

# Currency Misalignments and Optimal Monetary Policy: A Reexamination

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- What is optimal monetary policy when there are currency misalignments?
  - “currency misalignments” is my term for inefficient deviations from the law of one price.
- Specifically, should monetary policy target exchange rates if it already targets inflation and the output gap?
- I examine this in a simple model – the famous Clarida-Gali-Gertler (2002) – but with deviations from the law of one price. In particular, local-currency pricing (LCP)
- Currency misalignments are inefficient in much the same way as misaligned prices are in a sticky price model with asynchronized price setting.

## Why is CGG such a prominent paper?

- The optimality conditions are simple tradeoffs between the output gaps in each country and the (PPI) inflation rates
- The model is simple
- The optimal policy exactly the same as for the closed economy!

1. My narrow objective is to change one assumption (LCP instead of PCP, or more generally price wedges between countries) in CGG and see how these results hold up.

- I'm striving for something just as simple as in CGG.

2. I hope the paper also lays the groundwork for future research.

- More realistic (perhaps) price setting
- Richer and more empirically plausible structure

What is the contribution of this paper, especially since the literature has examined monetary policy in much richer models, even under LCP?

It is useful to place this work relative to three contributions:

1. Relative to CGG, this introduces LCP. It also allows home bias in preferences, though this turns out not to alter optimal policy.
2. Relative to Devereux and Engel (2003) and Corsetti and Pesenti (2005), it introduces a richer model.
  - a. In DE, optimal policy simultaneously set CPI inflation to zero, and fixed the exchange rate. And there is no tradeoff between these goals and that of minimizing the output gap.
  - b. The model was so simple that optimal policy did not face a tradeoff among these goals. The model here is enriched by introducing cost-push shocks (as in CGG) and home bias.
  - c. Also has Calvo-pricing, and monetary policy set as an interest rate rule (as in CGG).

3. What about the contribution relative to the large literature that develops rich models?

a. Many are small country or have ad hoc welfare functions.

But some examine the optimal interest rate reaction function (Taylor rule) or “instrument rule”. However, the instrument rule relies on more assumptions than the targeting rule.

In decreasing order of generality:

1. Loss function – can be derived without specifying price-setting behavior.

2. Targeting rules – can be derived without specifying stochastic process for exogenous variables.

3. Instrument rules

I derive these rules for optimal cooperative policy under commitment and discretion.

Model is the same as CGG except:

Add possibility of home bias in preferences.

Consider LCP as well as PCP.

Do not have different country sizes (which adds little in CGG.)

Model is 2-country, sticky-price model.

Households:

Cobb-Douglas preferences over Home and Foreign aggregates

CES preferences over continuum of varieties of Home and Foreign goods.

Monopolistic suppliers of a unique type of labor to all firms.

But or sell state contingent nominal assets – complete set.

## Firms:

Produce output according to a function that is linear in a labor aggregate, with productivity shocks.

Home firms use Home labor, e.g.

Labor aggregate is CES over different types of labor.

Price setting by Calvo rule. In producer's currency under PCP, under importer's currency under LCP.

## Shocks:

Country-specific productivity shocks.

Country-specific “cost push” shocks – shocks to the elasticity of demand for types of labor.

## Policy:

Monetary policy sets nominal interest rates.

Fiscal policy imposes optimal constant subsidy rate to monopolists to offset monopoly power of firms and workers in non-stochastic steady state.

Government lacks the instruments needed to deal with time-varying monopoly power of workers.

Monetary policy then must work to allay:

Distortions from “cost-push” shocks

Sticky-price distortion

Under LCP, violations of law of one price arising from pricing identical goods in different currencies.

Model:

Consumers maximize

$$U_t(h) = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}(h)^{1+\phi} \right] \right\}, \sigma > 0, \phi \geq 0$$

$$C_t(h) = (C_{Ht}(h))^{\frac{\nu}{2}} (C_{Ft}(h))^{1-\frac{\nu}{2}}, \quad 0 \leq \nu \leq 2.$$

If  $\nu = 1$ , Home and Foreign preferences are identical as in CGG.

