

- To evaluate projects, we will calculate a present value of returns from that project.
 - But what rate do we discount at?
 - Risky projects sometimes have a higher average return, but how do we take into account the risk?
 - We will use a larger, risk-adjusted discount rate.
- But how do we evaluate the riskiness of a project?
 - Is it the volatility of returns?
 - No, we want to use a measure of how the market measures the riskiness of a project.
 - What matters is whether adding an investment to our portfolio increases the volatility of our portfolio
 - For example, an investment that has volatile returns but which are uncorrelated with our portfolio are not considered risky.

Sources of risk on an international investment:

Volatility of the return in foreign currency

Volatility of the foreign exchange rate

Comovement of foreign return and foreign exchange rate

Recall: $1 + r(t + 1, \$) = \frac{S(t + 1)}{S(t)} \times (1 + r(t + 1, FC))$

Write $\frac{S(t + 1)}{S(t)} = 1 + s(t + 1)$, where $s(t + 1) = \frac{S(t + 1) - S(t)}{S(t)}$.

Then $r(t + 1, \$) = s(t + 1) + r(t + 1, FC) + s(t + 1) \times r(t + 1, FC)$
 $\approx s(t + 1) + r(t + 1, FC)$

The dollar rate of return is the sum of the foreign currency rate of return and the rate of appreciation of the foreign currency.

We will use Var to denote “variance”, and Vol to denote “volatility” (as measured by the standard deviation, which is the square root of the variance.) Cov is the “covariance”. ρ is the correlation.

$$\begin{aligned}\text{Var}[r(t+1, \$)] &= \text{Var}[s(t+1) + r(t+1, FC)] \\ &= \text{Var}[s(t+1)] + \text{Var}[r(t+1, FC)] \\ &\quad + 2\text{Cov}[s(t+1), r(t+1, FC)]\end{aligned}$$

$$\begin{aligned}\text{Var}[r(t+1, \$)] &= \text{Var}[s(t+1)] + \text{Var}[r(t+1, FC)] \\ &\quad + 2\rho\text{Vol}[s(t+1)]\text{Vol}[r(t+1, FC)]\end{aligned}$$

	Means			Volatilities		
	Market Return	Currency Return	Dollar Return	Market Return	Currency Return	Dollar Return
U.S.	13.38	0.00	13.38	15.01	0.00	15.01
Canada	11.51	0.32	12.18	16.86	5.31	19.34
Japan	7.92	3.43	11.59	18.83	11.53	23.17
UK	14.54	-0.17	14.15	16.33	10.54	18.37
France	14.79	-0.25	14.09	20.17	10.93	21.23
Germany	12.28	1.15	13.09	21.04	11.04	22.22
Italy	17.62	-1.73	15.36	24.58	10.66	25.10

Country	Correlation
Canada	0.35
Japan	0.08
UK	-0.13
France	-0.16
Germany	-0.17
Italy	-0.15

$$\text{Sharpe ratio} = \frac{Er - r_f}{\text{Vol}(r)}$$

Investors want higher Sharpe ratios on their portfolios.

That is intuitive.

It is the assumption underlying CAPM.

But it is incorrect to conclude that to achieve the highest possible Sharpe ratio on our portfolio, we want to acquire investments with high Sharpe ratios.

Acquiring an asset with a lower Sharpe ratio than our current portfolio may increase the Sharpe ratio of our portfolio!!

Let the return on some foreign investment (in dollars) be r^* .
 r is the return on our risky portfolio of domestic assets.

Will adding the foreign asset to our portfolio increase the Sharpe ratio of our portfolio?

The appendix of Chapter 13 shows it will as long as:

$$\frac{Er^* - r_f}{Vol(r^*)} > \rho \frac{Er - r_f}{Vol(r)}$$

ρ is the correlation of r and r^* .

As long as $\rho < 1$, the foreign asset can increase the Sharpe ratio of our portfolio even if its Sharpe ratio is less than the Sharpe ratio of our domestic portfolio.

We can rewrite this a formula for determining a “hurdle rate” for whether we should add the asset to our portfolio.

