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# Introducing Financial Frictions and Unemployment into a Small Open Economy Model

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# Introducing Financial Frictions and Unemployment into a Small Open Economy Model\*

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## Abstract

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? In order to address this question we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b) in three important dimensions. First, we introduce the feature that exports are produced by using imported goods in addition to domestically produced goods. Second, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). Third, we include the search and matching framework of Mortensen and Pissarides (1994), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007) into a small open economy model. As a first step, we calibrate the model and analyze the effects of a monetary tightening. It turns out that the introduction of financial and labor market frictions allow for additional interesting insights about the effects of monetary policy.

*Keywords:* small open economy, DSGE, financial frictions, unemployment.

*JEL codes:* E0, E3, F0, F4, G0, G1, J6.

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# 1. Introduction

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? In order to address this question we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b) in three important dimensions.

First, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce ‘entrepreneurs’. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, ‘banks’, can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are ‘bankrupt’. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

Second, we include the search and matching framework of Mortensen and Pissarides (1994), Hall (2005a,b,c), Shimer (2005a,b), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007) into the small open economy model. We integrate this into our specific framework - which includes capital and monetary factors - following the version of the Gertler, Sala and Trigari (2006) (henceforth GST). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an

employer in recruiting new employees. In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employed, it has unemployment and vacancies.

The labor market in our model is a slightly modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model.

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services. The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by ‘employment agencies’.

Each employment agency retains a large number of workers. At the beginning of the period a fraction of workers is randomly selected to separate from the firm and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every  $N$  periods. Each employment agency is permanently allocated to one of  $N$  different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort,  $1/N$  of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same indexing rule used in our baseline model. The intensity of labor effort is determined by equating the worker’s marginal cost to the agency’s marginal benefit.

Furthermore, the employment agency in the  $i^{th}$  cohort determines how many employees it will have in period  $t + 1$  by choosing vacancies subject to quadratic vacancy posting costs.

Third, we also introduce the feature that exports are produced by using imported goods in addition to domestically produced goods in our baseline model as this appears to be an important feature of the data.

As a first step, we calibrate the model to Swedish data and analyze the effects of a monetary tightening in our small open economy. It turns out that the introduction of financial and labor market frictions allow for additional interesting insights about the effects of monetary policy.

As a next step, we are currently estimating our small open economy model which includes financial and labor market frictions jointly by using Bayesian techniques.

The paper is organized as follows. In section 2 we describe the baseline small open economy model. Section 3 introduces financial frictions while section 4 incorporates search and matching frictions to the model. Section 5 contains an quantitative analysis of our model. Finally, section 6 summarizes the paper.

## 2. The Baseline Small Open Economy Model

This manuscript describes an extension of the model presented in Adolfson, Laséen, Lindé and Villani (2005) (henceforth ALLV) as well as Adolfson, Laséen, Lindé and Villani (2007a, 2007b), and presents a way to introduce financial frictions and search and matching in the labor market. Our baseline model makes some changes on the ALLV model:

- Exports are produced by using homogenous imported goods in addition to homogenous domestically produced goods.
- The price of investment goods is treated as a random variable with a unit root. Thus, growth in the model is driven by two independent unit root processes, one for neutral technology shocks and the other for technology shocks in the production of investment goods.
- Capital maintenance costs are deducted from capital income taxes, and physical depreciation is deducted at historic cost.
- The capital income tax rate is realized at the time the investment decision is made, not at the time when the payoff on investment is realized.
- Wages are indexed to the steady state growth rate of the economy, rather than to the current realization of technology shocks.
- All producers of specialized goods are assumed to require working capital loans.

### 2.1. Firms

A homogeneous domestic good,  $Y_t$ , is produced using

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right]^{\lambda_{d,t}}, \quad 1 \leq \lambda_{d,t} < \infty. \quad (2.1)$$

The domestic good is produced by a competitive, representative firm which takes the price of output,  $P_t$ , and the price of inputs,  $P_{i,t}$ , as given.

The  $i^{th}$  intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^\alpha - z_t^+ \phi,$$

where  $K_{i,t}$  denotes the labor services rented by the  $i^{th}$  intermediate good producer. Firms must borrow a fraction of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu_t^f R_t + 1 - \nu_t^f, \quad (2.2)$$

where  $W_t$  is the aggregate wage rate,  $R_t$  is the interest rate on working capital loans, and  $\nu_t^f$  corresponds to the fraction that must be financed in advance.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by  $mc_t$  :

$$\begin{aligned} mc_t &= \frac{\left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^\alpha (r_t^k P_t)^\alpha \left(W_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}}{z_t^{1-\alpha} P_t} \\ &= \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^\alpha (\bar{r}_t^k)^\alpha \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t} \end{aligned} \quad (2.3)$$

where  $r_t^k$  is the nominal rental rate of capital scaled by  $P_t$ . See appendix A on how variables are scaled in the model. Productive efficiency dictates that another expression for marginal cost must also be satisfied:

$$\begin{aligned} mc_t &= \frac{1}{P_t} \frac{W_t R_t^f}{MP_{i,t}} \\ &= \frac{1}{P_t} \frac{W_t R_t^f}{(1-\alpha) z_t^{1-\alpha} (k_{i,t} z_{t-1}^+ \Psi_{t-1} / H_{i,t})^\alpha} \\ &= \frac{(\mu_{\Psi,t})^\alpha \bar{w}_t R_t^f}{(1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t}} / H_{i,t}\right)^\alpha} \end{aligned} \quad (2.4)$$

The  $i^{th}$  firm is a monopolist in the production of the  $i^{th}$  good and so it sets its price. Price setting is subject Calvo frictions. With probability  $\xi_d$  the intermediate good firm cannot change its price, in which case,

$$P_{i,t} = (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d} P_{i,t-1},$$

where  $\kappa_d$  is a parameter,  $\pi_{t-1}$  is the lagged inflation rate and  $\bar{\pi}_t^c$  is the central bank's target inflation rate. With probability  $1 - \xi_d$  the firm can change its price. When we combine the

optimization conditions of the  $1 - \xi_d$  intermediate good firms which can optimize their price with the usual cross-firm consistency condition on price, we obtain (after linearizing about steady state):

$$\begin{aligned} \hat{\pi}_t - \widehat{\pi}_t^c &= \frac{\beta}{1 + \kappa_d \beta} E_t (\hat{\pi}_{t+1} - \widehat{\pi}_{t+1}^c) + \frac{\kappa_d}{1 + \kappa_d \beta} (\hat{\pi}_{t-1} - \widehat{\pi}_t^c) \\ &\quad - \frac{\kappa_d \beta (1 - \rho_\pi) \widehat{\pi}_t^c}{1 + \kappa_d \beta} \\ &\quad + \frac{1}{1 + \kappa_d \beta} \frac{(1 - \beta \xi_d)(1 - \xi_d)}{\xi_d} (\widehat{m}c_t + \hat{\lambda}_{d,t}). \end{aligned} \quad (2.5)$$

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + \frac{1}{\Psi_t} [I_t^d + a(u_t) \bar{K}_t] + \int_0^1 X_{i,t}^d. \quad (2.6)$$

Here,  $C_t^d$  denotes intermediate goods used (together with foreign consumption goods) to produce final household consumption goods. The term in square brackets corresponds to domestic investment expenditures. These are allocated to two activities. One,  $I_t^d$ , is used in combination with imported foreign investment goods to produce a final investment good which can be used to add to the physical stock of capital,  $\bar{K}_t$ . The other is  $a(u_t) \bar{K}_t$  which is used for maintenance costs arising from the utilization of physical capital. Here,  $u_t$  denotes the utilization rate of capital, with capital services being defined by:

$$K_t = u_t \bar{K}_t.$$

We adopt the following functional form for  $a$  :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b(1 - \sigma_a)u + \sigma_b((\sigma_a/2) - 1), \quad (2.7)$$

where  $\sigma_a$  and  $\sigma_b$  are the parameters of this function. Finally, the integral in (2.6) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

## 2.2. Exports and Imports

This section reviews the structure of imports and exports. Both activities involve Calvo price setting frictions, and so require the presence of market power. In each case, we follow the Dixit-Stiglitz strategy of introducing a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export and import sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which converts a homogeneous domestically produced good and a

homogeneous good derived from imports into specialized exports. The exports are sold to foreign, competitive retailers which create a homogenous export good that is sold to foreign citizens.

In the case of imports, specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. See figure 1 for a graphical illustration.

We emphasize two features of this setup. First, before passing to final domestic users, imported goods must first be combined with domestic inputs. This is consistent with the view emphasized by Burstein, Eichenbaum and Rebelo (2005, 2007), that there are substantial distribution costs associated with imports. Second, the pricing behavior of firms is described by local currency pricing ('pricing to market') as opposed to producer currency pricing. Hence, the model features limited nominal exchange rate pass-through.

### 2.2.1. Exports

We assume there is a total demand by foreigners for domestic goods, which takes on the following form:

$$X_t = \left( \frac{P_t^x}{P_t^*} \right)^{-\eta_f} Y_t^*.$$

Here,  $Y_t^*$  is foreign GDP and  $P_t^*$  is the foreign currency price of foreign homogeneous goods. Also,  $P_t^x$  is an index of export prices, whose determination is discussed below. The goods,  $X_t$ , are produced by a representative, competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[ \int_0^1 X_{i,t}^{\frac{1}{\lambda_{x,t}}} di \right]^{\lambda_{x,t}}. \quad (2.8)$$

Here,  $X_{i,t}$ ,  $i \in (0, 1)$ , are exports of specialized goods. The retailer that produces  $X_t$  takes its output price,  $P_t^x$ , and its input prices,  $P_{i,t}^x$ , as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left( \frac{P_{i,t}^x}{P_t^x} \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} X_t. \quad (2.9)$$

Combining (2.8) and (2.9), we obtain:

$$P_t^x = \left[ \int_0^1 (P_{i,t}^x)^{\frac{1}{1-\lambda_{x,t}}} di \right]^{1-\lambda_{x,t}}.$$

The  $i^{th}$  specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[ \omega_x^{\frac{1}{\eta_x}} (X_{i,t}^m)^{\frac{\eta_x-1}{\eta_x}} + (1 - \omega_x)^{\frac{1}{\eta_x}} (X_{i,t}^d)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x-1}},$$

where  $X_{i,t}^m$  and  $X_{i,t}^d$  are the  $i^{th}$  exporter's use of the imported and domestically produced goods, respectively. We derive the marginal cost associated with the CES production function from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d + \lambda \left\{ X_{i,t} - \left[ \omega_x^{\frac{1}{\eta_x}} (X_{i,t}^m)^{\frac{\eta_x-1}{\eta_x}} + (1 - \omega_x)^{\frac{1}{\eta_x}} (X_{i,t}^d)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x-1}} \right\},$$

where  $P_t^{m,x}$  is the price of the homogeneous import good and  $P_t$  is the price of the homogenous domestic output good. Nominal marginal cost is  $\lambda$ , so that real (in terms of the homogeneous final export good) marginal cost,  $mC_t^x$ , is

$$mC_t^x = \frac{\lambda}{S_t P_t^x} = \frac{R_t^x}{S_t P_t^x} \left[ \omega_x (P_t^{m,x})^{1-\eta_x} + (1 - \omega_x) (P_t)^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}},$$

where

$$R_t^x = \nu_t^x R_t + 1 - \nu_t^x. \quad (2.10)$$

The  $i^{th}$ ,  $i \in (0, 1)$ , domestic exporting firm takes (2.9) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. In a given period,  $1 - \xi_x$  producers can reoptimize their price and  $\xi_x$  cannot. The firms that cannot optimize price, do so as follows:

$$P_{i,t}^x = (\pi_{t-1}^x)^{\kappa_x} (\pi^x)^{1-\kappa_x} P_{i,t-1}^x.$$

This leads to the following Phillips curve for export prices:

$$\begin{aligned} \hat{\pi}_t^x &= \frac{\beta}{1 + \kappa_x \beta} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1 + \kappa_x \beta} \hat{\pi}_{t-1}^x \\ &+ \frac{1}{1 + \kappa_x \beta} \frac{(1 - \beta \xi_x)(1 - \xi_x)}{\xi_x} \left( \widehat{mC}_t^x + \hat{\lambda}_{x,t} \right). \end{aligned} \quad (2.11)$$

