

Our model resembles a standard probability distribution that is called a "mixture of normal distributions." This distribution is a superposition of two (or more) simple normal distributions. A histogram of data drawn from such a distribution would represent the sum of two overlapping bell-shaped curves. The parameters of the distribution would be the mean and variance of each of the simple normal distributions, and a weight given to the first distribution to represent the fraction of realizations that were likely to have been drawn from it. One could use these parameters to calculate the probability that any given observation came from the first distribution. The difference between our model and this mixture of normals is that the draws of y_t in our model are not independent. When we infer the odds that a particular y_t comes from the first distribution, that probability depends on the realizations of y at other times.

II. Maximum Likelihood Estimation of Parameters

The probability law for the data $\{y_t\}$ is summarized by six population parameters,

$$\boldsymbol{\theta} = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})'.$$

These parameters are sufficient to describe (a) the distribution of y_t given s_t, (b) the distribution of s_t given s_{t-1} in equations (1), and (c) the unconditional distribution of the state of the first observation:

$$(2)\quad p(s_1=1;\boldsymbol{\theta}) \equiv \rho = \frac{(1-p_{22})}{(1-p_{11})+(1-p_{22})};$$

of course $p(s_1=2;\boldsymbol{\theta})=1-\rho$. The joint probability distribution of the observed data for a sample of size T $(y_1,\dots,y_T)$ along with the unobserved states $(s_1,\dots,s_T)$ is then

$$(3)\quad \begin{aligned} &p(y_1,\dots,y_T,s_1,\dots,s_T;\boldsymbol{\theta}) \\ &= p(y_T|s_T;\boldsymbol{\theta})\cdot p(s_T|s_{T-1};\boldsymbol{\theta}) \\ &\cdot p(y_{T-1}|s_{T-1};\boldsymbol{\theta})\cdot p(s_{T-1}|s_{T-2};\boldsymbol{\theta})\cdot\dots\cdot \\ &p(s_2|s_1;\boldsymbol{\theta})\cdot p(y_1|s_1;\boldsymbol{\theta})\cdot p(s_1;\boldsymbol{\theta}). \end{aligned}$$

The sample likelihood function could be thought of as the summation of (3) over all possible values of $(s_1,\dots,s_T)$:

$$(4)\quad p(y_1,\dots,y_T;\boldsymbol{\theta}) = \sum_{s_1=1}^{2}\cdots\sum_{s_T=1}^{2} p(y_1,\dots,y_T,s_1,\dots,s_T;\boldsymbol{\theta}).$$

In practice we use Hamilton's (1989a) simpler algorithm for evaluation of (4) that does not require 2^T summations.

Given knowledge of the population parameters $\boldsymbol{\theta}$, it is straightforward to characterize the probability that the process was in some particular regime s_t at date t on the basis of information available at the time

$$p(s_t|y_1,\dots,y_t;\boldsymbol{\theta}).$$

We refer to this as the "filter" inference about the probable regime at date t. Alternatively, one can use the full sample of *ex post* available information $(y_1,\dots,y_T)$ to draw an inference about the historical state the process was in at some date t:

$$p(s_t|y_1,\dots,y_T;\boldsymbol{\theta}),$$

which we refer to as the "smoothed" inference about the regime at date t.

Note that unlike the model of a mixture of normal distributions in which the y_t are independent, these probabilities depend at each time on y's that occur at other times. For example, if $s_{t-1}=1$ and p_{11} is high, then y_t is more likely to have been generated from distribution 1; on the other hand, if $s_{t-1}=2$ and p_{22} is high, then y_t is more likely to have been drawn from distribution 2.

Hamilton (forthcoming) showed that first-order conditions for maximization of (4) with respect to $\boldsymbol{\theta}$ characterize the MLE $\hat{\boldsymbol{\theta}}$ as satisfying

$$(5)\quad \hat{\mu}_j=\frac{\sum_{t=1}^{T} y_t\cdot p(s_t=j|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})}{\sum_{t=1}^{T} p(s_t=j|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})}\qquad j=1,2$$

$$(6)\quad \hat{\sigma}_j^2=\frac{\sum_{t=1}^{T} (y_t-\hat{\mu}_j)^2\cdot p(s_t=j|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})}{\sum_{t=1}^{T} p(s_t=j|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})}\qquad j=1,2$$

$$(7)\quad \hat{p}_{11}=\left\{\sum_{t=2}^{T} p(s_t=1,s_{t-1}=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})\right\}\div\left\{\sum_{t=2}^{T} p(s_{t-1}=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})+\hat{\rho}-p(s_1=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})\right\}$$

$$(8)\quad \hat{p}_{22}=\left\{\sum_{t=2}^{T} p(s_t=2,s_{t-1}=2|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})\right\}\div\left\{\sum_{t=2}^{T} p(s_{t-1}=2|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})-\hat{\rho}+p(s_1=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})\right\}.$$

Consider first the intuition behind equation (5). Suppose that the econometrician knows with certainty which observations came from regime 1 and which from regime 2. Then $p(s_t=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})$ is either one or zero for all observations, and the estimated mean for regime 1, $\hat{\mu}_1$, is simply the average value of y_t for those observations known to come from regime 1 (that is, the average of y_t for those dates t for which $p(s_t=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})=1$). If the designation of regimes is not known with certainty, then an observation t whose smoothed probability of coming from regime 1 is 0.3 ($p(s_t=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})=0.3$) is given a weight of 0.3 in constructing the estimate of μ_1 and of 0.7 in constructing the estimate of μ_2.

Similarly, the variance imputed to regime 1 ($\hat{\sigma}_1^2$ in equation 6) is a weighted sum of squared deviations of the observations around the imputed population mean ($\hat{\mu}_1$), with weights again proportional to the probability that any date t's datum was indeed generated from regime 1.

Finally, the estimate of the Markov transition probability (7) is again best understood by first considering the case where designation of regimes is known with certainty ($p(s_t=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})=1$ or 0). Then the estimated Markov transition probability ($\hat{p}_{11}$) is essentially the number of times the transition was made from 1 to 1 as a fraction of the number of times the process had been in state 1 the previous period. In addition, there is a slight adjustment in the denominator for initial conditions. If the process seems to have been in state 1 at date 1 ($p(s_1=1|y_1,\dots,y_T;\hat{\boldsymbol{\theta}})$ large), and yet an ergodic draw from the Markov process (1) is unlikely to be in state 1 ($\hat{\rho}$ in equation

2 is small), then the adjustment favors choosing a larger value for $\hat{p}_{11}$.

We found solutions to equations (5)–(8) by using the EM algorithm developed in Hamilton (forthcoming).

A well-known problem (for example, B. S. Everitt and D. J. Hand, 1981) with estimating parameters for i.i.d. mixtures of normal distributions is that a singularity in the likelihood function arises when, for example, the mean of regime 1 is imputed to equal the value of the realization of the first observation in the sample ($\mu_1 = y_1$) and the variance of regime 1 is permitted to vanish ($\sigma_1 \to 0$). At such a singularity, the likelihood function (4) blows up to infinity. This paper follows Hamilton (1988b) in incorporating a Bayesian prior for the parameters of the two regimes, replacing (5) and (6) with

$$(9)\quad \hat{\mu}_j = \frac{\sum_{t=1}^{T} y_t \cdot p(s_t = j | y_1, \ldots, y_T; \hat{\boldsymbol{\theta}})}{\nu + \sum_{t=1}^{T} p(s_t = j | y_1, \ldots, y_T; \hat{\boldsymbol{\theta}})}$$

$$(10)\quad \hat{\sigma}_j^2 = \left[\frac{1}{\alpha + (1/2) \cdot \sum_{t=1}^{T} p(s_t = j | y_1, \ldots, y_T; \hat{\boldsymbol{\theta}})}\right] \times \left[\beta + (1/2) \cdot \sum_{t=1}^{T} (y_t - \hat{\mu}_j)^2 \cdot p(s_t = j | y_1, \ldots, y_T; \hat{\boldsymbol{\theta}}) + (1/2) \cdot \nu \cdot (\hat{\mu}_j)^2\right].$$

This Bayesian approach reproduces the MLE as a special case of the diffuse prior $\nu = \alpha = \beta = 0$. In general, (9) shrinks $\hat{\mu}_j$ toward zero for $j = 1,2$, as if one had, in addition to the observed data ($y_1, \ldots, y_T$), ν additional observations from each regime that took on the value zero. Equation (10) adjusts $\hat{\sigma}_j^2$ toward (β/α), as though one had 2α observations relevant toward this adjustment. Equations (7) and (8) are left as is. The prior thus shifts the MLE estimates in the direction of concluding that there is no difference between the two regimes.

A numerically equivalent way to think about this prior is that one is seeking to maximize not the likelihood function (4) but rather the generalized objective function

$$(11)\quad z(\boldsymbol{\theta}) = \log p(y_1, \ldots, y_T; \boldsymbol{\theta}) - \left[(\nu \cdot \mu_1^2)/(2\sigma_1^2)\right] - \left[(\nu \cdot \mu_2^2)/(2\sigma_2^2)\right] - \alpha \log \sigma_1^2 - \alpha \log \sigma_2^2 - \beta/\sigma_1^2 - \beta/\sigma_2^2.$$

Unlike the likelihood function (4), the singularities described above are not a feature of the objective function (11) for α, β, and $\nu > 0$. Monte Carlo simulations reported in Hamilton (1988b) suggest that very modest priors can consistently improve mean squared errors.

