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Business cycle analysis without much theory A look at structural VARs

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Abstract

This paper examines the usefulness of applying structural vector autoregressions (SVARs) to the study of business cycles. The SVAR approach aims to provide robust inferences, by imposing only weak theoretical restrictions. We illustrate that the robustness of conclusions drawn from SVAR exercises are questionable. We also examine the problem of identification failure in structural VAR models. © 1998 Elsevier Science S.A.

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1. Introduction

Carl Christ is one of the important figures in the development of econometrics, particularly macroeconometrics. In the early 1950s Carl made important contributions to the early development of econometric models. While his subsequent research has spanned many areas, he has also made an important, ongoing contribution in repeatedly directing the attention of the profession back to the coherent

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views developed at the Cowles Commission. A recent example of this is his (Christ, 1994) historical retrospective on the early days of the Cowles Commission. An important element of the many contributions of the Cowles economists was their work on the marriage of economic theory and empirical methods, in particular their work on identification. The last four decades have not been easy for this marriage. It has weathered many trial separations. The key point of contention in recent years has been over the nature of restrictions used to identify econometric models. It seems appropriate at a conference honoring Carl to consider some aspects of the current status of this relationship. In this paper we do this by considering the structural VAR approach to the study of business cycles. This approach to developing the empirical characteristics of business cycles assigns a very small role to economic theory.

Beginning in the early 1980s vector autoregressions (hereafter VARs) emerged as an important vehicle for the empirical analysis of macroeconomic times series. VARs have been attractive research tools for at least three reasons. First, they offer a convenient way to characterize data without having to invoke economic theory to restrict the dynamic relations among variables. Second, many completely specified economic models give rise to VAR representations of the variables in the model. As a result VARs have been widely exploited for both data description and model characterization. Third, VARs can be readily transformed to interpret the evolution of the system's variables as a function of orthogonalized 'innovations' in any of these variables. Cooley and LeRoy (1985), among others, describe the relation between identification and notions of causality and exogeneity as they apply to VARs.

Ordinary VARs have the drawback that the impulse responses they generate cannot be given any structural interpretation because their innovations are not identified with the underlying structural errors. A response to this problem has been the development of *structural* VARs (hereafter SVARs) which have proliferated in the past few years. SVARs solve the problem of interpreting VARs by introducing restrictions sufficient to identify the underlying shocks. The SVAR approach we examine here introduces just enough restrictions to permit a coherent interpretation of the shocks to the system. This identification is achieved in two stages.

First, a set of atheoretical restrictions are imposed. These assumptions typically specify time series models of the data, and restrictions on the interactions of structural innovations. We refer to these assumptions as *auxiliary*. To complete the identification process, this SVAR method then imposes additional 'theoretical' restrictions upon this first set of a priori assumptions. The method justifies this second set of restrictions by making reference to theories that imply them, but which are not fully articulated in the sense that they do not operate at the level of preferences, technologies and explicit equilibrium concepts. The justification of these identifying restrictions is of the casual sort that originally led Sims (1980) to brand them as 'incredible' and to advocate dispensing with them entirely.

The types of restrictions used to identify VARs have also been criticized as being empirically misleading by Canova et al. (1993), Mellander et al. (1992) and Faust and Leeper (1994) to mention just a few.

While models which impose over-identifying restrictions can be tested, just-identifying restrictions are what they are; there is no way to test them. Thus alternative, just-identified models must be observationally equivalent, in terms of the reduced form behavior they describe. Pagan (1994) emphasizes this point. One motivation for the recent reluctance to impose over-identifying restrictions is the perception that all economic models employ gross simplifications and thereby must be false. The presumption is that rejections of over-identifying restrictions primarily reflect these simplifying assumptions, and may not be economically significant. We will maintain this perspective for the purpose of providing a fair examination of the SVAR approach. Notice that the SVAR paradigm imposes restrictions of two types – 'auxiliary' and 'theoretical'. First, we question what is gained by taking this bimodal approach to model specification. In particular, is this approach to identifying a model any more robust than one in which all restrictions are derived from a completely specified economic model?

By substituting atheoretical restrictions for theoretical ones, the SVAR approach attempts to offer a degree of robustness with respect to model misspecification. Consequently, this methodology has been put forward as a way of deciding on the relative importance of real and monetary shocks, or demand and supply shocks, for the business cycle. Practitioners have argued this issue can be resolved empirically with SVARs, using only a minimal amount of theory to identify the models. Increasingly, the dynamic responses to shocks implied by SVAR identifications are treated as part of the stylized facts of the business cycle that any fully articulated business cycle model must account for. Perhaps the best example of this is the 'hump shaped' response of output to money – a feature of the data that is cited by King (1995), Cochrane (1994) and many others as something that ought to be accounted for in dynamic general equilibrium business cycle models. For these reasons, it is important to examine the purported robustness of the SVAR method.

To this end, we present several examples which demonstrate that this robustness is largely illusory. We emphasize that SVARs impose only enough economic restrictions for identification *conditional* on a set of auxiliary, atheoretical restrictions. The SVAR approach implicitly assumes that these latter restrictions are innocuous. First we show that even when economic processes satisfy the weak theoretical restrictions imposed by an SVAR model, the subsequent identifications induced by the SVAR can substantially misrepresent the true dynamic responses of those processes. This occurs precisely because the implied dynamics of the SVAR model are quite sensitive to misspecification of these auxiliary assumptions.

These assumptions are of two types – testable, and untestable. With regard to the former, SVAR methods frequently make assumptions about the type of nonstationarity exhibited by the data. Assumptions regarding whether data are difference or trend stationary are testable. Unfortunately, most such tests lack power, and there is a large literature which argues that trying to make such distinctions is a fruitless exercise. [1] While for many purposes, the near observational equivalence of trend and difference stationary specifications is inconsequential, it is of crucial importance here, because the structural identification relies upon the distinction.

While some of these auxiliary assumptions are notoriously difficult to verify, others are impossible, such as the orthogonality of structural shocks. Again because these assumptions are used to identify the model, misspecifications have substantial impact on the dynamic responses implied by SVAR-type identifications.

