Valuation Effects and the Dynamics of Net External Assets*

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Abstract

The traditional current account can be an inaccurate measure of the change in the net foreign asset (NFA) position. Using gross asset and liability positions at the country level, a number of 'valuation effects' have been identified which contribute to changes in NFA but do not enter the reported current account. This paper uses new developments in the analysis of portfolio allocation in general equilibrium to investigate valuation effects in a simple two-country model. The model can be used to analyze both qualitatively and quantitatively the role of valuation effects that have been observed in empirical studies. Broadly speaking, the valuation effects in the model correspond to those in the data, and have the effect of enhancing cross country risk sharing. But there is a key distinction between 'unanticipated' and 'anticipated' valuation effects. Unanticipated effects can be large, dominating the movement in NFA, but anticipated effects arise only at higher orders of approximation and are small for reasonable parameterisations. The paper also contributes to the study of the determinants of international portfolio positions, and their role in generating valuation effects from asset price and terms of trade changes.

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1 Introduction

Open economy macroeconomic models typically pay close attention to the current account as a measure of the evolution of an economy's net external assets. The growth of current account imbalances, and in particular the US current account deficit, has recently brought this linkage to the forefront of economic policy discussion. Since countries must satisfy intertemporal budget constraints, large and growing current account deficits will reduce net external assets and require the establishment of future trade surpluses.

This traditional view of the current account has been put into question more recently, however. A series of detailed and careful data construction studies suggest that traditional measures of the current account may give a highly inaccurate measure of the movement of an economy's net external wealth (Lane and Milesi Ferretti 2001, 2006). These studies show that corrected measures of net external assets must incorporate changes in asset prices, returns, and foreign exchange rates that impact on the value of an economy's net external wealth through separate 'valuation effects' on gross assets and liabilities. Moreover, given the explosive growth in cross-border capital flows since the mid 1990's, leading to huge increases in the scale of gross external assets and liabilities, these previously unmeasured valuation effects on net external assets have risen dramatically relative to the traditional measures of the current account. A number of studies have emphasized the empirical relevance of these valuation effects (Tille 2003, Higgens et al. 2005, Lane and Milesi Ferretti, 2005, Gourinchas, 2007).

By now, economists have recognized the importance of correctly measuring the impact of valuation changes on net external assets. Until recently however, there has been little impact of these new empirical findings on the traditional modeling of the current account and net external asset movements in open economy macro models. One of the key reasons for this is that it has proven difficult to incorporate classic principles of portfolio choice into the conventional dynamic general equilibrium open economy model. Recent developments in the literature, however, now provide techniques for making progress in combining portfolio choice with general equilibrium open macro models[1]. This paper makes use of these new techniques to provide a qualitative and quantitative analysis of the ability of theoretical models to account for valuation effects in the evolution of net external assets,

[1] See, for instance Coeurdacier (2005), Evans and Hnatkovska, (2005), Kollmann (2006), Engel and Matsumoto (2006), Devereux and Sutherland (2006), (2007), Tille and Van Wincoop, (2007).

and to explore the interaction between valuation effects and traditional measures of the current account.

We begin by developing a highly simplified two-country endowment economy model in which each country faces two sources of risk - one from capital income, which is assumed to be internationally diversifiable through equity sales, and the other from labor income, which cannot be directly diversified. Although the model is simple, it allows us to illustrate in an analytical example the main elements of the dichotomy between the traditionally measured current account and the valuation channel in determining the movement of net external assets. Defining the valuation channel as the gap between the movement of net external assets and the standard measure of the current account, we show that the valuation effect may be broken into anticipated and unanticipated components. The anticipated component of the valuation effect captures expected excess returns on a country's portfolio due to differences in the covariance risk associated with each country's traded equity. Such country risk premia allow, in principle, for permanent imbalances in national current accounts. In addition, there may be time-varying anticipated excess returns that are associated with current account adjustment. The unanticipated component of the valuation effect captures the way in which national portfolios are structured so as to hedge against consumption risk. In this model, a basic property of the unanticipated valuation component is that it should co-vary negatively with the traditional current account. The model also allows for a decomposition of unanticipated valuation effects into those coming from movements in rates of return on assets, and those coming from movements in the portfolio holdings.

Having defined these different components of valuation effects, we go on to provide a quantitative account of the importance of each component in the evolution of net assets. We show that the model indicates that anticipated valuation effects are extremely small, except for counterfactually high values of risk aversion and differences in country endowment volatilities. But unanticipated valuation components may represent a large fraction of the volatility of net external assets, even when the model is calibrated to realistic sizes of gross national portfolios. Moreover, unanticipated valuation effects in the model behave in quite a similar fashion to those imputed from the net foreign assets (NFA) data - in particular, they are large as compared to traditional macro shocks, they dominate the movements in NFA, they tend to be negatively correlated with the current account, and they are approximately i.i.d.

One aspect of the recent portfolio discussion emphasizes the difference between the effects of shocks to returns for a given portfolio, and the effects of adjustment in the portfolio itself, sometimes called 'portfolio rebalancing' (see Hau and Rey, 2007). In our model, both effects form part of the dynamics of NFA. Unanticipated valuation channels involve both shocks to returns, and movements in portfolio holdings. But in our quantitative decomposition of the volatility of net external assets, the latter channel plays at best a small role. The biggest driver of the volatility of net external assets is the unanticipated movement in returns, holding the portfolio constant. Portfolio adjustment and movements in expected returns can also create anticipated valuation effects. But our analysis suggests that these effects arise only at higher orders of approximation and are quantitatively very small.

The main results of the paper are presented in the context of a one-good world economy with stochastic endowments. In a later section, we show how the decomposition of valuation effects extends to a context with differentiated home and foreign goods. In this section, we also emphasize the important role of bonds as well as equities in risk sharing, and both asset prices and terms of trade changes in generating unanticipated valuation effects. With stochastic endowment shocks to capital and labor income and endogenous terms of trade, we find that bond trading *alone* can achieve full risk-sharing across countries. As a by-product then, the model introduces a new source of 'home equity bias'. In this case, all valuation effects come from movements in the terms of trade. Moreover, valuation effects in this context are substantially larger than in the model without terms of trade volatility.

The paper's contribution is also pedagogical. We document how valuation effects enter in the evolution of net foreign assets, and at what order of approximation each effect is important. To this extent, the paper can be seen as a theoretical underpinning for some traditional 'portfolio balance' modeling, which combined goods and asset market modeling in one framework, but based on assumed rules of thumb behaviour with respect to portfolio composition. At the same time, our analysis naturally places a limit on the potential importance of each component of valuation effects. In one sense, our results suggest that in order to support the importance of some key elements of portfolio balance models, it would be necessary to develop models of risk-bearing that differ substantially from those of the standard intertemporal stochastic model that underlies the traditional open economy macro framework used in this paper.

There is a large and growing literature on valuation effects and current account dynamics in general equilibrium models. Notable recent papers are Cavallo and Tille (2006), Ghironi, Lee and Rebucci (2006), and Pavlova and Rigibon (2007). Cavallo and Tille (2006) and Ghironi, Lee and Rebucci (2006) provide a careful quantitative accounting of the impact of valuation effects in models in which the portfolio structure is calibrated to match the data. Pavlova and Rigibon (2007) present a rich continuous time dynamic model in which the portfolio rules can be obtained in closed form, but follow a different line of inquiry from that considered here.

The paper is structured as follows. The next section discusses some properties of the data on the current account and net external assets. We then set out a simple model of the current account in the face of capital and labour income risk. Following this, we discuss the properties of the solution method for portfolio choice. We then explore some analytical results on valuation effects. After this, we present quantitative results on the importance of anticipated and unanticipated valuation effects. The main sections of the paper are based on a simple single-good model with trade in equities. In the last section of the paper we extend the analysis to a two-good model with trade in both equities and bonds.

2 The Current Account and Valuation Effects

Here, we provide a brief description of the evolution of net external assets and their decomposition in terms of the conventional measure of the current account, and those driven by valuation effects[2]. We focus on a subset of OECD countries. Start with a simple decomposition of net external assets into the conventional current account, as measured in balance of payments accounting, and valuation terms. Thus, for country i at time t, we have:

$$NFA_{it} - NFA_{it-1} = CA_{it} + VAL_{it} \tag{1}$$

[2] Similar discussion is provided in Kollmann (2006), Gourinchas (2007), Lane and Milesi Ferretti (2006) among others.

We compute these using the IMF/Lane-Milesi-Ferretti External Wealth of Nations (EWN) dataset on international investment positions, and from balance of payments data on the current account. As discussed in Lane and Milesi-Ferretti (2006), Tille (2003), and Gourinchas (2007), movements in VAL_{it} are driven by asset price and exchange rate changes which cause revisions in the value of gross external assets and liabilities, but are not incorporated in the income account as returns paid or received on gross external liabilities or assets.

We derive VAL_{it} indirectly, since NFA_{it} is reported in the EWN data-base (and updated using the IMF IIP), and CA_{it} is observable from Balance of Payments data. To make the VAL_{it} variable comparable with our model, we scale by GDP. Thus, we define

$$val_{it} = \frac{(NFA_{it} - NFA_{it-1})}{GDP_{it}} - \frac{CA_{it}}{GDP_{it}} \equiv \Delta nx_{it} - ca_{it}, \tag{2}$$

Since NFA_{it} and CA_{it} are reported in US dollars, we use US dollar GDP_{it} from the OECD database. The variable val_{it} is constructed for a sample of 23 OECD countries for the period 1980-2006. Table 1 describes the characteristics of val_{it}. The first column of the table reports the standard deviation of val_{it} for each country. As noted in Lane and Milesi-Ferretti (2006), the valuation term is highly volatile, with an average standard deviation of 0.07 across countries. The second column illustrates the fraction of the total variation in Δnx_{it} accounted for by valuation effects; $VR_i = var(val_i)/var(\Delta nx_i)$ over the sample. For most countries, this is well above 50 percent. The average value is 0.90, and the US is highest at 1.39. In terms of accounting for the variation in net external assets, for most countries the valuation effects completely dominate the share attributable to the current account in the variation of net external assets.

The valuation term is of course not independent of the current account itself. The third column of Table 1 reports the correlation coefficient between val_{it} and ca_{it} for each country. In the endowment economies explored below, this correlation is negative. The results in the data are mixed. For 14 of the 23 countries in the sample, $corr(ca_i, val_i)$ is negative, with the highest negative correlation being for the US.

Kollman (2006) has previously noted that Δnx_{it} is approximately $i.i.d.$ for most countries, while the current account displays substantial persistence. Here, when we impute the valuation effect as the difference between the two, we find that val_{it} inherits the persistence properties of the Δnx_{it} series. The measured val_{it} has no serial correlation for almost all countries. Table 1 reports the results from the AR(1) regression $val_{it} = c_1 + c_2 val_{it-1}$

for each country. The AR(1) coefficient is insignificant for almost all countries. Below, we show that val_{it} as defined in the theoretical model should be *i.i.d.*

Since the model's predictions for risk-sharing are mainly concerned with the trade account, we also report the following decomposition:

$$vai_{it} = \Delta nx_{it} - ta_{it} \tag{3}$$

where ta_{it} is the trade account to GDP ratio. Thus vai_{it} is the sum of the valuation term to GDP ratio, plus the income account to GDP ratio. In practice, vai_{it} and ta_{it} behave very similarly. Table 2 reports the identical results to Table 1 for this decomposition. As before, the variance of the valuation term is very high relative to the variance of net external assets - the average value is again about .9. Thus, a large component of nx is driven by portfolio effects, rather than trade balance effects. In addition, we find that the correlation between vai and ta is negative now for most countries. Finally, constructed in this way, vai is transitory - the AR(1) coefficient is again insignificant for most countries.

