

# Foreign Exchange Futures

In many ways like forward contracts, but some important differences:

- Futures contracts can be traded in smaller amounts than forward contracts
- Futures contracts are traded on exchanges such as the Chicago Mercantile Exchange. Forward contracts are over-the-counter.
- Futures contracts are traded in fixed contract sizes. (Standards are JPY12,500,000; EUR125,000; CAD100,000; GBP62,500; CHF125,000; AUD100,000; and MXN500,000).

- Futures contracts have fixed maturity dates (e.g., the third Wednesday of March, June, September, and December.) Forward contracts have standard maturities of 30, 60, 90, 180, and 360 days.
- Parties on forward contracts must assess each other's riskiness. Licensed brokers make contracts with customers in futures market.
- There are margin requirements in futures markets, but not in forward market.
  - These margins pay interest.
  - There is an initial margin
  - The contract is marked to market. If the margin falls below the "maintenance margin", there is a margin call and the account must be brought up to the original margin.

## Exhibit 20.1

<b>Day</b>	<b>Futures Price (\$/€)</b>	<b>Change in Futures Price (\$/€)</b>	<b>Gain or Loss</b>	<b>Cumulative Gain or Loss</b>	<b>Margin Account</b>
<i>t</i>	1.3321	0	0		\$2,000.00
<i>t</i> + 1	1.3315	−\$0.0006	−\$75.00	−\$75.00	\$1,925.00
<i>t</i> + 2	1.3304	−\$0.0011	−\$137.50	−\$212.50	\$1,787.50
<i>t</i> + 3	1.3288	−\$0.0016	−\$200.00	−\$412.50	\$1,587.50
<i>t</i> + 4	1.3264	−\$0.0024	−\$300.00	−\$712.50	\$2,000.00
<i>t</i> + 5	1.3296	+\$0.0032	+\$400.00	−\$312.50	\$2,400.00
<i>t</i> + 6	1.3301	+\$0.0005	+\$62.50	−\$250.00	\$2,462.50

Note that because futures contracts are only traded at a very limited number of maturity dates, there is a greater possibility of misalignment of the maturity date and the date at which you need to hedge FX risk.

Also, note that because futures are traded in fixed amounts, there is risk of mismatch between the size of the contract and what you need.

## Basics of foreign exchange options

- Foreign exchange options are both exchange traded and over the counter.
- A call option gives the buyer the right, but not the obligation, to buy foreign exchange at an exchange rate stated in the contract.
- A put option gives the buyer the right, but not the obligation, to sell foreign exchange at the exchange rate stated in the contract.
- A European option can only be exercised at the maturity date. An American option can be exercised early.
- The exchange rate in an option contract is the option's strike price or exercise price.

- If the option holder could make money by exercising the option immediately, the option is “in the money”.
- If the option holder cannot make money by exercising the option immediately, the option is “out of the money”.
- If the option holder would break even by exercising the option immediately – that is, if the strike price equals the current spot price – the option is “at the money”.
- Would you necessarily exercise an American option that is “in the money”?

- The immediate revenue that could be obtained by exercising an option is called the option's "intrinsic value".
  - So, let  $S$  be the current spot rate, and  $K$  be the strike price.
  - The intrinsic value of a call option is  $\max[S-K, 0]$
  - The intrinsic value of a put option is  $\max[K-S, 0]$
- Maturity dates of options contracts on the Chicago Mercantile Exchange correspond to those of futures contracts.
- Banks and other financial institutions sell options over the counter.
- Longer-maturity currency options are called currency warrants.

## Example 20.2

Buy a European call option that allows us to buy €1,000,000 at the price of \$1.20/€.

Suppose at the maturity date, the spot exchange rate is \$1.25/€. Then exercising the option gains  
 $(\$1.25/€ - \$1.20/€) \times €1,000,000 = \$50,000$

The right to buy €1,000,000 for \$1.20 is the same as the right to sell \$1,200,000 for €0.83333/\$.

So, the call option above that allows the buyer of the option to buy euros is equivalent to a put option that allows the buyer of the option to sell dollars.

Suppose a firm expects to receive payment in a foreign currency. Buying a put option (a contract that allows it to sell the foreign currency at the strike price) buys insurance against a big weakening of that currency.

A put option puts a floor on the losses that the firm can suffer from an adverse foreign exchange movement. It allows the firm to reap the benefits of a strengthening of the foreign currency.

In contrast, hedging with a forward or futures contract eliminates both the up-side and down-side risk.

## Example 20.7

Pfimerc will receive £500,000 in 1 month (32 days.) There are several put options it can buy. We focus on one that gives the firm the option to sell pounds at \$1.5250/£.

The cost of such a put is \$.0228/£. For £500,000, the cost is  $£500,000 \times \$.0228/£ = \$11,400$ .

The interest rate is 3.75% on an annualized basis, which is 0.35% for the one month. Since the put contract must be bought now, the cost including foregone interest in one month is  $\$11,400 \times 1.0035 = \$11,439.90$

What does Pfimerc get for this put contract? If the pound depreciates to an exchange rate lower than \$1.5250/£, Pfimerc can sell its pounds for  $£500,000 \times \$1.5250/£ = \$762,500$ .

If the exchange rate is above \$1.5250, it does not exercise its option and instead sells its pounds at the spot rate.

But in either case, it has the \$11,439.90 cost of the option. So its net revenue has a floor of  $\$762,500 - \$11,439.90 = \$751,060.10$

Per pound that it is due, the floor in  $\$/\pounds$  is  $\$751,060.10/\pounds 500,000 = 150.21\text{¢}/\pounds$ .