The domestic resources used by specialized exporters are equal to:

$$\int_0^1 X_{i,t}^d di,$$

which after some tedious algebra can be expressed in terms of aggregate variables in the following way:

$$\begin{aligned}
\int_0^1 X_{i,t}^d di &= (mc_t^x q_t p_t^c p_t^x)^{\eta_x} (1 - \omega_x) X_t \left( \frac{\bar{P}_t}{P_t^x} \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \\
&= (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) \left( \frac{\bar{P}_t}{P_t^x} \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^*
\end{aligned} \tag{2.12}$$

with

$$\bar{P}_t = \left[ \int_0^1 (P_{i,t}^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di \right]^{\frac{\lambda_{x,t}-1}{\lambda_{x,t}}}.$$

$\bar{P}_t/P_t^x$  denotes the ratio of two linear homogeneous functions of  $P_{i,t}^x$ , where each weights  $P_{i,t}^x$  for different  $i \in (0, 1)$  in different ways. As argued in a similar context by Yun (1999),  $\bar{P}_t/P_t^x$  can be replaced by unity when studying the first order properties of this model about its steady state. This is guaranteed by our assumption about the form of the price updating equation, which implies that there are no price distortions in steady state, that is, that  $\bar{P}_t/P_t^x = 1$  in steady state.

### 2.2.2. Imports

We now turn to a discussion of imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input and supply that input monopolistically to domestic retailers. Importers are subject to Calvo price setting frictions. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment goods, and (iii) one produces an intermediate good used for the production of an input into the production of export goods.

Consider (i) first. The production function is:

$$C_t^m = \left[ \int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_t^{m,C}}} di \right]^{\lambda_t^{m,C}},$$

where  $C_{i,t}^m$  is the output of the specialized producer and  $C_t^m$  is an intermediate good used in the production of consumption goods. Let  $P_t^{m,c}$  denote the price index of  $C_t^m$  and let  $P_{i,t}^{m,c}$  denote the price of the  $i^{th}$  intermediate input. The marginal cost, in domestic currency units, of the firm that produces  $C_{i,t}^m$  is

$$S_t P_t^* R_t^{\nu,*}, \tag{2.13}$$

where

$$R_t^{\nu,*} = \nu_t^* R_t^* + 1 - \nu_t^*, \tag{2.14}$$

and  $R_t^*$  is the foreign nominal, intratemporal rate of interest. The notion here is that the firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce  $C_{i,t}^m$ . There is no risk to this firm, because all shocks are realized at the beginning of the period, and so there is no uncertainty within the duration of the working capital loan about the realization of prices and exchange rates.

Now consider (ii). The production function for,  $I_t^m$ , the intermediate good used in the production of investment goods is:

$$I_t^m = \left[ \int_0^1 (I_{i,t}^m)^{\frac{1}{\lambda_t^{m,I}}} di \right]^{\lambda_t^{m,I}},$$

where  $I_{i,t}^m$  is the output of the specialized producer. The marginal cost of the specialized producer is also (2.13). Note that we implicitly assume the importing firm's cost is  $P_t^*$  (before borrowing costs and exchange rate conversion), which is the same cost for the specialized inputs used to produce  $C_t^m$ . This may seem inconsistent with the property of the domestic economy that domestically produced consumption and investment goods have different relative prices. We assume that (2.13) applies to both types of producer in order to simplify notation. Below, we suppose that the efficiency of imported investment goods grows over time, in a way that makes our assumptions about the relative costs of consumption and investment, whether imported or domestically produced.

Now consider (iii). The production function for,  $X_t^m$ , the intermediate good used in the production of exports goods is:

$$X_t^m = \left[ \int_0^1 (X_{i,t}^m)^{\frac{1}{\lambda_t^{m,X}}} di \right]^{\lambda_t^{m,X}}.$$

This importer is competitive, and takes the prices of  $X_t^m$  and  $X_{i,t}^m$  as given. This importer's marginal cost is (2.13).

Each of the above three types of intermediate good firm is subject to Calvo price-setting frictions. With probability  $1 - \xi_{m,j}$ , the  $j^{th}$  type of firm can reoptimize its price and with probability  $\xi_{m,j}$  it sets price according to the following relation:

$$P_{i,t}^{m,j} = (\pi_{t-1}^{m,j})^{\kappa_{m,j}} (\widehat{\pi}_t^c)^{1-\kappa_{m,j}} P_{i,t-1}^{m,j},$$

for  $j = c, i, x$ .

The usual Phillips curve argument applies to each of the above producers, so that,

$$\begin{aligned} \widehat{\pi}_t^{m,j} - \widehat{\pi}_t^c &= \frac{\beta}{1 + \kappa_{m,j}\beta} E_t (\widehat{\pi}_{t+1}^{m,j} - \widehat{\pi}_{t+1}^c) + \frac{\kappa_{m,j}}{1 + \kappa_{m,j}\beta} (\widehat{\pi}_{t-1}^{m,j} - \widehat{\pi}_t^c) \\ &\quad - \frac{\kappa_{m,j}\beta(1 - \rho_\pi)}{1 + \kappa_{m,j}\beta} \widehat{\pi}_t^c \\ &\quad + \frac{1}{1 + \kappa_{m,j}\beta} \frac{(1 - \beta\xi_{m,j})(1 - \xi_{m,j})}{\xi_{m,j}} (\widehat{m}c_t^{m,j} + \widehat{\lambda}_t^{m,j}), \end{aligned} \quad (2.15)$$

for  $j = c, i, x$ . Real marginal cost is

$$mc_t^{m,j} = \frac{S_t P_t^*}{P_t^{m,j}} R_t^{\nu,*} \text{ for } j = c, i, x.$$

### 2.3. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[ \zeta_t^c \ln(C_t - bC_{t-1}) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1 + \sigma_L} \right], \quad (2.16)$$

where

$$C_t = \left[ (1 - \omega_c)^{\frac{1}{\eta_c}} (C_t^d)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} (C_t^m)^{\frac{(\eta_c-1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}. \quad (2.17)$$

The price of  $C_t$  is

$$P_t^c = \left[ (1 - \omega_c) (P_t)^{1-\eta_c} + \omega_c (P_t^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}.$$

The rate of inflation of the consumption good is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[ \frac{(1 - \omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}}{(1 - \omega_c) + \omega_c (p_{t-1}^{m,c})^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \quad (2.18)$$

Households do the economy's investment, using the following technology:

$$I_t = \left[ (1 - \omega_i)^{\frac{1}{\eta_i}} (I_t^d)^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} (\Psi_t I_t^m)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}.$$

To obtain the demand for the two inputs we use the fact that this technology is operated by a representative, competitive firm which takes the output price,  $P_t^i$ , and the prices of  $I_t^d$  and  $I_t^m$ ,  $(P_t/\Psi_t)$  and  $P_t^{m,i}$  respectively, as given. Profit maximization results in the following expression:

$$P_t^i = (\Psi_t)^{-1} \left[ (1 - \omega_i) \left( \frac{1}{P_t} \right)^{\eta_i-1} + \omega_i \left( \frac{1}{P_t^{m,i}} \right)^{(\eta_i-1)} \right]^{-\frac{1}{\eta_i-1}}.$$

Then, the inflation in the price of investment goods is:

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[ \frac{(1 - \omega_i) + \omega_i (p_t^{m,i})^{1-\eta_i}}{(1 - \omega_i) + \omega_i (p_{t-1}^{m,i})^{1-\eta_i}} \right]^{\frac{1}{1-\eta_i}}. \quad (2.19)$$

The law of motion of the physical stock of capital is:

$$\bar{K}_{t+1} = (1 - \delta) \bar{K}_t + \Upsilon_t F(I_t, I_{t-1}),$$

where

$$F(I_t, I_{t-1}) = \left( 1 - \tilde{S} \left( \frac{I_t}{I_{t-1}} \right) \right) I_t,$$

and  $\tilde{S}$  in scaled form is given by:

$$\begin{aligned} \tilde{S}(x) &= \frac{1}{2} \left\{ \exp \left[ \sqrt{\tilde{S}''} (x - \mu_{z^+} \mu_{\Psi}) \right] + \exp \left[ -\sqrt{\tilde{S}''} (x - \mu_{z^+} \mu_{\Psi}) \right] - 2 \right\} \\ &= 0, \quad x = \mu_{z^+} \mu_{\Psi}. \end{aligned}$$

The household's first order conditions are as follows. The first order condition for consumption in scaled form is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1} \frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1} \mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c (1 + \tau_t^c) = 0. \quad (2.20)$$

To define the intertemporal Euler equation associated with the household's capital accumulation decision, we need to define the rate of return on a period  $t$  investment in a unit of physical capital,  $R_{t+1}^k$ :

$$R_{t+1}^k = \frac{(1 - \tau_t^k) \left[ u_{t+1} r_{t+1}^k - \frac{1}{\Psi_{t+1}} a(u_{t+1}) \right] P_{t+1} + (1 - \delta) P_{t+1} P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}} \quad (2.21)$$

Here,  $P_{k',t}$  denotes the price of a unit of newly installed physical capital, which operates in period  $t + 1$ . This price is expressed in units of the homogeneous good, so that  $P_t P_{k',t}$  is the domestic currency price of physical capital. The numerator in the expression for  $R_{t+1}^k$  represents the period  $t + 1$  payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets in (2.21) captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. Capital is a good hedge against inflation, except for the way depreciation is treated. A rise in inflation effectively raises the tax rate on capital because

of the practice of valuing depreciation at historical cost. The first order condition for capital implies:

$$\psi_{z^+,t} = \beta E_t + \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1} \mu_{z^+,t+1}}.$$

We differentiate the Lagrangian representation of the household's problem as displayed in ALLV with respect to  $I_t$  :

$$-v_t P_t^i + \omega_t \Upsilon_t F_1(I_t, I_{t-1}) + \beta \omega_{t+1} \Upsilon_{t+1} F_2(I_{t+1}, I_t) = 0,$$

where  $v_t$  denotes the multiplier on the household's nominal budget constraint and  $\omega_t$  denotes the multiplier on the capital accumulation technology. In addition, the price of capital is the ratio of these multipliers:

$$P_t P_{k',t} = \frac{\omega_t}{v_t}.$$

Our first order condition for  $I_t$  appears to differ slightly from the first order condition in ALLV but the two actually coincide when we take into account the definition of  $F$ .

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'(u_t).$$

The tax rate on capital income does not enter here because of the deductibility of maintenance costs. The first order condition associated with foreign bond holdings in scaled form is:

$$\begin{aligned} & -\psi_{z^+,t} + \beta E_t \left[ \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1} \pi_{t+1}} (s_{t+1} R_t^* \Phi(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t)) \right. \\ & \left. - \tau_{t+1}^k s_{t+1} \left( R_t^* \Phi(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t) - 1 \right) - \tau_{t+1}^k (s_{t+1} - 1) \right] = 0, \end{aligned} \quad (2.22)$$

where

$$\Phi(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t) = \exp \left( -\tilde{\phi}_a (a_t - \bar{a}) - \tilde{\phi}_s (E_t s_{t+1} s_t - 1) + \tilde{\phi}_t \right). \quad (2.23)$$

This expression is zero in steady state. This reflects that in the model,  $S_t$  is constant in steady state and our assumption that  $\tilde{\phi}_t$  is zero in steady state.