In the model below, the presence of valuation effects is critically tied to the size of a country's gross asset and liability position. Figures 1 and 2 illustrate the relationship between gross positions and valuation effects. In the figure, gross positions are measured by the average value of assets plus liabilities to GDP over the sample.[3] Figure 1 suggests that there is a positive relationship between the gross position and the standard deviation of $\sigma(VAL)$. Countries with higher gross positions have higher volatility of the valuation residual. But Figure 2 indicates that this is not true for the VR measure. That is, there is no clear relationship between the gross positions and the degree to which net foreign asset changes are accounted for by VAL. For most countries, VR is close to 1.

These stylized facts are 'first-order' in nature. Interpreted this way, as we discuss below, val and vai can thus be thought of as the result of an optimal risk-sharing portfolio, because they can be interpreted as implicit insurance against business cycle shocks. Gourinchas (2007) refers to these as 'unpredictable' valuation terms. But other recent discussion of valuation effects in international financial data stress the presence of 'predictable' valuation effects at the national level, meaning that there are predictable excess returns on some component of a country's gross assets relative to the same component in its gross liabilities. As a rough measure of this, Table 1 computes the average valuation

[3] Clearly this is an imperfect statistic since both measures have been distinctly trending upwards over the sample.

effect over the sample for each country. If valuation changes were just attributable to first-order risk-sharing, then this should be a very small number. In fact, it is negative, and a relatively large share of GDP for many countries. For the US, it is positive and 1.4 percent of GDP.

Gourinchas and Rey (2005) estimate a substantial excess return on US assets relative to liabilities, for all components of its international portfolio. For portfolio equity and debt securities, Curcuru, Dvorak, and Warnock (2008) argue that the actual excess return to the US is quite small. But for FDI, Higgins, Klitgaard and Tille (2006) find a 2-3 percent persistently higher return on US assets abroad than foreign assets held in the US. Lane and Milesi Ferretti (2007) provide an overview of some of the measurement problems inherent in these estimates. Lane and Milesi Ferretti (2005) take a larger sample of countries, and find that average rates of return on assets and liabilities have had significant differences over substantial periods of time for many countries.

Gourinchas and Rey (2007) highlight a somewhat different predictable valuation effect. They find that, conditional on an increase in the US trade balance deficit, the US experiences a predictable persistent increase in the excess return on its international investment portfolio, thereby reducing the required increase in the future trade balance surplus required to achieve overall intertemporal budget balance.

While unpredictable valuation gains or losses are relatively easy to model, in terms of an optimal insurance arrangement, it has proven much more difficult to integrate the findings of predictable excess returns into general equilibrium modeling. This is because these effects are of a 'higher order' nature. In our analysis below, we examine higher order approximations of portfolio choice within a standard general equilibrium framework, and explore the degree to which they give rise to predictable valuation effects on the evolution of net external assets.

3 A Simple Example Model

We first illustrate how the decomposition of the measured current account and valuation effects interact in a simple two-country endowment model with only two traded assets, and a single world consumption good. A later section extends the analysis to two goods.

Agents in the home country have a utility function of the form

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_\tau) \tag{4}$$

where C is consumption and $u(C_\tau) = (C_\tau^{1-\rho})/(1-\rho)$.

The budget constraint for home agents is given by

$$\alpha_{1,t} + \alpha_{2,t} = \alpha_{1,t-1} r_{1,t} + \alpha_{2,t-1} r_{2,t} + Y_t - C_t \tag{5}$$

where Y is the endowment received by home agents, $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ are the real holdings of the two assets (purchased at the end of period $t-1$ for holding into period t) and $r_{1,t}$ and $r_{2,t}$ are gross real returns. The stochastic process determining endowments and the nature of the assets and the properties of their returns are specified below.

We define $W_t = \alpha_{1,t} + \alpha_{2,t}$ to be the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets, NFA, of home agents). Defining $r_{x,t} = r_{1,t} - r_{2,t}$ as the "excess return" on asset 1, the budget constraint can then be re-written as

$$W_t = Y_t - C_t + r_{2,t} W_{t-1} + \alpha_{1,t-1} r_{x,t}. \tag{6}$$

At the end of each period agents select the portfolio of assets to hold into the following period. The first-order condition for the choice of $\alpha_{1,t}$ can be written in the following form

$$E_t \left[u'(C_{t+1}) r_{1,t+1} \right] = E_t \left[u'(C_{t+1}) r_{2,t+1} \right] \tag{7}$$

Foreign agents face a similar consumption and portfolio allocation problem with an analogous utility function and budget constraint.

Assets are assumed to be in zero net supply, so market clearing in asset markets implies $\alpha_{1,t-1} + \alpha^*_{1,t-1} = 0$ and $\alpha_{2,t-1} + \alpha^*_{2,t-1} = 0$. To simplify notation in this example, we can drop the subscript from $\alpha_{1,t}$ and simply refer to α_t. Note that $\alpha_{1,t} = -\alpha^*_{1,t-1} = \alpha_t$, $\alpha_{2,t} = W_t - \alpha_t$ and $\alpha^*_{2,t} = W^*_t + \alpha_t$, where W^* is foreign net external assets, and $W_t + W^*_t = 0$.

Assume that endowments are the sum of 'capital income' components, $Y_{K,t}$ and 'labour income' components $Y_{L,t}$, so that

$$Y_t = Y_{K,t} + Y_{L,t}. \tag{8}$$

The two countries may trade assets representing claims on capital income, but labour income is non-diversifiable. Endowments are determined by the following stochastic processes

$$\log Y_{K,t} = \mu \log Y_{K,t-1} + \varepsilon_{K,t}, \qquad \log Y_{L,t} = \mu \log Y_{L,t-1} + \varepsilon_{L,t} \tag{9}$$

where $\varepsilon_{K,t}$, $\varepsilon_{L,t}$, are zero-mean i.i.d. shocks which are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_K] = \sigma_K^2$, $Var[\varepsilon_L] = \sigma_L^2$. In what follows, we make the further assumption that $\sigma_K^2 = \sigma_L^2$, and define ζ =corr$[\varepsilon_K, \varepsilon_L]$ as the correlation between capital and labour income shocks. Foreign income processes are defined analogously, and we assume zero covariance between home and foreign income shocks.

The two traded assets are equity claims on the home and foreign capital income. The real payoff on a unit of home equity is $Y_{K,t}$ and the real price is $Z_{E,t-1}$. Thus the gross real rate of return is

$$r_{1,t} = \frac{Y_{K,t} + Z_{E,t}}{Z_{E,t-1}} \tag{10}$$

The real return on foreign equity is defined analogously, where $Z_{E,t-1}^{*}$ is the price of the foreign equity.

The first-order condition for home consumption is

$$C_t^{-\rho} = \beta E_t \left[C_{t+1}^{-\rho} r_{2,t+1} \right] \tag{11}$$

Finally, equilibrium consumption plans must satisfy the resource constraint

$$C_t + C_t^{*} = Y_t + Y_t^{*} \tag{12}$$

A competitive equilibrium in this example is defined by (6), (7) and its foreign counterpart, (10) and the analogous equation for r_{2t}, (11) and its foreign counterpart, and (12). These implicitly give the solutions for the equilibrium values of C, C^{*}, r_1, r_2, $Z_{E,t}$, $Z_{E,t}^{*}$, W_t and α_t.

4 Portfolio Holdings and Excess Returns

We first discuss the nature of portfolio solutions within this model. In particular, we define some terms relating to the true and approximated solutions for gross portfolio holdings and equilibrium asset returns.

Consider an approximation of α up to order N

$$\alpha_t = \bar{\alpha} + \hat{\alpha}_t^{(1)} + \hat{\alpha}_t^{(2)} + ... + \hat{\alpha}_t^{(N)} + O\left(\epsilon^{N+1}\right) \tag{13}$$

where $\bar{\alpha}$ is the *zero-order component* (i.e. α at the point of approximation) and $\hat{\alpha}^{(i)}$ is the *order-i component* of the deviation of α from $\bar{\alpha}$. In what follows we will confine attention to the first two terms in this approximation, $\bar{\alpha}$ and $\hat{\alpha}_t^{(1)}$. Notice that, by definition, $\bar{\alpha}$ is constant and therefore captures the average or steady-state element of portfolio holdings, while the (first-order) time varying element in portfolio holdings is captured by $\hat{\alpha}_t^{(1)}$.

Agents make their portfolio decisions at the end of each period and are free to rearrange their portfolios each period. In a recursive equilibrium, the equilibrium asset allocation will be some function of the state of the system in each period - which is summarised by the state variables. We therefore postulate that the true portfolio (i.e. the equilibrium portfolio in the non-approximated model) is a function of state variables, $\alpha_t = \alpha(Z_t)$ where Z is the vector of state variables. We can therefore deduce that $\hat{\alpha}_t^{(1)}$ is a linear function of the first-order deviation of Z from $\bar{Z}$, i.e.

$$\hat{\alpha}_t^{(1)} = \varrho \hat{Z}_t^{(1)}$$

where ϱ is a vector of coefficients (which are tied down by a third-order approximation of the portfolio optimality condition, (7)).

When analysing a DSGE model up to first-order accuracy, the standard solution approach is to use the non-stochastic steady-state of the model as the approximation point, (i.e. the zero-order component of each variable) and to use a first-order approximation of the model's equations to solve for the first-order component of each variable. Neither of these steps can be used in the above model. This is because in the non-stochastic equilibrium, the portfolio optimality conditions imply $r_{1,t+1} = r_{2,t+1}$ i.e. both assets pay the same rate of return. This implies that any value for α is consistent with equilibrium. A similar problem arises in a first-order approximation of the model. First-order approximation of equations the portfolio optimality conditions imply certainty equivalence, so that $E_t[\hat{r}_{1,t+1}^{(1)}] = E_t[\hat{r}_{2,t+1}^{(1)}]$. Again, any value of α is consistent with equilibrium. So neither the non-stochastic steady state nor a first-order approximation of the model provide enough equations to tie down the zero or first-order components of α.

It is clear that the risk characteristics of assets only show up in the second-moments of model variables, and it is only by considering higher-order approximations of the model that the effects of second-moments can be captured. In Devereux and Sutherland (2006) we show that a second-order approximation of the portfolio optimality conditions provides a condition which makes it possible to tie down the zero-order component of α. Having established this starting point, it is relatively straightforward to extend the procedure to higher-order components on α. In Devereux and Sutherland (2007) we show that the solution for the first-order component of α can be derived from third-order approximations of the portfolio optimality conditions.[4]

Now consider equilibrium returns, $r_{1,t}$ and $r_{2,t}$ or, more specifically, the *excess* return, $r_{x,t}$. Consider an approximation of $r_{x,t}$ up to order N

$$r_{x,t} = \bar{r}_x + \hat{r}_{x,t}^{(1)} + \hat{r}_{x,t}^{(2)} + ... + \hat{r}_{x,t}^{(N)} + O\left(\epsilon^{N+1}\right) \tag{14}$$

What can our solution approach tell us about equilibrium excess returns at different orders of approximation? First, notice that the properties of the non-stochastic steady state and the first-order system tell us immediately that $\bar{r}_x = E_t[\hat{r}_{t+1}^{(1)}] = 0$. It follows therefore that expected excess returns only deviate from zero at orders 2 and higher. In Devereux and Sutherland (2006) we show that $E_t[\hat{r}_{t+1}^{(2)}]$ can be solved in conjunction with $\bar{\alpha}$. Furthermore, we show that $E_t[\hat{r}_{t+1}^{(2)}]$ can be written as a function of one-period-ahead conditional second moments of first-order realised asset returns and consumption. Because these one-period-ahead conditional second moments are non-time-varying, $E_t[\hat{r}_{t+1}^{(2)}]$ will also be non-time-varying. In fact $E_t[\hat{r}_{t+1}^{(2)}]$ can naturally be thought of as the steady-state equilibrium expected excess return which corresponds to steady-state equilibrium asset holdings, $\bar{\alpha}$.