We add the asset to our portfolio if

$$\frac{Er^* - r_f}{Vol(r^*)} > \rho \frac{Er - r_f}{Vol(r)},$$

or

$$Er^* > \rho \frac{Er - r_f}{Vol(r)} Vol(r^*) + r_f$$

Exhibit 13.6

Correlations Between Foreign and U.S. Equity Market Returns, 1980-2006

Country	Correlation
Canada	0.73
Japan	0.31
UK	0.61
France	0.54
Germany	0.52
Italy	0.33

Exhibit 13.7

Hurdle Rates for Foreign Investments

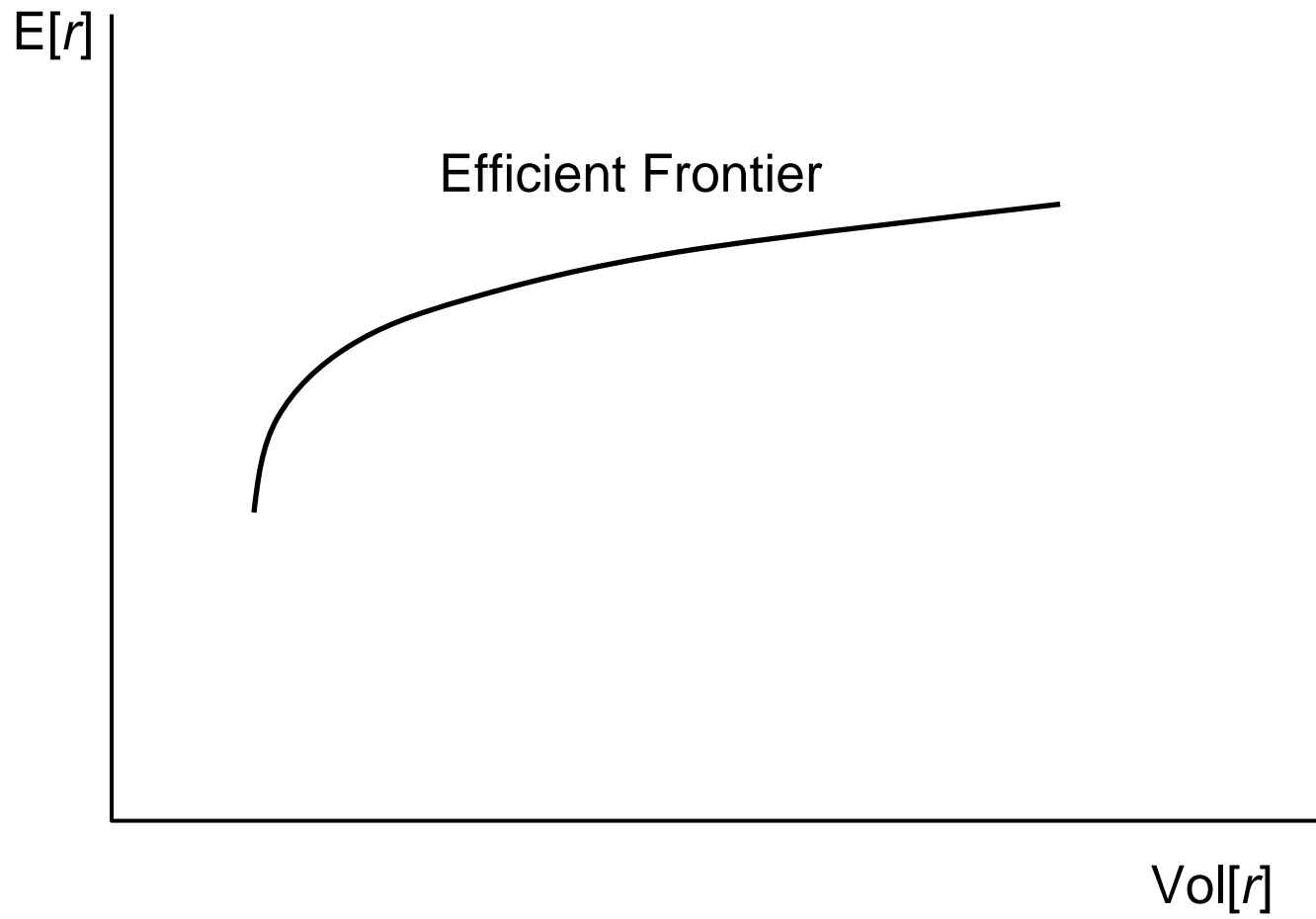
Country	$E[r] = 10\%$	$E[r] = 12\%$
Canada	9.74%	11.62%
Japan	7.91%	8.88%
UK	8.97%	10.45%
France	9.08%	10.62%
Germany	9.08%	10.62%
Italy	8.22%	9.33%

How does an investor find his/her optimal portfolio of assets?

