

The model is solved algebraically, except for a cubic root which is solved numerically. The method of solution is undetermined coefficients. The notation in this note corresponds to the notation in the program.

The model is given by:

$$(1) \quad \lambda_t = \alpha \gamma_{t+1} - \eta_t$$

Here, λ_t is the ex ante excess return on the foreign deposit (relative to the home deposit) as in the paper. γ_{t+1} is the home less foreign interest rate differential (expressed as $i_t - i_t^*$ in the paper. The $t+1$ subscript is useful because we can think of this as a predetermined variable if we were to use the Blanchard-Kahn solution technique.)

$$(2) \quad \lambda_t = E_t q_{t+1} - q_t - \gamma_{t+1} + E_t \pi_{t+1}$$

This is the definition of the excess return. π_{t+1} is home less foreign inflation.

$$(3) \quad \pi_t = \delta (q_t - \bar{q}_t) + \beta E_t \pi_{t+1}$$

This is the (home less foreign) Phillips curve.

$$(4) \quad \gamma_{t+1} = \sigma \pi_t + \rho \gamma_t$$

This is the (home less foreign) Taylor rule.

The following equations are the stochastic processes for the exogenous variables:

$$(5) \quad \bar{q}_t = \xi \bar{q}_{t-1} + \varepsilon_t$$

$$(6) \quad \eta_t = \mu \eta_{t-1} + u_t$$

The program takes as input the values of the parameters α , δ , β , σ , ρ , ξ , μ , and the variances of \bar{q}_t and η_t , which are entered in **lines 18 through 26** in the program.

We posit an undetermined coefficients solution for the interest differential, the real exchange rate and the relative inflation rates:

$$(7) \quad \gamma_{t+1} = a \bar{q}_t + b \eta_t + c \gamma_t$$

$$(8) \quad q_t = d\bar{q}_t + e\eta_t + f\gamma_t$$

$$(9) \quad \pi_t = g\bar{q}_t + h\eta_t + k\gamma_t$$

Using equations (5)-(9), we have:

$$(10) \quad q_{t+1} = d\xi\bar{q}_t + d\varepsilon_{t+1} + e\mu\eta_t + eu_{t+1} + fa\bar{q}_t + fb\eta_t + fc\gamma_t$$

Taking expectations:

$$(11) \quad E_t q_{t+1} = (d\xi + fa)\bar{q}_t + (e\mu + fb)\eta_t + fc\gamma_t$$

Then again using equations (5)-(9), we have:

$$(12) \quad \pi_{t+1} = g\xi\bar{q}_t + g\varepsilon_{t+1} + h\mu\eta_t + hu_{t+1} + ka\bar{q}_t + kb\eta_t + kc\gamma_t$$

Taking expectations:

$$(13) \quad E_t \pi_{t+1} = (g\xi + ka)\bar{q}_t + (h\mu + kb)\eta_t + kc\gamma_t$$

Now use (7) and (9) to make substitutions into equation (4):

$$a\bar{q}_t + b\eta_t + c\gamma_t = \sigma(g\bar{q}_t + h\eta_t + k\gamma_t) + \rho\gamma_t$$

Matching coefficients, we have:

$$(14) \quad a = \sigma g$$

$$(15) \quad b = \sigma h$$

$$(16) \quad c = \sigma k + \rho$$

Next use (7), (9) and (13) to make substitutions into equation (3):

$$g\bar{q}_t + h\eta_t + k\gamma_t = \delta(d\bar{q}_t + e\eta_t + f\gamma_t) - \delta\bar{q}_t + \beta((g\xi + ka)\bar{q}_t + (h\mu + kb)\eta_t + kc\gamma_t)$$

Matching coefficients, we have:

$$(17) \quad g = \delta(d-1) + \beta(g\xi + ka)$$

$$(18) \quad h = \delta e + \beta(h\mu + kb)$$

$$(19) \quad k = \delta f + \beta kc$$

Eliminate λ_t by equating (1) and (2), then use equations (7), (8), (11) and (13):

$$\alpha(a\bar{q}_t + b\eta_t + c\gamma_t) - \eta_t = (d\xi + fa)\bar{q}_t + (e\mu + fb)\eta_t + fc\gamma_t - (d\bar{q}_t + e\eta_t + f\gamma_t) - (a\bar{q}_t + b\eta_t + c\gamma_t) + (g\xi + ka)\bar{q}_t + (h\mu + kb)\eta_t + kc\gamma_t.$$

Matching coefficients, we have:

$$(20) \quad (1 + \alpha)a = \xi d + fa - d + g\xi + ka$$

$$(21) \quad (1 + \alpha)b - 1 = \mu e + fb - e + \mu h + kb$$

$$(22) \quad (1 + \alpha)c = fc - f + kc$$

Equations (14)-(22) are nine equations that allow for us to solve for the nine undetermined coefficients: a, b, c, d, e, f, g, h and k .