In a similar way, in Devereux and Sutherland (2007) we show that the third-order component of excess returns, $E_t[\hat{r}_{t+1}^{(3)}]$, can be solved in conjunction with the first-order component of asset holdings, $\hat{\alpha}_t^{(1)}$. We show there that $E_t[\hat{r}_{t+1}^{(3)}]$ can be written in terms of expected products of first and second-order realised asset returns and consumption. Furthermore, just as $\hat{\alpha}_t^{(1)}$ is time varying, it follows that $E_t[\hat{r}_{t+1}^{(3)}]$ is also time varying and it is possible to show that $E_t[\hat{r}_{t+1}^{(3)}]$ is a linear function of the first-order component of state

[4] The fundamental insights underlying our solution approach have existed in the literature for many years. They were first formalised by Samuelson (1970), who states a general principle that, in order to derive the Nth-order component of the portfolio, it is necessary to approximate the portfolio problem up to order $N+2$. A general numerical approach to the solution which parallels ours in found in Tille and Van Wincoop (2007).

variables, i.e.

$$E_t[\hat{r}_{t+1}^{(3)}] = \delta \hat{Z}_t^{(1)}$$

where δ is a vector of coefficients which are functions of one-period-ahead conditional second moments (i.e. δ is order-2 and thus the right-hand side is order-3). $E_t[\hat{r}_{t+1}^{(3)}]$ can naturally be thought of as the time varying element of excess returns that corresponds to the first-order time varying element of portfolio holdings.

5 Valuation Effects in the Example Model

Now focus on the definition of the current account and valuation effects within the simple model outlined above. Equation (6) can be rearranged to give a representation for the change in net external wealth as

$$\Delta W_t = Y_t - C_t + (r_{2,t} - 1)W_{t-1} + \alpha_{t-1} r_{x,t} \tag{15}$$

where $\Delta W_t = W_t - W_{t-1}$ and, as previously noted, we simplify notation by dropping the subscript from $\alpha_{1,t-1}$. There are two ways to define valuation effects in the context of (15). The first approach, and the clearest way within this model, is to take the first term $Y_t - C_t + (r_{2,t} - 1)W_{t-1}$, as a measure of the conventional current account, and the second term $\alpha_{t-1} r_{x,t}$, as a measure of valuation effects which impact on net external assets, but do not directly enter into the current account. To the extent that the current account does not directly incorporate dividend payments from equity, this is consistent with standard balance of payments accounting. An alternative approach is to decompose r_{1t} and r_{2t} into dividend and capital gains terms, and to assume that the dividend payments are incorporated in the measurement of the current account. As we show below, both decompositions have very similar implications, so we focus only on the first one. We therefore rewrite (15) as follows

$$\Delta W_t = CA_t + VAL_t \tag{16}$$

where

$$CA_t = Y_t - C_t + (r_{2,t} - 1)W_{t-1} \tag{17}$$

$$VAL_t = \alpha_{t-1} r_{x,t} \tag{18}$$

Our analysis focuses on the behaviour of VAL as defined in (18). In the course of our analysis it will become apparent that VAL can be decomposed into anticipated and unanticipated components. These two components play different roles in the dynamics of NFA and these roles can be clearly identified and quantified within our model. It will also become apparent that the anticipated component of VAL can be further decomposed into a non-time-varying and a time-varying component. Again, the roles played by these two components in the dynamics of NFA can be clearly identified and quantified within the model.

Our analysis is conducted using an approximate solution for VAL which is constructed using the approximate solutions for α and r_x given in (13) and (14). Using these expressions, together with $\bar{r}_x = 0$, it is simple to show that an expression for VAL (up to third-order is) is given by

$$VAL_t = \widehat{VAL}_t^{(1)} + \widehat{VAL}_t^{(2)} + \widehat{VAL}_t^{(3)} + O\left(\epsilon^4\right)$$

where

$$\widehat{VAL}_t^{(1)} = \tilde{\alpha}\hat{r}_{x,t}^{(1)} \tag{19}$$

$$\widehat{VAL}_t^{(2)} = \tilde{\alpha}\hat{r}_{x,t}^{(2)} + \hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)} \tag{20}$$

$$\widehat{VAL}_t^{(3)} = \tilde{\alpha}\hat{r}_{x,t}^{(3)} + \hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(2)} + \hat{\alpha}_{t-1}^{(2)}\hat{r}_{x,t}^{(1)} \tag{21}$$

where $\hat{r}_{x,t} = \hat{r}_{1t} - \hat{r}_{2t}$, $\hat{r}_{1t}$, and $\hat{r}_{2t}$ are log-deviations from the non-stochastic steady state and $\tilde{\alpha}$ and $\hat{\alpha}_t$ are defined as $\tilde{\alpha} = \bar{\alpha}/(\beta\bar{Y})$ and $\hat{\alpha}_t = \frac{1}{\beta\bar{Y}}(\alpha_t - \bar{\alpha}) = \frac{\alpha_t}{\beta\bar{Y}} - \tilde{\alpha}$. We analyse the first, second and third-order components of VAL, given by (19), (20) and (21) in turn and show how the unanticipated, anticipated and time-varying components of VAL arise at these different orders of approximation.

5.1 First-order Valuation Effects

We first compute the importance of valuation effects at the first-order, i.e. as measured by (19). This is the standard approach taken in much of the earlier literature on valuation effects.

The first and most obvious feature of $\widehat{VAL}_t^{(1)}$ which can be deduced from (19) is that it is a zero mean i.i.d. random variable. This follows from the properties of $\hat{r}_{x,t}^{(1)}$. It therefore follows that $\widehat{VAL}_t^{(1)}$ has no predictable component. In order to analyse the properties of $\widehat{VAL}_t^{(1)}$ in more detail however, it is necessary to solve for its components $\tilde{\alpha}$ and $\hat{r}_{x,t}^{(1)}$.

Following Devereux and Sutherland (2006), it is easy to compute the first-order solutions for consumption, asset prices, and asset returns[5]. Given this, we may then compute $\tilde{\alpha}$ as follows

$$\tilde{\alpha} = -\frac{1}{2(1-\beta)} \frac{\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 + \zeta^*\sigma_K^{2*})}{\sigma_K^2 + \sigma_K^{2*}} \tag{22}$$

where $\phi = \bar{Y}_K/\bar{Y}$ is the share of capital income in the total endowment in the non-stochastic steady state. The optimal steady state portfolio, $\tilde{\alpha}$, takes a negative weight on home equity, and a positive weight on foreign equity, so long as $\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 + \zeta^*\sigma_K^{2*}) > 0$. A fully diversified portfolio would have each country holding half of the other's equity to GDP ratio, or $\tilde{\alpha}^F = -\frac{\phi}{2(1-\beta)}$. The degree of home bias in equity holdings will depend on the correlation between capital and labour income in each country. Thus, if ζ and ζ^* are less than zero, we have $|\tilde{\alpha}| < |\tilde{\alpha}^F|$, and there is home bias in equity holdings. Note that even with home bias in equity holdings, there can be significant gross positions. For instance, when the home country holds only 10 percent of foreign equity, $\tilde{\alpha} = -\frac{0.1\phi}{2(1-\beta)}$. With a discount factor of 0.96 and a capital income share of $\phi = 0.36$, this is equivalent to a gross asset and liability position of 0.45 times GDP.

In addition to solving for the steady-state portfolio, it is simple to derive the following expression for the first-order behaviour of the excess return

$$\hat{r}_{x,t}^{(1)} = \frac{(1-\beta)}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{23}$$

so the first-order component of the valuation effect, $\widehat{VAL}_t^{(1)}$, is given by

$$\widehat{VAL}_t^{(1)} = \tilde{\alpha}\frac{(1-\beta)}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{24}$$

We may compare the behaviour of $\widehat{VAL}_t^{(1)}$ to the first-order behaviour of the current account, which is given by

$$\widehat{CA}_t^{(1)} = \frac{\beta}{2}\frac{(1-\mu)}{(1-\beta\mu)}\left[\phi(\hat{Y}_{k,t} - \hat{Y}_{k,t}^*) + (1-\phi)(\hat{Y}_{l,t} - \hat{Y}_{l,t}^*)\right] - \tilde{\alpha}\frac{(1-\beta)^2}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{25}$$

[5] To be precise, expression (22) is derived by solving the condition $E_t\left[(\widehat{C}_t - \widehat{C}_t^*)\widehat{r}_{x,t}\right] = 0$, (which comes from a second order expansion of (7)), where both terms inside the square brackets are evaluated up to the first order.

This expression contains two terms. The first term is the familiar textbook definition of the current account. When there is a rise in home relative to foreign income, whether capital or labour income, the current account will improve, so long as $\mu < 1$. The second term captures the impact of portfolio valuation effects on consumption, and therefore the current account. The valuation term represents the income gain or loss due to unanticipated changes in the excess return on assets. The scale of this will depend on the portfolio position $\tilde{\alpha}$, given in (22).

How large are the first-order valuation effects within this model? We measure both the volatility of $\widehat{VAL}_t^{(1)}$ directly, and the volatility relative to the volatility of net foreign assets. Take the case where $\sigma_K^2 = \sigma_K^{2*}$ and $\zeta = \zeta^*$. In addition, to make the discussion simpler, assume that $\zeta > -\frac{\phi}{1-\phi}$. Then, from (22) and (24), the standard deviation of $\widehat{VAL}_t^{(1)}$ is

$$\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = |\tilde{\alpha}| \frac{(1-\beta)}{(1-\beta\mu)}\sqrt{2\sigma_K^2} = \frac{\phi + (1-\phi)\zeta}{\sqrt{2}(1-\beta\mu)}\sigma_K$$

The volatility of $\widehat{VAL}_t^{(1)}$ depends positively on the size of the gross asset position. This is consistent with the evidence in Figure 1. In addition $\sigma_{t-1}(\widehat{VAL}_t^{(1)})$ is increasing in the persistence of endowment shocks, and the volatility of shocks. A higher μ has no effect on $\widetilde{\alpha}$, but increases excess returns volatility.

Using (22), (24), and (25), we can define the ratio of the variance of $\widehat{VAL}_t^{(1)}$ to that of the variance in the change in net foreign assets as:

$$VR = \frac{\sigma_{t-1}^2(\widehat{VAL}_t^{(1)})}{\sigma_{t-1}^2(\Delta\hat{W}_t^{(1)})} = \frac{(\phi + (1-\phi)\zeta)^2}{\beta^2[(1-\mu)^2(1-\phi)^2(1-\zeta^2) + \mu^2(\phi + (1-\phi)\zeta)^2]} \qquad (26)$$

Theoretically, this can take any value in the range between zero and infinity. When $\phi = 1$ or $\zeta = 1$, there are effectively complete markets, and the right hand side of (26) is $1/\mu^2$, which always exceeds unity. If shocks are quite transitory, then the optimal portfolio keeps net external assets very stable, and the valuation ratio is very high. On the other hand, for low or negative ζ, the optimal portfolio stance is small, due to home bias, and the valuation ratio may be very small.

Now let us roughly quantify these expressions. Let the discount factor be $\beta = 0.96$. The portfolio size $\widetilde{\alpha}$ depends on the value of ζ, the correlation between labour and capital income, ϕ, the share of capital in income, and the discount factor. For ϕ we take the conventional measure for the US economy of $\phi = 0.36$. We set $\mu = 0.9$, with $\sigma_K^2 = .02^2$, which is approximately the volatility of annual US GDP growth. Empirical estimates of ζ have varied quite a lot (see Bottazzi et al (1996), and Engel and Matsumoto (2008)). The correct measure of ζ should compare the overall returns to physical capital with those to human capital. Following this procedure, Bottazzi et al (1996) find a range of estimates both negative and positive. In this model, to allow for home bias in equity holdings within this example, it is necessary to have $\zeta < 0$. In section 7 below, we show that in the presence of endogenous terms of trade and bond holdings, we can obtain home equity bias for any value of ζ. Here however, we simply choose a range of values of ζ which give rise to different values for the gross asset and liability positions $\widetilde{\alpha}$. For $\zeta = -.4625$, the home country holds a gross asset and liability position equal to 80 percent of GDP (approximately that of the US), so $\widetilde{\alpha} = -0.8$. This implies an 18 percent holding of foreign equity.