III. Empirical Results

A. Maximum Likelihood Estimates

The raw data for this project are an arithmetic average of the bid and asked prices for the exchange rate (in dollars per unit of foreign currency) for the last day of the quarter, beginning with the third quarter of 1973 and ending with the first quarter of 1988. The data are expressed in units of percentage change, denoted y_t^{WG}, y_t^{FR}, and y_t^{UK}.[4]

We estimated the parameter vector $\boldsymbol{\theta}$ for each currency in isolation from the others. Table 1 reports maximum likelihood estimates; the Appendix provides further details.

[4]All series were taken from the data banks compiled by Data Resources, Inc., as of June 1988. The raw data have the DRI series names WGCOOA, WGCOOB, FRCOOA, FRCOOB, UKCOOA, and UKCOOB. Natural logarithms were taken. The data were then first-differenced and multiplied by 100 to express in units of percentage change. The resulting quarterly series (y_t^{WG}, y_t^{FR}, and y_t^{UK}) run from 1973:IV to 1988:I.

TABLE 1—ESTIMATES FIT TO INDIVIDUAL COUNTRY DATA, $y_t = e_t - e_{t-1}$, $t = 73{:}\text{IV}$ TO $88{:}\text{I}$, $e_t = 100$ TIMES THE LOG OF THE EXCHANGE RATE (IN DOLLARS PER UNIT OF FOREIGN CURRENCY)

Parameter	Germany	France	U.K.
μ_1	3.987	3.256	2.627
	(1.230)	(0.967)	(0.872)
μ_2	−1.183	−2.712	−3.752
	(1.480)	(1.367)	(1.139)
p_{11}	0.848	0.822	0.927
	(0.122)	(0.105)	(0.057)
p_{22}	0.928	0.908	0.913
	(0.066)	(0.063)	(0.073)
σ_1^2	17.652	9.991	16.918
	(9.351)	(5.001)	(4.660)
σ_2^2	42.166	36.921	20.247
	(11.242)	(10.252)	(5.841)
$\hat{\rho}$	0.322	0.342	0.542
$p(s_1 = 1 \mid y_1, \ldots, y_T; \hat{\theta})$	0.004	0.000	0.373

Note: Standard errors are in parentheses.

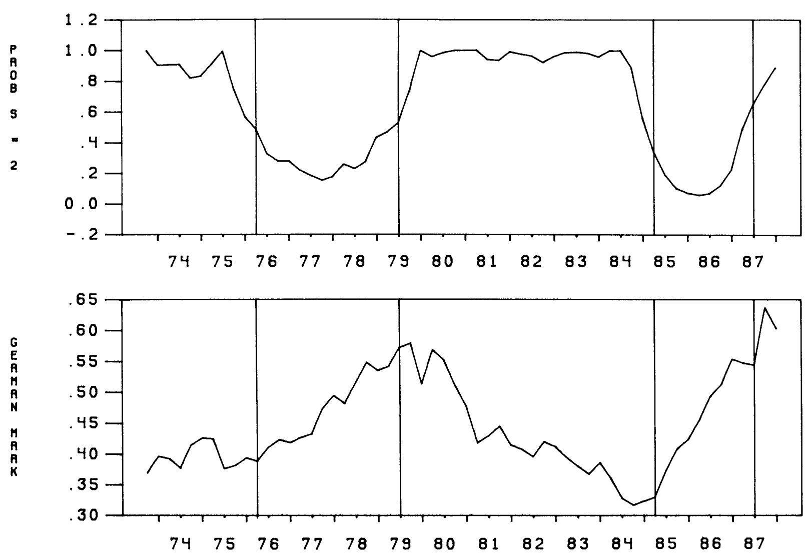

FIGURE 2

The maximum likelihood estimates associate state 1 with a 4 percent quarterly rise in the German mark, a 3.3 percent rise in the franc, and a 2.6 percent rise in the pound. In state 2 the currencies fall by −1.2 percent, −2.7 percent, and −3.8 percent, respectively, with considerably more variability in the exchange rate apparent in state 2 than in state 1.

The bottom panel of Figure 2 plots the exchange rate for the German mark (in \$/mark). The top panel plots the smoothed probability that the process was in regime 2 at each date in the sample; that is, $p(s_t = 2 \mid y_1^{WG}, \ldots, y_T^{WG}; \hat{\theta}^{WG})$ is plotted as a function of t. This inference uses the full sample of observations for Germany $(y_1^{WG}, \ldots, y_T^{WG})$ and the maximum likelihood estimates of

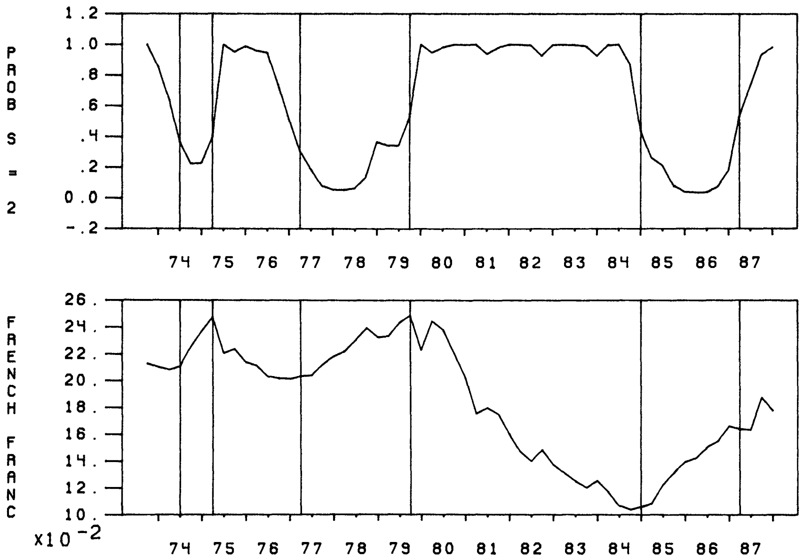

FIGURE 3

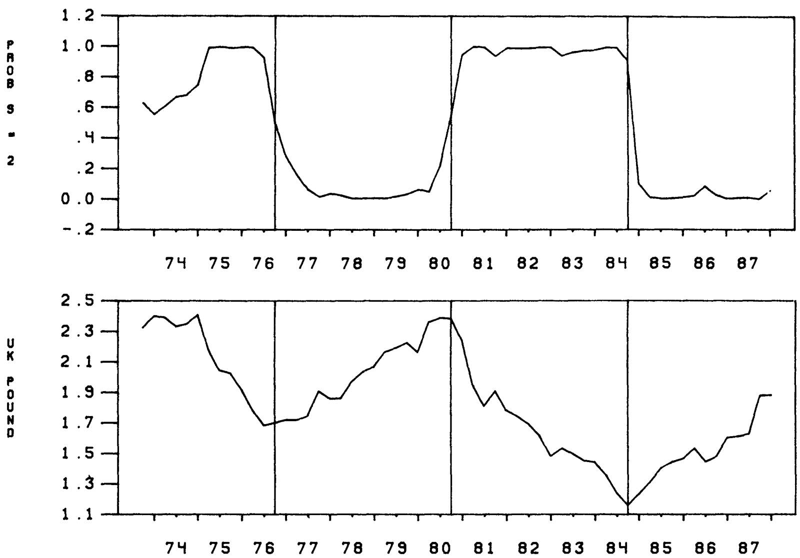

FIGURE 4

parameters $\hat{\theta}^{WG}$ to draw an inference about the state of the process at each date t. The dates at which the econometrician would conclude that the process had switched between regimes (based on $p(s_t = 2|y_1^{WG},\dots,y_T^{WG};\hat{\theta}^{WG}) \gtrless 0.5$) are shown as vertical bars. Similar diagrams for France and the U.K. appear as Figures 3 and 4.

The estimates in Table 1 show that movements in the exchange rate are characterized by long swings. The point estimates of p_{11} range from 0.822 to 0.927, while the estimates of p_{22} go from 0.908 to 0.928. These probabilities indicate that if the system is in either state 1 or state 2, it is likely to stay in that state. Inspection of Figures 2–4 shows that by our estimates the switches between states are infrequent. All of the currencies were in a state of appreciation of the dollar (that is, they were in state 2) from 1980–1984, and were in a state of depreciation of the dollar (state 1) from the end of 1984 to 1987. Thus, our model of long swings tends to match closely what one might be led to believe from casual inspection of Figure 1.[5]

States 1 and 2 are differentiated not only by their means but also by the variances of the conditional distributions. The exchange rate seems to be much more variable when the dollar is appreciating. For the mark and franc, our estimates show that the dollar entered the appreciation stage in the middle of 1987. This assessment is based on the unusual volatility in the exchange rate during that year.