We have in mind that the investor can buy the “risk free” asset, that pays r_f with certainty. Or, they can buy any of a large number of risky assets.

A portfolio of risky assets will generally offer a higher Sharpe ratio than any single asset.

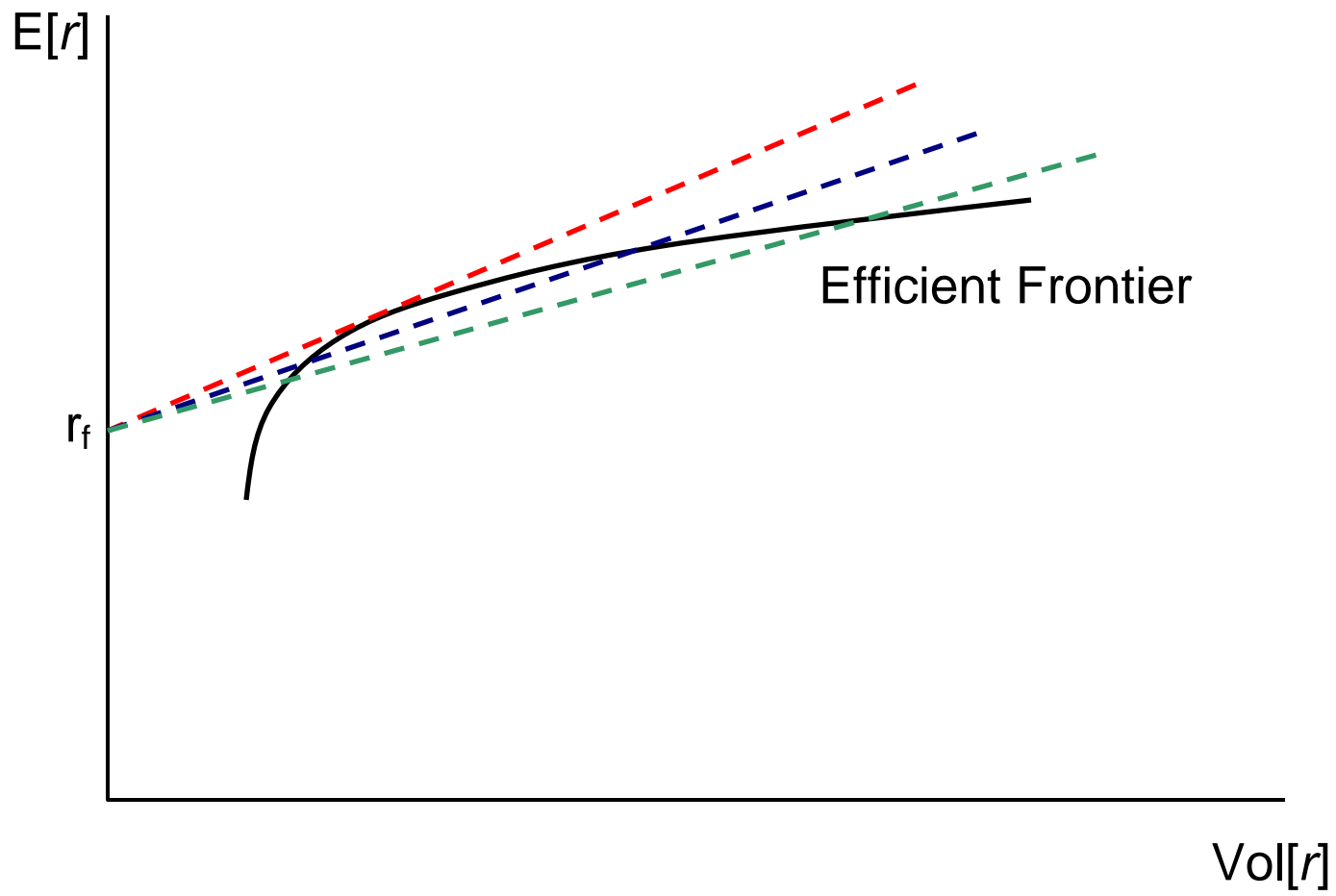
We can derive the “efficient” portfolios of risky assets. These are the portfolios that offer the highest expected return for any given volatility.