Note that the interest rate on foreign bonds acquired in period  $t$  is:

$$R_t^* \Phi(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t).$$

Recall that  $R_t^*$  is the intratemporal rate of interest. The intertemporal rate of interest on bonds is different from  $R_t^*$  because there is uncertainty between the time that bonds are purchased and the time that they pay off. In the case of intratemporal loans (i.e., the working capital loans) there is no risk because no uncertainty is realized during the duration of the loan.

The Fisher equation in scaled form is:

$$-\psi_{z^+,t} + \beta E_t \left[ \frac{\psi_{z^+,t+1} R_t - \tau_{t+1}^k (R_t - 1)}{\mu_{z^+,t+1} \pi_{t+1}} \right] = 0, \quad (2.24)$$

where  $R_t$  is the state non-contingent return on a domestic bond acquired in period  $t$ , which pays off in period  $t + 1$ . Finally, we consider wage setting. We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t = \left[ \int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty,$$

where  $h_j$  denotes the  $j^{\text{th}}$  household supply of labor services. Households are subject to Calvo wage setting frictions as in Erceg, Henderson and Levin (2000). With probability  $1 - \xi_w$  the  $j^{\text{th}}$  household is able to reoptimize its wage and with probability  $\xi_w$  it sets its wage according to:

$$W_{j,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w)} \mu_{z^+} W_{j,t}. \quad (2.25)$$

If we combine the first order optimality condition of optimizing households with the cross household wage restriction, we obtain the familiar dynamic expression for the scaled wage rate:

$$E_t \left[ \begin{array}{l} \eta_0 \widehat{w}_{t-1} + \eta_1 \widehat{w}_t + \eta_2 \widehat{w}_{t+1} + \eta_3 (\widehat{\pi}_t - \widehat{\pi}_t^c) + \eta_4 (\widehat{\pi}_{t+1} - \rho_{\widehat{\pi}^c} \widehat{\pi}_t^c) \\ + \eta_5 (\widehat{\pi}_{t-1}^c - \widehat{\pi}_t^c) + \eta_6 (\widehat{\pi}_t^c - \rho_{\widehat{\pi}^c} \widehat{\pi}_t^c) \\ + \eta_7 \widehat{\psi}_{z^+,t} + \eta_8 \widehat{H}_t + \eta_9 \widehat{\tau}_t^y + \eta_{10} \widehat{\tau}_t^w + \eta_{11} \widehat{\zeta}_t^h \\ + \eta_{12} \widehat{\mu}_{z^+,t} + \eta_{13} \widehat{\mu}_{z^+,t+1} \end{array} \right] = 0, \quad (2.26)$$

where

$$b_w = \frac{[\lambda_w \sigma_L - (1 - \lambda_w)]}{[(1 - \beta \xi_w)(1 - \xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w \xi_w \\ (\sigma_L \lambda_w - b_w (1 + \beta \xi_w^2)) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ (1 - \lambda_w) \\ -(1 - \lambda_w) \sigma_L \\ -(1 - \lambda_w) \frac{\tau^y}{(1 - \tau^y)} \\ -(1 - \lambda_w) \frac{\tau^w}{(1 + \tau^w)} \\ -(1 - \lambda_w) \\ -b_w \xi_w \\ b_w \beta \xi_w \end{pmatrix}.$$

With one exception, this reduced form expression was obtained from ALLV. The exception stems from the fact that ALLV index the wage to current realized technology growth, while in our specification the wage is indexed to steady state technology growth. Our indexation strategy necessitates adding technology growth to the dynamic wage equation. In doing this, we followed the formula in the technical appendix of Altig, Christiano, Eichenbaum and Lindé (2004), which shows that technology growth appears in the manner indicated when the wage is not indexed to realized technology (and which also shows that technology growth does not appear in the event that there is full indexation). We did not re-derive this dynamic equation because both ALLV and Altig, Christiano, Eichenbaum and Lindé (2004) address essentially the same environment.

## 2.4. Fiscal and Monetary Authorities

We suppose that the central bank pursues the following Taylor rule:

$$\begin{aligned} \widehat{R}_t = & \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left[ \widehat{\pi}_t^c + \frac{r_\pi}{n} E_t \left[ (\widehat{\pi}_{t+1}^c - \widehat{\pi}_{t+1}^c) + \dots + (\widehat{\pi}_{t+n}^c - \widehat{\pi}_{t+n}^c) \right] \right. \\ & \left. + r_y \widehat{y}_{t-1} + r_q \widehat{q}_{t-1} \right] + r_{\Delta\pi} \Delta \widehat{\pi}_t^c + r_{\Delta y} \Delta \widehat{y}_t + \varepsilon_{R,t}. \end{aligned} \quad (2.27)$$

The fiscal authorities have access to lump sum taxes which are used to redistribute the revenues from distortionary taxes,  $\tau^c, \tau^y, \tau^w, \tau^k$ , and raise revenues to cover government consumption,  $g_t$ .

## 2.5. Resource Constraints

The market clearing condition for the homogeneous domestic output good is, using (2.12),

$$\begin{aligned} Y_t = & G_t + C_t^d + \frac{1}{\Psi_t} \left[ I_t^d + a(u_t) \bar{K}_t \right] \\ & + (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) \left( \frac{\bar{P}_t}{P_t^x} \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^*. \end{aligned}$$

Substituting the production function:

$$\begin{aligned} [\vartheta(\cdot) (z_t H_t)^{1-\alpha} \epsilon_t K_t^\alpha - z_t^+ \phi] = & G_t + C_t^d + \frac{1}{\Psi_t} \left[ I_t^d + a(u_t) \bar{K}_t \right] \\ & + (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) \left( \frac{\bar{P}_t}{P_t^x} \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^*. \end{aligned}$$

where  $\vartheta(\cdot)$  is a function of wage and price dispersion. Following the logic of Yun (1996), we ignore these in our analysis of the first order approximation of the model. So, the resource

constraint that we work with is:

$$(z_t H_t)^{1-\alpha} \epsilon_t K_t^\alpha - z_t^+ \phi = G_t + C_t^d + \frac{1}{\Psi_t} [I_t^d + a(u_t) \bar{K}_t] \\ + (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (p_t^x)^{-\eta_x} Y_t^*.$$

where

$$K_t = \bar{K}_t u_t. \quad (2.28)$$

We obtain the current account by combining the resource constraint, the government budget constraint (which says that  $G = \text{taxes} + \text{seignorage}$ ) and the household's budget constraint. According to the resource constraint and the government budget constraint, household accumulation of foreign assets plus acquisition of foreign goods must equal foreign acquisition of domestic output:

$$S_t B_{t+1}^* - R_{t-1}^* \Phi \left( a_{t-1}, \frac{E_{t-1} S_t}{S_{t-2}}, \tilde{\phi}_{t-1} \right) S_t B_t^* + \text{expenses on imports}_t = \text{receipts from exports}_t.$$

The expenses on imports is the amount spent by the three types of domestic importers of specialized import goods. These three types make zero profits, so that the amount that they spend on imports equals the receipts from their sales. Their sales are the sum of sales to households of consumption goods,  $C_t^m$ , sales to businesses of investment goods,  $I_t^m$ , and sales to domestic exporters,  $X_t^m$ :

$$\text{expenses on imports}_t = S_t P_t^* R_t^{\nu,*} (C_t^m + I_t^m + X_t^m).$$

A similar reasoning based on zero profits implies that

$$\text{receipts from exports}_t = S_t P_t^x X_t.$$

We conclude that the current account can be written as follows:

$$S_t B_{t+1}^* - R_{t-1}^* \Phi \left( a_{t-1}, \frac{E_{t-1} S_t}{S_{t-2}}, \tilde{\phi}_{t-1} \right) S_t B_t^* + S_t P_t^* R_t^{\nu,*} (C_t^m + I_t^m + X_t^m) = S_t P_t^x X_t.$$

We define GDP as the sum of final consumption, investment, government consumption and net exports:

$$\text{nominal } GDP_t = P_t^c C_t + P_t^i I_t + P_t G_t + NX_t,$$

where  $NX_t$  denotes net exports and

$$NX_t = S_t P_t^x X_t - S_t P_t^* (C_t^m + I_t^m + X_t^m).$$

Gross exports is  $S_t P_t^x X_t$  and gross imports is  $S_t P_t^* (C_t^m + I_t^m + X_t^m)$ . We model real GDP as nominal GDP divided by  $P_t$ .

In addition to the above equations we also include the restrictions across inflation rates implied by the relative price formulas. In particular, there are restrictions implied by  $p_t^{m,j}/p_{t-1}^{m,j}$ ,  $j = x, c, i$ , and  $p_t^x$  and  $q_t/q_{t-1}$  as indicated in appendix A. Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \quad (2.29)$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \quad (2.30)$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \quad (2.31)$$

$$\frac{p_t^x}{p_{t-1}^x} = \frac{\pi_t^x}{\pi_t^*} \quad (2.32)$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^c}{\pi_t^c}. \quad (2.33)$$

This completes the derivation of the baseline open economy model. We solve for the deterministic steady state as well as for the dynamics using a first order Taylor series expansion.

### 3. Introducing Financial Frictions into the Model

A number of the activities in the model of the previous section require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. In the model of the previous section financing requirements affect the allocations, but not very much. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household, who puts up the finances and later reaps the rewards. When real-world financial frictions are introduced into a model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

Although the literature shows how to introduce financial frictions much more extensively, here we proceed by assuming that only the accumulation and management of capital involves frictions. We will continue to assume that working capital loans are frictionless. At the end of this introduction, we briefly discuss the idea of introducing financial frictions into working

capital loans. Our strategy of introducing frictions in the accumulation and management of capital follows the variant of the Bernanke, Gertler and Gilchrist (1999) (henceforth BGG) model implemented in Christiano, Motto and Rostagno (2003) The discussion here borrows heavily from the derivation in Christiano, Motto and Rostagno (2007) (henceforth CMR).

The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce ‘entrepreneurs’. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, ‘banks’, can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. An alternative arrangement that is more efficient is one in which banks extend entrepreneurs a ‘standard debt contract’, which specifies a loan amount and a given interest payment. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are ‘bankrupt’. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed.

The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneur’s net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneur’s assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

The ultimate source of funds for lending to entrepreneurs is the household. The standard debt contracts extended by banks to entrepreneurs are financed by issuing liabilities to households. Although individual entrepreneurs are risky, banks themselves are not. We suppose that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. Extensions of the model that introduce risk into banking have been developed, but it is not clear that the added complexity is justified.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth

is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

At the level of abstraction of the model, the capital stock includes both housing and business capital. As a result, the entrepreneurs can also be interpreted as households in their capacity of homeowners. An expanded version of the model would pull apart the household and business sectors to study each individually. Another straightforward expansion of the model would apply the model of financial frictions to working capital loans.