There is home bias in preferences when  $\nu > 1$ .

$$C_{Ht}(h) = \left( \int_0^1 C_{Ht}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \quad C_{Ft}(h) = \left( \int_0^1 C_{Ft}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}}.$$

$N_t(h)$  is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

$$N_t(i) = \int_0^1 N_t(i, f) df.$$

The budget constraint is given by:

$$P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Z(\nabla^{t+1} | \nabla^t) D(h, \nabla^{t+1}) = W_t(h) N_t(h) + \Gamma_t - T_t + D(h, \nabla^t),$$

$$P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)}, \quad k = (1 - (\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}.$$

$$P_{Ht} = \left( \int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \quad \text{and} \quad P_{Ft} = \left( \int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}.$$

Household  $h$  faces demand for its labor services given by:

$$N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t, \quad \text{where} \quad W_t = \left( \int_0^1 W_t(h)^{1-\eta_t} dh \right)^{\frac{1}{1-\eta_t}}.$$

1<sup>st</sup>-order conditions:

$$P_{Ht} C_{Ht} = \frac{\nu}{2} P_t C_t,$$

$$P_{Ft} C_{Ft} = \left( 1 - \frac{\nu}{2} \right) P_t C_t,$$

$$C_{Ht}(f) = \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left( \frac{P_{Ft}(f)}{P_{Ft}} \right)^{-\xi} C_{Ft},$$

$$\beta \left( C(s^{t+1}) / C(s^t) \right)^{-\sigma} (P_t / P_{t+1}) = \ddot{Q}(s^{t+1} | s^t).$$

$$\frac{W_t(h)}{P_t} = (1 + \mu_t^W)(C_t(h))^\sigma (N_t(h))^\phi, \text{ where } \mu_t^W \equiv \frac{1}{\eta_t - 1}.$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have  $W_t = W_t(h)$  and  $N_t = N_t(h)$ .

## Firms

Each Home good,  $Y_t(f)$  is made according to:

$$Y_t(f) = A_t N_t(f).$$

$$N_t(f) = \left( \int_0^1 N_t(h, f)^{\frac{\eta_t-1}{\eta_t}} dh \right)^{\frac{\eta_t}{\eta_t-1}},$$

where the technology parameter,  $\eta_t$ , is stochastic and common to all Home firms.

Firms receive a subsidy that is  $\tau_t$  percentage of its total labor costs.

Profits are given by:

$$\Gamma_t(f) = P_{Ht}(f)C_{Ht}(f) + E_t P_{Ht}^*(f)C_{Ht}^*(f) - (1 - \tau_t)W_t N_t(f).$$

$$C_{Ht}(f) = \int_0^1 C_{Ht}(h, f)dh.$$

$C_{Ht}^*(f)$  is defined analogously.

$$Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f).$$

## Equilibrium

$$Y_t = C_{Ht} + C_{Ht}^* = \frac{\nu}{2} \frac{P_t C_t}{P_{Ht}} + \left(1 - \frac{\nu}{2}\right) \frac{P_t^* C_t^*}{P_{Ht}^*} = k^{-1} \left( \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right) (S_t^*)^{-\nu/2} C_t^* \right)$$
$$Y_t^* = C_{Ft} + C_{Ft}^* = \left(1 - \frac{\nu}{2}\right) \frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2} \frac{P_t^* C_t^*}{P_{Ft}^*} = k^{-1} \left( \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t \right)$$

.

We have used  $S_t$  and  $S_t^*$  to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

$$S_t = P_{Ft} / P_{Ht},$$

$$S_t^* = P_{Ht}^* / P_{Ft}^*.$$

Familiar complete-markets condition:

$$\left( \frac{C_t}{C_t^*} \right)^\sigma = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry:

$$N_t = \int_0^1 N_t(f) df = A_t^{-1} \int_0^1 Y_t(f) df = A_t^{-1} (C_{Ht} V_{Ht} + C_{Ht}^* V_{Ht}^*).$$

where

$$V_{Ht} \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df, \quad \text{and} \quad V_{Ht}^* \equiv \int_0^1 \left( \frac{P_{Ht}^*(f)}{P_{Ht}^*} \right)^{-\xi} df.$$

$$W_t / P_{Ht} = (1 + \mu_t^W) C_t^\sigma N_t^\phi S_t^{1-(\nu/2)}.$$

We can derive the loss function without making assumptions about the price setting process. That is, we do not make assumptions about price stickiness, Calvo pricing, etc.

We do assume workers are homogeneous, which rules out staggered wage setting, but allow firm pricing heterogeneity.

These loss functions will approximate the loss when there are deviations from the law of one price that arise in models with other types of pricing rules.

## Deviations from the law of one price

Define:

Currency misalignment:

$$m_t \equiv \frac{1}{2}(e_t + p_{Ht}^* - p_{Ht} + e_t + p_{Ft}^* - p_{Ft})$$

This is the average deviation from the law of one price

Relative price differences

$$z_t \equiv \frac{1}{2}(p_{Ft} - p_{Ht} - (p_{Ft}^* - p_{Ht}^*))$$

This turns out to be zero in symmetric Calvo-pricing model

## Period Loss Functions with and without deviations from law of one price

Law of one price holds (as in PCP):

$$X_t = \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\xi}{2} (\sigma_{P_H^t}^2 + \sigma_{P_F^{*t}}^2)$$

(Notation:  $\zeta_t^R \equiv \frac{1}{2}(\zeta_t - \zeta_t^*)$  and  $\zeta_t^W \equiv \frac{1}{2}(\zeta_t + \zeta_t^*)$ .)