It is straightforward conceptually to generalize this approach to vector processes $\mathbf{y}_t$ (see Hamilton, 1988b). Here we posit that $\mathbf{y}_t|s_t \sim N(\boldsymbol{\mu}_{s_t}, \boldsymbol{\Omega}_{s_t})$. Equations (7), (8), and (9) continue to hold with y_t and μ_j interpreted as the corresponding vectors. Equation (10) is replaced by

$$(12)\quad \hat{\boldsymbol{\Omega}}_j = \left(\frac{1}{\alpha + (1/2)\cdot \sum_{t=1}^{T} p(s_t = j|\mathbf{y}_1,\ldots,\mathbf{y}_T;\hat{\boldsymbol{\theta}})}\right) \times \Big[\boldsymbol{\Lambda} + (1/2)\cdot \sum_{t=1}^{T} (\mathbf{y}_t - \hat{\boldsymbol{\mu}}_j)(\mathbf{y}_t - \hat{\boldsymbol{\mu}}_j)' \cdot p(s_t = j|\mathbf{y}_1,\ldots,\mathbf{y}_T;\hat{\boldsymbol{\theta}}) + (1/2)\cdot \nu \cdot \hat{\boldsymbol{\mu}}_j \hat{\boldsymbol{\mu}}_j'\Big]$$

where $(\alpha, \boldsymbol{\Lambda})$ is a multivariate generalization of (α, β) based on the Wishart distribution.

[5]Kaminsky (1988) fit Hamilton's model to monthly data on the pound. She assumed constant variances and arrived at parameter estimates for the means and transition probabilities, as well as inference about historical switch points, that are comparable to those we find for quarterly data.

Unfortunately, we had little success in using these equations to fit all three currencies to a process driven by a single scalar state variable s_t. The estimates did not correspond well with the individual inferences of any of the three currencies. The behavior of individual exchange rates is determined, of course, not only by events in the United States but also by events in each of the corresponding countries. It appears that treating these three exchange rates as a group is inappropriate because country-specific developments played an important role in the evolution of exchange rates in the 1970s. For this reason, we proceed in our analysis of each of the three countries in insolation from the others.

B. *Testing the Null Hypothesis That Exchange Rates Follow a Random Walk*

An alternative to the segmented trends model is the simple random walk. Michael Mussa (1979), Richard Meese and Kenneth Singleton (1982), Meese and Kenneth Rogoff (1983a, 1983b), and Francis Diebold and James Nason (forthcoming) have all produced evidence that the log of the exchange rate follows a random walk. David Hsieh (1989), however, found evidence consistent with both the earlier random walk conclusions and the predictions of our model, asserting that while there was little or no linear serial dependence in the log of the change in daily exchange rates, there seems to be general nonlinear serial dependence.

There are some knotty methodological problems in testing the null hypothesis that exchange rates follow a random walk against the segmented trends alternative. If one views the null hypothesis as the claim that $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$, then under the null hypothesis, the parameters p_{11} and p_{22} are unidentified. Moreover, at the constrained MLE

$$\left(\hat{\mu}_1 = \hat{\mu}_2 = \bar{y} = \sum_{t=1}^{T} y_t / T,\quad \hat{\sigma}_1^2 = \hat{\sigma}_2^2 = \sum_{t=1}^{T} (y_t - \bar{y})^2 / T\right),$$

TABLE 2—TESTS OF THE NULL HYPOTHESIS THAT EXCHANGE RATES FOLLOW A MARTINGALE AGAINST THE ALTERNATIVE OF SEGMENTED TRENDS

	H_0': $p_{11}=1-p_{22}$		H_0'': $\mu_1=\mu_2$	
Country	Wald Test	Likelihood Ratio Test	Wald Test	Likelihood Ratio Test
Germany	24.70	2.49	8.64	3.00
	(0.00)	(0.11)	(0.00)	(0.08)
France	29.45	5.70	12.65	6.10
	(0.00)	(0.02)	(0.00)	(0.01)
U.K.	61.26	6.15	25.80	8.85
	(0.00)	(0.01)	(0.00)	(0.00)

Note: All statistics are asymptotically $\chi^2(1)$. Asymptotic p-values are in parentheses.

the derivative of the likelihood function with respect to μ_1 or σ_1 is identically zero. These difficulties combine features of the problems discussed by Mark Watson and Robert Engle (1985) and Lung-Fei Lee and Andrew Chesher (1986). The information matrix is singular under the null, and the standard regularity conditions for establishing asymptotically valid tests of H_0 do not hold in this case.

In this paper we sidestep these issues by focusing on the following slightly more general null hypothesis:

$$H_0': \quad p_{11}=1-p_{22} \qquad \mu_1\neq\mu_2 \qquad \sigma_1\neq\sigma_2.$$

Note that under H_0', the distribution of s_t is independent of s_{t-1}; [from (1), the probability that $s_t=1$ is p_{11} regardless of whether $s_{t-1}=1$ or 2]. Changes in the exchange rate under H_0' thus comprise an i.i.d. sequence with individual densities given by the following mixture of two normals:

$$p(y_t;\boldsymbol{\theta})=\frac{p_{11}}{\sqrt{2\pi}\,\sigma_1}\exp\left[\frac{-(y_t-\mu_1)^2}{2\sigma_1^2}\right]+\frac{(1-p_{11})}{\sqrt{2\pi}\,\sigma_2}\exp\left[\frac{-(y_t-\mu_2)^2}{2\sigma_2^2}\right].$$

We can then hope to test H_0' against the alternative that $p_{11}\neq1-p_{22}$, using standard distribution theory, since under H_0' the parameters μ_1, μ_2, σ_1, σ_2, p_{11}, and p_{22} are all identified.

We report two tests of H_0', the fist being a Wald test. Let $\hat{\text{var}}(\hat{p}_{jj})$ denote the asymptotic variance of $\hat{p}_{jj}$ (as estimated from the inverse of the negative of the matrix of second derivatives of (11)) and $\hat{\text{cov}}(\hat{p}_{11},\hat{p}_{22})$ the asymptotic covariance. Then under H_0',

$$\text{(13)}\qquad \frac{[\hat{p}_{11}-(1-\hat{p}_{22})]^2}{[\hat{\text{var}}(\hat{p}_{11})+\hat{\text{var}}(\hat{p}_{22})+2\,\hat{\text{cov}}(\hat{p}_{11},\hat{p}_{22})]}\approx\chi^2(1).$$

Column 1 of Table 2 reports this Wald test statistic for the three currencies. The 5 percent critical value for a $\chi^2(1)$ variate is 3.84, implying overwhelming rejection of H_0' for all three currencies.

We also tested H_0' by using a likelihood ratio test.[6] This statistic compares the value of the objective function achieved by the estimates in Table 1 with the largest value achievable when estimated subject to the constraint $p_{11}=(1-p_{22})$. The latter estimation is a straightforward application of estimating parameters for an i.i.d. mixture of two normals; we used the EM algorithm described in Everitt and Hand (1981, pp.

[6]A. Ronald Gallant (1987, p. 219) argues that the likelihood ratio test is apt to be more robust than Wald tests in a nonlinear model such as this one.

36–37) for this purpose, with Bayesian correction as in (9) and (10). Twice the difference in the objective function (11) between the constrained and unconstrained estimates is reported in column 2 of Table 2, and is presumed asymptotically to have a $\chi^2(1)$ distribution. The magnitude of the difference between the Wald test statistics and the likelihood ratio test statistics is disconcerting.[7] Still, at least in the case of France and the U.K., the rejection of H_0' continues to be fairly decisive.

It is also interesting to test the hypothesis H_0'': $\mu_1 = \mu_2$. Under this hypothesis, the exchange rate follows a stochastic process as described in Section I, with the mean rate of depreciation the same in both states. If $\mu_1 = \mu_2$, but $\sigma_1 \neq \sigma_2$, the states have the same mean rate of depreciation but different variances. A Wald statistic for testing H_0'' is given by

$$\frac{(\hat{\mu}_1 - \hat{\mu}_2)^2}{\hat{\text{var}}(\hat{\mu}_1) + \hat{\text{var}}(\hat{\mu}_2) - 2\hat{\text{cov}}(\hat{\mu}_1, \hat{\mu}_2)} \approx \chi^2(1).$$

The statistics are reported in column 3 of Table 2. Column 4 reports an analogous likelihood ratio test. The means of the two states are different.

We thus conclude that movements in the dollar are described by long swings. The dollar enters stages in which it appreciates or depreciates and it remains in such stages for years. The expected length of state i is $1/(1 - p_{ii})$. State 1 is expected to persist for seven quarters for Germany, six for France, and fourteen for the U.K. On average state 2 lasts fourteen quarters for the mark, eleven for the franc, and twelve for the pound.

C. *Forecasting*

As further evidence on the random walk hypothesis, we calculated the in-sample and post-sample forecast errors for the segmented trends model in comparison to those of the random walk specification.