Using this calibration, at $\widetilde{\alpha} = -0.8$, we find $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.007$, about quarter that in the data for the US. To match the US estimate of $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.03$, we would need $\widetilde{\alpha} = -3.6$. If shocks were much more persistent, $\sigma_{t-1}(\widehat{VAL}_t^{(1)})$ would be higher. For $\mu = 0.95$, for instance, we have $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.01$.

Figure 3 reports the value for VR for various ranges of $\tilde{\alpha}$. We use the same calibration as above, varying ζ to generate different values for $\tilde{\alpha}$. For $\mu = 0.9$, we find that VR is quite low initially, but rises with $\tilde{\alpha}$, exceeding 1 after $\tilde{\alpha} = 1.5$. With higher persistence, however VR is much higher initially, and relatively insensitive to the gross asset position. Intuitively, from (24) and (25), when μ is closer to unity, both $\widehat{VAL}_t^{(1)}$ and $\Delta\hat{W}_t^{(1)}$ are proportional to $\tilde{\alpha}$, so that their ratio is relatively insensitive to the size of gross positions.

This example suggests that in principle, a model of efficient risk-sharing can account for the properties of the valuation shocks, the absence of persistence in these shocks, and their large size relative to overall fluctuations in NFA. But we also saw that for the majority of countries, the correlation between the conventional current account measure and the valuation term is negative. This also extends to the alternative, trade balance measure of VAL. In the model, using the solution for $\tilde{\alpha}$, we may establish that:

$$\text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)}) = -\frac{(\phi + (1-\phi)\zeta)(1-\mu\beta)}{\sqrt{\beta^2(1-\mu)^2(1-\phi)^2(1-\zeta^2) + (1-\beta\mu)^2(\phi + (1-\phi)\zeta)^2}} < 0. \tag{27}$$

Hence, the theoretical correlation in this example is always negative. When either $\zeta = 1$,

$\phi = 1$, or $\mu = 1$, this correlation is equal to -1. In the first and second case, this is because equity holdings allow for effectively complete markets. The effects of any endowment shock on the current account precipitate a movement in the valuation term which is exactly proportional to the shock. In the case $\mu = 1$, the correlation is -1 because the only source of movement in the measured current account (25) is due to movements in the portfolio itself. In Table 1, the average value for this correlation is -0.11. In the model, the most important determinant of $\text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)})$ is ζ. When we calibrate ζ to achieve a value for $\tilde{\alpha}$ equal to -0.9, using the same values for other parameters as in Figure 3, we find $\text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)}) = -0.17$.

To illustrate how risk sharing works in the model, take a one unit positive home endowment shock in the special case where $\phi = 1$. In the absence of valuation effects, measured income would rise by 1, while consumption would rise by $\frac{(1-\frac{\beta(1+\mu)}{2})}{(1-\beta\mu)}$, leading to a current account surplus equal to $\frac{0.5\beta(1-\mu)}{(1-\beta\mu)}$, consistent with (25). When equities are chosen optimally however, there is a simultaneous negative payoff from the portfolio return, as the excess return on home equity is positive, and the home household holds a negative gross position. Consumption is adjusted downwards by $(1-\beta)$ times the valuation effect, or $\tilde{\alpha}\frac{(1-\beta)^2}{(1-\beta\mu)}$. Given $\phi = 1$,and $\tilde{\alpha} = 0.5/(1-\beta)$, consumption rises only by $\frac{(1-\frac{\beta(1+\mu)}{2})}{(1-\beta\mu)} - \frac{0.5(1-\beta)}{(1-\beta\mu)} = 0.5$, and the net effect on the measured current account is 0.5. Thus, the home endowment shock is shared equally among home and foreign consumption. The sum of the measured current account and the direct negative valuation term, then leads to a *fall* in net foreign assets for the home country equal to $0.5 - \tilde{\alpha}\frac{(1-\beta)}{(1-\beta\mu)} = -\frac{0.5\beta\mu}{(1-\beta\mu)}$. In the next period, the combination of higher income but lower net foreign assets leads home consumption to rise by 0.5μ, again leading to an optimal sharing across countries. In this way, the initial (unanticipated) valuation effect leads to an evolution of net foreign assets following the shock such that the consumption response is equalized across the home and foreign country, facilitating an optimal risk-sharing response to the original shock.

In this one-good model, valuation effects come from two sources - direct dividend payments (associated with capital income shocks), and from capital gains (associated with fluctuations in equity prices). How much variation is attributable to each source? We may define $\sigma_{t-1}(\widehat{VAL}_{Dt}^{(1)})$ and $\sigma_{t-1}(\widehat{VAL}_{Qt}^{(1)})$ as the standard deviation in $\widehat{VAL}_t^{(1)}$ attributable to each source. In this model, we have:

$$\sigma_{t-1}(\widehat{VAL}_{Dt}^{(1)}) = |\tilde{\alpha}|\,(1-\beta)\sqrt{2}\sigma_K \tag{28}$$

$$\sigma_{t-1}(\widehat{VAL}_{Qt}^{(1)}) = |\tilde{\alpha}| \frac{\mu\beta(1-\beta)}{(1-\mu\beta)}\sqrt{2}\sigma_K \tag{29}$$

With substantial persistence in shocks, the second term will dominate the first. For example, take the same numerical calibration used with respect to (**??**) above, where $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.007$. The decomposition here indicates that $\sigma_{t-1}(\widehat{VAL}_{Qt}^{(1)}) = 0.0065$, while $\sigma_{t-1}(\widehat{VAL}_{Dt}^{(1)}) = 0.001$.[6]

5.2 Second-order Valuation Effects: Anticipated Excess Returns

Now consider the properties of the second-order component of VAL, given by expression (20). It is useful to break the analysis of $\widehat{VAL}_t^{(2)}$ into two stages. First we consider the mean, or expected, value of $\widehat{VAL}_t^{(2)}$. In the next section we consider the stochastic behaviour of $\widehat{VAL}_t^{(2)}$. Taking expectations of (20) yields

$$E_{t-1}[\widehat{VAL}_t^{(2)}] = \tilde{\alpha}E_{t-1}[\hat{r}_{x,t}^{(2)}] \tag{30}$$

Notice that, given the properties of $\hat{r}_{x,t}^{(2)}$ this will be constant and may be non-zero.

Following Devereux and Sutherland (2006), we obtain the following expression for the second-order component of the expected excess return

$$E_{t-1}[\hat{r}_{x,t}^{(2)}] = \frac{\rho}{2}\frac{(1-\beta)}{(1-\beta\mu)}\left(\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 - \zeta^*\sigma_K^{2*})\right) + O\left(\epsilon^3\right) \tag{31}$$

The expected excess return on the home country asset is negative if the volatility of the foreign capital income shock exceeds that of the home shock, and the covariance of capital and labour income shocks in the foreign country exceed those in the home country. Intuitively, if $\sigma_K^2 < \sigma_K^{2*}$, then the foreign capital income shock is more responsible for world consumption volatility than the home shock. Investors in both countries then must receive a higher expected return on the foreign asset. Even if $\sigma_K^2 = \sigma_K^{2*}$, however, if $\zeta < \zeta^*$, then again world consumption volatility is more correlated with the foreign asset return, and there is a risk premium on the foreign asset.

A risk premium on the foreign asset translates into an expected long run current account imbalance in the following way. Take expectations of a second-order approximation of (16) yields

$$E_{t-1}[\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}] = E_{t-1}[\widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}] + \tilde{\alpha}E_{t-1}[\hat{r}_{x,t}^{(2)}]$$

[6] This supports the argument made above that it makes little difference if we define VAL as is done here, or using an alternative definition excluding dividend payments.

The first term on the right hand side is the expected current account surplus, evaluated up to second-order, while the second term is the expected excess return on the external portfolio. If a country holds an external portfolio which commands a positive risk premium, so that $\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(2)}] > 0$, then it can sustain a permanent average current account deficit, and yet keep $E_{t-1}[\Delta \hat{W}_t^{(1)} + \Delta \hat{W}_t^{(2)}] = 0$. For instance, if $\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\sigma_{KL} - \sigma_{KL}^*) < 0$, then country 1's asset is less correlated with world consumption risk. Since $\tilde{\alpha} < 0$, we then have $\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(2)}] > 0$, and country 1 can have a permanent current account deficit equal to this. By acting as a 'safe haven', a country with a low volatility of output can on average consume more than its income, if it is willing to hold more risky foreign assets.

How big can this safe haven effect on the current account be within our simple example? To estimate this, we must combine the solution for $\tilde{\alpha}$ with the expected excess return within the model to obtain:

$$-\frac{\rho}{4} \frac{[\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta \sigma_K^2 + \zeta^* \sigma_K^{2*})]\,[\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\zeta \sigma_K^2 - \zeta^* \sigma_K^{2*})]}{(1-\beta\mu)(\sigma_K^2 + \sigma_K^{2*})}$$

The two key parameters determining the size of this expression are the coefficient of relative risk aversion, and the degree of persistence in endowment shocks. Figure 4 illustrates the excess return and the current account effect. The figure assumes that $\sigma_K^2 = 0.01^2$, and $\sigma_K^{2*} = 0.04^2$, indicating that the foreign country has a much more volatile endowment process. The correlation between capital and labor income in each country is varied in order to allow variation in the value of $\tilde{\alpha}$. We again assume that $\beta = 0.96$, $\phi = 0.36$, $\mu = 0.9$, and assume that $\rho = 8$, indicating a high rate of risk aversion, but still well within the range used in asset pricing studies (e.g. Bansal and Yaron, 2004). Again, $-\tilde{\alpha}$ is on the horizontal axis.

For values of $\tilde{\alpha}$ in the range of 0 to -1, the effect of differential risk on the current account is very small. A 'safe haven' country in this range could expect to have a current account deficit of 0.02 percent of GDP. The excess return on the external portfolio is the same. As total leverage rises, the size of $E_{t-1}[\widehat{VAL}_t^{(2)}]$ rises. For gross asset positions equal to 5 times GDP (as seen for some countries), the safe haven effect could finance a current account deficit of 0.25 of a percent of GDP, under this calibration. For higher values of μ, this effect is magnified. But even for $\tilde{\alpha} = -5$ and $\mu = .95$, the implied current account deficit is less than 0.5 of a percent of GDP.

5.3 Second-order valuation effects: Portfolio Rebalancing

Now let us examine the stochastic properties of $VAL^{(2)}$. These are captured by the second term in (20), given by $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$. This term picks up the impact of portfolio adjustment on the volatility of VAL. Notice, however, that portfolio adjustment does not give rise to *predictable* time variation in VAL at the second-order level. Given the properties of $\hat{r}_{x,t}^{(1)}$, the term $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$ is a mean zero i.i.d. random variable. Nevertheless, portfolio adjustment adds to the variance of VAL, evaluated to second-order.