We perform three types of sensitivity analysis with respect to the auxiliary assumptions of a canonical example of the SVAR approach, due to Blanchard and Quah (1989) (BQ). In each case the misspecification we consider is derived from a completely specified model economy. To focus attention on the auxiliary assumptions, all of these economies satisfy the long run *economic* restriction imposed by BQ. Our first model provides a local alternative to the trend dependence assumptions of BQ. In our second example, we consider variation in the dimensionality of the shocks underlying the system. The third economy we examine differs from the BQ example in both its alternative stationarity assumptions, and the non-orthogonality of its structural errors.

The use of completely specified economic models outlined above exemplifies the primary competing, just-identified approach to studying the relative importance of shocks for the business cycle. This dynamic general equilibrium approach (hereafter DGE) proceeds by constructing and computing the equilibria of fully specified artificial economies. The SVAR methodology and the dynamic general equilibrium methodology both view the imposition of overidentifying restrictions as inappropriate. Both approaches model fluctuations in output as driven by shocks to the system. Both have been used to try to determine the relative importance of productivity shocks (or real shocks) and other shocks (government spending, preference shocks, monetary policy shocks) for the fluctuations in output at business cycle frequencies.

Aside from having been used to address the same question, the methodologies have little in common. SVARs represent an empirical methodology that is only weakly grounded in economic theory. The dynamic general equilibrium approach studies business cycle fluctuations in fully articulated model economies that are consistent with long-term growth and competitive general equilibrium theory. It relies far more heavily on theory to determine the nature of business cycles. If both approaches gave the same answer to questions about the driving processes for business cycles that would be reassuring. Unfortunately, they seem to give very different answers regarding the relative importance of shocks. Accordingly,

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[1] See Blough (1990), Cochrane (1991), DeJong et al. (1992), and Zivot and Andrews (1992).

it is of some interest to know whether we can feel confident about the empirical findings derived from SVARs.

This paper argues that while DGE models impose strong assumptions, so do SVAR models. While all of the assumptions of a DGE model are interpretable with respect to an economic system, the majority of the assumptions of typical SVAR models are not. Atheoretical assumptions should not be considered weaker, or more plausible than theoretical ones. In our SVAR example, these auxiliary assumptions are either untestable, or effectively so. Without statistically compelling evidence for their adoption, we should be all the more suspicious of them, since we do not know what their economic implications are. If we are willing to maintain that all of our assumptions, both theoretical and atheoretical are likely to be misspecified, then the use of well motivated and economically interpretable restrictions as advanced in the DGE approach, should be particularly compelling.

Thus the first point of this paper is that the substitution of atheoretical restrictions for theoretical ones does not confer robustness. We then explore the reliability of SVAR restrictions via the issue of 'identification failure' as it is discussed by Phillips (1989). We argue that the strategy used to achieve identification in much of this literature may effectively rely upon weak instruments, and therefore lead to unreliable structural conclusions.

In the next section of the paper, we review the SVAR methodology using as an example the paper by Blanchard and Quah. In section three, we make use of the dynamic general equilibrium models alluded to above, to assess the robustness of the Blanchard–Quah conclusions. First we generate data from both a cash-in-advance model, and a simple real business cycle model, and subject these data to the SVAR identification strategy. We then consider the assumption that the VAR model is even the correct starting point for identification. Here we show that in a model with both productivity and preference shocks, a bivariate VAR constitutes a severe misspecification of the model's actual dynamics. We argue that the DGE-based approach to identification is closer to that envisioned by the Cowles Commission economists. Section four contains a discussion of the related but separate problem of identification failures in SVARs.

2. Identification using weak theory: the Blanchard–Quah model

We begin this discussion of the SVAR approach with an illustration of the well known identification scheme proposed by Blanchard and Quah (1989), hereafter BQ. This identification scheme is based on an arbitrary orthogonality restriction and a restriction on the long-run responses of the system. It has been criticized on a number of grounds by other authors (e.g. Canova et al., 1993).

BQ justify their long-run restriction using a simple model based on Fischer's (1977) nominal wage contracting theory. We characterize it as weak theory because it is not based on a specification of underlying preferences and technology and the equilibrium concept is not articulated. The contracting model takes the form:

$$Y_t = M_t - P_t + a\theta_t, \tag{1}$$

$$Y_t = N_t + \theta_t, \tag{2}$$

$$P_t = W_t - \theta_t, \tag{3}$$

$$W_t = W \mid \{E_{t-1}N_t = N^*\}, \tag{4}$$

where Y_t and N_t denote the logs of output, and employment, and θ is the realized productivity. N^* is full employment. P, W and M are the logs of the price level, the nominal wage, and the money supply respectively.

The evolution of M_t and θ_t is given by the following equations:

$$M_t = M_{t-1} + \varepsilon_{dt}, \tag{5}$$

$$\theta_t = \theta_{t-1} + \varepsilon_{st}. \tag{6}$$

Using the driving processes for productivity and money the model has a solution that can be written as

$$(1 - L)Y_t = (1 - L)\varepsilon_{dt} + (1 + (1 - L)a)\varepsilon_{st}, \tag{7}$$

$$U_t = -\varepsilon_{dt} - a\varepsilon_{st}, \tag{8}$$

where U_t represents the unemployment rate.

This leads to a structural model that has the form

$$A(L)\begin{pmatrix}\Delta Y_t \\ U_t\end{pmatrix} = \begin{pmatrix}\varepsilon_{dt} \\ \varepsilon_{st}\end{pmatrix}. \tag{9}$$

It has the implication that monetary shocks, interpreted as demand disturbances, have no long run effects on either output or unemployment but may have short-run effects. Productivity shocks, on the other hand, do have long-term effects on output.

One immediate drawback of the BQ empirical implementation is that instead of using a variable which is reasonably well measured, per-capita hours of work, they have used one that is less well measured – the unemployment rate. In order to implement their estimation methods we use hours, and we denote this by H_t.

Following BQ, we use the theory just described to justify one of the identifying restrictions imposed on a reduced form VAR in $X_t = [\Delta Y_t, H_t]'$.[2] The vector

[2] As in BQ, both series are linearly detrended.

moving average (VMA) representation of the structural VAR, e.g. $A(L)X_t = \varepsilon_t$ is,

$$X_t = C(L)\varepsilon_t \tag{10}$$

where $C(L) = A(L)^{-1}$. What makes this form structural is the ability to interpret the shocks as productivity and monetary shocks or supply and demand shocks: $\varepsilon_t = [\varepsilon_{dt} \;\; \varepsilon_{st}]'$.