Equations (16), (19) and (22) allow us to solve for c, f , and k . First, solve for k from (16):

$$k = \frac{c - \rho}{\sigma}.$$

Then substitute this into (19):

$$(1 - \beta c) \left(\frac{c - \rho}{\sigma} \right) = \delta f,$$

and solve for f :

$$f = (1 - \beta c) \left(\frac{c - \rho}{\delta \sigma} \right)$$

Then substitute this into equation (22) to get:

$$(1 + \alpha)c = (1 - \beta c) \left(\frac{c - \rho}{\delta \sigma} \right) (c - 1) + \left(\frac{c - \rho}{\sigma} \right) c$$

This gives us the equation:

$$0 = (1 - \beta c)(c - \rho)(1 - c) + \delta \sigma c(1 + \alpha) - \delta c(c - \rho).$$

The program solves this equation in **line 31**. There are three roots to this cubic equation. We use the solution that is less than one in absolute value, which is given in **line 34**.

Then from above, the program solves for k in **line 39** using $k = \frac{c - \rho}{\sigma}$. And in **line 40**, the program

$$\text{solves for } f \text{ using } f = (1 - \beta c) \left(\frac{c - \rho}{\delta \sigma} \right) = (1 - \beta c) \frac{k}{\delta}.$$

We have solved for c , f , and k . Then equations (14), (17) and (20) let us solve for a , d and g . From (14) we have $g = \frac{a}{\sigma}$. From (20), we have:

$$(\xi - 1)d = - \left(f - (1 + \alpha) + k + \frac{\xi}{\sigma} \right) a, \text{ so } d = \frac{\sigma(1 + \alpha - f - k) - \xi}{\sigma(\xi - 1)} a.$$

Substituting these expressions for g and d into (17), we get:

$$a = \frac{(\xi - 1)\sigma\delta}{(1 - \xi)(1 - \beta(\xi + \sigma k)) + \delta(\sigma(1 + \alpha - f - k) - \xi)}.$$

This is the solution for a in **line 41** of the program. Then **line 42** gives us d as

$$d = \frac{\sigma(1 + \alpha - f - k) - \xi}{\sigma(\xi - 1)} a, \text{ and } \text{line 43 gives us } g \text{ as } g = \frac{a}{\sigma}.$$

Next, equations (15), (18), and (21) will allow us to solve for b , h and e . From equation (15), we have $h = \frac{b}{\sigma}$. From (21) we get $(f + k - 1 - \alpha)b + 1 + \mu h = (1 - \mu)e$, so

$$e = \frac{(\sigma(f + k - 1 - \alpha) + \mu)b + \sigma}{\sigma(1 - \mu)}$$

Substituting these into (18) we get:

$$\frac{b}{\sigma} = \delta \frac{(\sigma(f + k - 1 - \alpha) + \mu)b + \sigma}{\sigma(1 - \mu)} + \beta \left(\frac{b}{\sigma} \mu + kb \right), \text{ or}$$

$$b = \frac{\delta\sigma}{(1 - \mu)(1 - \beta(\mu + \sigma k)) - \delta(\sigma(f + k - 1 - \alpha) + \mu)}$$

which is **line 44** of the program.

Line 45 gives us h as $h = \frac{b}{\sigma}$, and **line 46** gives us e as from (18) as

$$e = \frac{(1 - \beta\mu)h - \beta kb}{\delta} = \frac{(1 - \beta\mu - \sigma\beta k)b}{\sigma\delta}.$$

Next we solve for the real interest rate differential. We postulate a solution of the form

$$(23) \quad r_t = m\bar{q}_t + n\eta_t + p\gamma_t.$$

We can use the fact that $r_t = \gamma_{t+1} - E_t\pi_{t+1}$ and equations (7) and (13) to write:

$$m\bar{q}_t + n\eta_t + p\gamma_t = a\bar{q}_t + b\eta_t + c\gamma_t - \left((g\xi + ka)\bar{q}_t + (h\mu + kb)\eta_t + kc\gamma_t \right)$$