Consider first in-sample forecasts. If one takes the MLE $\hat{\boldsymbol{\theta}}$ (about which the full sample $y_1, \dots, y_T$ was used to draw inference) as known at date t, then the forecast one would make on the basis of observation of y through date t and on the basis of knowledge of $\hat{\boldsymbol{\theta}}$ is given by[8]

$$(14) \quad E\left[y_{t+j} | y_t, y_{t-1}, \dots, y_1; \hat{\boldsymbol{\theta}}\right] = \hat{\mu}_2 + \left\{\hat{\rho} + (-1 + \hat{p}_{11} + \hat{p}_{22})^j \cdot \left[p(s_t = 1 | y_1, \dots, y_t; \hat{\boldsymbol{\theta}}) - \hat{\rho}\right]\right\} \cdot \{\hat{\mu}_1 - \hat{\mu}_2\},$$

with $\hat{\rho}$ given in (2). Letting $\hat{y}_{t+j|t}$ denote the expression in (14), we calculated k-period ahead forecasts of the level of the log of the exchange rate

$$\hat{e}_{t+k|t} = e_t + \hat{y}_{t+1|t} + \hat{y}_{t+2|t} + \cdots + \hat{y}_{t+k|t}$$

and calculated the average squared value of the forecast error,

$$\sum_{t=1}^{T-k} (\hat{e}_{t+k|t} - e_{t+k})^2 / (T-k),$$

for forecast horizons (k) of one through four quarters.

The top panel of Table 3 compares these forecast errors with those of a random walk specification, whose forecasts are given by $\hat{e}_{t+k|t} = e_t + k \cdot \bar{y}$, where $\bar{y} = \sum_{t=1}^{T} y_t / T$. Note that the variance of the latter forecast error should, if the random walk specification is correct, rise linearly with the forecast horizon k. The actual MSE's for the random walk specification in Table 3 perform more poorly than this, owing to positive autocorrelation in y_t at lags one through

[7] This seems due in part to asymmetry in the likelihood surface for increases and decreases in p_{jj}. For values of p_{jj} above $\hat{p}_{jj}$, the Wald approximation slightly understates the true curvature of the likelihood function, whereas for $p_{jj} < \hat{p}_{jj}$, the likelihood function quickly becomes much flatter than the Hessian evaluated at $\hat{p}_{jj}$ predicts.

[8] See equation (3.2) in Hamilton (1989a).

TABLE 3—IN-SAMPLE AND POST-SAMPLE MEAN SQUARED FORECAST ERROR AT HORIZONS FROM ONE TO FOUR QUARTERS OF SEGMENTED TRENDS MODEL AND RANDOM WALK WITH DRIFT

A. In-Sample Mean Squared Forecast Errors Country (Model)	Forecast Horizon (Quarters) 1	 2	 3	 4
Germany				
(Random Walk)	38.01	83.79	130.84	199.18
(Segmented Trend)	36.48	76.39	113.55	174.93
(Percent Improvement)	4 Percent	9 Percent	13 Percent	12 Percent
France				
(Random Walk)	34.37	82.96	143.99	220.88
(Segmented Trend)	31.27	72.13	124.31	194.76
(Percent Improvement)	9 Percent	13 Percent	14 Percent	12 Percent
United Kingdom				
(Random Walk)	29.10	76.06	124.45	187.26
(Segmented Trend)	25.11	65.95	107.82	170.36
(Percent Improvement)	14 Percent	13 Percent	13 Percent	9 Percent

B. Post-Sample Mean Squared Forecast Errors Country (model)	Forecast Horizon (Quarters) 1	 2	 3	 4
Germany				
(Random Walk)	54.58	141.33	251.62	406.49
(Segmented Trend)	50.44	133.37	245.77	409.85
(Percent Improvement)	8 Percent	6 Percent	2 Percent	−1 Percent
France				
(Random Walk)	52.47	145.59	266.34	426.76
(Segmented Trend)	46.80	134.32	255.33	427.01
(Percent Improvement)	11 Percent	8 Percent	4 Percent	−0 Percent
United Kingdom				
(Random Walk)	42.35	117.54	186.68	270.43
(Segmented Trend)	35.11	98.40	161.28	252.61
(Percent Improvement)	17 Percent	16 Percent	14 Percent	7 Percent

Notes: A. In-Sample Forecast Errors. In each case, the population parameters were estimated by using data from $t = 1973{:}\text{IV}$ to 1988:I and mean squared errors are those associated with forecasts for dates $t = 1973{:}\text{IV} + k$ to 1988:I where k is the forecast horizon.

B. Post-Sample Forecast Errors. In each case, the population parameters were estimated by using data from $t = 1973{:}\text{IV}$ to 1983:IV and mean squared errors are those associated with forecasts for dates $t = 1984{:}\text{I}$ to 1988:I.

three[9] and to the fact that $\bar{y}$, the mean of observations 1 through T, is not quite the same as the mean of observations k though T. The improvement in forecasting at horizons of two to four quarters offered by the segmented trends specification is 9–14 percent for all three currencies.

It is worth noting that our model is doing more than just mimicking an AR(1) specification for exchange rate changes. An AR(1) model has an in-sample one-quarter ahead R^2 of 8 percent for the U.K., 4 percent for France, and less than 1 percent for Germany and offers virtually no improvement in forecasting at longer horizons.

To evaluate the post-sample forecasting performance of the model, we reestimated

[9]Recall John Cochrane's (1988, p. 906) result that

$$\text{Var}(e_{t+k} - e_t) = k\left[\text{Var}(e_{t+1} - e_t) + 2\sum_{j=1}^{k-1}((k-j)/k)\cdot\text{Cov}(y_t, y_{t-j})\right].$$

the parameters with data only up to the end of 1983. We chose the end of 1983 because the major turning point in the dollar that occurred in 1985 had not yet happened. Hence, the entire period of the dollar depreciation of 1985–1987 was not used for estimating parameters. Furthermore, with our out-of-sample forecasts our model must meet the challenge of picking out the turning point.

The parameter estimates for the truncated sample are similar to those of the full sample; using only data through 1983, there is evidence in favor of the long swings hypothesis.

The bottom panel of Table 3 compares the post-sample mean squared error of the forecasts of our model with that of a random walk (with the drift term estimated from data through the end of 1983). The forecasts for the segmented trends model were calculated as in equation (14), but using the parameter estimates from the restricted sample. We found that our model generally outperformed the random walk, particularly at short forecasting horizons.

Table 3 follows Mussa (1979) and Meese and Singleton (1982) in including a drift term in the random walk; by contrast, Meese and Rogoff (1983a, 1983b) and Diebold and Nason (1989) set the drift term a priori to zero. The zero-drift random walk specification has a forecasting performance over 1984:I–1988:I that significantly beats both our specification and the random walk with drift. Some would interpret this as evidence in favor of the random walk hypothesis. We would instead argue that the superiority during 1984–1988 of the random walk without drift over the random walk with drift offers conclusive evidence that exchange rates do *not* follow a random walk, with or without drift! If the data really followed a driftless random walk, then differences in post-sample forecasts between the two random-walk specifications should be entirely due to error in estimating the drift term. Conventional statistical tests lead to clear rejection of the null hypothesis that the exchange rate data come from a random walk with the same drift term before and after 1984 (see also Table 4 below). The driftless random walk is just a special case of this rejected hypothesis. Imposing a particular numerical value for the drift (in this case, zero) is of course going to improve the fit over selected subsamples, but cannot salvage the model as a specification that describes the complete sample. An apparent break in parameter values over particular subsamples may be an important feature that accounts for the results of Meese and Rogoff and Diebold and Nason, and it is precisely the feature of the data that our long swings representation is attempting to model.

We conclude that the accumulated evidence from the Wald tests, likelihood ratio tests, and forecasting performance favors the segmented-trends specification over the random walk.

D. *Specification Testing*

This section presents specification tests that fall into four broad groups. The first group explores the forecastability of the one-quarter-ahead in-sample forecast errors. Second, we consider tests based on the work of Whitney Newey (1985), George Tauchen (1985), and Halbert White (1987) that examine the null hypothesis that the score statistics are serially uncorrelated. Third, we perform Lagrange multiplier tests for various sorts of dynamic misspecification. Finally, we split the sample at the end of 1979 and again at the end of 1983 and perform likelihood ratio tests for changes in the stochastic process governing exchange rates.

Forecasting Tests. Our forecasting tests divide the one-quarter-ahead forecasts of our model (expression 14) by their conditional standard deviation:

$$\hat{\sigma}_{\hat{y}_{t+1|t}} = \Big\{\big\{\big[\hat{\mu}_1^2 + \hat{\sigma}_1^2\big]\cdot \hat{p}_{t+1|t} + \big[\hat{\mu}_2^2 + \hat{\sigma}_2^2\big]\cdot\big(1-\hat{p}_{t+1|t}\big)\big\} - \big\{\hat{\mu}_1\cdot\hat{p}_{t+1|t} + \hat{\mu}_2\cdot\big(1-\hat{p}_{t+1|t}\big)\big\}^2\Big\}^{1/2},$$

where

$$\hat{p}_{t+1|t} = (1-\hat{p}_{22}) + (-1+\hat{p}_{11}+\hat{p}_{22}) \cdot p(s_t = 1|y_1,\dots,y_t;\hat{\boldsymbol{\theta}}).$$

The resulting standardized one-period-ahead forecasts errors ($\hat{u}_{t+1}$) should be unforecastable with any time t variables. We calculated many regressions of these errors on their own lagged values, on lagged values of the log changes in exchange rates, and on various combinations of the squares and cross-products of these variables. In no case did a joint test of a zero intercept and zero slope coefficients reject the null hypothesis. For example, a regression of $\hat{u}_t$ on a constant, $\hat{u}_{t-1}$ and $\hat{u}_{t-1}^2$, for $t = 74\text{:II}-88\text{:I}$ yields $F(3,53)$ statistics whose p-values are (0.90), (0.84), and (0.70) for the three currencies. The smallest p-value in the two dozen regressions we looked at was (0.42) for the regression of $\hat{u}_t$ on $\hat{u}_{t-j}$ and $\hat{u}_{t-j}^2$ for $j = 1,2,3,4$ for the U.K.