To recover the structural parameters it is useful to consider first the reduced form VAR

$$B(L)X_t = v_t \tag{11}$$

where $B(0) = I$, and $\mathrm{E}(v_t v_t') = \Sigma$, where the v_t are the innovations. This is an unrestricted VAR of the sort proposed by Sims (1980). The VMA representation of this is

$$X_t = D(L)v_t \tag{12}$$

where $D(L) = B(L)^{-1}$. Now, to relate this to the structural form, observe that $B(L) = A(0)^{-1}A(L)$, $A(0)^{-1}\varepsilon_t = v_t$ and $C(L) = D(L)A(0)^{-1}$. Therefore, to recover $C(L)$ from $D(L)$ requires knowledge of $A(0)$. The first identifying assumption used is the assumption that $\Sigma = \mathrm{E}(\varepsilon_t\varepsilon_t')$ is diagonal, so supply and demand disturbances are uncorrelated. A further normalization is imposed, which is that $A(0)$ has ones on the diagonal.[3] From $A(0)^{-1}\varepsilon_t = v_t$ we get $A(0)^{-1}\Sigma A(0)^{-1'} = \Omega$. This gives us three equations in four unknowns. Therefore, in this bivariate system, one additional restriction is required. BQ use the weak theory above to justify the restriction

$$C_{11}(1) = 0, \tag{13}$$

where this is the long-run response of output growth to the demand shock. With this additional restriction, we can recover $C(L)$.

Pagan and Robertson (1994) point out that this identification strategy differs from the Cowles approach on two grounds. The Cowles approach relies on restrictions on the elements of A_j. It generally does not attempt to restrict Σ, and it usually generates overidentifying restrictions which opens the possibility for testing. The BQ approach, on the other hand, has the appearance of imposing minimal and plausible restrictions.

Following BQ, this decomposition is applied to the reduced form VAR of (11). The results of this decomposition, using per-capita hours instead of unemployment, are shown in Fig. 1. These results are qualitatively similar to what BQ

[3] The normalization to unity is inconsequential; the assumption that the shocks are uncorrelated is not. It is unclear why shocks would be uncorrelated since monetary shocks may well react to productivity shocks if the Fed pursues activist policies. We will return to this in subsections 3.2 and 3.3.

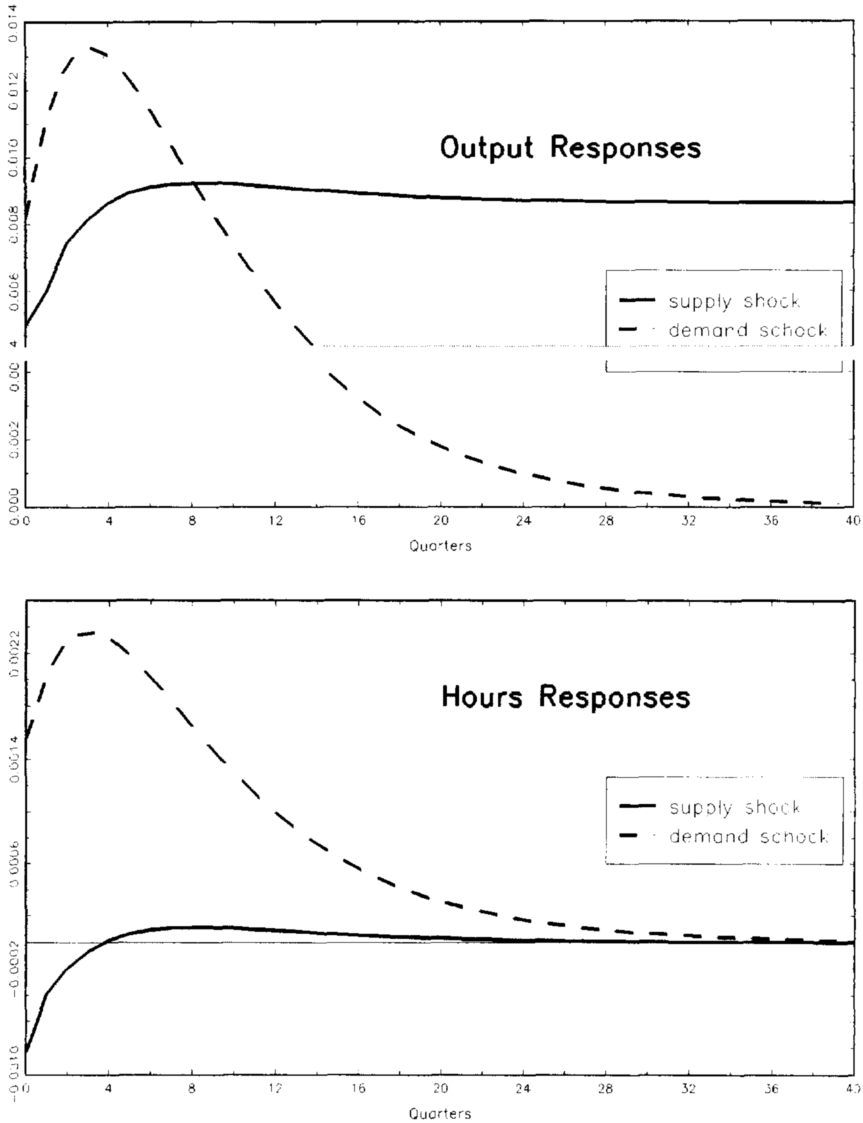

Fig. 1. Responses of output and hours identified via Blanchard and Quah's structural VAR.

found. They suggest that, at intermediate frequencies associated with the business cycle, demand shocks account for much of the variation in output and hours in this model, even though only supply shocks have long-term effects.

It is worthwhile being explicit about the connection between the dynamics of Fig. 1, and the nominal contracting model (1)–(6) that BQ appeal to. The assumptions of BQ's SVAR model are as follows.

(a) Detrended log output is difference stationary. Detrended hours are stationary.
(b) The structural innovations are contemporaneously uncorrelated.
(c) A bivariate VAR(p) appropriately represents the stationary dynamics of the series. Here p is assumed known.
(d) In the long run, demand shocks have no effect on output.