How important is the term $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$ in the determination of the variance of net external assets? That is, how much additional risk-sharing is offered by this portfolio adjustment term at the second-order level? In order to answer this question, we have to go beyond the solution (22), which gives the steady state (or zero order) component of the portfolio, to derive a solution for $\hat{\alpha}_t^{(1)}$. The resulting expression captures the way in which the portfolio is adjusted in response to movements in the underlying state variables of the economy. In Devereux and Sutherland (2007), it is shown that there is an analytical solution for this, which (for this model) can be written as:

$$\hat{\alpha}_t^{(1)} = \varrho_1 \hat{Y}_{K,t}^{(1)} + \varrho_2 \hat{Y}_{K,t}^{*(1)} + \varrho_3 \hat{Y}_{L,t}^{(1)} + \varrho_4 \hat{Y}_{L,t}^{*(1)} + \varrho_5 \hat{W}_t^{(1)} \tag{32}$$

where the ϱ_i coefficients are complicated functions of parameters and the moments of shocks.

What is the intuition for the dependence of α on the shocks and net wealth, as captured in (32)? To see this, go back to the portfolio selection condition (7), which indicates that the individual wishes to keep the expected product of marginal utility and excess returns equal to zero. When (7) is evaluated up to a second-order, this is equivalent to keeping the conditional one-step ahead covariance of log consumption and excess returns equal to zero. A constant portfolio $\tilde{\alpha}$ is sufficient to achieve this. But when we take a third-order approximation in order to obtain $\hat{\alpha}_t^{(1)}$, the conditional means of consumption and asset returns will affect overall portfolio risk, and agents will have to adjust their portfolio to hedge against this. These adjustments in turn affect the correct measure of valuation, evaluated up to a second-order. With some slight abuse of terminology, we call this a 'portfolio rebalancing' effect. In response to movements in the conditional means of consumption and asset returns, agents desire to adjust their portfolio holdings.

How important are these higher-order effects? In terms of variance decomposition,

Table 3, reports the results of the valuation terms when we solve the model up the second-order. We define the valuation ratios VR_1 and VR_2 respectively as

$$VR_1 = var(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)})/var(\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}),$$

and

$$VR_2 = var(\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)})/var(\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}).$$

Thus, VR_1 is a measure of the importance of the direct movement in excess returns on the portfolio on the volatility of net foreign assets (up to second-order approximation). VR_2 is a measure of the volatility in 'portfolio rebalancing' as a share of the volatility of net foreign assets. VR_1 is almost the same as the first-order solution VR from Figure 3. As gross asset positions rise, the importance of movements in excess returns on the portfolio grows larger. VR_2 was not measured before. But in fact, it is small. The adjustment of portfolios contributes is at most 3.5 percent of the variation in net external assets. Moreover, as the size of the gross asset positions rise, this falls. In the baseline calibration of the previous section, where $\tilde{\alpha} = -0.8$, movements in the portfolio contribute only 1.4 percent of the volatility of net foreign assets.

Despite the small size of the portfolio adjustment term in accounting for the movement in net external assets at the second-order level, it still exhibits the risk-sharing properties of the first-order solution. In Table 3,

$$corr_1 = corr(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)} + \hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

$$corr_2 = corr(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

$$corr_3 = corr(\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

Thus, the overall second-order valuation term, and the two subcomponents of the valuation term covary negatively with the current account. Portfolio rebalancing does play a role as part of the optimal portfolio. But relative to the first-order effect of having an optimally chosen fixed portfolio, this has only a minor impact on the evolution of net external assets[7].

[7] As before, these results are somewhat sensitive to the persistence of shocks. For a higher μ, VR_2 will be higher, at low values for $\tilde{\alpha}$. But it still remains small relative to VR_2 for empirically relevant values of $\tilde{\alpha}$.

5.4 Third-order Valuation Effects: Portfolio Adjustment

In the previous sections we have considered the valuation terms arising in the first and second-order approximations of VAL. Notice that these valuation terms depend on the zero-order and first-order component of gross asset holdings, $\tilde{\alpha}$, and the first and second-order components of excess returns. In fact the only anticipated valuation term that arises is the product of the zero-order component of α and the second-order component of expected excess returns. As explained, both the zero-order component of α and the second-order component of $E[r_x]$ are non-time-varying. So the *anticipated* valuation term in the second-order approximation of VAL is also non-time-varying.

Recent literature (in particular Gourinchas and Rey, 2007) has emphasised the possibility that portfolio adjustment may generate additional *predictable time-varying valuation* effects which affect VAL during the transitional phase that follows a shock. For instance, if home households respond to a shock by increasing α, an additional predictable valuation term will arise while α is above its steady-state value. Likewise, predictable valuation effects can be generated by predictable movements in expected excess returns following a shock. Gourinchas and Rey's evidence suggests that a negative shock to the US trade balance is followed by a predictable increase in the excess return on the US portfolio. If the expected excess return on the portfolio rises following a shock, an additional valuation effect will exist for as long as the expected excess return is above its steady state value. We now wish to examine whether these valuation terms can arise in our model. If so, how large are they? Moreover do they increase or decrease VAL during the transitional phase following a shock[8]?

Clearly, in order to answer these questions it is necessary to analyse the time varying behaviour of α and r_x. As discussed in section 4 this requires solving for higher-order components of α and r_x. More specifically it is necessary (at least) to solve for the first-order component of α and the *third-order* component of r_x. In the previous section we have already introduced the time-varying solution for $\hat{\alpha}_t^{(1)}$. In conjunction with this first-order solution for $\hat{\alpha}_t^{(1)}$ we may also derive the third-order component of r_x as a linear

[8] In Gourinchas and Rey's study, the key mechanism for predictable excess returns is the anticipated movement in the exchange rate. Our model has no exchange rate movements. Hence, this section should be seen as being motivated by Gourinchas and Rey rather than as a quantitative investigation of their empirical finding.

function of the state variables as follows

$$E_t[\hat{r}^{(3)}_{x,t+1}] = \delta_1 \hat{Y}^{(1)}_{K,t} + \delta_2 \hat{Y}^{*(1)}_{K,t} + \delta_3 \hat{Y}^{(1)}_{L,t} + \delta_4 \hat{Y}^{*(1)}_{L,t} + \delta_5 \hat{W}^{(1)}_t \tag{33}$$

where again the δ_i coefficients are complicated functions of parameters and the moments of shocks.

How do $\hat{\alpha}^{(1)}_t$ and $\hat{r}^{(3)}_{x,t+1}$ enter the expected evolution of NFA? The answer is that, at the level of first and second-order components of VAL, given in (19) and (20), neither of these terms has any (predictable) effect. While the term $\hat{\alpha}^{(1)}_{t-1}\hat{r}^{(1)}_{x,t}$ in equation (20) does depend on $\hat{\alpha}^{(1)}$, this does not give rise to a *predictable* valuation effect, because $\hat{\alpha}^{(1)}_{t-1}\hat{r}^{(1)}_{x,t}$ is mean zero i.i.d.. As shown in the previous section, such adjustments will contribute to the variance of net external assets, but by definition, any *predictable* time-variation in VAL will be zero at the second-order and will only emerge at the third-order or higher. These effects are likely to contribute a very small fraction of the overall variation of net external assets.

To see this more clearly, note that the expected value of the third-order component of VAL is given by

$$E_{t-1}[\widehat{VAL}^{(3)}_t] = \underbrace{\tilde{\alpha} E_{t-1}[\hat{r}^{(3)}_{x,t}]}_{\text{time variation in } E[r_x]} + \underbrace{\hat{\alpha}^{(1)}_{t-1} E_{t-1}[\hat{r}^{(2)}_{x,t}]}_{\text{time variation in } \alpha} \tag{34}$$

The first term in this expression, $\tilde{\alpha} E_{t-1}[\hat{r}^{(3)}_{x,t}]$, is time varying because $E_{t-1}[\hat{r}^{(3)}_{x,t}]$ is time-varying. And the second term, $\hat{\alpha}^{(1)}_{t-1} E_{t-1}[\hat{r}^{(2)}_{x,t}]$, is time varying because $\hat{\alpha}^{(1)}_{t-1}$ is time varying. Hence $\tilde{\alpha} E_{t-1}[\hat{r}^{(3)}_{x,t}]$ captures the effect of time varying expected returns on VAL while $\hat{\alpha}^{(1)}_{t-1} E_{t-1}[\hat{r}^{(2)}_{x,t}]$ captures the effect of time varying portfolio holdings on VAL. These two terms thus capture the predictable time-varying valuation effects arising from predictable movements in portfolios and excess returns..

Note that, while these terms only enter a third-order approximation of the VAL, it is not necessary to solve the model up to this order to be able to evaluate these terms. In fact once we have obtained the solutions of the form (32) and (33), these valuation effects can be evaluated directly from first-order impulse response of the state variables. The next section presents numerical calculations of impulse responses for the example model. These are used to construct numerical calculations for the two terms in (34).

6 Impulse Responses and Valuation Effects

To illustrate the role and potential magnitude of the different valuation effects, we consider some impulse responses following an innovation to capital income. These are shown in Figure 5. Again we set $\sigma_K^2 = \sigma_L^2 = \sigma_K^{*2} = \sigma_L^{*2} = .02^2$, $\beta = 0.96$, $\phi = 0.36$, and $\mu = 0.9$ and we choose $\zeta = \zeta^*$ so that $\tilde{\alpha} = -0.8$, i.e. home households hold a gross position in foreign equity equal to about 80% of GDP. Figure 5 shows the impact of a -1% shock to capital income in the home country (Y_K) in period 1. The impact on total income is shown in panel (a). Home country income falls by 0.36% on impact. Panel (b) shows that consumption in the home economy falls by approximately 0.22% in period 1. The impact effect of the shock is therefore to push the home economy into a trade deficit of approximately 0.14% of GDP. This deficit declines to zero as the effects of the shock dissipate.

While the home economy runs a trade and current account deficit following the shock, net foreign assets rise sharply in period 1 and then decline. The sharp rise in NFA in period 1 reflects the first-order unanticipated valuation effect that arises from the effects of the shock on realised equity returns. The shock to home country capital income implies a sharp unanticipated fall in the price of home equity so there is an unanticipated negative excess return on home equity (i.e. $\hat{r}_x$ is negative).[9] Home households have a negative external position in home equity (i.e. $\tilde{\alpha}$ is negative) so the negative excess return in home equity represents a positive valuation effect. The shock to $\hat{r}_x$ is approximately -0.3% so this first-order valuation effect is approximately 0.24% of GDP (i.e. -0.3×-0.8). This is illustrated in panel (k).

By evaluating the ϱ_i coefficients in (32) we are also able to trace out the dynamic effect of the shock on gross portfolio holdings. These are shown in panel (d) and panel (e). Here $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are home households' holdings of, respectively, home and foreign equity. Panels (d) and (e) show that, for this parameterisation of the model, the movements in gross equity holdings are significantly larger than the movement in NFA. The shock induces

[9] Notice from panel (g) that the prices of both home and foreign equity fall following the shock. The price of foreign equity falls because the expected future rate of return on all equity has to be above its steady state value to be consistent with the rising path of consumption. The price of home equity obviously falls more than the price of foreign equity because the persistent shock to home capital income reduces the income stream to holders of home equity.

home households to increase their gross holdings of home equity by over 1% of GDP while their holdings of foreign equity are reduced by an almost equivalent amount. As discussed in Devereux and Sutherland (2007), the response of gross assets and liabilities following the shock represents a combination of adjustment to overall wealth (the ϱ_5 coefficients) and direct responses to the income shock (the ϱ_1 coefficients).

Evaluation of the δ_i coefficients in (33) allows us also to plot the effects of the shock on the (third-order) expected path of the excess return (i.e. $E[\hat{r}_x]$). This is illustrated in panel (h). The shock leads to a persistent reduction in the expected excess return. The magnitude of this effect is very small however. $E[\hat{r}_x]$ falls by 0.000015% following the shock and gradually returns to zero as the effects of the shock fade.