Matching coefficients, we have:

$$m = a - (g\xi + ka), \text{ which is } \mathbf{line 47} \text{ of the program.}$$

$$n = b - (h\mu + kb), \text{ which is } \mathbf{line 48} \text{ of the program.}$$

$$p = c(1 - k), \text{ which is } \mathbf{line 49} \text{ of the program.}$$

Then we posit a solution for the ex ante excess return:

$$(24) \quad \lambda_t = q\bar{q}_t + r\eta_t + s\gamma_t$$

From equation (1), we have $\lambda_t = \alpha\gamma_{t+1} - \eta_t$. We can then substitute from (7) to get:

$$q\bar{q}_t + r\eta_t + s\gamma_t = \alpha a\bar{q}_t + \alpha b\eta_t + \alpha c\gamma_t - \eta_t$$

Matching coefficients, we have

$$q = \alpha a, \text{ which is } \mathbf{line 50} \text{ of the program.}$$

$$r = \alpha b - 1, \text{ which is } \mathbf{line 51} \text{ of the program.}$$

$$s = \alpha c, \text{ which is } \mathbf{line 52} \text{ of the program.}$$

$$\text{So we have } \lambda_t = \alpha a\bar{q}_t + r\eta_t + \alpha c\gamma_t.$$

$$\text{Then, } E_t\lambda_{t+1} = \alpha a\xi\bar{q}_t + (\alpha b - 1)\mu\eta_t + \alpha c\gamma_{t+1}$$

$$E_t \lambda_{t+2} = \alpha a \xi^2 \bar{q}_t + r \mu^2 \eta_t + \alpha c E_t \gamma_{t+2}, \text{ and so on.}$$

We have:

$$\Lambda_t = \frac{\alpha a}{1-\xi} \bar{q}_t + \frac{r}{1-\mu} \eta_t + s E_t (\gamma_t + \gamma_{t+1} + \gamma_{t+2} + \dots).$$

$$\text{Now, } \gamma_{t+1} = a \bar{q}_t + b \eta_t + c \gamma_t.$$

$$E_t \gamma_{t+2} = E_t (a \bar{q}_{t+1} + b \eta_{t+1} + c \gamma_{t+1}) = a \xi \bar{q}_t + b \mu \eta_t + c \gamma_{t+1}$$

$$E_t \gamma_{t+3} = E_t (a \bar{q}_{t+2} + b \eta_{t+2} + c \gamma_{t+2}) = (a \xi^2 + c a \xi) \bar{q}_t + (b \mu^2 + c b \mu) \eta_t + c^2 \gamma_{t+1}$$

$$E_t \gamma_{t+4} = E_t (a \bar{q}_{t+3} + b \eta_{t+3} + c \gamma_{t+3}) = (a \xi^3 + c a \xi^2 + c^2 a \xi) \bar{q}_t + (b \mu^3 + c b \mu^2 + c^2 b \mu) \eta_t + c^3 \gamma_{t+1}$$

etc. Then

$$\begin{aligned} E_t (\gamma_t + \gamma_{t+1} + \dots) = & \\ & a \left(1 + \xi + \xi^2 + c \xi + \xi^3 + c (\xi^2 + c \xi) + \xi^4 + c (\xi^3 + c (\xi^2 + c \xi)) + \dots \right) \bar{q}_t \\ & + b \left(1 + \mu + \mu^2 + c \mu + \mu^3 + c (\mu^2 + c \mu) + \mu^4 + c (\mu^3 + c (\mu^2 + c \mu)) + \dots \right) \eta_t \\ & + (1+c) \gamma_t + \frac{c}{1-c} \gamma_{t+1} \end{aligned}$$

We see that

$$E_t (\gamma_t + \gamma_{t+1} + \dots) = \frac{a}{1-\xi} (1 + \xi (c + c^2 + \dots)) \bar{q}_t + \frac{b}{1-\mu} (1 + \mu (c + c^2 + \dots)) \eta_t + (1+c) \gamma_t + \frac{c}{1-c} \gamma_{t+1}$$

which can be written as:

$$\begin{aligned}
E_t(\gamma_t + \gamma_{t+1} + \dots) &= \frac{a}{1-\xi} \left(1 + \xi \frac{c}{1-c}\right) \bar{q}_t + \frac{b}{1-\mu} \left(1 + \mu \frac{c}{1-c}\right) \eta_t + (1+c)\gamma_t + \frac{c}{1-c} \gamma_{t+1} \\
&= \frac{a}{1-\xi} \left(1 + \xi \frac{c}{1-c}\right) \bar{q}_t + \frac{b}{1-\mu} \left(1 + \mu \frac{c}{1-c}\right) \eta_t + (1+c)\gamma_t + \frac{c}{1-c} (a\bar{q}_t + b\eta_t + c\gamma_t) \\
&= \frac{a}{(1-\xi)(1-c)} \bar{q}_t + \frac{b}{(1-\mu)(1-c)} \eta_t + \frac{1}{1-c} \gamma_t
\end{aligned}$$