The SVAR approach maintains that economic theory motivates only assumption (d). Assumptions (a)–(c) constitute the atheoretical assumptions of the BQ model. Such auxiliary assumptions are the hallmark of the SVAR approach. This approach implies that these assumptions are not to be interpreted in terms of the nominal contracting model above, or in terms of any other specific, economic model.

The impact of these auxiliary assumptions (a)–(c) on the dynamic responses reported above can be clarified by considering what structural dynamics are implied by the nominal contracting model itself. Relations (7)–(8) constitute a vector MA(1) specification for $(\Delta Y_t, U_t)$. As such, there is no reason to estimate (11). Making the substitution of hours for employment, we rewrite this relation as

$$\begin{pmatrix} \Delta Y_t \\ H_t \end{pmatrix} = \begin{pmatrix} 1 & 1+a \\ 1 & a \end{pmatrix} \begin{pmatrix} e_{dt} \\ e_{st} \end{pmatrix} + \begin{pmatrix} -1 & -a \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e_{dt-1} \\ e_{st-1} \end{pmatrix}. \tag{14}$$

Then it is immediate that

$$Y_t = e_{dt} + (1+a)e_{st} + \sum_{j=1}^{\infty} e_{st-j}. \tag{15}$$

This reveals that the impulse response of output to a demand shock here is identically zero for all lags. Similarly the response of hours to both demand and supply shocks must be zero for all lags beyond one. These dynamics follow regardless of the correlation between the demand and supply shocks. The source of this discrepancy between the BQ model and this nominal contracting model is the auxiliary assumption (c).

This nominal contracting specification is admittedly stylized. But BQ use this model to motivate the long-run restriction they impose. In so doing, BQ give the impression that their conclusions should somehow be invariant to these discrepancies. By focusing on a particular restriction – that demand shocks have no long-run effect on output, the SVAR approach leads us to believe that this restriction is particularly relevant to the derived impulse response dynamics. Clearly this is not the case. Rather, this comparison dramatizes that SVAR auxiliary assumptions have a substantial impact on the nature of the structural dynamics inferred by the SVAR approach.

While the structural model (14) above does not rely on auxiliary assumptions, it does suffer from the limitation that it is detached from such economic fundamentals as preferences, technologies and equilibrium concepts. Because of this, its assumptions are difficult to compare with those of alternative theories. One virtue of the DGE approach to modelling is that alternative theories are expressed in a common language of assumptions on economic primitives. It is within this framework that we now continue our discussion of SVARs.

3. Identification using strong theory

The Blanchard and Quah approach yields exact identification and is not open to testing. One can still examine whether the conclusions drawn from their analysis are robust. We study this question via three examples that employ a lot of economic theory. These examples are based on dynamic general equilibrium models of the type used in real business cycle research. Since the BQ findings are widely regarded as a strong challenge to real business cycle theory, it seems appropriate to use such models. We use the first two model economies to generate data with known properties. We then see whether the BQ identification procedure, when applied to these data, reveals the true structural dynamics of these economies.

In the final subsection, we use the theory of the third model economy to identify an SVAR, and to decompose the data for the US economy. We contrast the impulse responses which arise from this identification with those of the BQ identification.

3.1. Data generation – two shock example

We consider first a basic business cycle model where there are two shocks, real and monetary. In this economy households have preferences defined over consumption and leisure. There are two types of consumption good; a 'cash' good, c_1, which can be purchased only with currency and a 'credit' good, c_2. The preferences of the representative household are summarized by the utility function

$$\max \mathrm{E} \sum_{t=1}^{\infty} \beta^t (\alpha \ln(c_{1t}) + (1-\alpha)\ln(c_{2t}) - \gamma h_t) \tag{16}$$

The fact that hours enter linearly in preferences captures the indivisible labor feature of Hansen (1985) and Rogerson (1988).[4] Purchases of the cash good must satisfy the cash-in-advance constraint

$$P_t c_{1t} \leqslant m_t + (1 + R_{t-1}) b_t + T_t - b_{t+1}. \tag{17}$$

[4] In this framework the competitive equilibrium involves households trading employment lotteries that specify the probability of working or not working, rather than hours of work directly.

Household allocations must satisfy the following sequence of budget constraints:

$$c_{1t} + c_{2t} + x_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} \leqslant w_t h_t + r_t k_t + \frac{m_t}{P_t} + \frac{(1 + R_{t-1})b_t}{P_t} + \frac{T_t}{P_t} \quad (18)$$

where x is investment, m is currency, b is government bonds, P is the price level and T_t are nominal transfers.

Aggregate output is produced according to constant returns-to-scale technology where, K_t and H_t are aggregate capital and labor respectively:

$$Y_t = \mathrm{e}^{z_t} K_t^{\theta} H_t^{1-\theta}, \quad 0 < \theta < 1. \quad (19)$$

The technology shock evolves according to the law of motion

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad 0 < \rho < 1. \quad (20)$$

The random variable ε is distributed normally with mean zero and standard deviation σ_{ε}.

The per-capita money supply is assumed to grow at the rate $\mathrm{e}^{\mu_t} - 1$ in period t. That is

$$M_{t+1} = \mathrm{e}^{\mu_t} M_t. \quad (21)$$

The random variable μ_t evolves according to

$$\mu_{t+1} = \eta \mu_t + \xi_{t+1}. \quad (22)$$

The random variable ξ_t is distributed normally with mean $(1 - \eta)\bar{\mu}$ where $\bar{\mu}$ is the average growth rate of money.

This is only the briefest description of a model economy that is elaborated more fully in Cooley and Hansen (1995). It has a well defined competitive equilibrium and it can be calibrated to match the features of US data using the principles described in Cooley and Prescott (1995). We use the calibrated parameters described in Cooley and Hansen (1995) and simulate the economy to produce data.

This model economy has the desirable feature that fluctuations in output are driven by two shocks, a productivity (supply) shock and a monetary (demand) shock. The monetary shock does have an effect on real allocations in this economy because inflation is a tax on holdings of cash balances. But we know these effects vanish asymptotically (Cooley and Hansen, 1989). Thus, this model satisfies the theoretical restriction that BQ impose in their identification procedure.