With this model, it is typically the practice to compare the net worth of entrepreneurs with a stock market quantity such as the Dow Jones Industrial Average. Whether this is really appropriate is uncertain. A case can be made that the ‘bank loans’ of entrepreneurs in the model correspond well with actual bank loans *plus* actual equity. It is well known that dividend payments on equity are very smooth. Firms work hard to accomplish this. For example, during the US Great Depression some firms were willing to sell their own capital in order to avoid cutting dividends. That this is so is perhaps not surprising. The asymmetric information problems with actual equity are surely as severe as they are for the banks in our model. Under these circumstances one might expect equity holders to demand a payment that is not contingent on the realization of uncertainty within the firm (payments could be contingent upon publicly observed variables). Under this vision, the net worth in the model would correspond not to a measure of the aggregate stock market, but to the ownership stake of the managers and others who exert most direct control over the firm. The ‘bank loans’ in this model would, under this view of things, correspond to the actual loans of firms (i.e., bank loans and other loans such as commercial paper) plus the outstanding equity. While this is perhaps too extreme, these observations highlight that there is substantial uncertainty over exactly what variable should be compared with net worth in the model. It is important to emphasize, however, that whatever the right interpretation is of net worth, the model potentially captures balance sheet problems very nicely.

Finally, we make some remarks on the introduction of financial frictions into working capital loans. It is possible to accomplish this with relatively little modification to the model, by following the strategy described in Fisher (1998). However, with this strategy, the effects of financial frictions are quite modest, because the firms in the model which use working capital do not have assets. As a result, the balance sheet channel does not operate. We conjecture that for financial frictions in working capital to be interesting, the borrowing firms would need to have assets. One way this could be accomplished would be to assume that they use and own capital that is specific to their firm. In this way, fluctuations in the value of that capital induced by changes in asset prices would change their ability to borrow, and hence to produce. This strategy is algebra-intensive because of the fact that these firms also set their prices subject to Calvo frictions.

### 3.1. Modifying the Baseline Model

As we shall see, entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumptions have been made in the model, which guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by an entrepreneur is proportional to his level of net worth. These are enough to guarantee the aggregation result.

#### 3.1.1. The Individual Entrepreneur

At the end of period  $t$  each entrepreneur has a level of net worth,  $N_{t+1}$ . The entrepreneur's net worth,  $N_{t+1}$ , constitutes his state at this time, and nothing else about his history is relevant. We imagine that there are many entrepreneurs for each level of net worth and that for each level of net worth, there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, and is derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with a particular level of net worth,  $N_{t+1}$ . The entrepreneur combines this net worth with a bank loan,  $B_{t+1}$ , to purchase new, installed physical capital,  $\bar{K}_{t+1}$ , from capital producers. The loan the entrepreneur requires for this is:

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \quad (3.1)$$

The entrepreneur is required to pay a gross interest rate,  $Z_{t+1}$ , on the bank loan at the end of period  $t+1$ , if it is feasible to do so. After purchasing capital the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital,  $\bar{K}_{t+1}$ , into  $\bar{K}_{t+1}\omega$ . Here,  $\omega$  is a unit mean, lognormally and independently distributed random variable across entrepreneurs. The variance of  $\log \omega$  is  $\sigma_t^2$ . The  $t$  subscript indicates that  $\sigma_t$  is itself the realization of a random variable. This allows us to consider the effects of an increase in the riskiness of individual entrepreneurs. We denote the cumulative distribution function of  $\omega$  by  $F_t$ .

After observing the period  $t+1$  shocks, the entrepreneur sets the utilization rate,  $u_{t+1}$ , of capital and rents capital out in competitive markets at nominal rental rate,  $P_{t+1}r_{t+1}^k$ . In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate  $u_{t+1}$  requires  $a(u_{t+1})$  of domestically produced investment goods for maintenance expenditures, where  $a$  is defined in (2.7). The entrepreneur then sells

the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic shock  $\omega$  earns a return (after taxes), of  $R_{t+1}^k \omega$ , where  $R_{t+1}^k$  is defined in (2.21). Because the mean of  $\omega$  across entrepreneurs is unity, the average return across all entrepreneurs is  $R_{t+1}^k$ .

After entrepreneurs sell their capital, they settle their bank loans. At this point, the resources available to an entrepreneur who has purchased  $\bar{K}_{t+1}$  units of physical capital in period  $t$  and who experiences an idiosyncratic productivity shock  $\omega$  are  $P_{t+1} P_{k',t+1} R_{t+1}^k \omega \bar{K}_{t+1}$ . There is a cutoff value of  $\omega$ ,  $\bar{\omega}_{t+1}$ , such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \quad (3.2)$$

Entrepreneurs with  $\omega < \bar{\omega}_{t+1}$  are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

to the bank, which is less than  $Z_{t+1} B_{t+1}$ . In this case, the bank monitors the entrepreneur, at cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where  $\mu \geq 0$  is a parameter.

We note briefly that the definition of  $R_{t+1}^k$  lacks some realism because it does not take into account the deductibility of interest payments. With the more realistic treatment of interest, the after tax rate of return on capital would be

$$\begin{aligned} R_{t+1}^k &= \frac{(1 - \tau_t^k) \left[ u_{t+1} r_{t+1}^k - \frac{1}{\Upsilon_{t+1}} a(u_{t+1}) - (Z_{t+1} - 1) \frac{B_{t+1}}{P_t P_{k',t} \bar{K}_{t+1}} \right] P_{t+1}}{P_t P_{k',t}} \\ &\quad + \frac{(1 - \delta) P_{t+1} P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}} \\ &= \frac{(1 - \tau_t^k) \left[ u_{t+1} r_{t+1}^k - \frac{1}{\Upsilon_{t+1}} a(u_{t+1}) - \bar{\omega}_{t+1} R_{t+1}^k + \frac{B_{t+1}}{P_t P_{k',t} \bar{K}_{t+1}} \right] P_{t+1}}{P_t P_{k',t}} \\ &\quad + \frac{(1 - \delta) P_{t+1} P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}}, \end{aligned}$$

by (3.2). With this representation,  $R_t^k$  is a function of features of the loan contract. This will change the choice of optimal contract, discussed below. We plan to explore the implications of this in future work.

Banks obtain the funds loaned in period  $t$  to entrepreneurs by issuing deposits to households at gross nominal rate of interest,  $R_t$ . The subscript on  $R_t$  indicates that the payoff to households in  $t + 1$  is not contingent on the period  $t + 1$  uncertainty. This feature of the relationship between households and banks is simply assumed. There is no risk in household

bank deposits, and the household Euler equation associated with deposits is exactly the same as (2.24).

We suppose that there is competition and free entry among banks, and that banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs.<sup>1</sup> It follows that the bank's cash flow in each state of period  $t + 1$  is zero, for each loan amount.<sup>2</sup> For loans in the amount,  $B_{t+1}$ , the bank receives gross interest,  $Z_{t+1}B_{t+1}$ , from the  $1 - F_t(\bar{\omega}_{t+1})$  entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is:

$$[1 - F_t(\bar{\omega}_{t+1})] Z_{t+1}B_{t+1} + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = R_t B_{t+1},$$

or, after making use of (3.2) and rearranging,

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \varrho_t = \varrho_t - 1 \quad (3.3)$$

where

$$\begin{aligned} G_t(\bar{\omega}_{t+1}) &= \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega). \\ \Gamma_t(\bar{\omega}_{t+1}) &= \bar{\omega}_{t+1} [1 - F_t(\bar{\omega}_{t+1})] + G_t(\bar{\omega}_{t+1}) \\ \varrho_t &= \frac{P_t P_{k',t} \bar{K}_{t+1}}{N_{t+1}}. \end{aligned}$$

The expression,  $\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})$  is the share of revenues earned by entrepreneurs that borrow  $B_{t+1}$ , which goes to banks. Note that  $\Gamma'_t(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1}) > 0$  and  $G'_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} F'_t(\bar{\omega}_{t+1}) > 0$ . It is thus not surprising that the share of entrepreneurial revenues accruing to banks is non-monotone with respect to  $\bar{\omega}_{t+1}$ . BGG argue that the expression on the left of (3.3) has an inverted 'U' shape, achieving a maximum value at  $\bar{\omega}_{t+1} = \omega^*$ , say. The expression is increasing for  $\bar{\omega}_{t+1} < \omega^*$  and decreasing for  $\bar{\omega}_{t+1} > \omega^*$ . Thus, for any given value of  $\varrho_t$  and  $R_{t+1}^k/R_t$ , generically there are either no values of  $\bar{\omega}_{t+1}$  or two that satisfy (3.3). The value of  $\bar{\omega}_{t+1}$  realized in equilibrium must be the one on the left side of the inverted 'U' shape. This is because, according to (3.2), the lower value of  $\bar{\omega}_{t+1}$  corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. As discussed below, the equilibrium contract is one that maximizes entrepreneurial welfare subject to the zero profit condition

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<sup>1</sup>If banks also had access to state contingent securities, then free entry and competition would imply that banks earn zero profits in an ex ante expected sense from the point of view of period  $t$ .

<sup>2</sup>Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state by state zero profit condition quoted in the text.

on banks. This reasoning leads to the conclusion that  $\bar{\omega}_{t+1}$  falls with a period  $t + 1$  shock that drives  $R_{t+1}^k$  up. The fraction of entrepreneurs that experience bankruptcy is  $F_t(\bar{\omega}_{t+1})$ , so it follows that a shock which drives up  $R_{t+1}^k$  has a negative contemporaneous impact on the bankruptcy rate. According to (2.21), shocks that drive  $R_{t+1}^k$  up include anything which raises the value of physical capital and/or the rental rate of capital.

As just noted, we suppose that the equilibrium debt contract maximizes entrepreneurial welfare, subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date  $t$  debt contract specifies a level of debt,  $B_{t+1}$  and a state  $t + 1$ -contingent rate of interest,  $Z_{t+1}$ . We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\begin{aligned} & \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} [R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1}] dF_t(\omega)}{R_t N_{t+1}} \\ &= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} [\omega - \bar{\omega}_{t+1}] dF_t(\omega) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1}}{R_t N_{t+1}} \\ &= E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t, \end{aligned}$$

after making use of (3.1), (3.2) and

$$1 = \int_0^{\infty} \omega dF_t(\omega) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF_t(\omega) + G_t(\bar{\omega}_{t+1}).$$

We can equivalently characterize the contract by a state- $t + 1$  contingent set of values for  $\bar{\omega}_{t+1}$  and a value of  $\varrho_t$ . The equilibrium contract is the one involving  $\bar{\omega}_{t+1}$  and  $\varrho_t$  which maximizes entrepreneurial welfare (relative to  $R_t N_{t+1}$ ), subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t, \{\bar{\omega}_{t+1}\}} E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \varrho_t + \lambda_{t+1} \left( [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 \right) \right\},$$

where  $\lambda_{t+1}$  is the Lagrange multiplier which is defined for each period  $t + 1$  state of nature. The first order conditions for this problem are:

$$\begin{aligned} E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \lambda_{t+1} \left( [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} - 1 \right) \right\} &= 0 \\ -\Gamma'_t(\bar{\omega}_{t+1}) \frac{R_{t+1}^k}{R_t} + \lambda_{t+1} [\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} &= 0 \\ [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 &= 0, \end{aligned}$$

where the absence of  $\lambda_{t+1}$  from the complementary slackness condition reflects that we assume  $\lambda_{t+1} > 0$  in each period  $t+1$  state of nature. Substituting out for  $\lambda_{t+1}$  from the second equation into the first, the first order conditions reduce to:

$$E_t \left\{ \begin{aligned} & [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \\ & + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})} \left( [\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} - 1 \right) \end{aligned} \right\} = 0 \quad (3.4)$$

$$[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)] \frac{R_t^k}{R_{t-1}} \varrho_{t-1} - \varrho_{t-1} + 1 = 0,$$

for  $t = 0, 1, 2, \dots, \infty$ .