The dynamic responses of $\hat{\alpha}$ and $E[\hat{r}_x]$ provide us with the information necessary to calculate the two third-order valuation effects in (34). These are illustrated in panel (l). The plot labelled val(α) represents the value of the second term in (34) while the first term in (34) is labelled val(rx). It can be seen from panel (l) that $\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]$ is zero in this parameterisation of the model. This reflects the symmetric nature of the parameterisation, which implies that $E_{t-1}[\hat{r}_{x,t}^{(2)}] = 0$. Dynamic adjustment of $\hat{\alpha}_t^{(1)}$ therefore does not generate any predictable valuation effect. Panel (l) shows however that $\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(3)}]$ is positive following the shock. This reflects the fact that $E[\hat{r}_x]$ is negative (see panel (h)) while $\tilde{\alpha}$ is also negative. The persistent negative value of $E[\hat{r}_x^{(3)}]$ therefore represents a positive valuation effect for home households. This effect is, however, minute. At its largest it is only 0.000012 % of GDP! This should be compared to the trade deficit, which is 0.14% of GDP in the period of the shock.

As a further illustration of the size of the third-order valuation effects consider an asymmetric case where $\sigma_K^2 = 0.01^2$ and $\sigma_K^{*2} = .04^2$, i.e. a case where foreign capital income is more volatile than home capital income. This implies a steady-state risk premium in foreign equity of 0.0079% (i.e. $E_{t-1}[\hat{r}_{x,t}^{(2)}] = -0.0079\%$). In this case time variation in $\hat{\alpha}_t^{(1)}$ generates a non-zero valuation effect via the term $\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]$. Impulse responses (not reported) show that this valuation effect is negative (because $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ is negative and $\hat{\alpha}_t^{(1)}$ is positive) and it has a maximum absolute value of 0.0001 following a -1% shock to home capital income. Again this is minute in comparison to the trade deficit created by the shock.

We may therefore conclude that time varying expected returns do act so as to stabilize the impact of a fall in the trade balance in the model. But in practice, this mechanism plays essentially no role at all in the adjustment process. To obtain an economically meaningful pattern of time varying expected valuation effects through movements in excess returns, we would need a model in which time-varying risk played a much bigger role than it does here.

7 Real Exchange Rate Valuation Effects

Up to now, all the valuation effects we have analysed are due to changes in asset prices (and to a minor extent, dividend payments). But Lane and Milesi Ferretti (2007) emphasize the importance of exchange rate changes in valuation gains and losses. To do full justice to the role of nominal exchange rate variation on the valuation of net external assets is beyond the scope of the present paper. However, we can easily extend the model to incorporate the impact of terms of trade and *real* exchange rate movements in generating valuation effects. In this section, we do this by allowing for differentiation between home and foreign goods. As a by-product, this extension introduces gains from bond trade. As we show, this simple extension has quite dramatic implications for the structure of external portfolios as well as the source of valuation effects.

Again, agents in the home country have utility functions of the form given by (4). Now however, C is a consumption index defined across all home and foreign goods, given by:

$$C_t = \left[\gamma^{\frac{1}{\theta}} C_{Ht}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{35}$$

where C_H and C_F are aggregators over individual home and foreign produced goods. The parameter θ in (35) is the Armington elasticity of substitution between home and foreign goods. The parameter γ measures the importance of consumption of the home good in preferences. For $\gamma > 0.5$, we have 'home bias' in preferences.

The aggregate CPI for home agents is therefore:

$$P_t = \left[\gamma P_{Ht}^{1-\theta} + (1-\gamma) P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{36}$$

where P_H and P_F are the aggregate price indices for home and foreign goods.

The budget constraint of the home country agent is then:

$$P_t C_t + P_t W_{t+1} = P_{Ht} Y_t + P_t \sum_{k=1}^{N} \alpha_{k,t-1} r_{kt} \tag{37}$$

where W_t denotes home country net external assets in terms of the home consumption basket. The final term represents the total return on the home country portfolio. We now allow for trade in up to $N = 4$ assets. We compare an equilibrium where agents trade only in equities to one where there is trade in both equities and bonds. Bond payoffs are denominated in either home or foreign goods.

The conditions for the foreign economy are similar. The foreign CPI is

$$P_t^* = \left[(1-\gamma)P_{Ht}^{1-\theta} + \gamma P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{38}$$

so that γ measures the weight on the foreign consumption in the foreign consumption index.

Equity prices and returns are as described above, except that home equity is now a claim on the capital income return on the home good, Y_{Kt}, and similarly for foreign equity. Thus, the return on the home equity, in terms of the home CPI, is given by

$$r_{et} = \frac{P_{Ht}Y_{Kt}/P_t + Z_{Et}}{Z_{Et-1}} \tag{39}$$

where Z_{Et} is now the price of the home equity in terms of the home consumption basket. The return on a home country bond is written as

$$r_{bt} = \frac{P_{Ht}/P_t}{Z_{Bt-1}} \tag{40}$$

where Z_{Bt} is the price of the home bond in terms of the home consumption basket. The foreign equity and foreign good-denominated bond are defined analogously.

From (37), we define the evolution of net foreign assets, evaluated up to the first-order, as:

$$\hat{W}_t - \hat{W}_{t-1} = \frac{1-\beta}{\beta}\hat{W}_{t-1} + \widehat{P}_{Ht} - \widehat{P}_t + \hat{Y}_t - \hat{C}_t + \tilde{\alpha}'\hat{r}_{x,t} + O\left(\epsilon^2\right) \tag{41}$$

where $\tilde{\alpha}'$ represents the vector of portfolio holdings. In case of equity trade alone, as before, $\tilde{\alpha}$ is the real holding of home equity, and $\hat{r}_{x,t}$ is the excess return on home equity relative to foreign equity. When both equities and bonds can be traded, defining the home bond as the residual asset, $\tilde{\alpha}'$ is the vector of real holdings of home equity, foreign equity, and foreign bonds, given by:

$$\tilde{\alpha}' = \begin{bmatrix} \tilde{\alpha}_e & \tilde{\alpha}_e^* & \tilde{\alpha}_b^* \end{bmatrix}$$

The excess return is then defined as:

$$\hat{r}'_{x,t} = \begin{bmatrix} (\hat{r}_{e,t} - \hat{r}_{b,t}) & \left(\hat{r}^*_{e,t} - \hat{r}_{b,t}\right) & \left(\hat{r}^*_{b,t} - \hat{r}_{b,t}\right) \end{bmatrix} = \begin{bmatrix} \hat{r}_{x,1,t} & \hat{r}_{x,2,t} & \hat{r}_{x,3,t} \end{bmatrix}.$$

The zero-order optimal portfolio may be constructed using the same procedure as before. In equilibrium, households choose a portfolio of home and foreign equity, and home and foreign bonds so as satisfy a portfolio selection equation coming from a second-order approximation of a condition akin to (7) above.

7.1 Equity Trade Only

First, look at the case of equity trade only. The solution for $\tilde{\alpha}$ is then a complicated function of parameters and shock variance and covariances. In the special case where $\gamma = 0.5$ however, we may express the solution as

$$\tilde{\alpha} = -\frac{(\theta-1)}{2}\frac{\theta\phi - 1 + (1-\phi)[2\phi(1-\zeta) + \theta\zeta]}{(1-\beta)[(1-\theta)^2 + 2(1-\phi)(1-\zeta)(\theta-\phi)]} \tag{42}$$

When $\theta \to \infty$, this recovers (22), the equilibrium equity holding in the one good model. On the other hand, when $\theta = 1$ equity holdings are zero, since, with identical preferences in both countries, relative price movements achieve full risk sharing, for familiar reasons. In addition, when $\phi = 1$, we again have $\tilde{\alpha} = -0.5/(1-\beta)$, so there is perfect pooling, as before.

7.2 Bond and Equity Trade

Now extending the asset menu to allow for bond trading, we may solve for the equilibrium vector of portfolio holdings of equities and bonds. In this case, we find that

$$\tilde{\alpha} = \begin{bmatrix} 0 & 0 & \frac{(1-\gamma)(2\gamma-1-\rho(2\gamma\theta-1)}{\rho(1-\mu\beta)} \end{bmatrix} \tag{43}$$

The striking feature of (43) is that equilibrium equity holdings are zero! In terms of gross equity positions, this implies that home agents hold no foreign equity, and take no measures to diversify away their status quo holding of all domestic equity. This means that, when agents can trade in real bonds, there is *complete home bias* in the equity portfolio. This holds independently of the size of shocks, the covariance of capital and labour income shocks ζ, the value of the elasticity θ, or any other parameters of the model.[10] Moreover, we can easily establish that (43) achieves full cross country risk

[10] To be clear, there is a singularity at the points $\phi = 1$, and/or $\theta = 1$, where perfect pooling of equity would achieve the same allocation as (43).

sharing (up to first-order) in the model with endogenous terms of trade movements. That is, an optimal bond portfolio supports complete assets markets, with no need for any foreign equity portfolio[11].

Why is it that bonds allow for full cross country risk sharing? The key feature of the bond asset is that up to first-order, the relative rate of return on bonds (i.e. foreign bonds relative to home bonds) is equal to the movement in the terms of trade. The terms of trade is driven only by country-specific fluctuations in capital or labour income. From (40), we find that:

$$\left(\widehat{r}^{*}_{b,t} - \widehat{r}_{b,t}\right) = \widehat{p}_{ft},$$

where $\widehat{p}_{ft} = \widehat{P}_{Ft} - \widehat{P}_{Ht}$.

In the economy with a zero portfolio (i.e. $\widetilde{\alpha} = 0$), we may describe the first-order terms of trade movement as

$$\widehat{p}_{ft} = \lambda_1(\widehat{Y}_{K,t} - \widehat{Y}^{*}_{K,t}) + \lambda_2(\widehat{Y}_{H,t} - \widehat{Y}^{*}_{H,t}),$$

where the λ_1 and λ_2 are coefficients which come from solving a first-order approximation to the model. An equity portfolio has an excess return that may also be written as

$$\left(r_{e,t} - r^{*}_{e,t}\right) = \delta_1(\widehat{Y}_{K,t} - \widehat{Y}^{*}_{K,t}) + \delta_2(\widehat{Y}_{H,t} - \widehat{Y}^{*}_{H,t}),$$

where δ_1 and δ_2 are the analogous coefficients in this case. In each case, the optimal portfolio is being chosen so as to attempt to achieve cross country risk sharing. Cross country risk sharing does not require that consumption movements are equated across countries, but rather that consumption movements adjusted for real exchange rate movements are equalized. In the zero-portfolio case, we may write

$$\rho(\widehat{C}_t - \widehat{C}^{*}_t) + \widehat{P}_{H,t} - \widehat{P}_{F,t} = \pi_1(\widehat{Y}_{K,t} - \widehat{Y}^{*}_{K,t}) + \pi_2(\widehat{Y}_{H,t} - \widehat{Y}^{*}_{H,t}). \tag{44}$$

In both the equity-trade economy and the bond-trade economy, a portfolio is a claim on a linear combination of relative capital and labour income shocks. But the key difference is that in the case of bond trade, the relative effect of capital and labour income shocks

[11] In a different context, with multiple shocks and capital accumulation, Coeurdacier et al. (2008) show how a combination of bond holdings and equity holdings can support complete risk sharing. Devereux and Saito (2007), in a very different context, also show that bond holding may substitute for international trade in equity.

on excess returns is identical to the relative effect of these shocks on the real-exchange-rate-adjusted consumption deviation. That is:

$$\frac{\lambda_1}{\lambda_2} = \frac{\pi_1}{\pi_2} = \frac{\phi}{1-\phi}$$

Hence, holding a bond portfolio of an appropriate size can offer a perfect hedge against real-exchange-rate-adjusted consumption risk. By contrast, for the equity portfolio, the impact of capital and labour income shocks on returns differs from their relative impact on adjusted consumption deviations. In particular,

$$\frac{\delta_1}{\delta_2} = -0.5\frac{2\gamma(\theta-1+\phi)+1-2\phi}{(1-\phi)(1-\gamma)}. \tag{45}$$

As a result, the equity portfolio cannot be used as a perfect hedge against real-exchange-rate-adjusted consumption risk. Only in the case where there is no labour income risk at all (i.e. $\lambda_2 = \delta_2 = \pi_2 = 0$), can we use equities to achieve perfect risk sharing in the same way that bonds can be used[12]. Hence, the critical feature of the bond portfolio is that in the zero portfolio economy, the variation in the terms of trade is proportional to the variation in adjusted consumption deviations. This allows for bonds to achieve full risk sharing in the economy driven by independent capital and labour income shocks.