Then we get:

$$\begin{aligned}
\Lambda_t &= \frac{\alpha a}{1-\xi} \bar{q}_t + \frac{r}{1-\mu} \eta_t + \alpha c \left(\frac{a}{(1-\xi)(1-c)} \bar{q}_t + \frac{b}{(1-\mu)(1-c)} \eta_t + \frac{1}{1-c} \gamma_t \right) \\
&= \frac{\alpha a}{(1-\xi)(1-c)} \bar{q}_t + \frac{1}{1-\mu} \left(r + \frac{\alpha c b}{1-c} \right) \eta_t + \frac{\alpha c}{1-c} \gamma_t \\
&= \frac{\alpha a}{(1-\xi)(1-c)} \bar{q}_t + \frac{r+c}{(1-\mu)(1-c)} \eta_t + \alpha c \left(\frac{1}{1-c} \gamma_t \right)
\end{aligned}$$

So we can write

$$(25) \quad \Lambda_t = v\bar{q}_t + w\eta_t + x\gamma_t, \text{ where}$$

$$v = \frac{\alpha a}{(1-\xi)(1-c)}, \text{ which is **line 53** of the program.}$$

$$w = \frac{r+c}{(1-\mu)(1-c)}, \text{ which is **line 54** of the program}$$

$$x = \frac{\alpha c}{1-c}, \text{ which is **line 55** of the program.}$$

Next, we calculate some variances and covariances.

$$\text{cov}(\gamma_{t+1}, \bar{q}_{t+1}) = \text{cov}(a\bar{q}_t + b\eta_t + c\gamma_t, \xi\bar{q}_t + \varepsilon_{t+1}) = a\xi \text{var}(\bar{q}_t) + c\xi \text{cov}(\gamma_t, \bar{q}_t).$$

$$\text{Hence, } \text{cov}(\gamma_t, \bar{q}_t) = \frac{a\xi}{1-c\xi} \text{var}(\bar{q}_t), \text{ which is **line 56** of the program.}$$

Then, $\text{cov}(\gamma_{t+1}, \eta_{t+1}) = \text{cov}(a\bar{q}_t + b\eta_t + c\gamma_t, \mu\eta_t + u_{t+1}) = b\mu \text{var}(\eta_t) + c\mu \text{cov}(\gamma_t, \eta_t)$.

Hence, $\text{cov}(\gamma_t, \eta_t) = \frac{b\mu}{1-c\mu} \text{var}(\eta_t)$, which is **line 57** of the program.

$$\begin{aligned} \text{var}(\gamma_{t+1}) &= \text{var}(a\bar{q}_t + b\eta_t + c\gamma_t) \\ &= c^2 \text{var}(\gamma_t) + a^2 \text{var}(\bar{q}_t) + b^2 \text{var}(\eta_t) + 2ac \text{cov}(\bar{q}_t, \gamma_t) + 2bc \text{cov}(\eta_t, \gamma_t) \\ &= \frac{a^2}{1-c^2} \text{var}(\bar{q}_t) + \frac{b^2}{1-c^2} \text{var}(\eta_t) + \frac{2ac}{1-c^2} \text{cov}(\bar{q}_t, \gamma_t) + \frac{2bc}{1-c^2} \text{cov}(\eta_t, \gamma_t) \end{aligned}$$

This is **line 58** of the program.

$$\begin{aligned} \text{cov}(\lambda_t, r_t) &= \text{cov}(q\bar{q}_t + r\eta_t + s\gamma_t, m\bar{q}_t + n\eta_t + p\gamma_t) \\ &= qm \text{var}(\bar{q}_t) + rn \text{var}(\eta_t) + ps \text{var}(\gamma_t) + (pq + ms) \text{cov}(\bar{q}_t, \gamma_t) + (pr + ns) \text{cov}(\eta_t, \gamma_t) \end{aligned}$$

which is **line 59** of the program.

$$\begin{aligned} \text{cov}(\Lambda_t, r_t) &= \text{cov}(v\bar{q}_t + w\eta_t + x\gamma_t, m\bar{q}_t + n\eta_t + p\gamma_t) \\ &= vm \text{var}(\bar{q}_t) + wn \text{var}(\eta_t) + xp \text{var}(\gamma_t) + (pv + mx) \text{cov}(\bar{q}_t, \gamma_t) + (pw + nx) \text{cov}(\eta_t, \gamma_t) \end{aligned}$$

which is **line 60** of the program.