Since  $N_{t+1}$  does not appear in the last two equations, we conclude that  $\varrho_t$  and  $\bar{\omega}_{t+1}$  are the same for all entrepreneurs, regardless of their net worth. The results for  $\varrho_t$  implies that

$$\frac{B_{t+1}}{N_{t+1}} = \varrho_t - 1,$$

that an entrepreneur's loan amount is proportional to his net worth. Rewriting (3.1) and (3.2) we see that the rate of interest paid by the entrepreneur is

$$Z_{t+1} = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{N_{t+1}}{P_t P_{k',t} \bar{K}_{t+1}}} = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{1}{\varrho_t}}, \quad (3.5)$$

which is the same for all entrepreneurs, regardless of their net worth.

### 3.1.2. Aggregation Across Entrepreneurs and the Risk Premium

Let  $f(N_{t+1})$  denote the density of entrepreneurs with net worth,  $N_{t+1}$ . Then, aggregate average net worth,  $\bar{N}_{t+1}$ , is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1}) dN_{t+1}.$$

We now derive the law of motion of  $\bar{N}_{t+1}$ . Consider the set of entrepreneurs who in period  $t-1$  had net worth  $N$ . Their net worth after they have settled with the bank in period  $t$  is denoted  $V_t^N$ , where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \quad (3.6)$$

where  $\bar{K}_t^N$  is the amount of physical capital that entrepreneurs with net worth  $N_t$  acquired in period  $t-1$ . Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t) dN_t.$$

Integrating (3.6) over all entrepreneurs,

$$\begin{aligned}
V_t &\equiv \int_{N_t} N_t f(N_t) dN_t = \int_{N_t} f(N_t) \{R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N\} dN_t \\
&= \int_{N_t} f(N_t) \{R_t^k \varrho_{t-1} N_t - \Gamma_{t-1}(\bar{\omega}_t) R_t^k \varrho_{t-1} N_t\} dN_t \\
&= R_t^k \varrho_{t-1} \bar{N}_t - \Gamma_{t-1}(\bar{\omega}_t) R_t^k \varrho_{t-1} \bar{N}_t.
\end{aligned}$$

Because  $\varrho_{t-1}$  is the same for all entrepreneurs, it follows that

$$\varrho_{t-1} = \frac{P_{t-1} P_{k',t-1} \bar{K}_t}{\bar{N}_t}, \quad (3.7)$$

so that

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t$$

Writing this out more fully:

$$\begin{aligned}
V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F_{t-1}(\bar{\omega}_t)] \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\
&= (1 + R_t^k) P_{t-1} P_{k',t-1} \bar{K}_t \\
&\quad - \left\{ [1 - F_{t-1}(\bar{\omega}_t)] \bar{\omega}_t + (1 - \mu) \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) + \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.
\end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal  $R_t(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$ . Substituting:

$$V_t = R_t^k P_{t-1} P_{k',t-1} - \left\{ R_t + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) (1 + R_t^k) P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t).$$

After  $V_t$  is determined, each entrepreneur faces an identical and independent probability  $1 - \gamma_t$  of being selected to exit the economy. With the complementary probability,  $\gamma_t$ , each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is simply  $\gamma_t \bar{V}_t$ . A fraction,  $1 - \gamma_t$ , of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer,  $W_t^e$ . This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the  $W_t^e$  transfers have been made and exits and entry have occurred, is  $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$ , or,

$$\begin{aligned}
\bar{N}_{t+1} &= \gamma_t \left\{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[ R_t + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \right\} \\
&\quad + W_t^e.
\end{aligned} \quad (3.8)$$

The resource constraint in the financial frictions model contains an additional term,  $d_t$ , that reflects monitoring costs. In terms of scaled variables we obtain:

$$y_t = g_t + c_t^d + i_t^d \tag{3.9}$$

$$+ (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) (p_t^x)^{-\eta_f} y_t^* + d_t,$$

where

$$d_t = \frac{\mu G_{t-1}(\bar{\omega}_t) R_t^k p_{k',t-1} \bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

Account has also to be taken of the consumption by exiting entrepreneurs. The net worth of these entrepreneurs is  $(1 - \gamma_t) V_t$  and we assume a fraction,  $1 - \Theta$ , is taxed and transferred in lump-sum form to households, while the complementary fraction,  $\Theta$ , is consumed by the exiting entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta \frac{1 - \gamma_t}{\gamma_t} (n_{t+1} - w_t^e) z_t^+ P_t$$

from the right side of (2.17). In practice we do not make this adjustment because we assume  $\Theta$  is sufficiently small that the adjustment is negligible.

We now turn to the risk premium on entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate,  $R_t$ , which he loses by applying it to capital rather than just depositing it in the bank. The average payment by all entrepreneurs to the bank is the entire object in square brackets in equation (3.8). So, the term involving  $\mu$  represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the risk premium in the model. Another is the excess of the interest rate paid by entrepreneurs who are not bankrupt, over  $R_t$  :

$$Z_{t+1} - R_t = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t} k_{t+1}}} - R_t,$$

according to (3.5).

## 4. Introducing Unemployment into the Model

This section replaces the model of the labor market in our baseline model with the search and matching framework of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005a,b). We integrate the framework into our specific framework - which includes capital and monetary factors - following the version of the Gertler, Sala and Trigari (2006) (henceforth GST) strategy implemented in Christiano, Ilut, Motto, and Rostagno (2007). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting

frictions have an impact on the effort of an employer in recruiting new employees. In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employment, it has unemployment and vacancies.

The labor market in our alternative labor market model is a slightly modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model. In what follows, we first provide an overview and after that we present the detailed decision problems of agents in the labor market.

#### 4.1. Sketch of the Model

As in the discussion of section 2.1, we adopt the Dixit-Stiglitz specification of homogeneous goods production. A representative, competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists, who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions in the baseline model.

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services (see section 2.1). The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by ‘employment agencies’. See figure 2 for a graphical illustration. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected.<sup>3</sup>

Each employment agency retains a large number of workers. At the beginning of the period a fraction,  $\rho$ , of workers is randomly selected to separate from the firm and go into unemployment.<sup>4</sup> Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which

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<sup>3</sup>An alternative (perhaps more natural) formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem.

<sup>4</sup>We thus specify that the job separation rate is constant. This is consistent with the findings reported in Hall (2005b,c) and Shimer (2005a,b), who report that the job separation rate is relatively acyclical.

occurs once every  $N$  periods. Each employment agency is permanently allocated to one of  $N$  different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort,  $1/N$  of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same rule used in our baseline model.

Once a wage rate is determined - whether by Nash bargaining or not - we assume that each matched worker-firm pair finds it optimal to proceed with the relationship in that period. In our calculations, we verify that this assumption is correct, by confirming that the wage rate in each worker-agency relationship lies inside the bargaining set associated with that relationship.

Next, the intensity of labor effort is determined according to a particular efficiency criterion. To explain this, we discuss the implications of increased intensity for the worker and for the employment agency. The utility function of the household in the present labor market model is a modified version of (2.16):

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left\{ \zeta_{t+l}^c \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \frac{\varsigma_{t+l}^{1+\sigma_L}}{1+\sigma_L} L_{t+l} \right\}, \quad (4.1)$$

where  $L_t$  is the fraction of members of the household that are working and  $\varsigma_t$  is the intensity with which each worker works. As in GST, we follow the family household construct of Merz (1995) in supposing that each household has a large number of workers. Although the individual worker's labor market experience - whether employed or unemployed - is determined in part by idiosyncratic shocks, the household has sufficiently many workers that the total fraction of workers employed,  $L_t$ , as well as the fractions allocated among the different cohorts,  $l_t^i$ ,  $i = 0, \dots, N - 1$ , is the same for each household. We suppose that all the household's workers are supplied inelastically to the labor market (i.e., labor force participation is constant). Each worker passes randomly from employment with a particular agency to unemployment and back to employment according to the endogenous probabilities described below.

The household's currency receipts arising from the labor market are:

$$(1 - L_t) P_t^c b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i l_t^i \varsigma_t \quad (4.2)$$

where  $W_t^i$  is the nominal wage rate earned by workers in cohort  $i = 0, \dots, N - 1$ . The index,  $i$ , indicates the number of periods in the past when bargaining occurred most recently. Note that we implicitly assume that labor intensity is the same in each employment agency, regardless of cohort. This is explained below. The presence of the term involving  $b^u$  indicates

the assumption that unemployed workers receive a payment of  $b^u z_t^+$  final consumption goods. The unemployment benefits are financed by lump sum taxes.

Let the price of labor services,  $W_t$ , denote the marginal gain to the employment agency that occurs when an individual worker raises labor intensity by one unit. Because the employment agency is competitive in the supply of labor services,  $W_t$  is taken as given and is the same for all agencies, regardless of which cohort it is in. Labor intensity equates the worker's marginal cost to the agency's marginal benefit:

$$W_t = \zeta_t^h A_L \varsigma_t^{\sigma_L} \frac{1}{v_t}. \quad (4.3)$$

To understand the expression on the right of the equality, note that the marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is  $\zeta_t^h A_L \varsigma_t^{\sigma_L}$ . This is converted to currency units by dividing by the multiplier,  $v_t$ , on the household's nominal budget constraint.

Labor intensity is the same for all cohorts because none of the variables in (4.3) is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (4.3).

Finally, the employment agency in the  $i^{th}$  cohort determines how many employees it will have in period  $t + 1$  by choosing vacancies,  $v_t^i$ . The vacancy posting costs associated with  $v_t^i$  are:

$$\frac{\kappa z_t^+}{2} \left( \frac{Q_t^i v_t^i}{l_t^i} \right)^2 l_t^i,$$

units of the domestic homogeneous good. Here,  $l_t^i$  denotes the number of employees in the  $i^{th}$  cohort and  $\kappa z_t^+/2$  is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate. Also,  $Q_t$  is the probability that a posted vacancy is filled. The functional form of our cost function nests GT and GST when  $\iota = 1$ . With this parameterization the cost function is in terms of the number of people hired, not the number of vacancies per se. We interpret this as reflecting that the GT and GST specifications emphasize internal costs (such as training and other) of adjusting the work force, and not search costs. In models used in the search literature (see, e.g., Shimer (2005a)), vacancy posting costs are independent of  $Q_t$ , i.e.,  $\iota = 0$ . We also plan to investigate this latter case. We suspect that the model implies less amplification in response to expansionary shock in the case,  $\iota = 0$ . In a boom,  $Q_t$  can be expected to fall, so that with  $\iota = 1$ , costs of posting vacancies decrease in the GT specification.

## 4.2. Model Details

An employment agency in the  $i^{\text{th}}$  cohort which does not renegotiate its wage in period  $t$  sets the period  $t$  wage,  $W_{i,t}$ , as in (2.25):

$$W_{i,t} = \tilde{\pi}_{w,t} \mu_{z^+} W_{i-1,t-1}, \quad \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}^c\right)^{\kappa_w} \left(\bar{\pi}_t^c\right)^{(1-\kappa_w)}, \quad (4.4)$$

for  $i = 1, \dots, N - 1$  (note that an agency that was in the  $i^{\text{th}}$  cohort in period  $t$  was in cohort  $i - 1$  in period  $t - 1$ ). After wages are set, employment agencies in cohort  $i$  supply labor services,  $l_t^i \varsigma_t$ , into competitive labor markets. In addition, they post vacancies to attract new workers in the next period.