What determines the size and sign of the bond exposure? In general, $\tilde{\alpha}_b = \frac{(1-\gamma)(2\gamma-1-\rho(2\gamma\theta-1)}{\rho(1-\gamma\beta)}$ may be positive or negative. In the case $\gamma = 0.5$, the sign of $\tilde{\alpha}$ is determined only by the size of θ. When $\theta > 1$, home agents hold a negative position in foreign denominated bonds. This is because relative home consumption rises as relative home endowments increase, when $\theta > 1$, while simultaneously, the return on the home bond tends to fall, as the terms of trade appreciates, so that home bonds represent a good hedge against consumption risk. When $\gamma > 0.5$, it is no longer true that the sign of $\tilde{\alpha}_b$ is determined only by the size of θ. But, for reasonable values of parameters, we would anticipate that $\tilde{\alpha}_b < 0$, as before.

7.3 Valuation Effects

What implications do the different asset market specifications have for valuation effects? We may compute the analogous valuation terms that we constructed in the one-good

[12]Note that when $\theta = 1$ and $\gamma = 0.5$, the coefficients π_1 and π_2 in (44), are zero in any case, so (45) is not relevant for risk sharing.

model above. Table 4 reports the results, using the benchmark calibration. We also report the breakdown of valuation effects into dividend payments, asset price movements, and relative price movements. In the baseline calibration, we set all parameters as in Table 3, except $\theta = 1.5$, and $\gamma = 0.75$, which are close to standard consensus values for these parameters in the literature. Again, we choose ζ so as to reproduce a gross position whereby the home country holds 10 percent of foreign equity, in the equity-trade only case[13].

Table 4 shows that the standard deviation of the valuation term is 0.5 percent for this case. This is less than the value in the one-good model for the analogous calibration. In the previous model, without terms of trade adjustment, excess returns are more variable in response to capital income shocks, leading to a higher volatility of the valuation term. Despite the lower volatility of the valuation term, the valuation ratio is higher in this model. When $\tilde{\alpha}_e = -.8$, $VR = 0.93$, compared with 0.82 in the model without terms of trade adjustment (see Figure 3). The intuitive reason behind this is that terms of trade adjustments act so as to stabilize the current account in response to shocks, and hence the volatility of net foreign assets is lower, relative to the volatility of the valuation effects

How are the valuation effects decomposed between dividend movements, asset price movements, and terms of trade movements? Table 4 shows that in the model with only equity trade, most of the variance of the valuation effect is accounted for by movements in asset prices. Equity price volatility accounts for over twice as much in terms of overall valuation movements as does dividend movements. Also, the share of volatility accounted for by movements in the terms of trade is very small.

How do these results compare with the model with bond trade? When agents can trade in real bonds, as we know from above, there is no equity holding at all. Choosing a coefficient of relative risk aversion $\rho = 2$, we find that gross foreign bond holdings are about -1.8 times GDP. This gives rise to significant valuation effects. The standard deviation of the valuation term is now 3 percent, three times as high as the model with equity trade alone, and in fact, approximately the same as in the data for many countries (see Table 1). The variance ratio is now 1.34. Furthermore, in this case, all valuation effects clearly are accounted for by terms of trade movements alone, since the volatility

[13] The value of ζ required for this is $\zeta = 0.14$. This is larger than in the one-good case, because here, terms of trade adjustment achieves some risk sharing, so that there can be more home bias in equity, for a given ζ.

in excess bond returns is driven only by relative price movements.

These results indicate that the size and composition of first-order valuation effects may be affected in important ways by the structure of the economy and the availability of assets. Nevertheless, the valuation mechanism still operates in the same way as in the simple model. Valuation effects act so as to enhance risk sharing between countries.

For brevity, we do not report higher order aspects of valuation effects for the extended model. As we would expect from the results of the previous section, in the current model, expected valuation effects (evaluated up to second-order) and time varying expected valuation effects (evaluated up to third-order) are very small.

8 Conclusion

This paper has shown how recent developments in the analysis of portfolio structure in open economy models may be applied to study the role of valuation effects in the movement of net external assets. While in traditional balance of payments theory, the change in net external assets should be equal to the current account, empirical evidence indicates that for most countries, the evolution of net external assets is dominated by valuation gains and losses coming from changes in asset prices and exchange rates, which do not enter into the measured current account. This gives rise to a valuation term, which can be measured as the gap between the change in net external assets and the current account. The paper shows that a simple model of risk-sharing based on optimal portfolio choice can provide a reasonable qualitative and quantitative account of the properties of this valuation term up to the first-order, where valuation effects are 'unanticipated'. The source of these valuation effects, the degree to which they act so as to provide cross country risk sharing, and their decomposition into asset price changes and terms of trade changes, will depend on the structure of international goods markets and the availability of international assets.

Recent literature has also suggested the presence of 'anticipated', or higher-order valuation effects, giving rise to anticipated average excess returns and anticipated time-varying excess returns. We show that these higher order effects do in principle play a role in the movement of net external assets. In practice however, for the benchmark international macro model with standard preferences and realistically calibrated consumption risk, these effects are quantitatively very small.

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Table 1: Valuation term based on Current Account data

	sd(val)	VR	corr(val,ca)	corr(val,gdp)	ar(1)	mean(val)
Australia	0.06	0.88	0.30	-0.03	0.13	0.00
Austria	0.04	1.12	-0.34	0.05	0.20	0.00
Canada	0.04	1.12	-0.39	0.05	0.61**	0.01
Denmark	0.05	0.62	0.22	-0.03	-0.08	-0.01
France	0.04	0.94	-0.06	0.00	-0.19	0.00
Germany	0.03	0.45	0.24	0.08	-0.13	0.00
Iceland	0.06	0.44	0.09	-0.18	-0.17	-0.02
Ireland	0.15	0.92	0.05	-0.23	-0.08	0.00
Italy	0.04	1.05	-0.26	-0.16	-0.01	0.00
Japan	0.03	0.98	-0.13	0.06	-0.39	-0.01
Korea	0.05	0.88	-0.31	0.30	0.07	-0.02
Mexico	0.04	1.28	-0.50	0.22	-0.16	-0.01
Netherlands	0.10	0.99	-0.08	0.04	-0.21	-0.05
New Z.	0.13	0.99	-0.08	-0.08	0.39*	-0.01
Norway	0.05	0.60	-0.35	-0.27	0.15	-0.01
Portugal	0.04	0.42	0.09	0.32	-0.01	0.00
Spain	0.04	0.70	0.11	-0.02	0.01	-0.01
Sweden	0.11	1.14	-0.35	-0.07	0.04	-0.02
Switzerland	0.15	1.14	-0.36	-0.06	0.09	-0.01
Turkey	0.04	0.71	-0.01	-0.19	0.10	-0.02
UK	0.05	0.89	0.01	-0.10	-0.24	0.00
US	0.03	1.40	-0.54	0.11	0.31	0.01

Table 2: Valuation term based on Trade Account data

	sd(vai)	VR	corr(vai,ca)	corr(vai,gdp)	ar(1)	mean(vai)
Australia	0.06	0.88	0.299	-0.019	0.146	-0.033
Austria	0.04	1.49	-0.577	0.132	0.383	0.017
Canada	0.04	1.07	-0.292	0.057	0.539**	-0.038
Denmark	0.05	0.78	0.124	-0.031	0.014	-0.030
France	0.04	0.93	0.006	0.035	-0.199	0.002
Germany	0.03	0.57	0.327	0.209	0.117	-0.024
Iceland	0.06	0.55	0.351	-0.257	-0.050	-0.058
Ireland	0.18	1.22	-0.452	-0.160	0.135	-0.152
Italy	0.07	1.05	-0.272	-0.083	-0.029	-0.014
Japan	0.03	1.00	-0.126	-0.019	0.447*	-0.007
Korea	0.05	1.02	-0.367	0.359	0.192	-0.028
Mexico	0.04	1.37	-0.578	0.339	-0.033	-0.034
Netherlands	0.10	1.02	-0.138	0.065	-0.268	-0.053
New Z.	0.13	0.99	-0.057	-0.072	0.408*	-0.076
Norway	0.05	0.57	-0.188	-0.299	0.118	-0.030
Portugal	0.06	0.67	0.089	0.371	0.520*	0.056
Spain	0.06	0.81	0.191	0.020	0.147	0.012
Sweden	0.10	0.96	-0.012	0.005	-0.121	-0.053
Switzerland	0.14	1.00	-0.095	0.002	0.040	0.067
Turkey	0.04	0.85	-0.115	-0.277	0.154	0.023
UK	0.05	0.86	0.052	-0.052	-0.255	0.017
US	0.03	1.29	-0.476	0.070	0.260	0.019

Table 3: Variances of 2nd-order valuation effects

α	VR_1	VR_2	$corr_1$	$corr_2$	$corr_3$
-0.30	0.18	0.035	-0.077	-0.064	-0.460
-0.43	0.32	0.028	-0.098	-0.089	-0.045
-0.55	0.45	0.023	-0.120	-0.113	-0.044
-0.68	0.57	0.018	-0.142	-0.137	-0.042
-0.80	0.68	0.014	-0.164	-0.160	-0.041
-0.93	0.77	0.011	-0.186	-0.182	-0.039
-1.05	0.85	0.009	-0.207	-0.204	-0.038
-1.17	0.91	0.007	-0.228	-0.226	-0.037
-1.30	0.96	0.006	-0.248	-0.246	-0.036
-1.42	1.01	0.005	-0.268	-0.264	-0.034

Table 4: Valuation effects in the Two Good Model

Equity Only	sd(val)	VR	corr(val,CA)
	0.005	0.93	-0.23
	sd(val_div)	sd(val_DQ)	sd(val_DP)
	0.001	0.004	0
Equity and	sd(val)	VR	corr(val,CA)
Bond Trade	0.032	1.34	-1
	sd(val_div)	sd(val_DQ)	sd(val_DP)
	0	0	0.032

Figure 1: Standard deviation of val

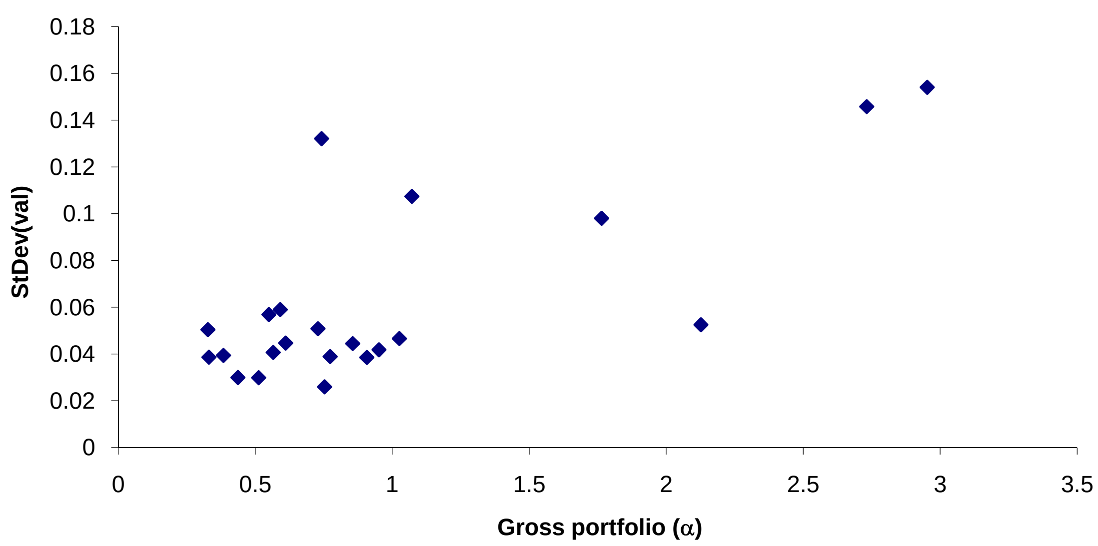


Figure 2: Variance ratio, vr=var(val)/var(ΔNFA)