If the exchange rate rises above  $152.5\text{¢}/\pounds$ , Pfimerc's revenue per pound rises 1-for-1 with the exchange rate.

If it does not hedge at all, there is no cost of the put contract, but its revenue rises 1-for-1 with the spot rate for all exchange rates.

Finally, a third alternative is to sell all of its revenues forward at  $152.92\text{¢}/\pounds$ , the 1-month forward rate. Then, no matter what the spot exchange rate, its revenue per pound is  $152.92\text{¢}/\pounds$

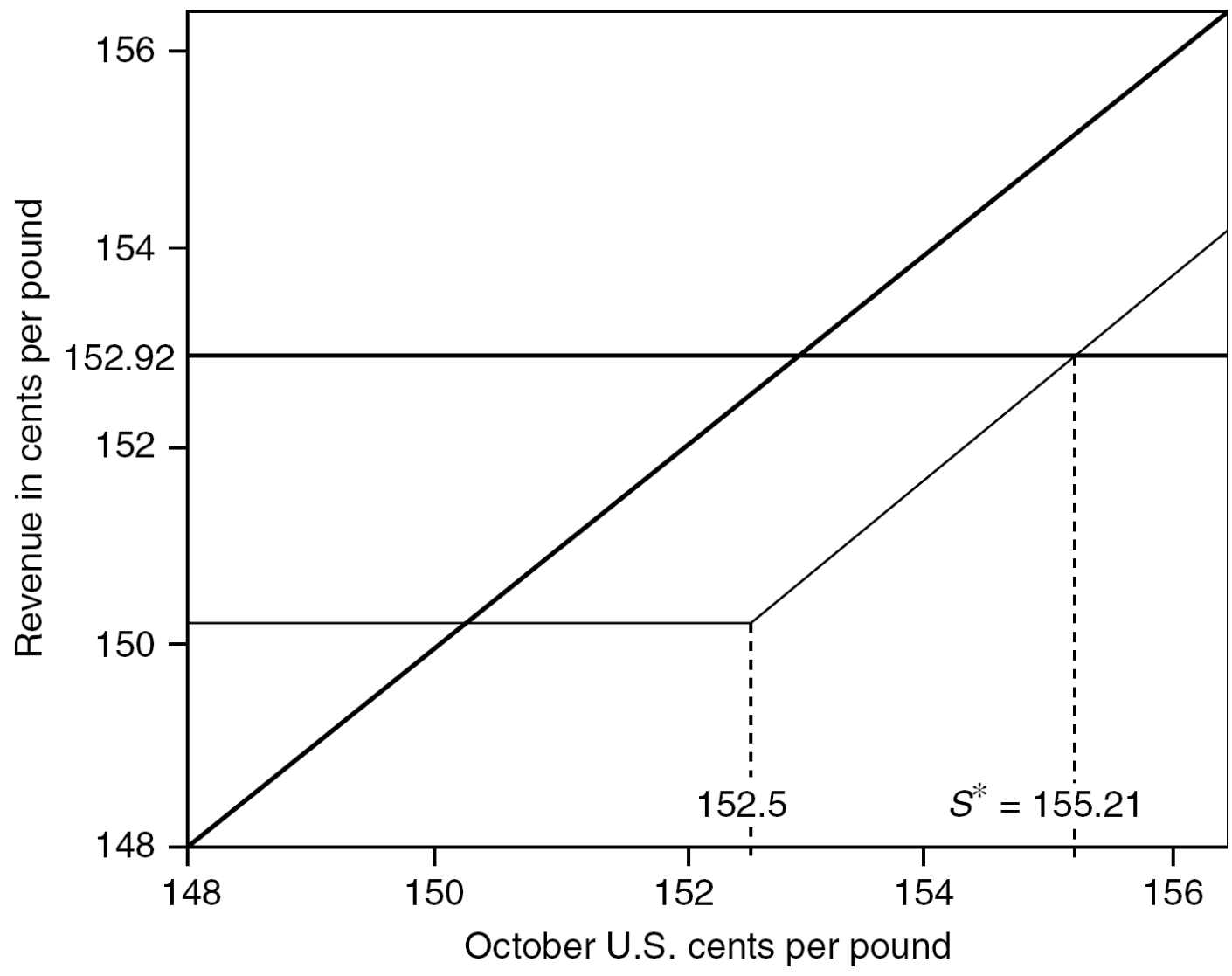
Exhibit 20.5 shows the payoffs per pound from each of these three strategies.

Note that when the exchange rate equals  $155.21\text{¢}/\text{£}$ , the payoff is exactly the same when buying the put option or hedging on the forward market.

The put option allows the firm to have the up-side of a pound appreciation, but puts a floor on its losses in the event of a pound depreciation.

It is like an insurance contract. However, the put option costs money to buy.

It is better than a forward contract the more likely it is that the actual spot rate will be more than  $155.21\text{¢}/\text{£}$



Example 20.8 – Now suppose a firm will be buying a product from Switzerland, and must pay CHF750,000. It can buy a call option that allows it to buy Swiss francs at the strike price.