Hsieh (1989) used the test of William Brock, W. Davis Dechert, and José Scheinkman (1987) to search for general nonlinear dependence in a time-series. He found evidence of significant dependence in daily data for several currencies. We repeated these tests on our standardized residuals.[10] We varied N (the dimension of "N-histories") between 2 and 6 (in steps of 1) and ℓ (the distance measure, in standard deviations of the data) between 0.5 and 1.5, in steps of 0.25. We found no evidence of serial dependence in the standardized residuals. However, in contrast to Hsieh's analysis of daily data, there is little evidence from this test of nonlinear dependence in the raw quarterly exchange rate changes.

Score tests. White (1987) noted that if a maximum likelihood model is correctly specified, the score statistics (the derivative of the conditional log likelihood of the t-th observation) should be serially uncorrelated. Hamilton (1989b) showed how White's results may be used to construct tests for possible alternatives to the Markov switching model. Table 4 applies these tests to the random walk specification, and Table 5 to our long swings model.

By considering the score with respect to the mean, a White test for autocorrelation can be constructed (which essentially tests for the correlation of the score at time t with respect to μ_i and the score at time $t-1$ with respect to μ_j, for $i,j = 1,2$.) Table 4 provides evidence of autocorrelation in the raw data for the U.K., while Table 5 finds no evidence of autocorrelation left over after fitting the long swings model.

An ARCH test can be implemented by examining the serial correlation properties of the scores with respect to σ_i^2, $i = 1,2$. Table 5 shows that the test for ARCH for the U.K. is significant at the 5 percent level. However, Hamilton (1989b) concluded from Monte Carlo simulations that "For a sample as small as 50 observations, one might be better off using the 1 percent critical value from the asymptotic distributions (rather than the 5 percent value) as a rough guide for a 5 percent small-sample test based on the Newey-Tauchen-White specification tests or Lagrange multiplier tests for misspecification of the variance." By this standard, the null hypothesis of no ARCH should not be rejected.

The Markov assumption that $p(s_t = i)$ depends only on the state at time $t-1$ can be tested against the alternatives that it depends on the state at earlier times or that it depends on the realizations of the data y_{t-1}. This test checks whether the score with respect to the transition probabilities can be predicted by the corresponding lagged score or the score with respect to the mean. Table 5 shows that the Markov specification cannot be rejected for any currency.

Lagrange Multiplier Tests. Tables 4 and 5 also present Lagrange multiplier tests of the random walk and long swings specifications. We tested against the alternatives that there is omitted autocorrelation only in state 1, autocorrelation only in state 2, and autocorrelation across regimes. These produced the same conclusions as the White tests for autocorrelation.

[10] We calculated these statistics by using computer code kindly distributed to us by W. Davis Dechert.

TABLE 4—TESTS OF NULL HYPOTHESIS THAT PERCENT CHANGES IN EXCHANGE RATES ARE i.i.d. GAUSSIAN

Test	Germany	France	U.K.
White Test for	0.28	2.15	**[6.02]**
Autocorrelation ($\chi^2(1)$)	(0.60)	(0.14)	(0.01)
White Test for	0.32	0.02	0.39
ARCH ($\chi^2(1)$)	(0.57)	(0.89)	(0.53)
LM Test for	0.28	2.14	**[6.02]**
Autocorrelation ($\chi^2(1)$)	(0.60)	(0.14)	(0.01)
LM Test for	0.68	0.00	0.32
ARCH ($\chi^2(1)$)	(0.41)	(1.00)	(0.57)
LM Test for Shift in Mean	**[4.14]**	**[6.70]**	**[3.92]**
79:IV–82:IV ($\chi^2(1)$)	(0.04)	(0.01)	(0.05)
LM Test for Shift in Mean	**[4.53]**	**[7.69]**	**[10.19]**
85:II–88:I ($\chi^2(1)$)	(0.03)	(0.01)	(0.00)

Notes: All statistics are asymptotically $\chi^2(1)$ [5 percent critical value = 3.84; 1 percent critical value = 6.63]. Brackets [ ] denote significant at 5 percent level. Asymptotic p-values are in parentheses.

TABLE 5—TESTS OF NULL HYPOTHESIS THAT EXCHANGE RATES FOLLOW THE LONG SWINGS MODEL

Test	Germany	France	U.K.
White Test for	2.56	4.23	4.61
Autocorrelation ($\chi^2(4)$)	(0.63)	(0.38)	(0.33)
White Test for	3.52	7.26	**[10.55]**
ARCH ($\chi^2(4)$)	(0.47)	(0.12)	(0.03)
White Test of Markov	2.73	4.47	1.59
Specification ($\chi^2(4)$)	(0.60)	(0.35)	(0.81)
LM Test for Autocorrelation	0.00	3.26	3.02
in Regime 1 ($\chi^2(1)$)	(1.00)	(0.07)	(0.08)
LM Test for Autocorrelation	1.44	0.24	0.30
in Regime 2 ($\chi^2(1)$)	(0.23)	(0.62)	(0.58)
LM Test for Autocorrelation	0.59	1.06	0.65
Across Regimes ($\chi^2(1)$)	(0.44)	(0.30)	(0.42)
LM Test for	1.27	0.10	**[4.47]**
ARCH ($\chi^2(1)$)	(0.26)	(0.75)	(0.03)
LM Test for Shift in Mean	1.66	1.92	1.50
79:IV–82:IV ($\chi^2(1)$)	(0.20)	(0.17)	(0.22)
LM Test for Shift in Mean	**[11.00]**	**[12.24]**	2.28
85:II–88:I ($\chi^2(1)$)	(0.00)	(0.00)	(0.13)

Notes: The first three statistics are asymptotically $\chi^2(4)$ [5 percent critical value = 9.49; 1 percent critical value = 13.28]. All other statistics are asymptotically $\chi^2(1)$ [5 percent critical value = 3.84; 1 percent critical value = 6.63]. Brackets [ ] denote significant at 5 percent level. Asymptotic p-values are in parentheses.

Tables 4 and 5 also report the results of LM tests for ARCH. For the alternative to the long swings model, the variance at time t, h_t, is modeled as

$$h_t = \gamma_{s_t}\left[1 + \frac{\xi(y_{t-1} - \mu_{s_{t-1}})^2}{\gamma_{s_{t-1}}}\right], \qquad i = 1,2$$

(see Hamilton, 1989b). Under the null hypothesis of no ARCH, $\xi = 0$, and $\gamma_{s_t} = \sigma_{s_t}^2$. Again, we found some evidence of ARCH for the U.K., but we would probably not consider it significant at the 5 percent level given our number of observations.

We can also use the Lagrange multiplier principle to test whether the mean of the

TABLE 6—LIKELIHOOD RATIO TESTS FOR WHETHER ALL PARAMETERS OF THE LONG SWINGS MODEL CHANGE AT SPECIFIED DATES

Date of Sample Break	Germany	France	U.K.
1979:IV	8.343	12.313	4.846
	(0.21)	(0.06)	(0.56)
1983:IV	4.303	8.050	3.775
	(0.64)	(0.23)	(0.71)

Notes: All variables are asymptotically $\chi^2(6)$. Asymptotic p-values are in parentheses.

process shifted over any subsample. When we applied this test to the random walk specification, we found evidence for all three currencies of a change in the drift associated with the change in U.S. Federal Reserve operating procedures during October 1979–October 1982 (see Table 4). By contrast, allowing for a separate mean for this subperiod does not make a statistically significant contribution to the long swings model (Table 5). Thus one feature of the data that is inconsistent with a random walk that the long swings model captures is the persistent tendency for the dollar to appreciate during the three-year period in which the Fed targeted nonborrowed reserves.

We also tested for a permanent break in the mean of the series for all possible change points in the sample. It is interesting that for all three currencies and for both the random walk and long swings specifications, the largest value of this statistic comes within one quarter of 1985:II. Table 4 reveals evidence of a break in the process after 1985 that is not captured by the random walk. Table 5 suggests that the long swings model is able to capture this break in the case of the U.K. but not in the case of Germany and France.

Likelihood Ratio Tests. We also tested for shifts in the stochastic process by performing likelihood ratio tests for joint changes in all the parameters at the end of 1979 and at the end of 1983. Table 6 shows that we cannot reject the null of no shift at the 5 percent level for any currency at either date.