Using this setup we generate 500 samples of length 251 quarters. We drop the first 100 quarters of each sample to eliminate the effects of initial conditions. We then apply the BQ identification and estimation strategy to these generated data. Fig. 2 shows the sample averaged impulse responses for log output and hours, implied by the BQ identification procedure. Both output and hours respond positively to a (money growth) demand shock. This is in sharp contrast to the

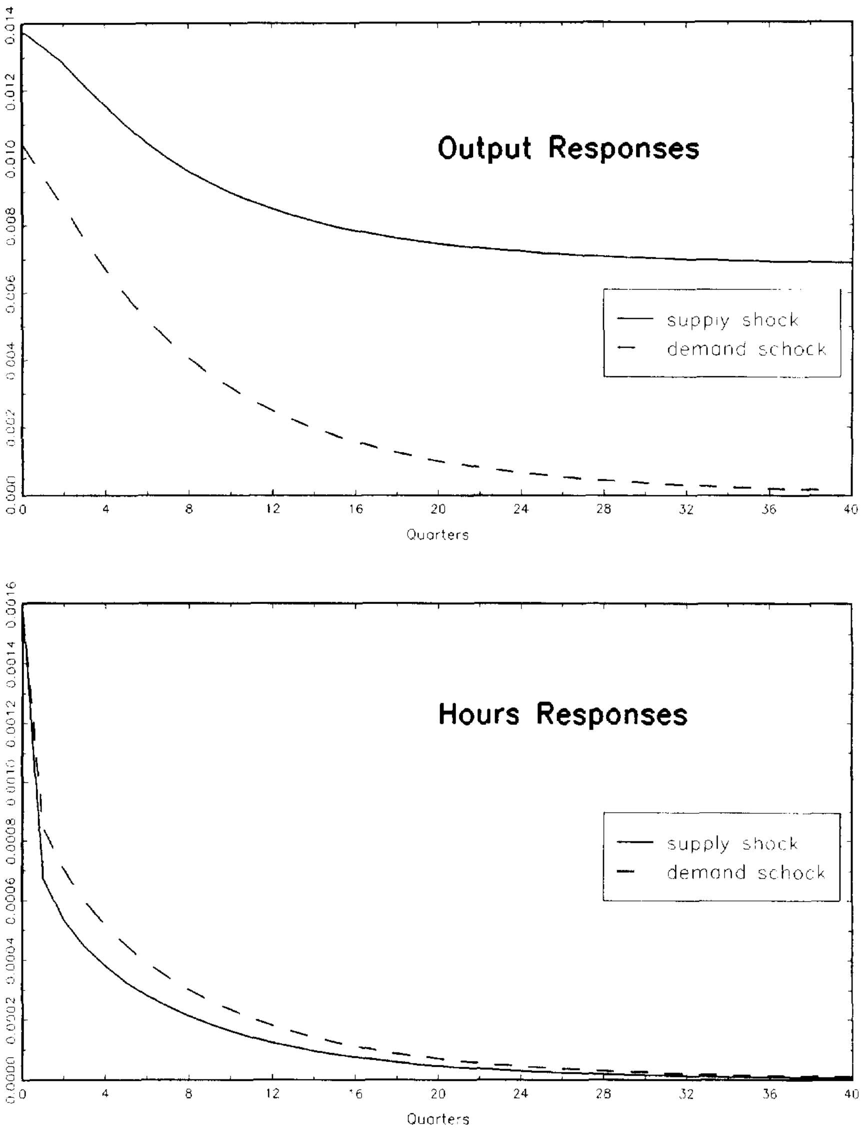

Fig. 2. Impulse responses using generated data: two-shock economy.

actual impulse responses, which are shown in Fig. 3. There, as predicted by the theory, the response of output and hours is negative.

What is the source of this dramatic distortion of the true economy's dynamics? Since these generated data obey the long-run restriction imposed by BQ, we must return to the auxiliary assumptions (a)–(c) underlying the BQ model. The real

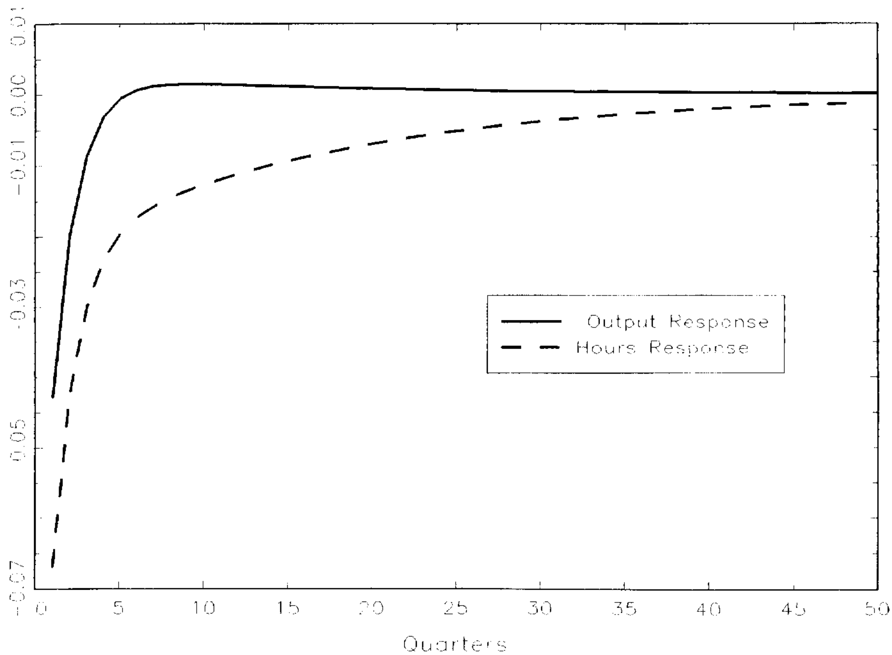

Fig. 3. True responses to demand: two-shock economy.

and monetary shocks of this simulated cash-in-advance model are independent. This focuses attention on assumptions (a) and (c). We have discussed the failure of assumption (c) in Section 2 above, and it is an issue that we will return to shortly. Here however, it is immediately apparent that the difference stationary assumption (a) is formally inconsistent with (20). For these generated data, $\rho = 0.95$. It is notoriously difficult to distinguish roots of this magnitude from unity, using postwar quarterly series. The implications of this near unit root alternative are quite substantial, for now the theoretical restriction (13) that demand shocks have no effect on the long-run growth rate of output, does not contribute to identification. This example illustrates the critical role played by the auxiliary assumptions of the SVAR approach. The *auxiliary* restriction – that log output is difference stationary, is necessary for translating the *theoretical* restriction (13), into an identifying restriction. Thus even under near alternatives to the auxiliary assumptions, the SVAR approach can fail to identify correctly the structural dynamics, despite the data actually satisfying the maintained theoretical restrictions.

