Econ 872
Spring 2012
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Homework 2
(Due Monday, February 20)

1. In class, we considered a two-country model of the current account and real exchange rates. Each country produced a nontraded good and a traded good. Households in each country consumed three goods – their country's nontraded good and each of the traded goods. Preferences were Cobb-Douglas over consumption of the nontraded good and the traded aggregate, with weight δ ($0<\delta<1$) on the nontraded good. The consumption index over traded goods was CES, with elasticity of substitution given by ε ($\varepsilon>0$). The weight households put on consumption of the good produced in their own country is θ. Home bias implies $\frac{1}{2}<\theta<1$.

Production of each good is linear in labor in both sectors in both countries. Only production of the traded good in the home country is subject to fluctuations in labor productivity, which is given by A_{Ht}.

The price of the foreign traded good is taken as numeraire. The price of the home traded good is P_{Ht}. We have the following price indexes:

$$P_t = P_{Nt}^{\delta} P_{Tt}^{1-\delta} \qquad\qquad P_t^* = (P_{Nt}^*)^{\delta}(P_{Tt}^*)^{1-\delta}$$

$$P_{Tt} = [\theta P_{Ht}^{1-\varepsilon} + 1-\theta]^{\frac{1}{1-\varepsilon}} \qquad\qquad P_{Tt}^* = [(1-\theta)P_{Ht}^{1-\varepsilon} + \theta]^{\frac{1}{1-\varepsilon}}$$

We derived the following equation in class that implicitly expresses P_{Ht} as a function of A_{Ht}:

$$\frac{P_{Ht}A_{Ht}}{1+P_{Ht}A_{Ht}} = \frac{1}{2}\left\{\delta + (1-\delta)\theta\left(\frac{P_{Ht}^{1-\varepsilon}}{\theta P_{Ht}^{1-\varepsilon}+1-\theta}\right) + (1-\delta)(1-\theta)\left(\frac{P_{Ht}^{1-\varepsilon}}{(1-\theta)P_{Ht}^{1-\varepsilon}+\theta}\right)\right\}$$

a. Log-linearize this equation to express p_{Ht} in terms of a_{Ht}, where $p_{Ht} \equiv \ln(P_{Ht})$ and $a_{Ht} = \ln(A_{Ht})$. (Do the log linearization around the point where $a_{Ht}=0$.) Show that when $\varepsilon>1$, $-\frac{dp_{Ht}}{da_{Ht}}<1$, so $a_{Ht}+p_{Ht}$ rises when a_{Ht} increases.

b. Derive an expression for p_t in terms of a_{Ht}, where $p_t \equiv \ln(P_t)$.

c. Derive an expression for $q_t \equiv p_t^* - p_t$ in terms of a_{Ht}. Show that if ε and δ are large enough, we may have $\frac{dq_t}{da_{Ht}}<0$.

2. Consider a small country that produces a traded and a nontraded good. It takes the interest rate, *r* (expressed in units of the traded good) as given.

a. Households maximize

$$\sum_{j=0}^{\infty}\beta^j\left(\frac{\sigma}{\sigma-1}\left(C_{t+j}^{\gamma}(1-L_{t+j})^{1-\gamma}\right)^{\frac{\sigma-1}{\sigma}}\right)$$, where $0<\gamma<1$, and $\sigma>0$.

C_t is a consumption aggregate at time *t*, and L_t is labor supply.

Households face the lifetime budget constraint:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j P_{t+j}C_{t+j}=(1+r)B_t+\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j w_{t+j}L_{t+j}\ ,$$

where B_t equals claims on foreigners carried into period *t*, and w_t equals real wages (in terms of the traded good.) P_t is the exact consumer price index, defined below.

Solve the household's optimization problem. Assume $\beta(1+r)=1$. Derive an equation for optimal consumption growth, $\frac{C_{t+1}}{C_t}$ in terms of $\frac{P_{t+1}}{P_t}$ and $\frac{w_{t+1}}{w_t}$.

Then let $C_t=C_{Nt}^{1-\alpha}C_{Tt}^{\alpha}$, where C_{Nt} is consumption of the nontraded good and C_{Tt} is consumption of the traded good. We have in each period $P_tC_t=p_tC_{Nt}+C_{Tt}$.

Rewrite the expression for $\frac{C_{t+1}}{C_t}$ in terms of $\frac{p_{t+1}}{p_t}$ and $\frac{w_{t+1}}{w_t}$, where $\frac{P_{t+1}}{P_t}=\left(\frac{p_{t+1}}{p_t}\right)^{1-\alpha}$.

Also, derive the demand for C_{Tt} as a function of C_t and p_t. Use this solution to solve for $\frac{C_{Tt+1}}{C_{Tt}}$ in terms of $\frac{p_{t+1}}{p_t}$ and $\frac{w_{t+1}}{w_t}$.

b. Assume production of the traded good is determined by $Y_{Tt}=A_{Tt}L_{Tt}$ and $Y_{Nt}=L_t-L_{Tt}$. Assuming labor is mobile between sectors, solve for p_t and w_t in terms of A_{Tt}. Use your solutions from part (a) to solve for $\frac{C_{Tt+1}}{C_{Tt}}$ in terms of $\frac{A_{Tt+1}}{A_{Tt}}$.