We now want to look at the theory of how the risk premium, or the excess return on an asset is determined.

The Capital Asset Pricing Model (CAPM) provides such a theory. It builds on the model we have examined of individual's optimal portfolio choice.

First, recall that we noted that under the following condition

$$\frac{Er_i - r_f}{Vol(r_i)} > \rho \frac{Er_p - r_f}{Vol(r_p)}$$

adding an asset with return given by r_i to our portfolio of risky assets (whose return is denoted r_p) will increase the Sharpe ratio of our portfolio.

How much of this new asset should we add?

As we add more and more of the asset, the correlation of the return on our portfolio with r_i (denoted ρ) increases.

We should keep adding more of the asset until

$$\frac{Er_i - r_f}{Vol(r_i)} = \rho \frac{Er_p - r_f}{Vol(r_p)}$$

This can be rewritten. Remember that

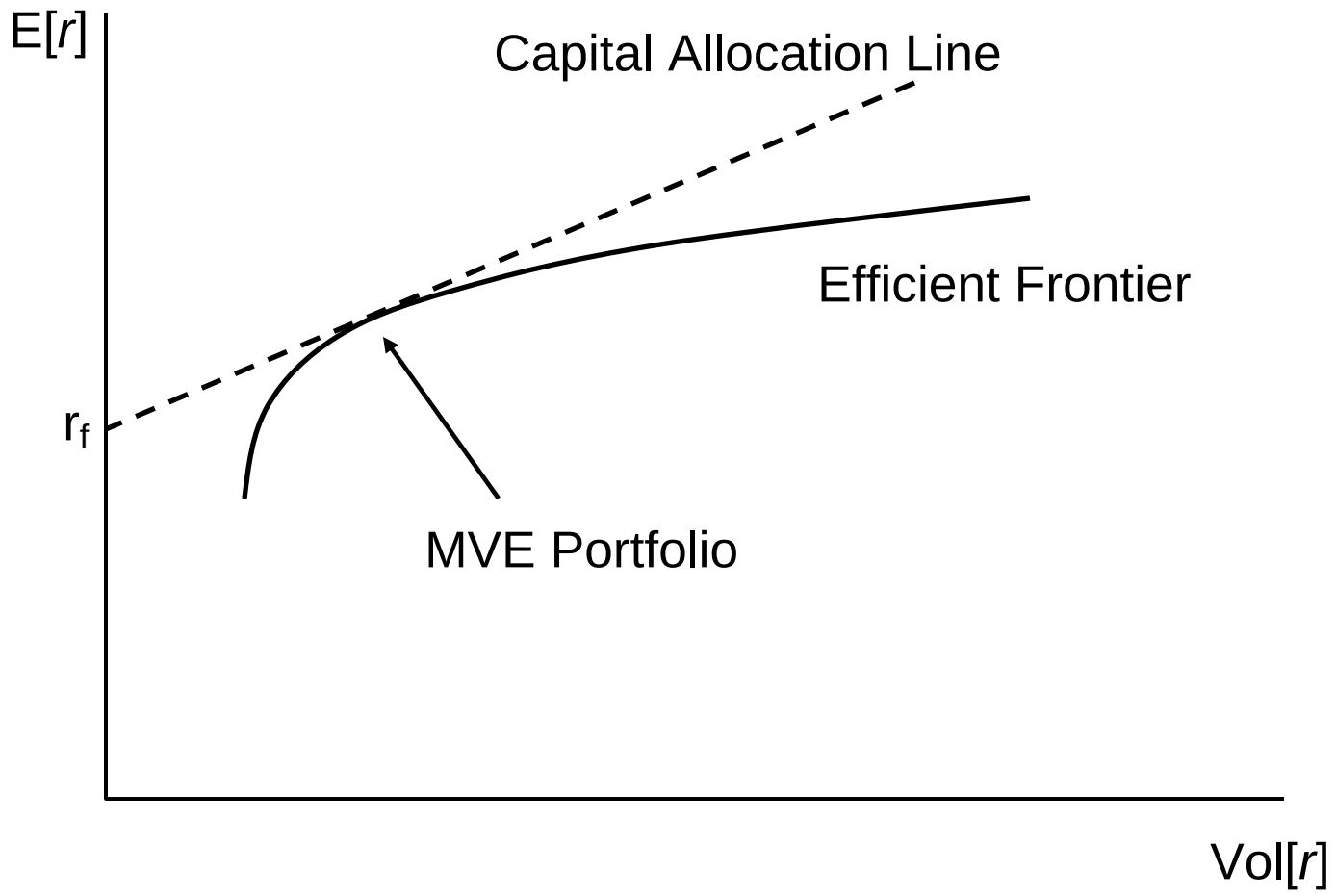
$$\rho = \frac{Cov(r_i, r_p)}{Vol(r_i)Vol(r_p)}.$$