### 4.2.1. The Employment-Agency Problem

To understand how agencies bargain and how they make their employment decisions, it is useful to consider  $F(l_t^0, \omega_t)$ , the value function of the representative employment agency in the cohort that negotiates its wage in the current period. The arguments of  $F$  are the agency's workforce after beginning-of-period separations and new arrivals,  $l_t^0$ , and an arbitrary value for the nominal wage rate,  $\omega_t$ . We are thus interested in the firm's problem after the wage rate has been set, when vacancy decisions remain to be made. To simplify notation, we leave out arguments of  $F$  that correspond to economy-wide variables. We find it convenient to adopt a change of variables. We suppose that the firm chooses a particular monotone transform of vacancy postings, which we denote by  $\tilde{v}_t^i$ :

$$\tilde{v}_t^i \equiv \frac{Q_t^i v_t^i}{l_t^i}.$$

The agency's hiring rate is related to  $\tilde{v}_t^i$  by:

$$\chi_t^i = Q_t^{1-\iota} \tilde{v}_t^i. \quad (4.5)$$

In this notation, the agency's objective is to solve:

$$F(l_t^0, \omega_t) = \sum_{j=0}^{N-1} \beta^j E_t \frac{v_{t+j}}{v_t} \max_{\tilde{v}_{t+j}^j} \left[ (W_{t+j} - \Gamma_{t,j}(\omega_t)) \varsigma_{t+j} - P_{t+j} \frac{\kappa_{z^+}}{2} (\tilde{v}_t^i)^2 \right] l_{t+j}^j \quad (4.6)$$

$$+ \beta^N E_t \frac{v_{t+N}}{v_t} F(l_{t+N}^0, \tilde{W}_{t+N}),$$

where  $\varsigma_t$  is assumed to satisfy (4.3). Here,

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1} \mu_{z^+}^j, & j > 0 \\ 1 & j = 0 \end{cases}. \quad (4.7)$$

Also,  $\tilde{W}_{t+N}$  denotes the Nash bargaining wage rate that will be negotiated when the agency next has an opportunity to do so. At time  $t$ , the agency takes  $\tilde{W}_{t+N}$  as given. The law of motion of an agency's work force is:

$$l_{t+1}^{i+1} = (\chi_t^i + \rho) l_t^i, \quad (4.8)$$

for  $i = 0, 1, \dots, N-1$ , with the understanding here and throughout that  $i = N$  is to be interpreted as  $i = 0$ . Expression (4.8) is deterministic, reflecting the assumption that the agency employs a large number of workers.

The firm chooses vacancies to solve the problem in (4.6). It is easy to verify:

$$F(l_t^0, \omega_t) = J(\omega_t) l_t^0, \quad (4.9)$$

where  $J(\omega_t)$  is not a function of  $l_t^0$ . The function,  $J(\omega_t)$ , is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is  $\omega_t$ . For convenience, we omit the expectation operator  $E_t$  below. Let

$$\begin{aligned} J(\omega_t) = & \max_{\{v_{t+j}^j\}_{j=0}^{N-1}} \left\{ (W_t - \omega_t) \varsigma_t - P_t z_t^+ \frac{\kappa}{2} (\tilde{v}_t^0)^2 \right. \\ & + \beta \frac{v_{t+1}}{v_t} \left[ (W_{t+1} - \Gamma_{t,1} \omega_t) \varsigma_{t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{2} (\tilde{v}_{t+1}^1)^2 \right] (\tilde{v}_t^0 Q_t^{1-\iota} + \rho) \\ & + \beta^2 \frac{v_{t+2}}{v_t} \left[ (W_{t+2} - \Gamma_{t,2} \omega_t) \varsigma_{t+2} - P_{t+2} z_{t+2}^+ \frac{\kappa}{2} (\tilde{v}_{t+2}^2)^2 \right] (\tilde{v}_t^0 Q_t^{1-\iota} + \rho) (\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho) \\ & + \dots + \\ & \left. + \beta^N \frac{v_{t+N}}{v_t} J(\tilde{W}_{t+N}) (\tilde{v}_t^0 Q_t^{1-\iota} + \rho) (\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho) \cdots (\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho) \right\}. \end{aligned}$$

The first order conditions expressed for a fixed date and different cohorts takes the following form:

$$\begin{aligned} P_t z_t^+ \kappa \tilde{v}_t^j \frac{1}{Q_t^{1-\iota}} = & \beta \frac{v_{t+1}}{v_t} \left[ (W_{t+1} - \Gamma_{t-j,j+1} \tilde{W}_{t-j}) \varsigma_{t+1} + P_{t+1} z_{t+1}^+ \kappa \left( \frac{(\tilde{v}_{t+1}^{j+1})^2}{2} + \frac{\tilde{v}_{t+1}^{j+1} \rho}{Q_{t+1}^{1-\iota}} \right) \right] \\ & \text{for } j = 0, \dots, N-2. \end{aligned}$$

The vacancy first order condition of agencies that are in the last period of their contract is:

$$\begin{aligned} P_t z_t^+ \kappa \tilde{v}_t^j \frac{1}{Q_t^{1-\iota}} = & \beta \frac{v_{t+1}}{v_t} \left[ (W_{t+1} - \tilde{W}_{t+1}) \varsigma_{t+1} + P_{t+1} z_{t+1}^+ \kappa \left( \frac{(\tilde{v}_{t+1}^{j+1})^2}{2} + \frac{\tilde{v}_{t+1}^{j+1} \rho}{Q_{t+1}^{1-\iota}} \right) \right] \\ & \text{for } j = N-1. \end{aligned}$$

We require the derivative of  $J$  with respect to  $\omega_t$ . By the envelope condition, we can ignore the impact of a change in  $\omega_t$  on the vacancy decisions and only be concerned with the

direct impact of  $\omega_t$  on  $J$  :

$$\begin{aligned}
J_{w,t} &= -\varsigma_t \\
&\quad -\beta \frac{v_{t+1}}{v_t} \Gamma_{t,1} \varsigma_{t+1} (\chi_t^0 + \rho) \\
&\quad -\beta^2 \frac{v_{t+2}}{v_t} \Gamma_{t,2} \varsigma_{t+2} (\chi_t^0 + \rho) (\chi_{t+1}^1 + \rho) \\
&\quad - \dots - \\
&\quad -\beta^{N-1} \frac{v_{t+N-1}}{v_t} \Gamma_{t,N-1} \varsigma_{t+N-1} (\chi_t^0 + \rho) (\chi_{t+1}^1 + \rho) \cdots (\chi_{t+1}^{N-2} + \rho).
\end{aligned}$$

The following is an expression for  $J_t$  evaluated at  $\omega_t = \tilde{W}_t$ , in terms of scaled variables. Dividing by  $P_t z_t^+$  :

$$\begin{aligned}
J_{z^+,t} &= \frac{J(\tilde{W}_t)}{P_t z_t^+} = \frac{(W_t - \tilde{W}_t)}{P_t z_t^+} \varsigma_t - \frac{\kappa}{2} (\tilde{v}_t^0)^2 \\
&\quad + \beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[ \frac{W_{t+1} - \Gamma_{t,1} \tilde{W}_t}{P_{t+1} z_{t+1}^+} \varsigma_{t+1} - \frac{\kappa}{2} (\tilde{v}_{t+1}^1)^2 \right] (\chi_t^0 + \rho) \\
&\quad + \beta^2 \frac{\psi_{z^+,t+2}}{\psi_{z^+,t}} \left[ \frac{W_{t+2} - \Gamma_{t,2} \tilde{W}_t}{P_{t+2} z_{t+2}^+} \varsigma_{t+2} - \frac{\kappa}{2} (\tilde{v}_{t+2}^2)^2 \right] (\chi_t^0 + \rho) (\chi_{t+1}^1 + \rho) \\
&\quad + \dots + \\
&\quad + \beta^N \frac{\psi_{z^+,t+N}}{\psi_{z^+,t}} J_{z^+,t+N} (\chi_t^0 + \rho) (\chi_{t+1}^1 + \rho) \cdots (\chi_{t+N-1}^{N-1} + \rho).
\end{aligned}$$

#### 4.2.2. The Worker Problem

We now turn to the worker. The period  $t$  value of being a worker in an agency in cohort  $i$  is  $V_t^i$  :

$$V_t^i = \Gamma_{t-i,i} \tilde{W}_{t-i} \varsigma_t - \zeta_t^h A_L \frac{\varsigma_t^{1+\sigma_L}}{(1+\sigma_L) v_t} + \beta E_t \frac{v_{t+1}}{v_t} [\rho V_{t+1}^{i+1} + (1-\rho) U_{t+1}], \quad (4.10)$$

for  $i = 0, 1, \dots, N-1$ . Here,  $\rho$  is the probability of remaining with the firm in the next period and  $U_t$  is the value of being unemployed in period  $t$ . The values,  $V_t^i$  and  $U_t$ , pertain to the beginning of period  $t$ , after job separation and job finding has occurred. For workers employed by agencies in cohort  $i = 0$ , the value function is  $V^0(\omega_t)$ , where  $\omega_t$  is an arbitrary value for the current period wage rate,

$$V^0(\omega_t) = \omega_t \varsigma_t - \zeta_t^h A_L \frac{\varsigma_t^{1+\sigma_L}}{(1+\sigma_L) v_t} + \beta E_t \frac{v_{t+1}}{v_t} [\rho V_{t+1}^1 + (1-\rho) U_{t+1}]. \quad (4.11)$$

The notation makes the dependence of  $V^0$  on  $\omega_t$  explicit to simplify the discussion of the Nash bargaining problem below. Below, we require the derivative of  $V^0(\omega_t)$  with respect to

$\omega_t$ , evaluated at  $\omega_t = \tilde{W}_t$  :

$$V_w^0(\omega_t) = \sum_{j=0}^{N-1} (\beta\rho)^j E_t \varsigma_{t+j} \Gamma_{t,j} \frac{v_{t+j}}{v_t}.$$

The value of being an unemployed worker is  $U_t$  :

$$U_t = P_t z_t^+ b^u + \beta E_t \frac{v_{t+1}}{v_t} [f_t V_{t+1}^x + (1 - f_t) U_{t+1}], \quad (4.12)$$

where  $f_t$  is the probability that an unemployed worker will land a job in period  $t + 1$ . Also,  $V_t^x$  is the period  $t + 1$  value function of a worker who finds a job, before it is known which agency it is found with:

$$V_t^x = \sum_{i=0}^{N-1} \frac{\chi_{t-1}^i l_{t-1}^i}{m_{t-1}} V_t^{i+1}, \quad (4.13)$$

where the total number of new matches is given by:

$$m_t = \sum_{j=0}^{N-1} \chi_t^j l_t^j. \quad (4.14)$$

In (4.13),

$$\frac{\chi_{t-1}^i l_{t-1}^i}{m_{t-1}}$$

is the probability of finding a job in an agency which was of type  $i$  in the previous period, conditional on being a worker who finds a job in  $t$ . Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m (1 - L_t)^\sigma v_t^{1-\sigma}, \quad (4.15)$$

where

$$L_t = \sum_{j=0}^{N-1} l_t^j. \quad (4.16)$$

Total hours worked is:

$$H_t = \varsigma_t \sum_{j=0}^{N-1} l_t^j. \quad (4.17)$$

The job finding rate is:

$$f_t = \frac{m_t}{1 - L_t}. \quad (4.18)$$

The probability of filling a vacancy is:

$$Q_t = \frac{m_t}{v_t}. \quad (4.19)$$

The  $i = 0$  cohort of agencies in period  $t$  solve the following Nash bargaining problem:

$$\max_{\omega_t} (V^0(\omega_t) - U_t)^\eta J(\omega_t)^{(1-\eta)}, \quad (4.20)$$

where  $V^0(\omega_t) - U_t$  is the match surplus enjoyed by a worker. We denote the wage that solves this problem by  $\tilde{W}_t$ . Note that  $\tilde{W}_t$  takes into account that intensity will be chosen according to (4.3) as well as (4.4). The first order condition associated with this problem follows accordingly.