IV. Testing the Hypothesis of Uncovered Interest Parity

We now turn to the second question posed by our paper—Is this apparent forecastability of exchange rates reflected in intercountry interest differentials? Uncovered interest parity posits that a three-month Eurodollar account should yield the same return expected by converting the dollars to marks, holding these marks in a Euromark account for three months, and converting back into dollars at the then-prevailing exchange rate:

$$(15)\quad i_t^{US} = i_t^{WG} + E_t\left(e_{t+1}^{WG} - e_t^{WG}\right) + u_t.$$

Here i_t is the return on a Eurocurrency account for the specified currency, e_t is the log of the exchange rate (in dollars per unit of foreign currency), and u_t is a disturbance term that reflects measurement and specification error.

Let the log of the forward exchange rate be f_t^{WG} dollars per mark. A pure arbitrage opportunity exists unless

$$(16)\qquad i_t^{US} = i_t^{WG} + f_t^{WG} - e_t^{WG}.$$

Thus, the hypothesis of uncovered interest parity (15) is essentially equivalent to the hypothesis that the forward rate is an unbiased predictor of the future spot rate

$$f_t^{WG} = E_t e_{t+1}^{WG} + u_t.$$

We report our results in terms of testing uncovered interest parity rather than of testing the risk neutrality of the forward currency market, though the two tests are conceptually the same.

Suppose that investors know the population parameter $\boldsymbol{\theta}$ of the segmented trends model and further observe the value of s_t, which governed the mean of the exchange rate change between $t-1$ and t. When $s_t = 1$, then the change in the exchange rate between t and $t+1$ will be drawn from a $N(\mu_1, \sigma_1^2)$ distribution with probability p_{11} and from a $N(\mu_2, \sigma_2^2)$ distribution with

TABLE 7—INTEREST DIFFERENTIALS AS PREDICTED BY (a) THE UNIVARIATE MLEs FOR EACH COUNTRY'S EXCHANGE RATE (TABLE 1 PARAMETERS), AND (b) THE BIVARIATE MLEs FOR EACH COUNTRY'S EXCHANGE RATE TOGETHER WITH THAT COUNTRY'S INTEREST DIFFERENTIAL (TABLE 8 PARAMETERS)

	Germany	France	U.K.
1. Predicted Value for $i_t^{US} - i_t^*$ When $s_t = 1$ Based on			
(a) Univariate Estimate of	3.203*	2.196*	2.159*
$p_{11}\mu_1 + (1 - p_{11})\mu_2$	(1.150)	(0.934)	(0.904)
(b) Bivariate Estimate of	0.542*	−0.249*	−0.282*
$\mu_1(2)$	(0.061)	(0.118)	(0.090)
2. Predicted Value for $i_t^{US} - i_t^*$ When $s_t = 2$ Based on			
(a) Univariate Estimate of	−0.811	−2.160	−3.199
$p_{22}\mu_2 + (1 - p_{22})\mu_1$	(1.335)	(1.214)	(1.001)
(b) Bivariate Estimate of	1.171	−1.228	−1.425
$\mu_2(2)$	(0.086)	(0.238)	(0.174)
3. Predicted Value for Change in $i_t^{US} - i_t^*$ When State Changes from 2 to 1 Based on			
(a) Univariate Estimate of	4.014*	4.356*	5.367*
$(-1 + p_{11} + p_{22})(\mu_1 - \mu_2)$	(1.597)	(1.447)	(1.182)
(b) Bivariate Estimate of	−0.629*	0.979*	1.143*
$\mu_1(2) - \mu_2(2)$	(0.102)	(0.249)	(0.193)

Notes: Standard errors are in parentheses. The standard error for a nonlinear function $h(\boldsymbol{\theta})$ of the $(p \times 1)$ parameter vector $\boldsymbol{\theta}$ was calculated as the square root of $[h_\theta(\hat{\boldsymbol{\theta}})]'\hat{\text{Var}}(\hat{\boldsymbol{\theta}})[h_\theta(\hat{\boldsymbol{\theta}})]$ where $h_\theta(\hat{\boldsymbol{\theta}})$ denotes the $(p \times 1)$ vector of derivatives of the function $h(\cdot)$ with respect to the elements of $\boldsymbol{\theta}$, evaluated at the MLE $\hat{\boldsymbol{\theta}}$, and $\hat{\text{Var}}(\hat{\boldsymbol{\theta}})$ denotes the $(p \times p)$ estimated variance-covariance matrix of $\hat{\boldsymbol{\theta}}$.

*Indicates that the 95 percent confidence intervals for (a) and (b) fail to overlap.

$\mu_1(j)$ denotes the jth element of the vector $\boldsymbol{\mu}_1$ in Table 8; thus $\mu_1(2)$ is the mean interest differential when the process is in state 1, and $\mu_2(2)$ is the mean interest differential when the process is in state 2.

probability $(1 - p_{11})$. Thus when $s_t = 1$, investors would forecast a change in the exchange rate between t and $t+1$ of

$$(17a)\quad E_t\left(e_{t+1}^{WG} - e_t^{WG}\right) = p_{11}\mu_1 + (1 - p_{11})\mu_2,$$

whereas when $s_t = 2$, their forecast would be

$$(17b)\quad E_t\left(e_{t+1}^{WG} - e_t^{WG}\right) = p_{22}\mu_2 + (1 - p_{22})\mu_1.$$

Substituting (17) into (15) gives

$$(18a)\quad i_t^{US} - i_t^{WG} = p_{11}\mu_1 + (1 - p_{11})\mu_2 + u_t \quad \text{when } s_t = 1$$

$$(18b)\quad = p_{22}\mu_2 + (1 - p_{22})\mu_1 + u_t \quad \text{when } s_t = 2.$$

Rows (1a) and (2a) of Table 7 present the predicted value for the interest differentials based on the univariate maximum likelihood estimates for each country's exchange rate.

The predictions for state 2 (row (2a)) are particularly interesting. State 2 is the state in which the dollar appreciates. During the period of the dollar appreciation of 1980–1984, the forward rate generally exceeded the current spot rate,[11] implying under uncovered interest parity that markets expected a depreciation of the dollar. One way to reconcile this finding with rationality of expectations is to argue that the econometrician faces a "peso problem" (see Robert Cumby and Maurice Obstfeld, 1984;

[11] See for example George Evans (1986), Jeffrey Frankel and Kenneth Froot (1987, 1988), Eduardo Borensztein (1987), and Robert Cumby (1988).

Robert Hodrick and Sanjay Srivastava, 1984; and Hodrick, 1987). When there is a small probability of a large depreciation, the forward rate may consistently predict a depreciation while none occurs.

This possibility is in principle allowed by equation (17b). Even when the process is in state 2, in which the dollar is more likely than not to appreciate ($\mu_2 < 0$, $p_{22} > 0.5$), the expected change in the exchange rate could be positive if the product of $(1 - p_{22})$ and μ_1 is sufficiently large. However, the probability of a depreciation could not have been large, because the appreciation stage lasted so long. The probability of a depreciation given that we are in state 2 is $1 - p_{22}$; our estimate is 0.072 for the mark. The value of μ_1 for Germany, 3.987, is large but not large enough to justify a positive interest differential. Our calculations suggest that a substantial negative differential of $0.928(-1.183) + 0.072(3.987) = -0.811$ was warranted despite the potential "peso" effect. From mid-1980 to mid-1984 the German mark was in state 2 and yet the U.S.-German interest differential was invariably positive. There is thus *prima facie* evidence against the joint hypothesis of uncovered interest parity and rational expectations. We have allowed for a "peso problem," but the evidence indicates that the probability of leaving state 2 once you are in it is so small that a positive interest differential is unwarranted.[12] Notwithstanding, a 95 percent confidence interval for the predicted interest differential does include positive values —the interval ranges from -3.427 to 1.805.

Essentially the same conclusion holds for the U.K. The predicted interest differential in state 2 is -3.20—indeed, the upper end of the 95 percent confidence interval for the predicted interest differential (-5.161 to -1.237) is negative—while the U.S.-U.K. interest differential was almost always positive from the end of 1980 to 1984. This is again a period when our estimates imply that the exchange rate was surely in state 2.

[12]This is reminiscent of the argument made by Frankel (1985) that a rational stochastic bubble could not explain the behavior of the dollar.

With France, we cannot make such a bold statement. It is still true that when we are in state 2 our univariate estimates of the exchange rate indicate the interest differential should be negative. However, the U.S.-French three-month interest differential was frequently negative during 1980–1984. So there is not a simple, clear-cut case against interest parity in the case of the dollar/franc relationship.

Of course, one could still try to salvage the peso story by postulating a possible depreciation of the dollar that is more dramatic than that associated with regime 1. According to this view, there is perhaps a third possible regime of violent depreciation, which was never observed in the sample. The problem with this view is that, under rational expectations, this massive depreciation has to be regarded as an extremely unlikely event—it did not happen once in 58 observations. Suppose we therefore take the probability of moving into this regime, p_{23}, as less than 0.02. For such a remote event to be able to change the calculations in row (2a) of Table 7 from negative to positive, the *quarterly* depreciation of the dollar in state 3 would have to be 40 percent (logarithmically) against the mark, 108 percent against the franc, and 160 percent agains the pound!