The loss depends on the relative and world output gaps, and the dispersion of PPI prices.

Why does the loss depend on PPI prices?

Law of one price does not hold (as in LCP):

$$X_t = \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\nu(2-\nu)}{4} z_t^2$$

$$+ \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{H^t}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{F^t}}^2 \right]$$

The first loss function is a special case of the second.

\*\*\*Currency misalignments lead to suboptimal output allocation even if output is produced efficiently.

\*\*\*Loss depends on dispersion of CPI prices rather than PPI prices.

Now assume Calvo pricing. From these, we can derive open-economy Phillips curves.

The policymaker wishes to minimize

$$(62) \quad E_t \sum_{j=0}^{\infty} \beta^j X_{t+j}.$$

Why do we derive a 2<sup>nd</sup>-order approximation in terms of output gaps, inflation and currency misalignment?

The policymaker wishes to minimize  $E_t \sum_{j=0}^{\infty} \beta^j X_{t+j}$ . If we assume

Calvo pricing, we can rewrite this objective as  $E_t \sum_{j=0}^{\infty} \beta^j \Psi_{t+j}$

PCP

We find:

$$\Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\xi}{2\delta} \left( (\pi_{Ht})^2 + (\pi_{Ft}^*)^2 \right).$$

This objective function extends the one derived in CGG to the case of home bias in preferences, or nontraded goods (i.e.,  $\nu \geq 1$  rather than  $\nu = 1$ .)

Policymakers face the log-linearized Phillips curves as constraints:

PCP

$$\pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta \mathbb{E}_t \pi_{Ht+1} + u_t,$$

where  $u_t = \delta \mu_t^W$ .

Similarly for foreign producer-price inflation, we have:

$$\pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta \mathbb{E}_t \pi_{Ft+1}^* + u_t^*.$$

Here  $D \equiv \sigma \nu (2 - \nu) + (\nu - 1)^2$ .

1<sup>st</sup>-order conditions or “Target Criteria” in PCP model:

**Commitment:**  $\tilde{y}_t^W - \tilde{y}_{t-1}^W + \frac{\xi}{2}(\pi_{Ht} + \pi_{Ft}^*) = 0$  and

$$\tilde{y}_t^R - \tilde{y}_{t-1}^R + \frac{\xi}{2}(\pi_{Ht} - \pi_{Ft}^*) = 0, \text{ or simply}$$

$$\tilde{y}_t - \tilde{y}_{t-1} + \xi\pi_{Ht} = 0 \quad \text{and} \quad \tilde{y}_t^* - \tilde{y}_{t-1}^* + \xi\pi_{Ft}^* = 0$$

**Discretion:**  $\tilde{y}_t^W + \frac{\xi}{2}(\pi_{Ht} + \pi_{Ft}^*) = 0$  and  $\tilde{y}_t^R + \frac{\xi}{2}(\pi_{Ht} - \pi_{Ft}^*) = 0$

or,  $\tilde{y}_t + \xi\pi_{Ht} = 0$ , and  $\tilde{y}_t^* + \xi\pi_{Ft}^* = 0$ .

Same as in CGG, and indeed same as in closed economy model. A tradeoff between the output gap and the *producer price* inflation level.

## LCP

$$\Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi)(\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{2\delta} \left( \frac{\nu}{2} (\pi_{Ht})^2 + \frac{2-\nu}{2} (\pi_{Ft})^2 + \frac{\nu}{2} (\pi_{Ft}^*)^2 + \frac{2-\nu}{2} (\pi_{Ht}^*)^2 \right).$$

Why does currency misalignment appear in the objective function?

Because currency misalignments lead to suboptimal output allocation even if output is produced efficiently.