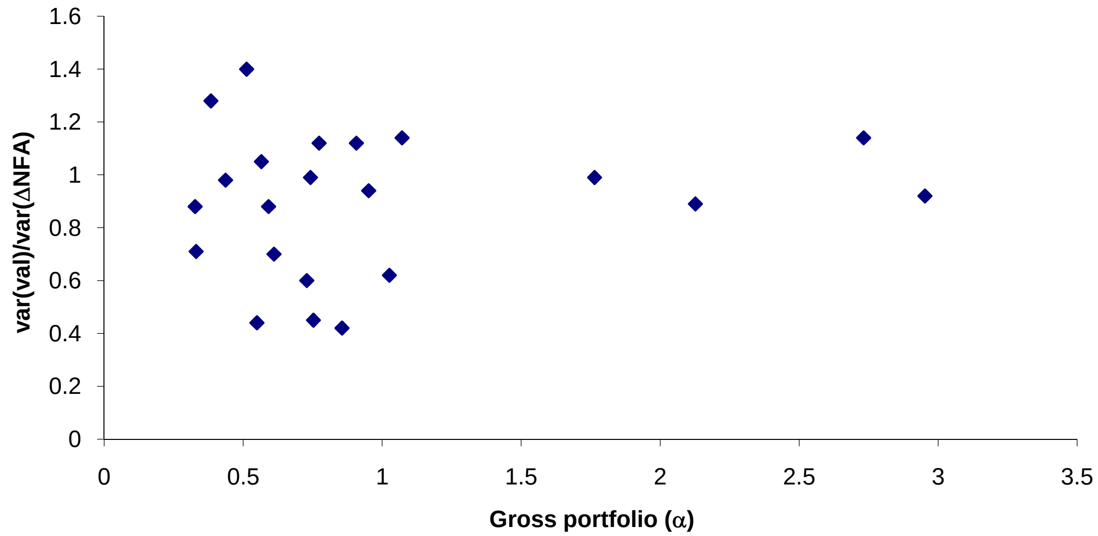
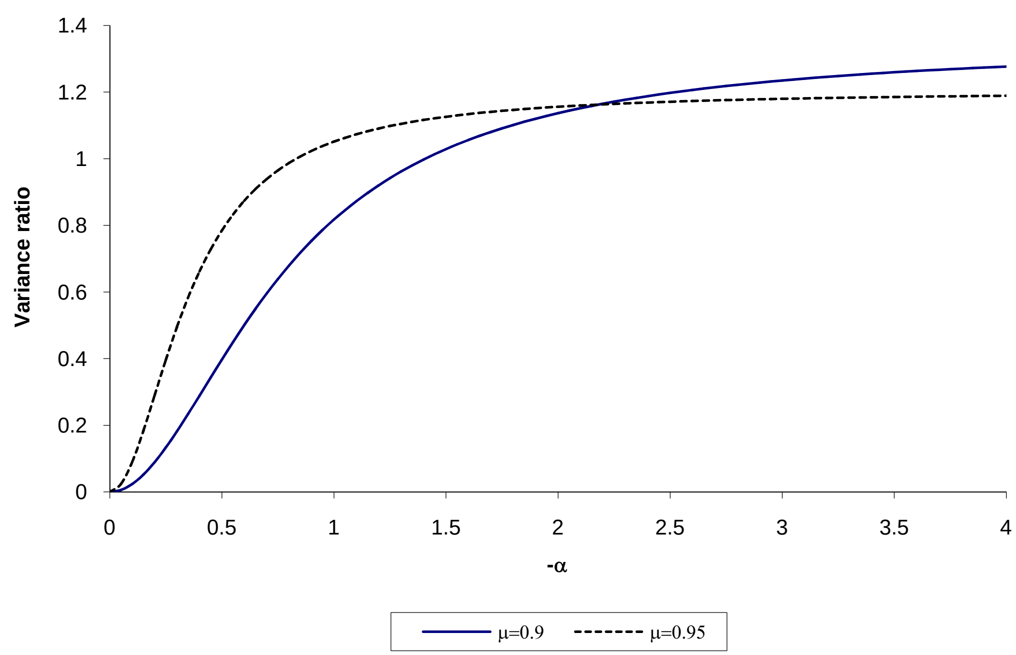


Figure 3: Variance Ratio



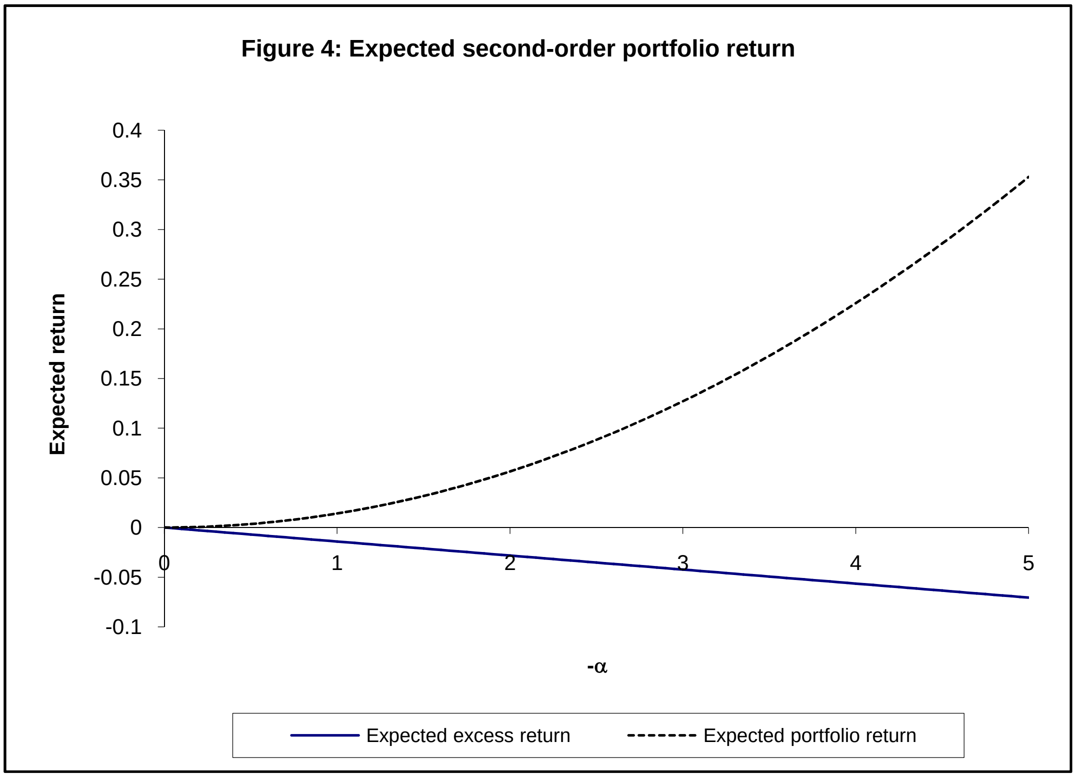


Figure 4: Expected second-order portfolio return
Expected return
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
1
2
3
4
5
-α
Expected excess return
Expected portfolio return

Figure 5: Impulse responses

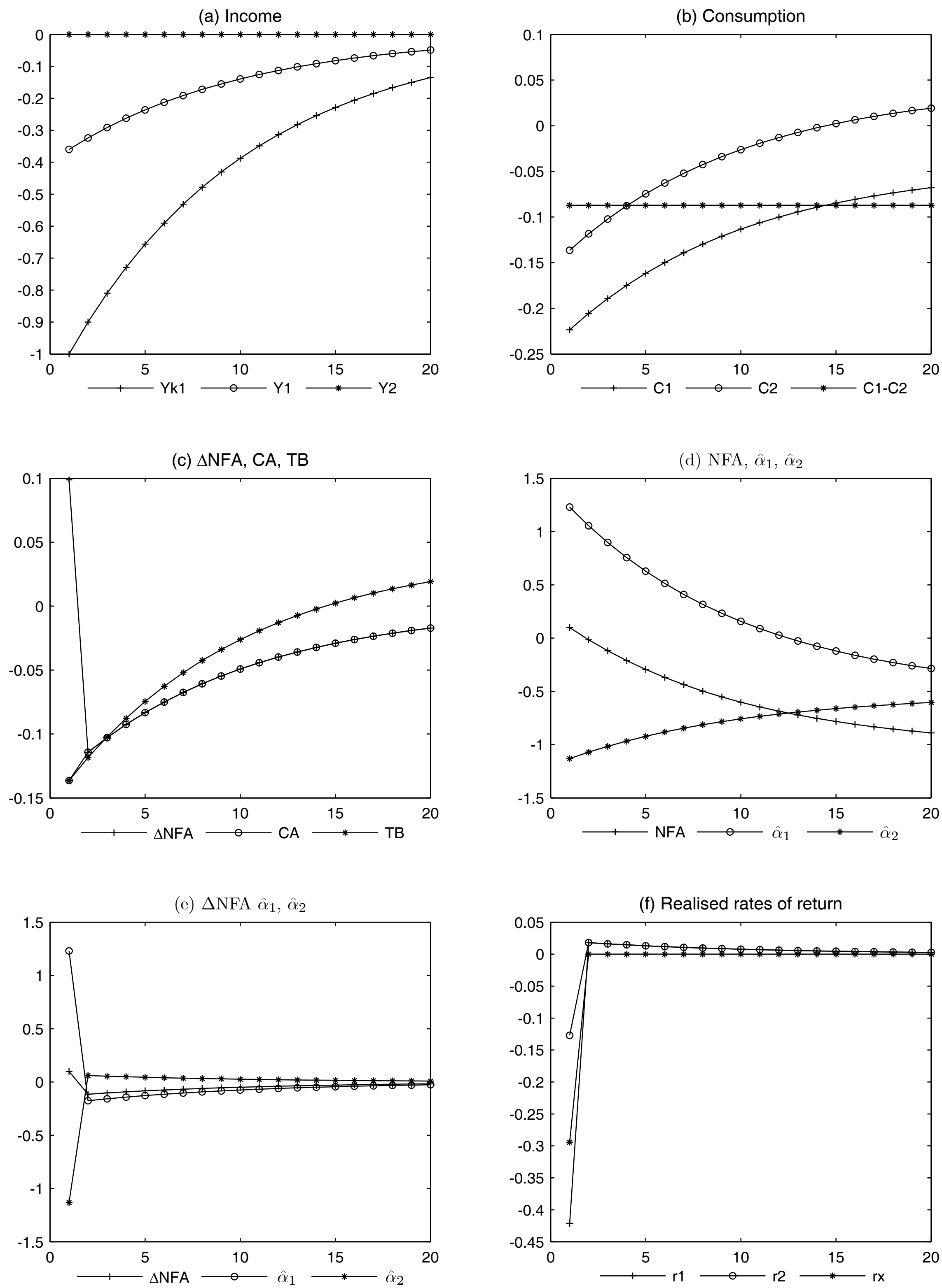


Figure 5 continued