3.2. Data generation – one shock example

This next example differs from the one considered above in that the data are generated from an economy where there is only one shock. The representative household maximizes the utility function

$$\max \mathrm{E} \sum_{t=1}^{\infty} \beta^t (\ln(c_t) - Ah_t^{\gamma}) \tag{23}$$

subject to

$$c_t + i_t \leqslant y_t, \tag{24}$$

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{25}$$

$$c_t \geqslant 0, \quad i_t \geqslant 0, \quad 0 \leqslant h_t \leqslant 1,$$

where c is consumption, i is investment, y is output and h is hours of work. The households in this economy rent capital and supply labor to a competitive firm which has access to a constant returns to scale technology:

$$y_t = k_t^{\theta}(\exp(a_t)h_t)^{1-\theta}. \tag{26}$$

The unique feature of this economy is that we assume technological progress is labor augmenting. We further assume that it consists of both a deterministic growth component and a difference stationary stochastic component; that is

$$a_t = z_t + gt, \tag{27}$$

$$\Delta z_t = \phi(L)\varepsilon_t, \tag{28}$$

where ε_t is white noise error. We add some structure to this difference stationary component by assuming a process in which technological shocks are absorbed gradually and in which the pattern of diffusion is the typical S-shaped response as in Jovanovic and Lach (1993) and Lippi and Reichlin (1994).[5] We assume a diffusion process with symmetric lag coefficients,

$$\phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + \phi_1 L^3 + \phi_0 L^4. \tag{29}$$

We require that $\phi_0 < \phi_1 < \phi_2$ for an S-shaped diffusion curve and $\phi(1) = 1$ implies that the long-run response to an innovation is the same as in the more usual case of a pure random walk technology shock.[6]

It is straightforward to define a competitive equilibrium for this economy. Since there are no distortions we can simply solve the social planner's problem. The solution method is the quadratic approximation described in Hansen and Prescott

[5] Whether the pattern of adoption of technologies is S-shaped at the aggregate level is an open empirical question. This is also similar to the framework considered by Andalfatto and MacDonald (1994).

[6] The reason for assuming the diffusion process is to produce more persistence in the response of output to technology shocks as is found in the data.

Table 1
Calibrated parameters

Preference		Technology		Shock process	
γ	1			ϕ_0	0.1
A	2.47	θ	0.36	ϕ_1	0.2
β	0.99	δ	0.02	ϕ_2	0.4
		g	0.05	$\sigma_\in$	0.01

(1995). Calibration of the model is also straightforward. The calibrated parameters are shown in Table 1.

The simulation procedure is the same as in the previous example. We generate 500 samples of length 251 quarters, and discard the first 100 periods of each sample to mitigate startup effects. We apply the BQ identification strategy to these generated data. Then we average across the impulse responses for each sample.

Fig. 4 shows the BQ identified impulse responses of each of the variables together with the BQ impulse responses from the actual data (of Fig. 1), and the two standard error bands of the latter.[7] The responses of output and hours to a supply (productivity) shock in this generated data are much stronger than in the actual data. This is as expected, since the only shock generating these data is the productivity shock.

It is surprising however that the BQ method still finds short-run effects of demand shocks in these data even though the data generating process is a one shock model. These demand shock effects are smaller and shorter lived than in the US data, but they appear to be significant.

It is useful to consider how this finding is possible given that all the simulated variables are functions of a common shock. Note that the S-shaped diffusion model of (29) follows an MA(4), which is only approximated by the VAR specification that BQ employ. This specification error is a weighted average of past supply shocks, and hence not perfectly correlated with the contemporaneous shock. This leads to differences between the short-run variations in hours and output. The result is something that looks like temporary fluctuations in output. This suggests that auxiliary assumption (c) again plays a role in the misidentification of the impulse responses.

The BQ auxiliary assumptions contribute to the misspecification in another way. Consider this single shock economy as the limit of a sequence of economies in which the variance of the demand shock is going to zero, while the correlation between the demand and supply shocks converges to unity. A correctly specified VAR would consistently estimate the demand innovation variance for these local alternatives. Unfortunately, by imposing orthogonality on the shocks via

[7] The error bands are computed by bootstrapping on the basis of 1000 draws.