So we can rewrite the condition above as:

$$Er_i - r_f = \frac{Cov(r_i, r_p)}{Var(r_p)}(Er_p - r_f)$$

The next important result to remember from that theory is that all individuals should have the ***same*** portfolio of risky assets. They all hold the "mean-variant efficient" (MVE) portfolio of risky assets.

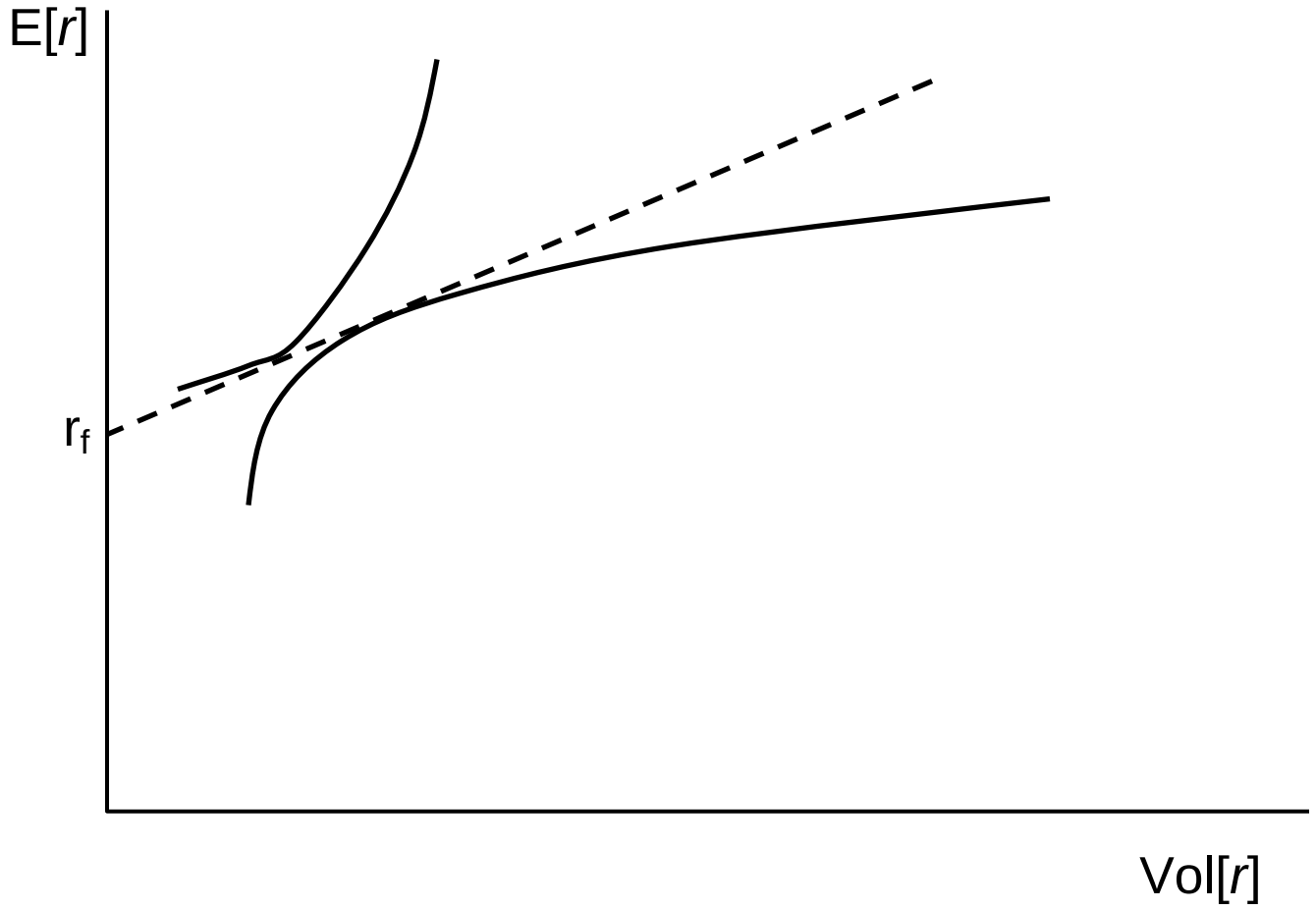
Investors may differ in their distaste for risk. But that gets reflected only in the balance in their holdings of the riskless asset and the MVE portfolio.