We now explore the hypothesis of uncovered interest parity by examining the joint behavior of exchange rates and interest rates. Expressions (18) predict that the interest differential at date t should have one of two means, selected by the same state variable s_t that governed the realization of the exchange rate change observed at t. Consider then the two-dimensional vector

$$\mathbf{y}_t \equiv \left[\left(e_t^{WG} - e_{t-1}^{WG}\right), \left(i_t^{US} - i_t^{WG}\right)\right]'.$$

The model holds that this vector comes from one of two distributions:

$$\mathbf{y}_t|(s_t = 1) \sim N\left(\begin{bmatrix} \mu_1 \\ p_{11}\mu_1 + (1 - p_{11})\mu_2 \end{bmatrix}, \mathbf{\Omega}_1\right)$$

$$\mathbf{y}_t|(s_t = 2) \sim N\left(\begin{bmatrix} \mu_2 \\ p_{22}\mu_2 + (1 - p_{22})\mu_1 \end{bmatrix}, \mathbf{\Omega}_2\right),$$

where we put no restrictions on the vari-

TABLE 8—ESTIMATES FIT TO $y_t = [(e_t - e_{t-1}),(i_t^{US} - i_t^{WG})]'$, $t = 73$:IV–88:I, $e_t = 100$ TIMES THE LOG OF THE EXCHANGE RATE (IN DOLLARS PER UNIT OF FOREIGN CURRENCY), i_t = INTEREST RATE (IN 100-BASIS POINTS AT QUARTERLY RATE)

Parameter	Germany		France		U.K.	
μ_1	2.407		1.319		0.216	
	(1.132)		(1.164)		(0.845)	
	0.542		−0.249		−0.282	
	(0.061)		(0.118)		(0.090)	
μ_2	−1.164		−3.042		−2.407	
	(1.178)		(1.163)		(1.057)	
	1.171		−1.228		−1.425	
	(0.086)		(0.238)		(0.174)	
p_{11}	0.972		0.916		0.983	
	(0.030)		(0.050)		(0.019)	
p_{22}	0.951		0.889		0.969	
	(0.039)		(0.071)		(0.044)	
Ω_1	36.553	0.269	34.423	−2.157	31.306	−1.950
	(9.639)	(0.358)	(8.819)	(0.733)	(6.702)	(0.577)
	0.269	0.095	−2.157	0.365	−1.950	0.342
	(0.358)	(0.026)	(0.733)	(0.096)	(0.577)	(0.073)
Ω_2	36.222	−0.920	25.068	−2.036	15.282	−0.335
	(9.816)	(0.528)	(7.586)	(0.974)	(5.733)	(0.680)
	−0.920	0.195	−2.036	0.603	−0.335	0.406
	(0.528)	(0.052)	(0.974)	(0.202)	(0.680)	(0.153)

Note: Standard errors are in parentheses.

ance-covariance matrices Ω_1 and Ω_2, whose properties are governed by the behavior of the specification error u_t in (15).

The unrestricted version of this model is thus a simple vector generalization of the process in Section I:

$$\text{(19)} \qquad \mathbf{y}_t | s_t \sim N(\boldsymbol{\mu}_{s_t}, \boldsymbol{\Omega}_{s_t}).$$

Our objective now is to maximize[13]

$$\begin{aligned}\text{(20)} \quad & \log p(\mathbf{y}_1,\dots,\mathbf{y}_T;\boldsymbol{\theta}) - (\nu/2) \\ & \cdot\left[\boldsymbol{\mu}_1'\boldsymbol{\Omega}_1^{-1}\boldsymbol{\mu}_1\right] - (\nu/2)\cdot\left[\boldsymbol{\mu}_2'\boldsymbol{\Omega}_2^{-1}\boldsymbol{\mu}_2\right] \\ & - \alpha\log|\boldsymbol{\Omega}_1| - \alpha\log|\boldsymbol{\Omega}_2| - 0.5\omega_1^{ee} - 0.1\omega_1^{ii} \\ & - 0.5\omega_2^{ee} - 0.1\omega_2^{ii},\end{aligned}$$

where ω_2^{ee}, for example, denotes the (1,1) element of $\boldsymbol{\Omega}_2^{-1}$.

We then fit the unrestricted bivariate model (19) to the exchange rate data along with interest rates. The series used for the latter were the average of bid and asked prices on three-month Eurocurrency rates (quarterly rates, in 100-basis points) as of the close of the London market on the last day of the quarter.[14]

The parameter estimates associated with the highest value for (20) are reported in Table 8 along with asymptotic standard errors. Figures 5 through 7 plot the data and imputed change points, with the two means for the interest differential shown as horizontal dashed lines.

[13]The prior $\boldsymbol{\Lambda} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}$ was used in (12) to weight the variance of the exchange rate innovations to five and the variance of interest rate innovations to one. A different scale variable is appropriate since exchange rates are considerably more variable than interest rate differentials.

[14]These are from the data banks of DRI (called WGD03A, WGD03B, FRD03A, FRD03B, UKD03A, and UKD03B). Data were converted from annual to quarterly rates as $100\cdot\{[1+(i/100)]^{0.25}-1\}$, with i the average of the bid and asked returns.

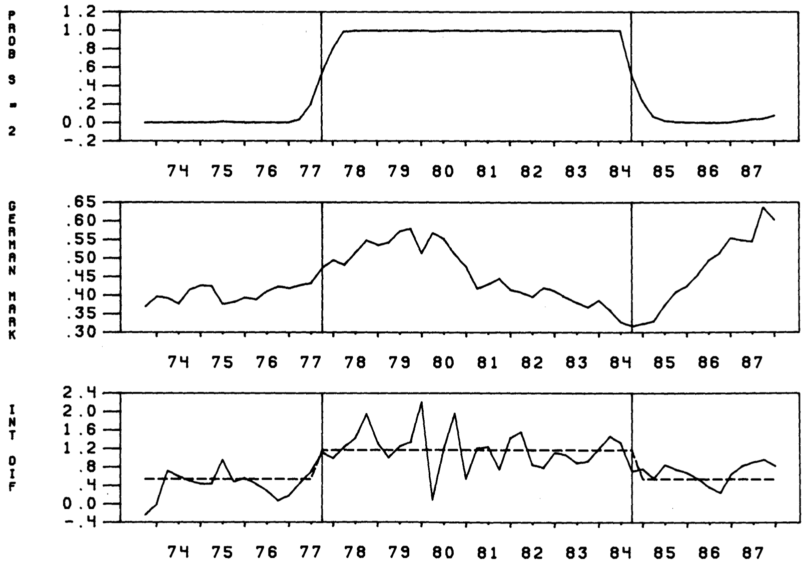

FIGURE 5

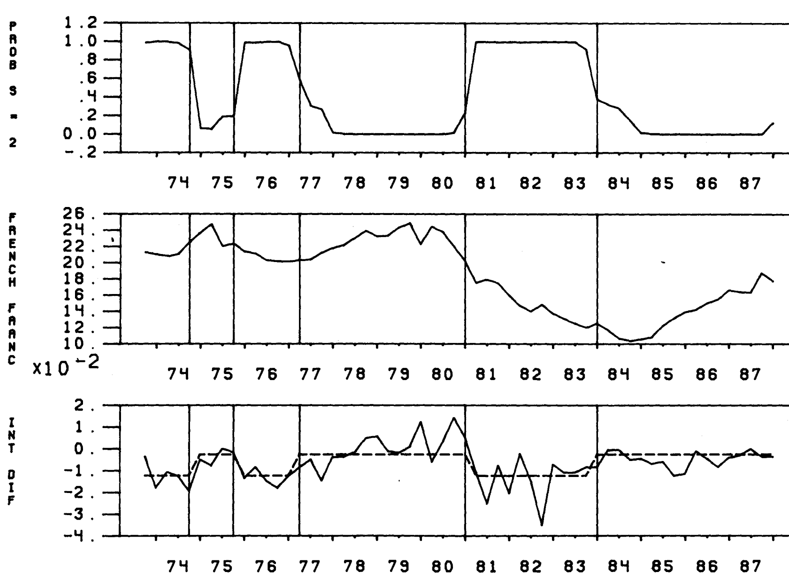

FIGURE 6

It is difficult to find much support for the hypothesis of uncovered interest parity in these results. Germany is the only country for which the segments identified by the bivariate system (the top panel of Figures 5–7) at all resemble those identified by the univariate process for exchange rates (Figures 2–4), and here the interest differential moves in the opposite direction from that predicted by the theory—the period when the mark was falling was a period when U.S. interest rates were unusually *high* relative

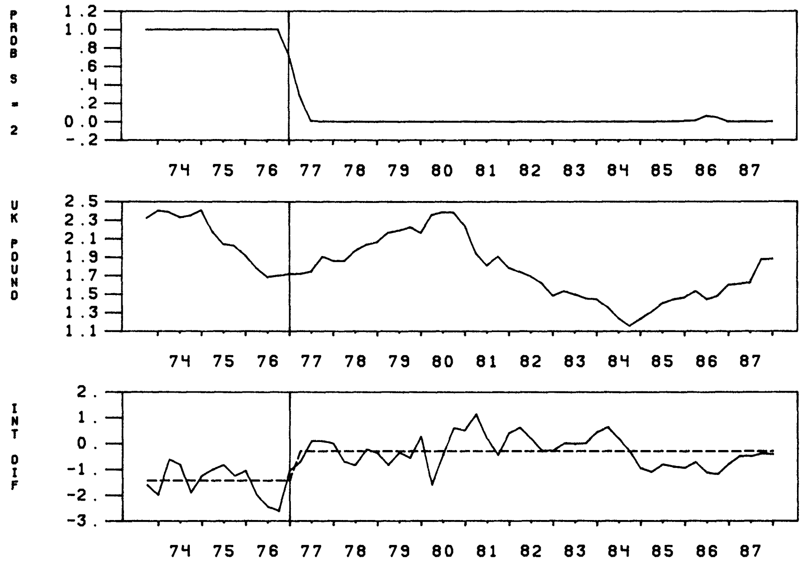

FIGURE 7

to Germany. The interest differential is 62.9 basis points higher in state 2 than in state 1 according to the estimates in Table 8 (1.171 − 0.542 = 0.629), rather than 400 basis points lower as predicted from the univariate estimates in row (3a) of Table 7.