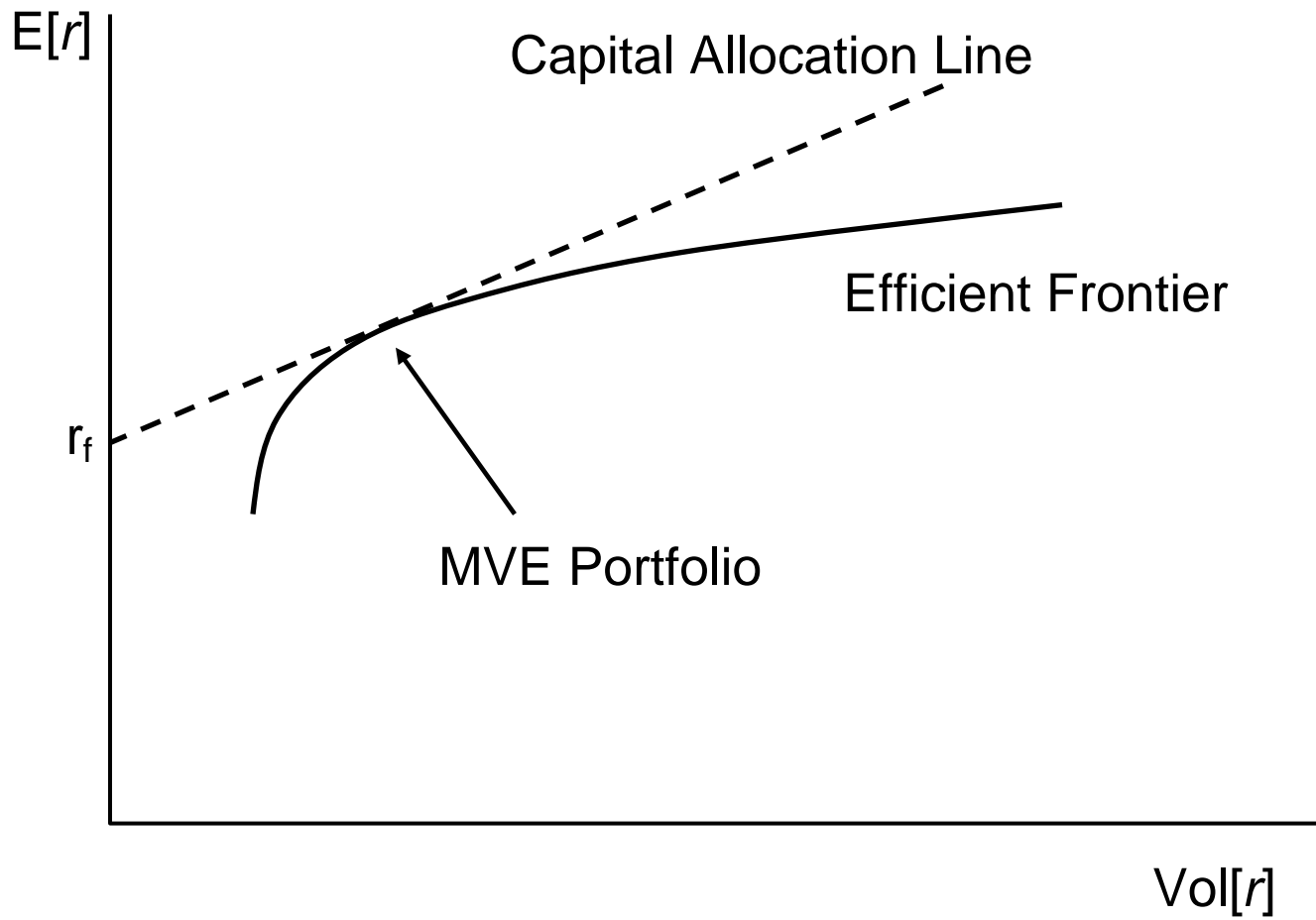


But we can obtain an even better trade-off between expected return and risk by holding a portfolio that is a mix of the riskless asset and a risky portfolio that is on the efficient frontier.

We find that the optimal portfolio is a mix between the riskless asset and the mean-variance efficient (MVE) portfolio.

We choose the portfolio we prefer on the Capital Allocation Line depending on our preferences for expected return vs. risk.