## Phillips curves under LCP

$$\pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D - (\nu - 1)}{2D} m_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$\pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D - (\nu - 1)}{2D} \right) m_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

$$\pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D + \nu - 1}{2D} \right) m_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

$$\pi_{Ft} = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D + \nu - 1}{2D} m_t \right] + \beta E_t \pi_{Ft+1} + u_t^*.$$

Recall  $m_t \equiv \frac{1}{2} (e_t + p_{Ht}^* - p_{Ht} + e_t + p_{Ft}^* - p_{Ft})$

An additional constraint in the LCP model besides the Phillips curves comes about because of the internal movements of terms of trade.

$$s_t = \frac{2\sigma}{D} y_t^R - \frac{(\nu - 1)}{D} m_t.$$

The whole model becomes much simpler if we assume  $\phi = 0$ . This makes the internal relative prices autonomous, and in turn greatly simplifies the form of the optimal policy rules:

$$s_t - s_{t-1} = -\delta \tilde{s}_t + \beta E_t(s_{t+1} - s_t) + u_t^* - u_t$$

We can simplify the loss function to:

$$\Psi_t \propto \frac{\sigma}{D} (\tilde{y}_t^R)^2 + \sigma (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{\delta} \left( (\pi_t^R)^2 + (\pi_t^W)^2 + \frac{\nu(2-\nu)}{4} (s_t - s_{t-1})^2 \right)$$

Then the target criteria are:

**Commitment:**  $\tilde{y}_t^W - \tilde{y}_{t-1}^W + \xi\pi_t^W = 0$

$$\frac{\nu-1}{D} (\tilde{y}_t^R - \tilde{y}_{t-1}^R) + \frac{\nu(2-\nu)}{2D} (m_t - m_{t-1}) + \xi\pi_t^R = 0$$

**Discretion:**  $\tilde{y}_t^W + \xi\pi_t^W = 0$

$$\frac{\nu-1}{D} (\tilde{y}_t^R) + \frac{\nu(2-\nu)}{2D} (m_t) + \xi\pi_t^R = 0$$

These rules involve CPI inflation

Consider some special cases.

First, suppose the two economies were closed, so that  $\nu = 2$ . Then we

$$\text{get: } \tilde{y}_t^R - \tilde{y}_{t-1}^R + \xi\pi_t^R = 0.$$

Together with the other criterion, we get  $\tilde{y}_t - \tilde{y}_{t-1} + \xi\pi_t = 0$  and

$$\tilde{y}_t^* - \tilde{y}_{t-1}^* + \xi\pi_t^* = 0.$$

Now assume no home bias in preferences, so  $\nu = 1$ . Then

$$\frac{1}{2\sigma}(m_t - m_{t-1}) + \xi\pi_t^R = 0.$$

We may tighten Home interest rates even when Home inflation is low in order to appreciate currency.

Note that there is no loss from ignoring currency misalignment as  $\nu \rightarrow 2$ .

There is no loss from ignoring relative output gap as  $\nu \rightarrow 1$ .

We can write the second criterion as:

$$\tilde{q}_t - \tilde{q}_{t-1} + 2\sigma\xi\pi_t^R = 0$$

$\tilde{q}_t$  is the real exchange rate (deviation from efficient level.)

For example, if  $\sigma = 2$  and  $\xi = 4$ , then optimally, we want the home currency to depreciate in real terms at a rate equal to 16 times the Foreign less Home inflation rates.

## Case of no mark-up shocks

- It is well-understood in the PCP model that in this case, optimal policy implies zero PPI inflation. With optimal subsidies in place, 1<sup>st</sup>-best efficient allocations achieved.
- Under LCP, impossible to achieve 1<sup>st</sup>-best because relative price of Foreign to Home goods cannot move freely.
  - But optimal policy still implies zero CPI inflation!
  - Optimal policy also sets the “world” output gap to zero, and the real exchange rate gap.
  - But output gaps in each country are not achieved, the terms of trade is not efficient, and LOOP deviations remain.

## Optimal Instrument Rules

- The paper shows how we can derive optimal interest rate rules, under the assumption that mark-up shocks are AR(1).
- The optimal rule sets the deviation of the nominal interest rate from the Wicksellian rate as a function of mark-up shocks and inflation within each country
- In the PCP model, the PPI inflation rate belongs in the rule
- In the LCP model, the CPI inflation rate is in the rule.
- In neither case does the output gap or currency misalignment appear in the interest rate rule.

However, this result is not general:

- The rules could be rewritten so that output gaps and currency misalignments do appear in the rule.
- As they are currently written, as functions of mark-up shocks, they are not easily implementable.
- In any case, the instrument rule depends on the stochastic process for the mark-up shocks, but the targeting rule does not.
- In particular, news about the future can influence the optimal instrument rule.

## Further Work

- Other models of pricing to market/currency misalignments.
  - E.g., sticky nominal wages and “latent competitors” as in Atkeson and Burstein (2007) and Burstein and Jaimovich (2008)
- Consumption misallocation is not the only problem from currency misalignments.
  - Misallocation between traded and non-traded sectors.
  - Misallocation of imported goods from 3<sup>rd</sup> country.