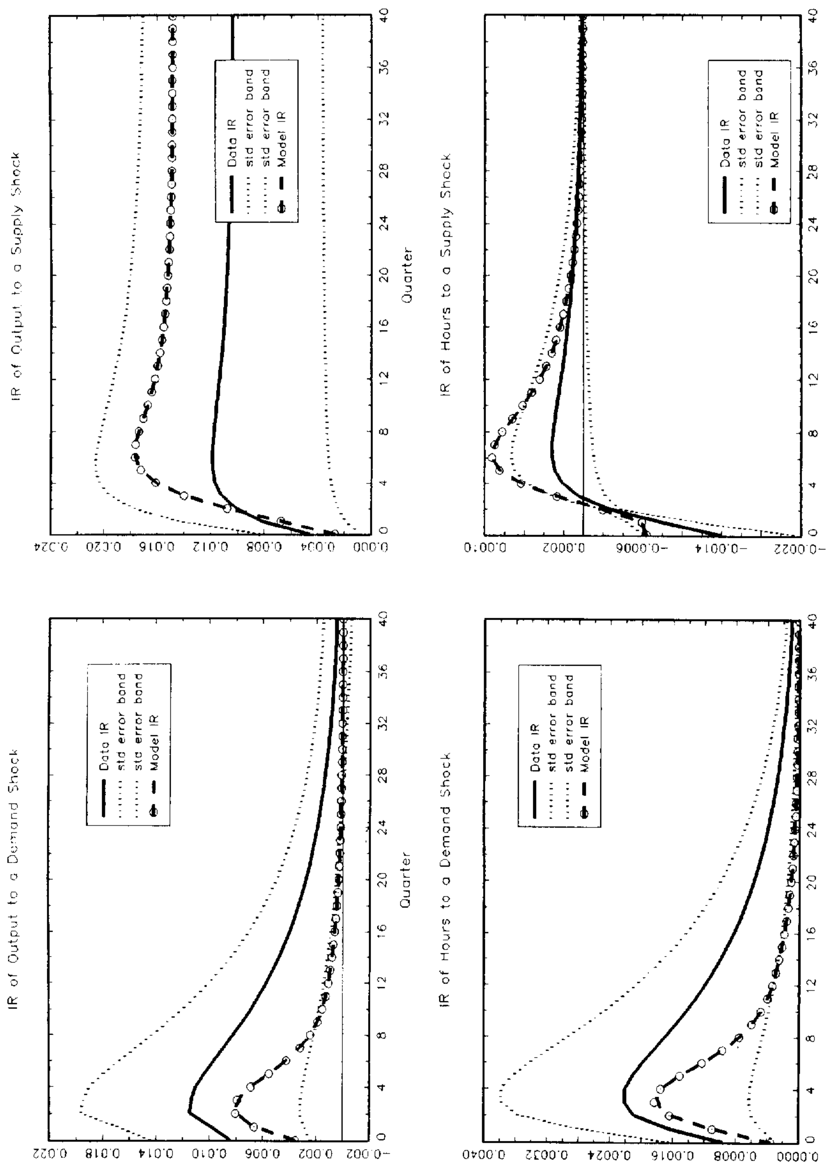

Fig. 4. Impulse responses using generated data: one-shock diffusion economy.

assumption (b), the SVAR procedure cannot detect correlations in the structural disturbances, necessary for correctly identifying these local alternatives.

3.3. Restrictions on short-run dynamics in the SVAR approach

In this section we question the initial starting presumption of the SVAR approach, namely that a reduced form VAR appropriately captures the dynamic relationship between variables of interest. We derive the dynamic specification of a DGE model, and show that it cannot be accommodated by the SVAR approach. This highlights the fact that SVARs impose auxiliary restrictions on the form of short-run dynamics as well.

Again, we start with a basic real business cycle model in which there are both permanent and temporary shocks. The former are shocks to technology, while the latter may be thought of as preference shocks (as in Bencivenga, 1992), or government spending shocks (as in Christiano and Eichenbaum, 1992). We adopt and extend the preference shock interpretation, viewing the preference shocks as the consequence of home production technology shocks as described by Greenwood et al. (1995). This latter interpretation is not the only possibility but it is an appealing one. Assume that the representative household has preferences defined over consumption and leisure but subject to temporary preference shocks:

$$\max \mathrm{E} \sum_{t=1}^{\infty} \beta^t \{U(c_t, h_t \exp z_{1t})\} \tag{30}$$

where z_{1t} is a preference shock. We assume that preference shocks are temporary of the form [8]

$$z_{1t} = \phi_1(L)\varepsilon_{1t}. \tag{31}$$

As before the households rent capital and supply labor to a firm which has access to a constant returns to scale technology of the form,

$$y_t = \exp(z_{2t}) h_t^{\theta} k_t^{1-\theta} \tag{32}$$

where θ is labor's share, and z_{2t} is the productivity shock. We can rewrite the technology as

$$\ln y_t = z_{2t} + \theta \ln h_t + (1-\theta) \ln k_t. \tag{33}$$

We assume that the productivity shocks have a permanent component:

$$\Delta z_{2t} = \phi_2(L)\varepsilon_{2t}. \tag{34}$$

[8] Bencivenga (1992) finds that preference shocks do appear to be temporary. She finds very different persistence of consumption and leisure shocks which suggests a more general specification than the one imagined here.

We further assume that the roots of $\phi_1(L)$, and $\phi_2(L)$ lie outside the unit circle. Once again, it is straightforward to define the competitive equilibrium for this economy and to find a solution to the corresponding social planners problem. From this solution we obtain a set of linear decision rules for hours, output, and capital, which give their values in terms of the state variables of the system.

$$\ln h_t = \beta_{11} z_{1t} + \beta_{12} z_{2t} + \beta_{13} \ln k_t, \tag{35}$$

$$\ln y_t = \beta_{21} z_{1t} + \beta_{22} z_{2t} + \beta_{23} \ln k_t, \tag{36}$$

$$\ln k_{t+1} = \beta_{31} z_{1t} + \beta_{32} z_{2t} + \beta_{33} \ln k_t. \tag{37}$$

To simplify the notation, let

$$X_t = (x_{1t}, x_{2t}, x_{3t})' = (\ln h_t, \ln y_t, \ln k_t)'.$$

Then so long as $|\beta_{33}| < 1$, these linear decision rules can be expressed as

$$x_{1t} = \beta_{11} z_{1t} + \beta_{12} z_{2t} + \beta_{13} x_{3t}, \tag{38}$$

$$x_{2t} = \beta_{21} z_{1t} + \beta_{22} z_{2t} + \beta_{23} x_{3t}, \tag{39}$$

$$x_{3t} = \zeta(L)(\beta_{31} z_{1t} + \beta_{32} z_{2t}), \tag{40}$$

where $\zeta(L) = (1 - \beta_{33} L)^{-1} L$. The capital equation (40), effectively introduces additional stationary, and difference stationary shocks

$$\tilde{z}_{1t} = \zeta(L) z_{1t},$$

$$\Delta \tilde{z}_{2t} = \zeta(L) \Delta z_{2t}.$$

The presence of the two unit root processes z_{2t} and $\tilde{z}_{2t}$ implies that there is a unique cointegrating vector for this trivariate system. The appendix provides common trend, and error correction representations of the system.

While the above constitutes a trivariate cointegrated system, there are only two fundamental shocks to the system, which we summarize in $U_t = (z_{1t}, \Delta z_{2t})'$. This implies that one of the variables of this system is a deterministic function of the other contemporaneous and lagged variables. Since our immediate interest focuses on the relationship between per capita hours and output, we consider capital (x_{3t}) as the completely determined variable. The appendix also shows explicitly how capital is determined in this conditionally nonstochastic sense.