Suppose it buys a call option that allow it to buy Swiss francs at \$0.72/CHF. The cost of such a contract is \$0.0155/CHF. The interest cost is .94%

The ceiling on the firm's costs are  
 $\$750,000 \times [\$0.72/\text{CHF} + (\$0.0155/\text{CHF}) \times (1.0094)]$   
 $= \$551,734.28.$

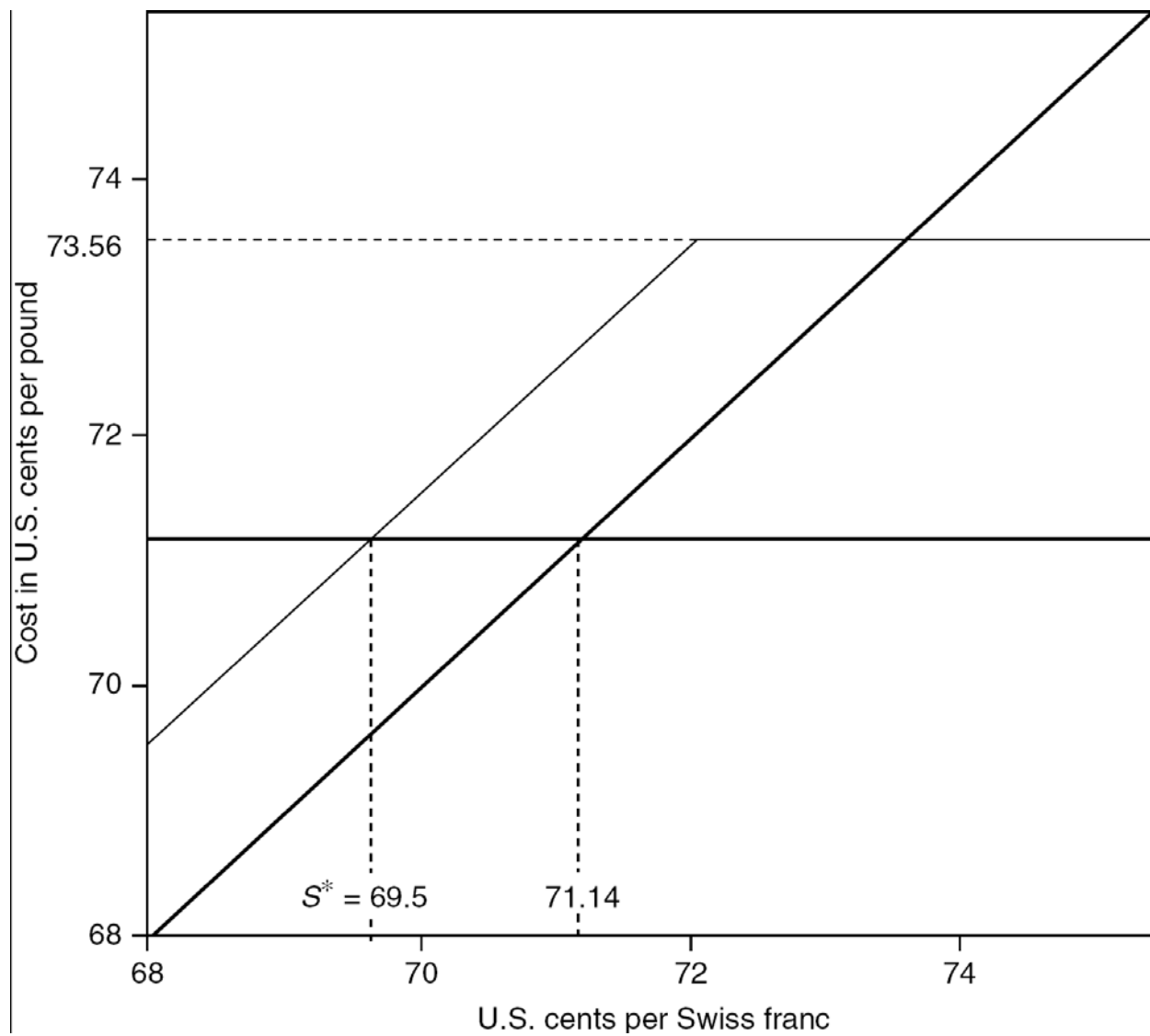
The per CHF cost is capped at  
 $\$551,734.28/\text{CHF}750,000 = 73.56\text{¢}/\text{CHF}.$

If the Swiss franc spot rate turns out to be less than the strike price of \$0.72/CHF, the firm does not exercise the option. It just buys Swiss francs on the spot market.

But, of course, it still has to bear the cost of having bought the option. So, its costs per Swiss franc are capped at 73.56¢/CHF and fall 1-for-1 as the Swiss franc falls below \$0.72/CHF.

Alternatively, it could buy Swiss francs on the forward market at 71.14¢/CHF.