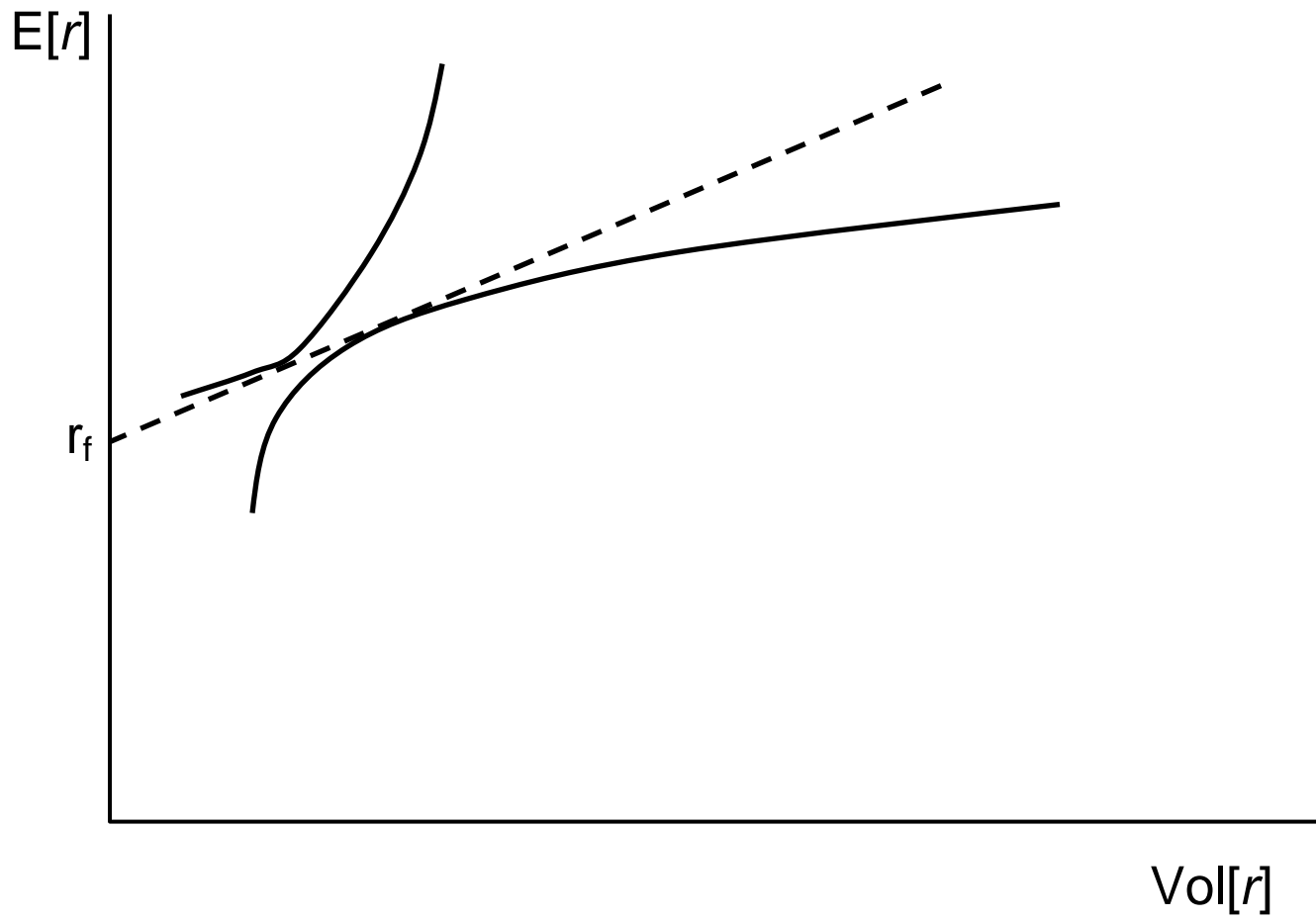




We can draw in indifference curves that show a person's preferred tradeoff between expected return and risk.

The best portfolio for the person is the one that achieves the highest indifference curve.

It is found at the indifference curve that is just tangent to the Capital Allocation Line.



Capital Asset Pricing Model (CAPM)

The CAPM is a theory or model of how the market determines the expected rate of return on a risky asset.

It is based on the underlying assumption that all investors act like investors who choose the optimal expected return/volatility tradeoff, just like we have been talking about.

CAPM also assumes

- There is a single-period investment horizon
- Individual investors are price-takers (their wealth is too small to individually influence the price.)
- All potential investments are traded.
- There are no taxes or transactions costs
- Information is costless and available to all investors.
- Investors all agree on their expectations – expected returns, volatility, and covariances among returns.

Why are we looking at CAPM?

- We want to understand how assets are priced
- But, also, we want to learn the appropriate way to discount future returns from a project.