$$\begin{aligned} \text{var}(r_t) &= \text{var}(m\bar{q}_t + n\eta_t + p\gamma_t) \\ &= m^2 \text{var}(\bar{q}_t) + n^2 \text{var}(\eta_t) + p^2 \text{var}(\gamma_t) + 2mp \text{cov}(\bar{q}_t, \gamma_t) + 2np \text{cov}(\eta_t, \gamma_t) \end{aligned}$$

which is **line 61** of the program.

$$\begin{aligned} \text{var}(\lambda_t) &= \text{var}(q\bar{q}_t + r\eta_t + s\gamma_t) \\ &= q^2 \text{var}(\bar{q}_t) + r^2 \text{var}(\eta_t) + s^2 \text{var}(\gamma_t) + 2qs \text{cov}(\bar{q}_t, \gamma_t) + 2rs \text{cov}(\eta_t, \gamma_t) \end{aligned}$$

which is **line 62** of the program.

Then the regression coefficient for regressing λ_t on r_t is given by $\frac{\text{cov}(\lambda_t, r_t)}{\text{var}(r_t)}$, which is

line 63 of the program. The regression coefficient for regressing Λ_t on r_t is given by $\frac{\text{cov}(\Lambda_t, r_t)}{\text{var}(r_t)}$

which is **line 64** of the program.

Next, we have:

$$\begin{aligned}\text{var}(\pi_t) &= \text{var}(g\bar{q}_t + h\eta_t + k\gamma_t) \\ &= g^2 \text{var}(\bar{q}_t) + h^2 \text{var}(\eta_t) + k^2 \text{var}(\gamma_t) + 2gk \text{cov}(\bar{q}_t, \gamma_t) + 2hk \text{cov}(\eta_t, \gamma_t)\end{aligned}$$

which is **line 65** of the program.

$$\begin{aligned}\text{cov}(\pi_{t+1}, \pi_t) &= \text{cov}(g\bar{q}_{t+1} + h\eta_{t+1} + k\gamma_{t+1}, g\bar{q}_t + h\eta_t + k\gamma_t) \\ &= \text{cov}(g\xi\bar{q}_t + g\varepsilon_{t+1} + h\mu\eta_t + hu_{t+1} + ka\bar{q}_t + kb\eta_t + kc\gamma_t, g\bar{q}_t + h\eta_t + k\gamma_t) \\ &= \text{cov}((g\xi + ka)\bar{q}_t + (h\mu + kb)\eta_t + kc\gamma_t, g\bar{q}_t + h\eta_t + k\gamma_t) \\ &= g(g\xi + ka) \text{var}(\bar{q}_t) + h(h\mu + kb) \text{var}(\eta_t) + k^2c \text{var}(\gamma_t) \\ &\quad + (k(g\xi + ka) + gkc) \text{cov}(\bar{q}_t, \gamma_t) + (h(h\mu + kb) + hkc) \text{cov}(\eta_t, \gamma_t)\end{aligned}$$

which is **line 66** of the program.

The serial correlation of inflation is given by $\frac{\text{cov}(\pi_{t+1}, \pi_t)}{\text{var}(\pi_t)}$, which is **line 67** of the program.

Then,

$$\text{cov}(\gamma_{t+1}, \gamma_t) = \text{cov}(a\bar{q}_t + b\eta_t + c\gamma_t, \gamma_t) = c \text{var}(\gamma_t) + a \text{cov}(\gamma_t, \bar{q}_t) + b \text{cov}(\gamma_t, \eta_t),$$

which is **line 68** of the program.

The serial correlation of the nominal interest rate is given by $\frac{\text{cov}(\gamma_{t+1}, \gamma_t)}{\text{var}(\gamma_t)}$, which is **line 69** of the program.

$$\begin{aligned}\text{cov}(\gamma_{t+1}, r_t) &= \text{cov}(a\bar{q}_t + b\eta_t + c\gamma_t, m\bar{q}_t + n\eta_t + p\gamma_t) \\ &= am \text{var}(\bar{q}_t) + bn \text{var}(\eta_t) + cp \text{var}(\gamma_t) + (pa + mc) \text{cov}(\bar{q}_t, \gamma_t) + (pb + nc) \text{cov}(\eta_t, \gamma_t)\end{aligned}$$

which is **line 70** of the program.