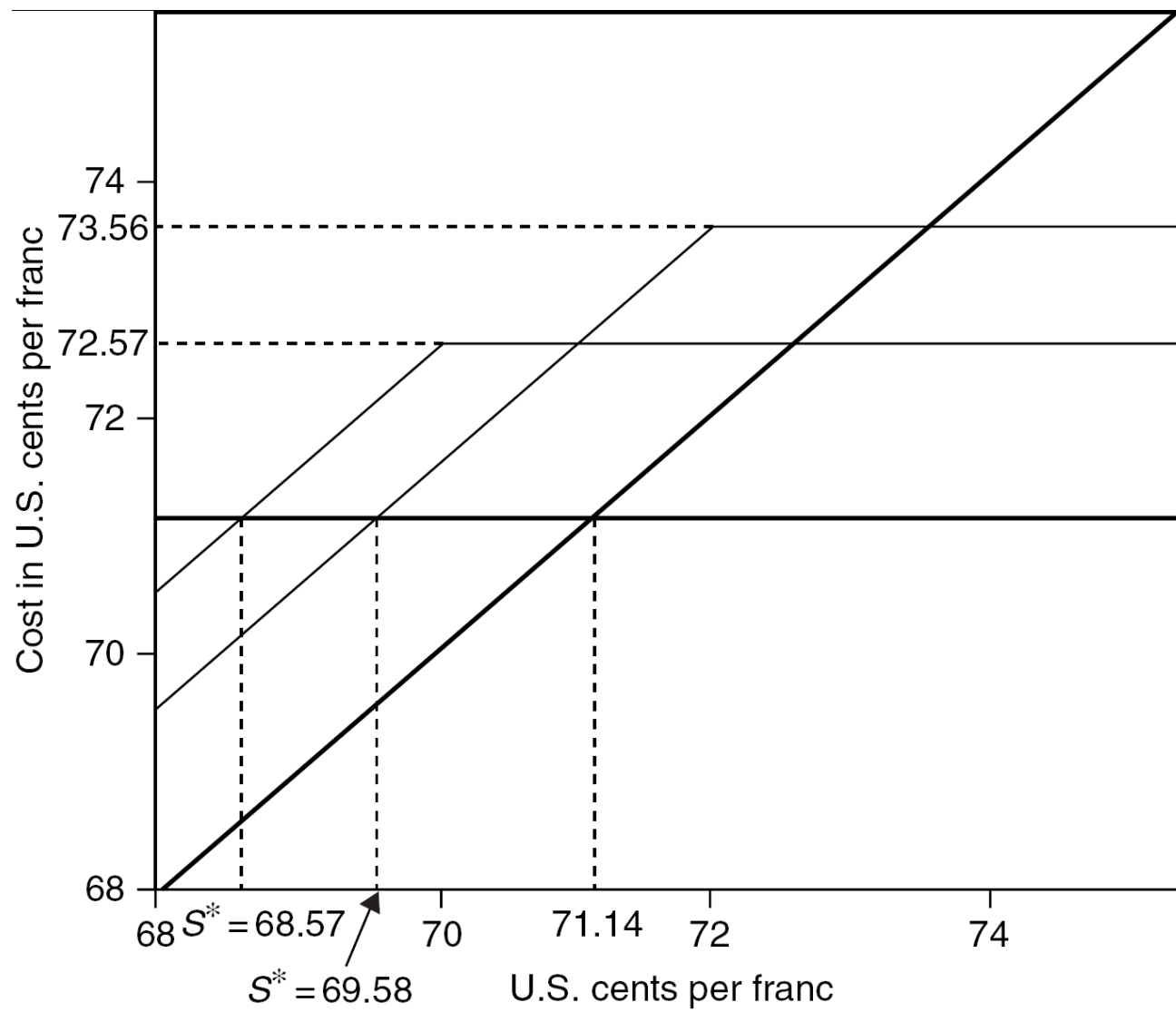
Or it could not hedge at all. Exhibit 20.6 displays the payoffs from the three strategies.



The forward contract and the option leave the firm with the same cost if the spot exchange rate turns out to be 69.5 ¢/CHF.

For lower values of the exchange rate, the option will turn out to be a better deal. The firm may buy the option instead of buying Swiss francs forward if it believes there is a high probability that the exchange rate will be less than 69.5¢/CHF.

What if instead the firm bought options that had a strike price of 70¢/CHF? Those give the firm more protection, but are more expensive – 2.55¢/CHF as compared to 1.55¢/CHF for the option that allows it to buy at 72¢/CHF. Exhibit 20.8 compares payoffs:



## Speculating with options

Suppose you expect to receive English pounds, as in our first example, that you want to sell for \$.

Buying a put option that allows you to sell pounds at the strike price puts a floor on the revenues you will receive. It is like insurance.

But you could sell a call option that allows the buyer to purchase pounds for dollars at the strike price. This is speculation. If you sell a call option, you are obligated to sell pounds at the strike price if the buyer of the option exercises the option.

Selling such an option would be risky for Pfimerc. It stands to lose the up-side of any pound appreciation, and it is not insuring against a drop in the pound.

Why would Pfimerc do it? If the price of the call option is high, it might be attractive to take such a gamble. Pfimerc receives the money from selling the option contract.

Suppose it sells a call option with a strike price of \$1.55/£. Pfimerc has £500,000 to sell. The most it can make is  $\$1.55/\text{£} \times \text{£}500,000 = \$775,000$ , which it will get if the spot rate rises above \$1.55/£

It sells these contracts for \$0.0150/£. And it earns interest of 0.35%.

So the most it can earn is:

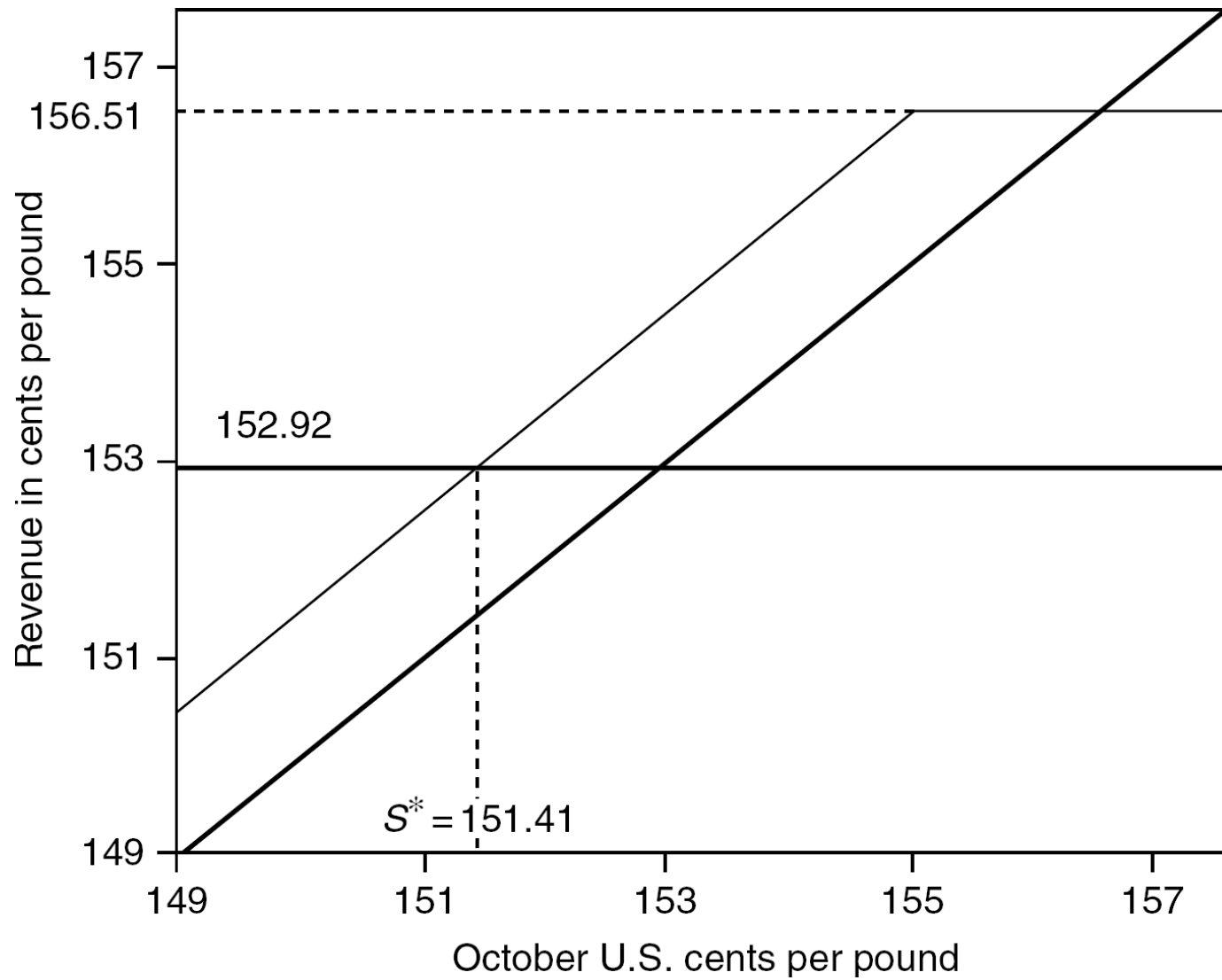
$$£500,000 \times [\$1.55/£ + (\$0.0150/£) \times (1.0035)] = \$782,526.25.$$

Per pound, it puts a cap on its revenue of  $\$782,526.25/£500,000 = 156.51¢/£$ . For spot rates less than 155¢/£, its revenue falls 1-for-1 with a fall in the exchange rate.

Exhibit 20.9 compares the payoffs from this strategy with the payoffs from selling pounds forward or not hedging.

If the spot rate turn out to be greater than  $156.51\text{¢}/\text{£}$ , Pfimerc would have been better off not hedging at all.

If the spot exchange rate is less than  $151.41\text{¢}/\text{£}$ , Pfimerc would have been better off selling pounds at the forward rate of  $152.92\text{¢}/\text{£}$



Conversely, a firm that has foreign exchange liabilities can speculate by selling put options.

That is, if a firm will have to pay Swiss francs (as in our second example), it could sell a put option that obligates it to sell dollars for Swiss francs at the strike price. This gives the firm a minimum cost it must pay for Swiss francs, but put no ceiling on its costs.

Alternatively, recall that by buying a call option to buy Swiss francs it put a ceiling on its costs, but imposed no floor.

Now, return to the case of Pfimerc. What would happen if it both bought a put option with a strike price of \$1.5250/£, and it sold a call option with a strike price of \$1.55/£.

It pays  $£500,000 \times \$.0228/£ \times 1.039 = \$11,439.90$  for the put option.

It earns  $£500,000 \times \$0.0150/£ \times 1.0035 = \$7,526.25$  from selling the call option.

So its option portfolio costs  $\$11,439.90 - \$7,526.25 = \$3913.65$ .

It puts a floor on the value of its sale of pounds of  $£500,000 \times \$1.5250/£ = \$762,500$ .

If the pound spot exchange rate is less than \$1.5250/£, the firm earns  $\$762,500 - \$3913.65 = \$758,586.35$

The per pound earnings are  $\$758,586.35 / \text{£}500,000 = 151.72\text{¢}/\text{£}$ .

If the pound rises above \$1.55/£, the firm must sell its pounds at \$1.55/£. The most it can earn is  $\text{£}500,000 \times \$1.55/\text{£} = \$775,000$ .

Its earnings less the cost of its option portfolio are then capped at  $\$775,000 - \$3913.65 = \$771,086.65$   
Per pound earnings are  $\$771,086.65 / \text{£}500,000 = 154.22\text{¢}/\text{£}$ .

So by buying a put and selling a call, the firm guarantees its earnings are between  $151.72\text{¢}/\text{£}$  and  $154.22\text{¢}/\text{£}$ .

If the spot exchange rate ends up between  $\$1.5250/\text{£}$  and  $\$1.55/\text{£}$ , the firm exercises neither option, and sells on the spot market. Its earnings per pound are the spot rate less  $\$3913.65/\text{£}500,000 = 0.78\text{¢}/\text{£}$

That is, the cost of keeping the exchange rate within the bands of  $\$1.5250/\text{£}$  and  $\$1.55/\text{£}$  is  $0.78\text{¢}/\text{£}$ .