There are some interesting statistics that help to reveal the inconsistencies between the univariate model of exchange rates and a bivariate model that imposes uncovered interest parity. From the standard errors in row (1a) of Table 7 we can construct 95 percent confidence intervals from the univariate estimates for the predicted level of the interest differential in state 1. That is, we can construct confidence intervals for $p_{11}\mu_1 + (1 - p_{11})\mu_2$, which is the predicted value for $i^{US} - i^{WG}$ if we are in state 1. These confidence intervals never overlap with the 95 percent confidence interval for the interest differential from the bivariate estimation for state 1 (row (1b), in Table 7). This is a conservative test in that the marginal significance level is strictly less than 0.05. This is because if the true parameter vector were in the gap between the two confidence intervals, two events (either of which alone has probability less than 0.05 of having occurred) would have to have *both* occurred. Even if the events were perfectly correlated, the probability of both occurring together could be no greater than 0.05, and in general it must be less than 0.05.

Row (3a) of Table 7 gives the change in the interest differential in moving from state 2 to state 1 predicted by the univariate estimates. Row (3b) compares these with the estimates of the actual change in the interest differential, $\mu_1(2) - \mu_2(2)$, as inferred from the bivariate system, where the subscript refers to the state and the "(2)" indicates the second element of the vector $\boldsymbol{\mu}$. In no case do the confidence intervals overlap. This offers evidence against not only the hypothesis of interest parity, but also of a constant risk premium.

The above calculations assumed that, unlike the econometrician, agents knew the state of the process s_t governing the most recent observation on exchange rates $(e_t - e_{t-1})$ with certainty at date t. Charles Engel (1985) and Karen Lewis (1989), for example, discussed models of the exchange rate in which individuals do not know the current monetary policy regime and learn about it gradually through Bayesian inference. Our results change little if we postulate that agents are learning about the state

s_t in the same way as the econometrician. The real-time forecast of the exchange rate change in this case would not be (17) but rather (14):

$$E\left[y_{t+1}|y_t, y_{t-1},\ldots,y_1;\hat{\boldsymbol{\theta}}\right]=\hat{\boldsymbol{\mu}}_2 +\left\{\hat{\rho}+(-1+\hat{p}_{11}+\hat{p}_{22})\cdot\left[p(s_t=1|y_1,\ldots,y_t;\hat{\boldsymbol{\theta}})-\hat{\rho}\right]\right\}\cdot\{\hat{\mu}_1-\hat{\mu}_2\},$$

which collapses to (17) in the special case when the econometrician has no uncertainty about the state ($p(s_t=1|y_1,\ldots,y_t;\hat{\boldsymbol{\theta}})=0$ or 1). Equation (18) then becomes

$$(21)\quad i_t^{US}-i_t^{WG}=\hat{\mu}_2 +\left\{\hat{\rho}+(-1+\hat{p}_{11}+\hat{p}_{22})\cdot\left[p(s_t=1|y_1,\ldots,y_t;\hat{\boldsymbol{\theta}})-\hat{\rho}\right]\right\}\cdot\{\hat{\mu}_1-\hat{\mu}_2\}+u_t.$$

Hamilton (1988a) showed how equation (21) could be estimated jointly with the process for exchange rates. Here we settle for a more modest descriptive statistic, obtained from the regression of the interest differential on the output of the filter from the univariate estimator

$$i_t^{US}-i_t^{WG}=\beta_0 +\beta_1\left[p(s_t=1|y_1^{WG},y_2^{WG},\ldots,y_t^{WG};\hat{\boldsymbol{\theta}})\right]+u_t.$$

This OLS regression has an R^2 of 0.01 for all three currencies, which we take as convincing evidence that uncovered interest parity can not explain much of the movements in interest differentials.

Thus neither the assumption that markets know the regime with certainty nor the assumption that they are learning about it through the rule $p(s_t=1|y_1,\ldots,y_t;\hat{\boldsymbol{\theta}})$ offers a very appealing account of time variation in cross-country interest differentials.

V. Conclusion

Movements in the dollar appear to be characterized by long swings. We have presented a formal statistical model of what it means for the dollar to follow a pattern of long swings, and we find that the model fits the data well. We conclude that the phenomenon of long swings deserves more attention from exchange rate theoreticians.

Can we offer an explanation of these exchange rate and interest rate movements? Dornbusch (1986, 1987), Bernard Dumas (1987), Stephan Schulmeister (1987), and Betty Daniel (1989) have suggested models that allow persistence in movements in the exchange rate. Robert Flood and Peter Garber (1983) have discussed models in which anticipated future events affect the current exchange rate, generating nonlinear behavior of the exchange rate akin to that described here. Hsieh (1988) has described a model in which monetary policy stochastically shifts between two regimes. Kaminsky (1988) has generated a simple model that also leads to nominal exchange rate movements of the type we describe in Section I, though empirically identifying the particular fundamentals that have shifted in the way postulated by her model poses a challenge for future research. Jeffrey Frankel and Kenneth Froot (1988) have described a model in which the behavior of irrational "chartists" interacts with rational agents to produce potentially long movements in one direction in the dollar and a failure of uncovered interest parity. These models seem able to account for some, but not all, of the empirical regularities uncovered here.

A model that allowed only rational investors would need to explain the pattern of the dollar and be able to generate risk premia that varied enough over time to explain the pattern of interest differentials. This is an imposing task (although see Cumby, 1988).

Earlier researchers found little evidence of linear serial dependence in exchange rate changes, supporting the conclusion that the exchange rate follows a random walk. We reproduce this result but nevertheless find compelling evidence of nonlinear serial dependence in the data characteristic of long swings. Our evidence indicates that movements in the dollar in one direction persist over long periods of time. Furthermore, interest differentials do not seem to take into account how long these movements are. Our estimation method provides a natural way of parameterizing the "peso problem," yet we still reject the uncovered interest parity hypothesis. In the absence of a plausible story about foreign exchange risk premia, we conclude that there are long swings in the dollar and that markets do not know it.

APPENDIX

Treating each currency separately, we began from an initial starting value for $\boldsymbol{\theta}^{(0)} = (\mu_1^{(0)}, \mu_2^{(0)}, p_{11}^{(0)}, p_{22}^{(0)}, \sigma_1^{(0)}, \sigma_2^{(0)})'$. We then iterated on equations (2), (9), (10), (7), and (8) until the largest element of $\hat{\boldsymbol{\theta}}^{(\ell+1)} - \hat{\boldsymbol{\theta}}^{(\ell)}$ was less than 1×10^{-8} in absolute value. For the Bayesian parameter ν that appears in (9), we specified $\nu = 0.1$, which roughly corresponds to proceeding as if we had observed one-tenth of an observation drawn from each regime that took on the value zero. We further specified $\alpha = 0.1$ and $\beta = 0.5$, as if we had two-tenths of an observation from each regime whose sum of squared deviations from the population mean of that regime was $\beta / \alpha = 5$.

For each currency we employed several hundred different starting values $\boldsymbol{\theta}^{(0)}$. These starting values all led to a single unique solution to the normal equations in the case of France. However, two local maxima were found for Britain and four for Germany. The maximum likelihood estimates reported in Table 1 are those that achieved the highest value of the objective function (11). When a diffuse Bayesian prior is used ($\nu = \alpha = \beta = 0$) and iteration is begun from the starting values in Table 1, the parameter estimates are changed very little. It further appears that, apart from the singularities, these correspond to the largest bounded local maxima of the raw likelihood function (4). Thus, use of the prior has essentially no consequences for any of the tests of conclusions reported in this paper. By contrast, other local maxima of (4) or (11) exhibit large changes in parameter estimates for slight changes in the priors. Hamilton (1988b) has argued that such a finding should be construed as an additional factor supporting selection of the global maxima reported in Table 1 for the U.K. and Germany.

Our bivariate analysis found four local maxima for France and Germany and six for the U.K.

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