We assume that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation unchanged, and we alter the resource constraint in scaled terms as follows:

$$y_t = g_t + c_t^d + i_t^d + (R_t^x)^{\eta_x} \left[ \omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) (p_t^x)^{-\eta_f} y_t^* + \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_t^j)^2 l_t^j.$$

Total vacancies  $v_t$  are related to vacancies posted by the individual cohorts as follows:

$$v_t = \frac{1}{Q_t} \sum_{j=0}^{N-1} \tilde{v}_t^j l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (4.19), (4.14) and (4.5).

## 5. Quantitative Analysis

### 5.1. Calibration and Parameterization

We calibrate the model to quarterly Swedish data. For the parameters not listed below, we use the parameter values in Adolfson, Laséen, Lindé and Villani (2007b). In the baseline model, we fix the following parameters:

$$\begin{aligned} \alpha = 0.3, \delta = 0.015, \eta_x = 6, \eta_y = y^*/y = 0.1, \kappa_{mx} = .5, \lambda_{mx} = 1.2, \lambda_x = 1.2, \lambda_d = 1.2, \nu^* = 0.5, \\ \nu_f = 0.5, \nu_x = 0.5, \omega_x = 0.5, \tilde{\phi}_a = .1, r_\pi = 1.5, r_q = .01, \rho_R = 0, \sigma_L = 2, \sigma_a = 1, S''' = 5, \\ \tau_k = 0.01, \xi_{mx} = .75. \end{aligned}$$

For the financial frictions we use the following additional parameters:

$$\gamma = .98, \mu = .1, we = .02, \bar{F}_\omega = 0.03.$$

Finally, for the labor market frictions we set the following parameters:

$$u = .06, N = 4, \rho = .96, \iota = 1, \sigma = .5, \kappa = 5, b_u = .6, \sigma_m = .25.$$

## 5.2. Impulse Responses

Figures 3, 4 and 5 show impulse responses of the baseline model, the financial frictions model as well as the labor market frictions model to a monetary policy shock for the above calibration. Similar to ALLV, in the baseline model, real quantities such as GDP, hours worked, investment, consumption, exports as well as inflation display a hump-shaped decrease in response to a contractionary monetary policy shock. In terms of prices, the real exchange rate and the price of capital show maximum downward reactions at impact before monotonically returning to the steady state.

In the financial frictions extension, the real quantities also display a hump-shaped response. Interestingly, the amplitude of investment in response to a monetary contraction has increased somewhat. The introduction of financial frictions allows us to study the implications of monetary policy on additional variables in an open economy model. Net worth of entrepreneurs falls in response to a monetary tightening due to two effects. First, the price of capital falls which implies a fall in the asset values of entrepreneurs. Second, on the liability side, the real value of outstanding debt increases as inflation drops. Since net worth has declined more than the value of the capital stock, a higher level of idiosyncratic technology is required for an entrepreneur to be able to survive in the economy. In other words, contractionary monetary policy leads to an increase in the bankruptcy rate in this economy. Finally, in general equilibrium, higher bankruptcy rates induce banks to increase the finance premium for loans to entrepreneurs.

Finally, in the search and matching frictions model real quantities such as GDP and total hours worked also show a hump-shaped pattern. Interestingly, inflation does not anymore. We suspect that the introduction of labor market frictions in conjunction with the current (preliminary) calibration (e.g. average Taylor wage contract length of one year) cause this result. After estimating the deep parameters of the model, especially those related to the labor market, we will be able to address this issue more thoroughly.

The search and matching framework allows for a decomposition of total hours into an intensive and an extensive margin. Interestingly, in the short-run variation of total hours worked is entirely driven by variations in the intensive margin, i.e. a fall in the hours per employee. In the medium-run, though, the extensive margin, i.e. the number of employees, becomes the more important determinant of total hours in the economy.

Given the current calibration, these examples show that the introduction of financial and labor market frictions allow for additional interesting insights about the effects of monetary

policy in an open economy.

## 6. Conclusion

This paper describes several important extensions of the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b). First, we introduce the feature that exports are produced by using imported goods in addition to domestically produced goods. Second, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). Third, we include the search and matching framework of Mortensen and Pissarides (1994), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007).

Given the current calibration, we have examined the effects of a monetary tightening in our open economy model. It turns out that the introduction of financial and labor market frictions allow for additional interesting insights about the effects of monetary policy in a small open economy.

We are working on a combined model with both financial and labor market frictions. In addition, we are also currently estimating the models using Bayesian techniques.

## References

- [1] Adolfson, Malin, Stefan Laséen, Jesper Linde and Mattias Villani (2005), “The Role of Sticky Prices in An Estimated Open Economy DSGE Model: A Bayesian Investigation”, *Journal of the European Economic Association Papers and Proceedings* Vol 3(2-3), 444-457.
- [2] Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2007a), “Evaluating An Estimated New Keynesian Small Open Economy Model”, *Journal of Economic Dynamics and Control*, forthcoming.
- [3] Adolfson, Malin, Stefan Laseén, Jesper Lindé and Mattias Villani (2007b), “Bayesian Estimation of Open Economy DSGE Model with Incomplete Pass-Through”, *Journal of International Economics* Vol 72, 481-511.
- [4] Altig, David, Christiano, Lawrence J, Martin Eichenbaum and Jesper Linde (2004), “Firm-Specific Capital, Nominal Rigidities and the Business Cycle”, manuscript, Northwestern University.
- [5] Barro, Robert (1977), “Long-Term Contracting, Sticky Prices and Monetary Policy,” *Journal of Monetary Economics* Vol 3(3), 305-316.
- [6] Bernanke, Ben, Mark Gertler and Simon Gilchrist. (1999), “The Financial Accelerator in a Quantitative Business Cycle Framework,” *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, 1341-1393. Amsterdam, New York and Oxford: Elsevier Science, North-Holland.
- [7] Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2005), “Large Devaluations and the Real Exchange Rate”, *Journal of Political Economy*, Vol 113, 742–784 .
- [8] Burstein, Ariel, Martin Eichenbaum and Sergio Rebelo (2007), “Modeling Exchange Rate Passthrough After Large Devaluations”, *Journal of Monetary Economics*, Vol 54(2), 346-368.
- [9] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2003). “The Great Depression and the Friedman-Schwartz Hypothesis,” *Journal of Money, Credit, and Banking*, Vol 35, 1119-1198.
- [10] Christiano, Lawrence J., Roberto Motto and Massimo Rostagno (2007), “Shocks, Structures or Monetary Policies? The Euro Area and US After 2001,” manuscript, European Central Bank.

- [11] Christiano, Lawrence J., Cosmin Ilut, Roberto Motto and Massimo Rostagno (2007), “Monetary Policy and Stock Market Boom-Bust Cycles”, manuscript, Northwestern University.
- [12] Erceg, Christopher J., Henderson, Dale, W., and Andrew T. Levin (2000), “Optimal Monetary Policy with Staggered Wage and Price Contracts,” *Journal of Monetary Economics*, Vol 46, 281-313.
- [13] Fisher, Irving (1933), “The Debt-Deflation Theory of Great Depressions,” *Econometrica* Vol 1, 337-357.
- [14] Fisher, Jonas (1998), “Credit market imperfections and the heterogeneous response of firms to monetary shocks”, Federal Reserve Bank of Chicago, Working Paper Series, 96-23, 1998.
- [15] Hall, Robert (2005a), “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, Vol 95(1), 50-65.
- [16] Hall, Robert (2005b), “Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market,” *Review of Economics and Statistics*, Vol 87(3), 397-407.
- [17] Hall, Robert (2005c), “Job Loss, Job Finding, and Unemployment in the U.S.Economy over the Past Fifty Years,” *NBER Macroeconomics Annual*, M.Gertler and K. Rogoff editors, MIT Press, 101-137.
- [18] Gertler, Mark, Luca Sala and Antonella Trigari (2006), “An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining,” manuscript, New York University.
- [19] Gertler, Mark and Antonella Trigari (2006), “Unemployment Fluctuations with Staggered Nash Bargaining,” manuscript, New York University.
- [20] Levin, A., Lopez-Salido, J.D. (2004), “Optimal Monetary Policy with Endogenous Capital Accumulation”, manuscript, Federal Reserve Board.
- [21] Levin, A., Onatski, A., Williams, J., Williams, N. (2005), “Monetary Policy under Uncertainty in Microfounded Macroeconometric Models,” *NBER Macroeconomics Annual 2005*, Gertler, M., Rogoff, K., eds. Cambridge, MA: MIT Press.
- [22] Merz, M. (1995), “Search In The Labor Market And The Real Business Cycle”, *Journal of Monetary Economics*, Vol. 36, 269–300.

- [23] Mortensen, Dale and Christopher Pissarides (1994), “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, Vol 61, 397-415.
- [24] Shimer, Robert (2005a), “The Cyclical Behavior of Equilibrium Unemployment Vacancies, and Wages: Evidence and Theory,” *American Economic Review*, Vol 95(1), 25-49.
- [25] Shimer, Robert (2005b), “Reassessing the Ins and Outs of Unemployment,” manuscript, University of Chicago.
- [26] Yun, Tack (1996), “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles,” *Journal of Monetary Economics*, Vol 37(2), 345-370.

## 7. Tables and Figures

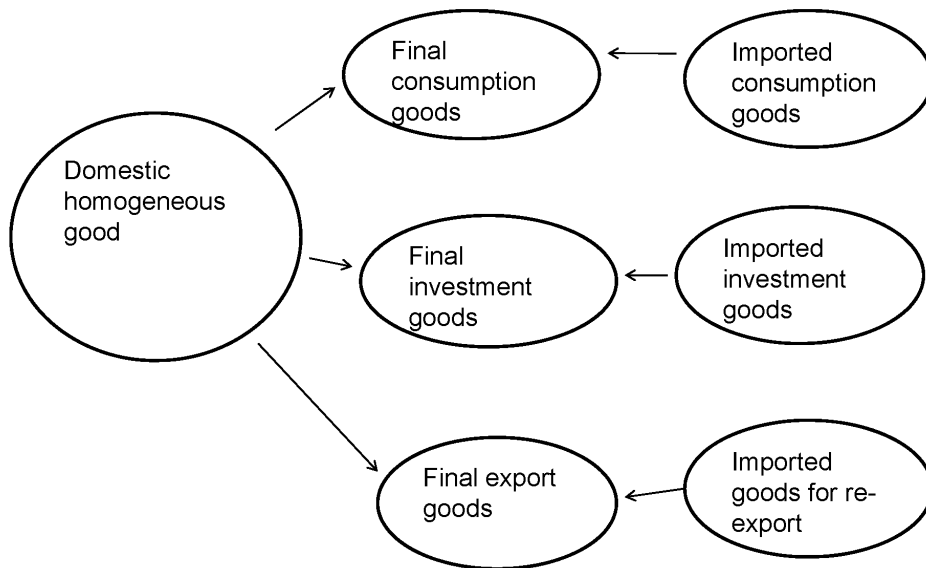


Figure 1: Inputs to consumption, investment and exports production in the model.