## Options Valuation

The Black-Scholes formula was derived to determine the prices of European put and call options on stocks. The formula and its derivation are too complicated for this class, but a nice simple derivation can be found on Wikipedia:

[http://en.wikipedia.org/wiki/Black\\_scholes](http://en.wikipedia.org/wiki/Black_scholes)

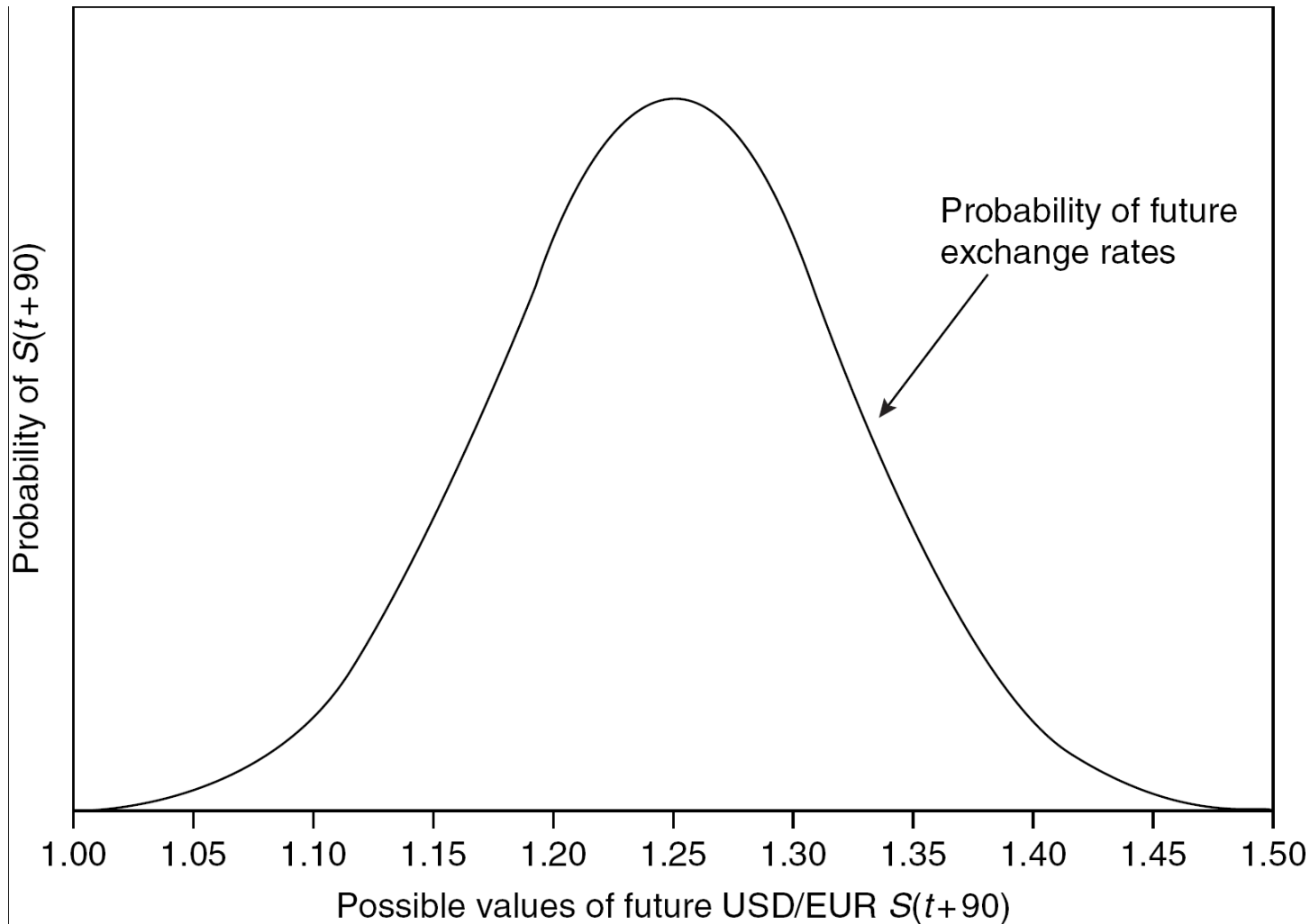
Here we will simply discuss the major factors that determine option prices for foreign exchange.

Recall that the intrinsic value of a call option is  $\max[S-K, 0]$ . ( $K$  is the strike price and  $S$  is the spot exchange rate.) That is, the value of a call is  $S-K$  if  $S-K > 0$ , and otherwise it is worth 0.

Now consider a European call option. At the time of maturity,  $T$ , the price of the call  $C(T)$  will be equal to its intrinsic value,  $\max[S(T)-K, 0]$ .

That is, at the time of maturity, the call option will have value  $> 0$  if  $S(T)-K > 0$ . Therefore, the call option is worth more today the more likely it is that  $S(T)-K > 0$ .