E[r]
Capital Allocation Line
Efficient Frontier
r_f
MVE Portfolio
Vol[r]

We can draw in indifference curves that show a person's preferred tradeoff between expected return and risk.

The best portfolio for the person is the one that achieves the highest indifference curve.

It is found at the indifference curve that is just tangent to the Capital Allocation Line.


E[r]
r_f
Vol[r]

Now let's model the entire market for risky assets.

If all investors hold the same portfolio of risky assets, what is that portfolio? It must be the "market portfolio".

The market portfolio contains all securities, and the proportion of each security is the market value as a percentage of total market value.

If everyone is holding the MVE portfolio of risky assets, then the MVE portfolio must be the market portfolio – that is the only way there can be no excess demand or supply for any security.

Let r_m denote the return on the market portfolio.

Putting things together, we have that the equilibrium excess return on an asset is given by:

$$Er_i - r_f = \frac{Cov(r_i, r_m)}{Var(r_m)}(Er_m - r_f)$$

The "beta" of the return on an asset is defined as:

$$\beta_i \equiv \frac{Cov(r_i, r_m)}{Var(r_m)}$$

We can write the CAPM theory of excess returns as:

$$Er_i - r_f = \beta_i (Er_m - r_f)$$

The risk premium of an asset is determined by the risk premium on the market, and the asset's beta.

- Should we use nominal or real returns?

  Real returns (though in practice there is little difference.)
- Should we use real returns for US investors or for some other investor?

  In principle the theory needs to be modified if US investors and Foreign investors earn different real returns.

  Real returns are only equal for US and Foreign investors if relative PPP holds.

  Also, the riskless real returns are only equal for US and Foreign investors if relative PPP and UIP hold.
- The International CAPM modifies the “world” CAPM to allow for deviations from the parities. In practice, little is changed if we simply use US returns.

A Recipe for the Cost of Equity Capital

$$Er_i - r_f = \beta_i (Er_m - r_f)$$

Step 1: Get data on the market portfolio returns, the equity returns on security *i*, and the T-bill interest rate.

Step 2: Determine the market risk premium – the expected excess return on a portfolio that approximates the market portfolio.

Step 3: Obtain an estimate of β_i

Step 4: Compute the expected return on investment *i* as:

$$Er_i = r_f + \beta_i (Er_m - r_f)$$

What is the market portfolio?

Since it is fairly costless to diversify our portfolio among assets from many countries, we want to use a measure of returns from a world portfolio of assets.

Really, the measure of the market portfolio should include returns from all assets that we can invest in – not only equities, but also bonds, real estate, gold, etc.

In practice, the MSCI (Morgan Stanley Capital International) Index is typically used.

The book gives an example of how risk could be mismeasured if we measure the market index incorrectly.

How do we estimate the beta for asset *i*?

We collect data on $r_i - r_f$ and $r_m - r_f$.

An estimate of β_i comes from a regression of $r_i - r_f$ on $r_m - r_f$.

If we don't have enough data, or if our data only comes from special periods of time (for instance, times when returns are not very volatile), we might mismeasure β_i.

In many emerging markets, there are barriers to free movement of capital across borders.

Therefore, the risk premium on an asset may be more closely related to the excess return on the domestic market portfolio, rather than the world market portfolio.

However, a multinational considering investment in an emerging market may still want to use the world market portfolio as the benchmark.

The Arbitrage Pricing Theory (APT) is an alternative model used for assessing the expected excess return or risk premium on an asset.

According to the APT, the excess return is related to several different risk factors:

$$Er_i - r_f = \beta_{i1} rp_1 + \beta_{i2} rp_2 + \ldots \beta_{ik} rp_k$$

CAPM is the special case with one risk factor, with $rp_1 = Er_m - r_f$.

There is not widespread agreement on how to measure the risk factors in the APT.