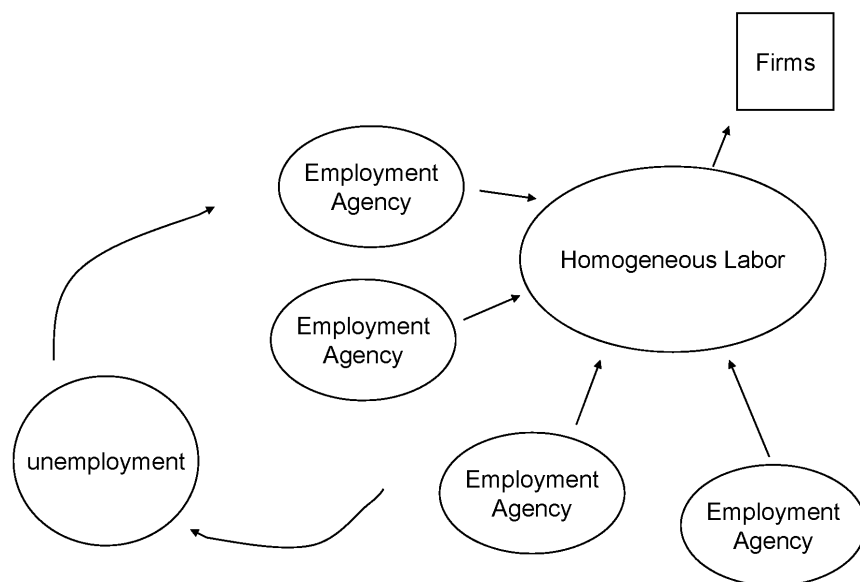


Figure 2: Illustration of the model with labor market frictions.

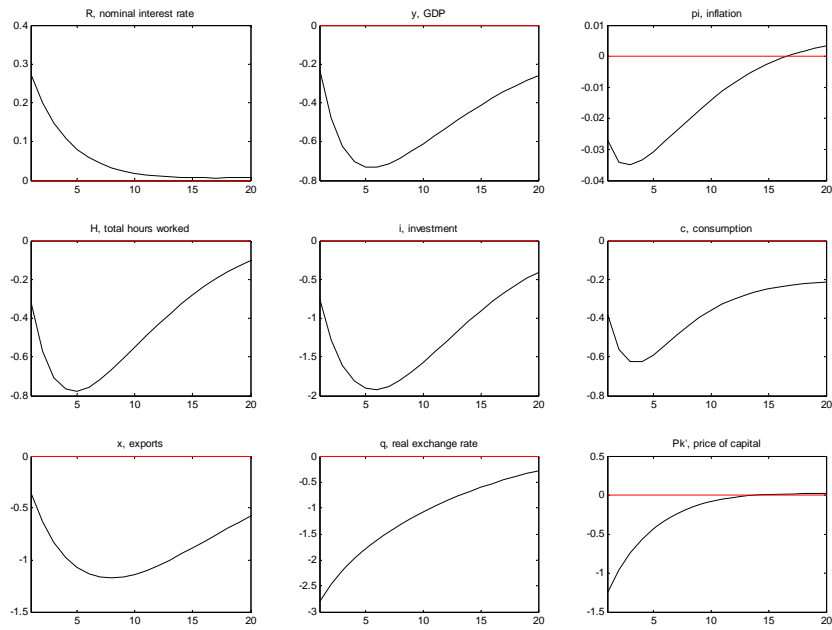


Figure 3. Baseline model: impulse response to a monetary policy shock.

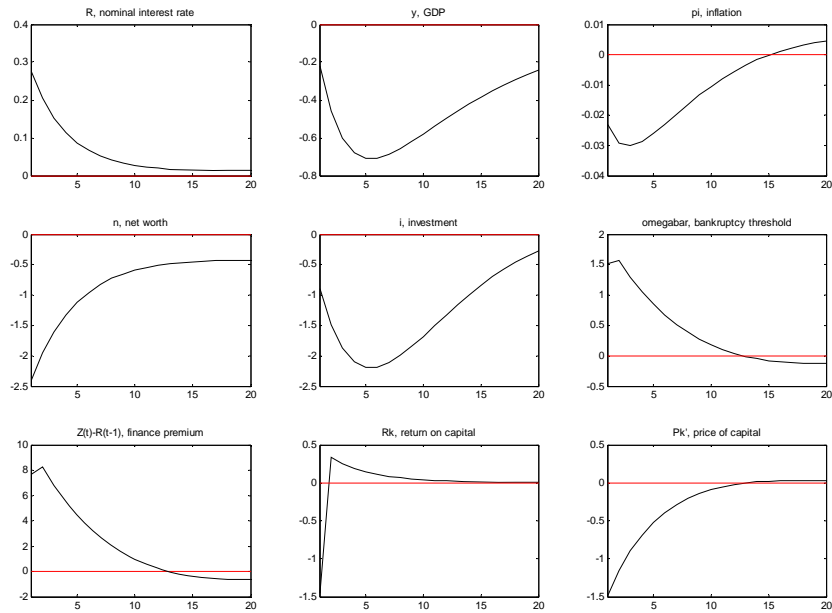


Figure 4. Financial frictions model: impulse response to a monetary policy shock.

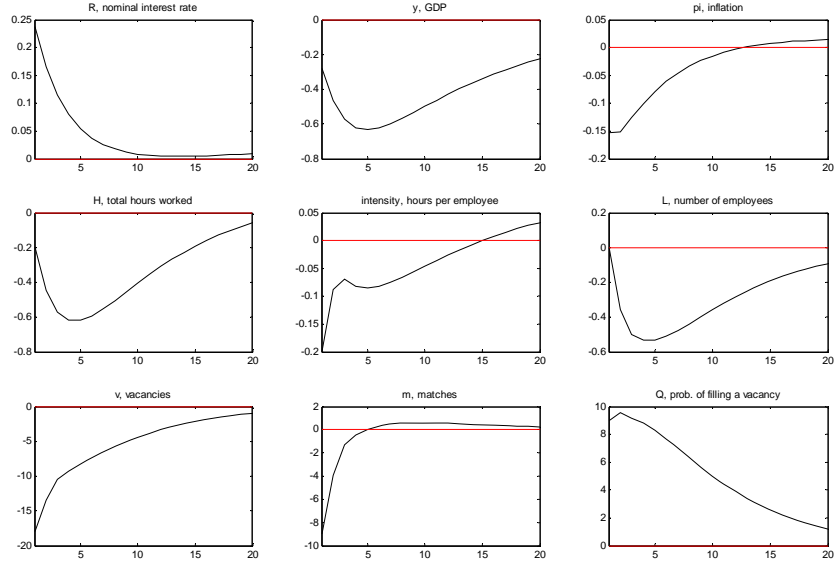


Figure 5. Search and matching frictions model: impulse response to a monetary policy shock

## A. Appendix: Scaling of Variables

We adopt the following scaling of variables. The nominal exchange rate is denoted by  $S_t$  and its growth rate is  $s_t$  :

$$s_t = \frac{S_t}{S_{t-1}}.$$

The neutral shock to technology is  $z_t$  and its growth rate is  $\mu_{z,t}$  :

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable,  $\Psi_t$ , is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \quad \mu_{z^+,t} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}.$$

Capital is scaled by  $z_t^+ \Psi_t$ . Final investment goods,  $I_t$ , and domestically produced intermediate investment goods,  $I_t^d$  are also scaled by  $z_t^+ \Psi_t$ . For reasons explained below, imported investment goods,  $I_t^m$ , are scaled by a different factor,  $z_t^+$ . Consumption goods ( $C_t^m$  are

imported intermediate consumption goods,  $C_t^d$  are domestically produced intermediate consumption goods and  $C_t$  are final consumption goods) are scaled by  $z_t^+$ . Government consumption, the real wage and real foreign assets are scaled by  $z_t^+$ . Exports ( $X_t^m$  are imported intermediate goods for use in producing exports and  $X_t$  are final export goods) are scaled by  $z_t^+$ . Also,  $v_t$  is the shadow value in utility terms to the household of domestic currency and  $v_t P_t$  is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by  $z_t^+$  to induce stationarity. Also, net worth, entrepreneurs first period wage, the value of a job to a worker as well as the value of being unemployed must be scaled by  $P_t z_t^+$ . Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t}, \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t}, i_t^d = \frac{I_t^d}{\Psi_t z_t^+}, i_t = \frac{I_t}{z_t^+ \Psi_t}, i_t^m = \frac{I_t^m}{z_t^+} \\ c_t^m &= \frac{C_t^m}{z_t^+}, c_t^d = \frac{C_t^d}{z_t^+}, c_t = \frac{C_t}{z_t^+}, g_t = \frac{G_t}{z_t^+}, \bar{w}_t = \frac{W_t}{z_t^+ P_t}, a_t \equiv \frac{S_t B_{t+1}^*}{P_t z_t^+}, \\ x_t^m &= \frac{X_t^m}{z_t^+}, x_t = \frac{X_t}{z_t^+}, \psi_{z^+,t} = v_t P_t z_t^+, n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+}, w_t^e = \frac{W_t^e}{P_t z_t^+}, \\ V_{z^+,t}^i &= \frac{V_t^i}{P_t z_t^+}, U_{z^+,t} = \frac{U_{z^+,t}}{P_t z_t^+}. \end{aligned}$$

We define the scaled date  $t$  price of new installed physical capital for the start of period  $t + 1$  as  $p_{k',t}$  and we define the scaled real rental rate of capital as  $\bar{r}_t^k$  :

$$p_{k',t} = \Psi_t P_{k',t}, \bar{r}_t^k = \Psi_t r_t^k.$$

where  $P_{k',t}$  is in units of the domestic homogeneous good. We define the following inflation rates:

$$\begin{aligned} \pi_t &= \frac{P_t}{P_{t-1}}, \pi_t^c = \frac{P_t^c}{P_{t-1}^c}, \pi_t^* = \frac{P_t^*}{P_{t-1}^*}, \\ \pi_t^i &= \frac{P_t^i}{P_{t-1}^i}, \pi_t^x = \frac{P_t^x}{P_{t-1}^x}, \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}}, \end{aligned}$$

for  $j = c, x, i$ . Here,  $P_t$  is the price of a domestic homogeneous output good,  $P_t^c$  is the price of the domestic final consumption goods (i.e., the ‘CPI’),  $P_t^*$  is the price of a foreign homogeneous good,  $P_t^i$  is the price of the domestic final investment good and  $P_t^x$  is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good,  $P_t$ . When the price is denominated in foreign currency units, we divide by  $P_t^*$ , the price of the foreign

homogeneous good. The exceptional case has to do with handling of the price of investment goods,  $P_t^i$ . This grows at a rate slower than  $P_t$ , and we therefore scale it by  $P_t/\Psi_t$ . Thus,

$$\begin{aligned} p_t^{m,x} &= \frac{P_t^{m,x}}{P_t}, \quad p_t^{m,c} = \frac{P_t^{m,c}}{P_t}, \quad p_t^{m,i} = \frac{P_t^{m,i}}{P_t}, \\ p_t^x &= \frac{P_t^x}{P_t^*}, \quad p_t^c = \frac{P_t^c}{P_t}, \quad p_t^i = \frac{\Psi_t P_t^i}{P_t}. \end{aligned} \tag{A.1}$$

Here,  $m, j$  means the price of an imported good which is subsequently used in the production of exports in the case  $j = x$ , in the production of the final consumption good in the case of  $j = c$ , and in the production of final investment goods in the case of  $j = i$ . When there is just a single superscript the underlying good is a final good, with  $j = x, c, i$  corresponding to exports, consumption and investment, respectively.

We denote the real exchange rate by  $q_t$  :

$$q_t = \frac{S_t P_t^*}{P_t^c}. \tag{A.2}$$

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