We can form a probability distribution over the various possibilities we think the spot rate will take at time  $T$ , as in Exhibit 20.10:



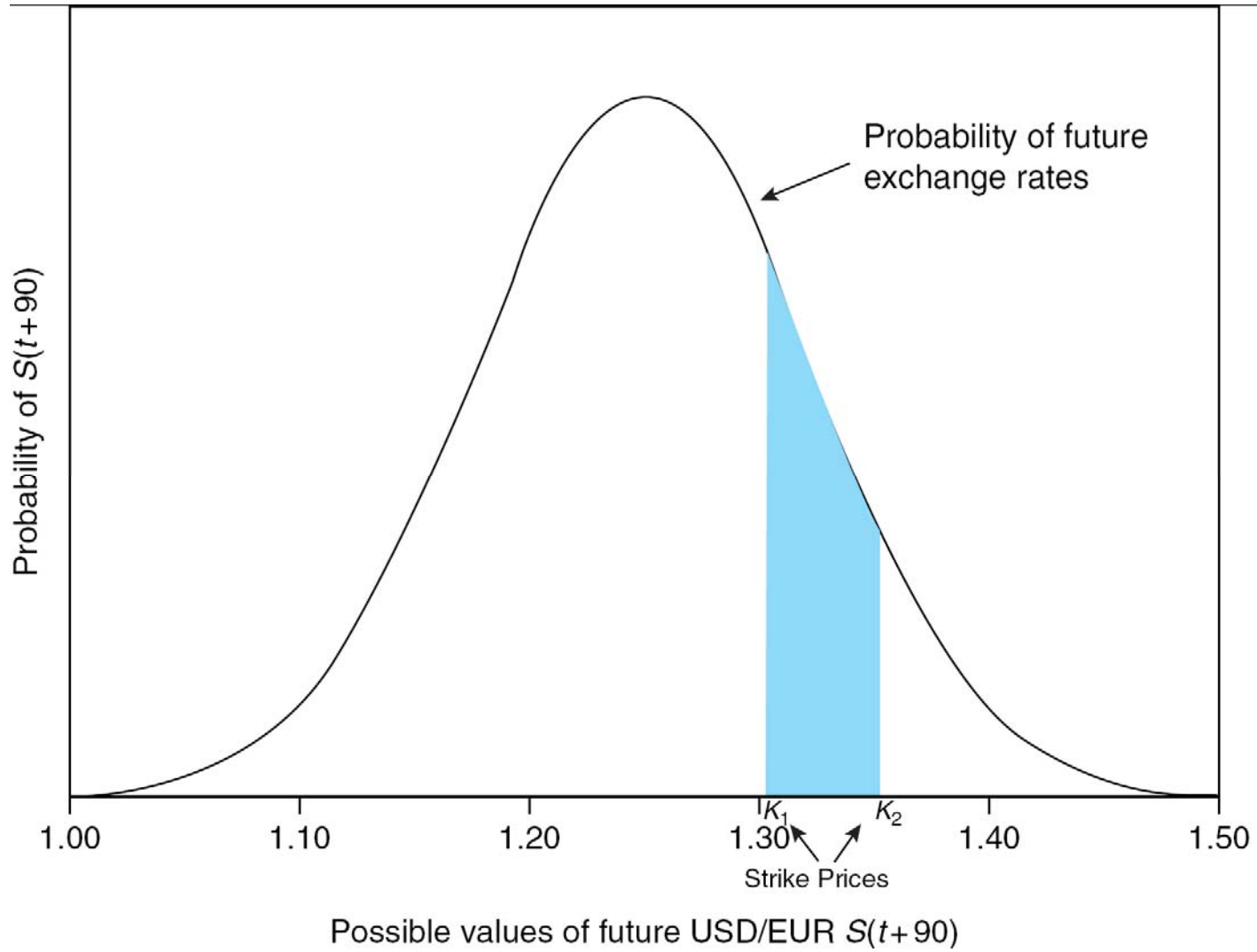
It is very likely, for example, that the exchange rate will be less than \$1.45/€.

If investors only cared about expected return, they would be willing to pay today the expected intrinsic value of the call option at time  $T$ , discounted back to today's time.

To get a sense of this, compare two call options. One has a strike price of  $K_1$  and the other  $K_2$ , where  $K_1 < K_2$ .

Which call option would investors pay more for? The one with strike price of  $K_1$ . That's because at time  $T$ , the option will have value for all spot rates  $> K_1$ , while the other option will only have value for spot rates  $> K_2$ . If there is a high probability that the spot rate will fall between  $K_1$  and  $K_2$ , the first option will have a much higher price today.

# Exhibit 20.11



Conversely, the intrinsic value of a put option is given by  $\max[K-S,0]$ . At time  $T$ , the option will only have value if  $K-S(T)>0$ .

Compare two put options. One has a strike price of  $K_1$  and the other  $K_2$ , where  $K_1 < K_2$ .

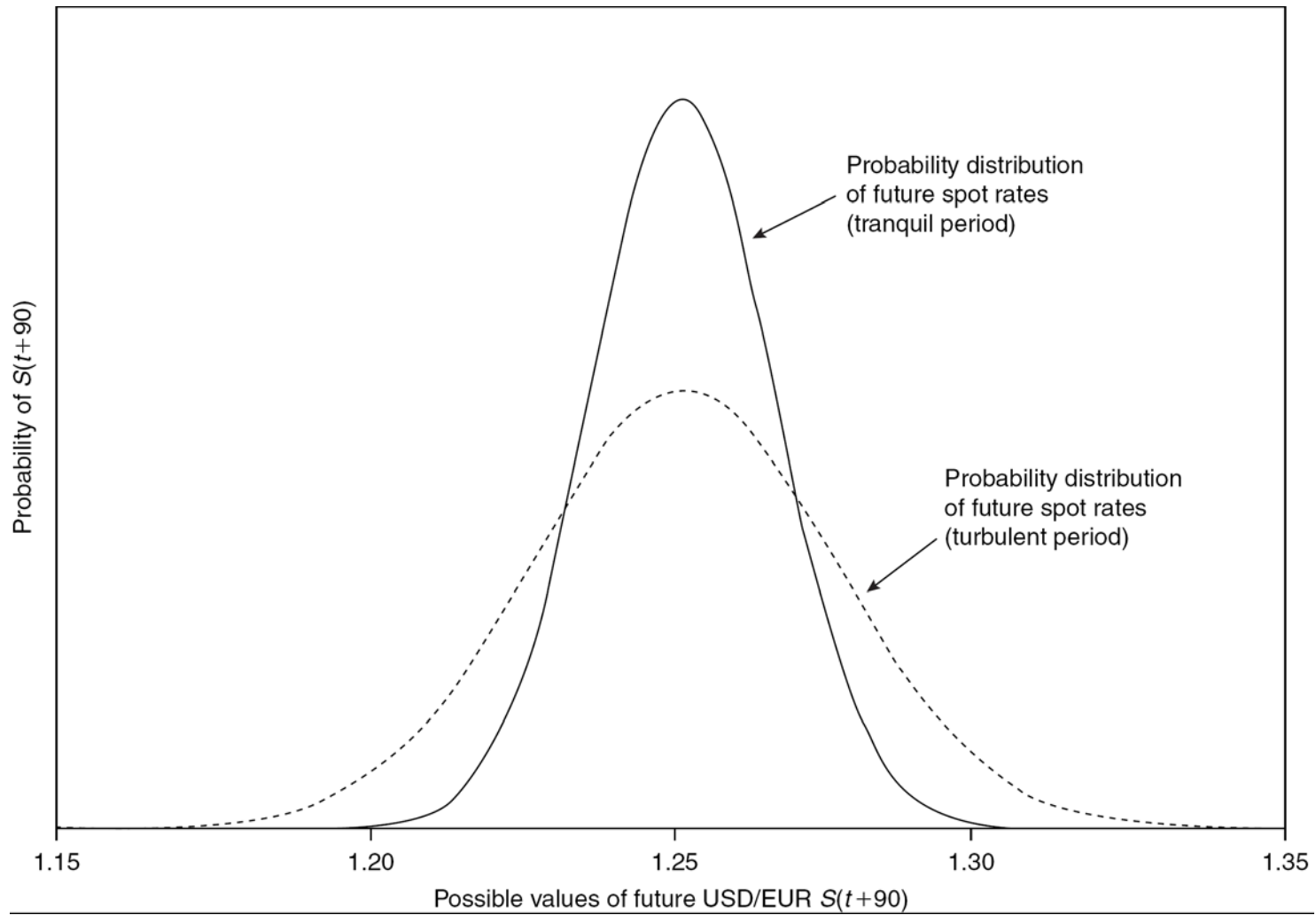
Which call option would investors pay more for? The one with strike price of  $K_2$ . That's because at time  $T$ , the option will have value for all spot rates  $< K_2$ , while the other option will only have value for spot rates  $< K_1$ . If there is a high probability that the spot rate will fall between  $K_1$  and  $K_2$ , the second option will have a much higher price today.

If there is more uncertainty about the future spot rate, the value of a given put or call option is higher.

That is, suppose that we don't change the mean of our probability distribution for the spot exchange rate at time  $T$ , but increase its spread.

Now, consider in Exhibit 20.12 a call option that has a strike price equal to the mean of the distribution, \$1.25/€.

The more spread-out distribution offers the possibility of higher payouts when the call option is in the money, so the price of the call option would be higher.



A similar logic applies to put options – the value of the put option that has a strike price of \$1.25/€ is higher for the more spread out distribution.

If the strike price of a call option is \$1.30/€, it is more likely the call option will end up in the money for the more spread out distribution – and there exist the possibility of very high payouts.

Note that if the strike price is \$1.20/€ that the probability of being in the money is higher when there is lower volatility. Nonetheless, according to the Black-Scholes formula, the price of the option is higher when volatility is higher – the possibility of high payouts outweighs the higher probability of ending up out of the money.

What if there is greater time to maturity for the option?

An American option is always worth more the longer to maturity. Why? Consider the price of a call option of \$1.25/€ at a 3-month and a 6-month horizon. We can always exercise the 6-month call option at the end of 3 months. But we have the option of holding onto it, hoping for an even bigger payout.

For a European option, generally the price is higher for a longer maturity. Longer maturity means more uncertainty about the possible spot rate at maturity, and volatility increases the value of the option.

Recall that with a European option, you have to wait until the maturity date to exercise the option. If the option is strongly in the money now, we might value the possibility of selling sooner rather than later to cash in. So the shorter maturity option could have higher value.

## Put-Call Parity

Suppose you bought a put that allowed you to sell euros at price  $K$  dollars per euro. You would exercise this put and sell euros at  $K$  as long as  $S(T) < K$ .

Also, suppose you sold a call that obligated you to sell euros at price  $K$ . You would have to sell euros at price  $K$  if  $S(T) > K$ .

If you hold both of these options, you will definitely sell euros at exchange rate  $K$ .

Let  $P$  be the price of the put, and  $C$  be the price of the call. The value of your portfolio of options is  $(C-P) \times [1+i(\$)]$  at the time of maturity.

Now this means that the gain from selling euros at price  $K$  is  $K + (C-P) [1+i(\$)]$ .

But another way to sell euros for a sure price is on the spot market, at price  $F$ . Arbitrage insures that these two methods of selling euros in advance yield the same profit, so

$$F = K + (C-P)[1+i(\$)].$$

So, we can derive a relationship between the difference of put and call rates to the forward rate, the strike price and the interest rate:

$$P-C = (K-F)/[1+i(\$)]$$

Or, recall from covered interest parity that

$$F = S[1+i(\$)]/[1+i(€)]$$

So we can rewrite the put-call parity relationship as:

$$P = C + K/[1+i(\$)] - S/[1+i(€)]$$

If we have calculated the price of all call options, then we can use this formula to calculate the price of put options.