A consequence of this is that we can substitute capital (x_{3t}) out of the model, and focus on the bivariate system in hours and output (x_{1t}, x_{2t}). We then have

$$\begin{aligned}(1 - \beta_{33} L)\Delta x_{jt} = {} & (1 - L)\{\beta_{j1} + (\beta_{j3}\beta_{31} - \beta_{j1}\beta_{33})L\} u_{1t} \\ & + \{\beta_{j2} + (\beta_{j3}\beta_{32} - \beta_{j2}\beta_{33})L\} u_{2t},\end{aligned} \tag{41}$$

Table 2
Tests of difference stationarity[a]

Statistics	Output		Hour 1		Hour 2		Hour 3	
DF	−1.72	(−3.44)	−1.84	(−3.44)	−3.70	(−3.45)	−1.51	(−3.45)
			−1.86	(−2.88)	−3.34	(−2.89)	−1.39	(−2.89)
ADF(1)	−2.45	(−3.44)	−2.45	(−3.44)	−3.16	(−3.45)	−3.45	(−3.45)
			−2.48	(−2.88)	−2.85	(−2.89)	−2.53	(−2.89)
ADF(2)	−2.89	(−3.44)	−2.63	(−3.44)	−2.70	(−3.45)	−2.83	(−3.45)
			−2.66	(−2.88)	−2.40	(−2.89)	−2.15	(−2.89)
ADF(3)	−2.62	(−3.44)	−2.56	(−3.44)	−2.69	(−3.45)	−2.87	(−3.45)
			−2.59	(−2.88)	−2.33	(−2.89)	−2.13	(−2.89)
ADF(4)	−2.79	(−3.44)	−2.75	(−3.44)	−3.19	(−3.45)	−2.49	(−3.45)
			−2.78	(−2.88)	−2.66	(−2.89)	−1.99	(−2.89)

[a] Hour 1 is per-capita hours (1948:1–1991:3). Hours from households survey (LHOURS) is divided by population over age 16(PO16), and then weekly hours are transformed into quarterly assuming that total available hours is 1369. Hour 2 is per-capita hours based on the household survey and adjusted into efficient units by Hansen (1993) (1955:3–1984:1). It is from Burnside (1993). Hour 3 is per-capita hours based on the establishment survey (1955:3–1984:1). It is from Burnside (1993). For each statistic, the statistic when trend is added is in the first row, without trend is in the second row. The numbers in the parentheses are 95% critical values.

for $j = 1, 2$. Thus, beginning from a real business cycle model with permanent and temporary shocks we arrive at a bivariate representation of output and hours in terms of the underlying preference and productivity shocks. As with the BQ specification, this model implies that preference (i.e. demand) shocks have no long-run effect on either hours or output. There are however two features of this representation that distinguish it from the BQ setup. First, the model implies that both output and hours should be difference stationary. Second, the model does not impose the orthogonality of the productivity and preference shocks. Indeed, if these preference shocks appear because they are shocks to home production technology then there is every reason to assume they may be correlated. Permitting nonzero correlation between shocks is much more in the spirit of the Cowles Commission approach, as has been argued by Pagan and Robertson (1994).

The difference stationarity implications of this model can be tested. Table 2 reports the results of testing for stationarity of both output and hours for three different measures of per-capita hours worked. The three hours measures are per-capita hours from the household survey (Hour 1), per-capita hours from the household survey adjusted into efficiency units as described by Hansen (1993) (Hour 2), and per-capita hours from the establishment survey (Hour 3). There is some weak evidence against the difference stationarity of the latter two measures but, in general, these results are not inconsistent with treating hours as difference stationary. Moreover, in contrast to the BQ stationarity assumptions which are not interpreted with respect to fluctuations in technologies and preferences, these difference stationary specifications are consistent with an economy with permanent technology shocks, and temporary preference shocks.

With the removal of capital from the system, cointegration no longer obtains. The conventional approach to modelling such systems relies upon a VAR(p) specification in first differences. Note however that if we treat U_t as white noise, the bivariate system in hours and output of (41) constitutes a VARMA(1,2) system. We reexpress (41) as

$$(1-\beta_{33}L)\begin{pmatrix}\Delta x_{1t}\\ \Delta x_{2t}\end{pmatrix} = B(L)\begin{bmatrix}1-L & 0\\ 0 & 1\end{bmatrix}U_t$$

$$= \begin{pmatrix}b_{11}(L) & b_{12}(L)\\ b_{21}(L) & b_{22}(L)\end{pmatrix}\begin{bmatrix}1-L & 0\\ 0 & 1\end{bmatrix}U_t, \qquad (42)$$

where each $b_{ij}(L)$ is linear in L, and $b_{ii}(L), i=1,2$ are invertible by assumption. Referring to our (31) and (34), we have

$$U_t = \Phi(L)\varepsilon_t = \begin{pmatrix}\phi_1(L) & 0\\ 0 & \phi_2(L)\end{pmatrix}\varepsilon_t,$$

where ε_t is the vector of underlying structural innovations. Assume that the u_{it} follow ARMA(p,q) specifications, so that we may write $\phi_i(L) = \phi_{i1}^{-1}(L)\phi_{i2}(L)$, where $\phi_{i1}(L)$ is order p, and $\phi_{i2}(L)$ is order q. Also note that we can express the inverse of $B(L)$ in terms of it's determinant $|B(L)|$ and cofactor matrix $C_B(L)$: $B(L)^{-1} = C_B(L)/|B(L)|$, where $C_B(L)$ is (2×2), and like $B(L)$, first order (in L) and the scalar $|B(L)|$ is second order. Applying these factorizations to (42) yields:

$$(1-\beta_{33}L)C_B(L)\begin{bmatrix}\phi_{11}(L) & 0\\ 0 & \phi_{21}(L)\end{bmatrix}\begin{pmatrix}\Delta x_{1t}\\ \Delta x_{2t}\end{pmatrix}$$

$$= |B(L)|\begin{bmatrix}1-L & 0\\ 0 & 1\end{bmatrix}\begin{pmatrix}\phi_{12}(L)\varepsilon_{1t}\\ \phi_{22}(L)\varepsilon_{2t}\end{pmatrix}, \qquad (43)$$

i.e., VARMA(p+2, q+3) specification for $(\Delta x_{1t}, \Delta x_{2t})'$.

This discussion emphasizes the fact that DGE models restrict short-run dynamics, as well as specifying trend dependencies. Reduced form VAR estimation cannot capture these VARMA specifications and is incompatible with them. While VARMA models involve additional estimation and identification issues, these complications do not justify *systematically* ignoring these moving average components, as in the SVAR approach. Though the direct contribution of these moving average components to the impulse response functions is finite lived, these terms alter the estimated autoregressive parameters, thereby affecting impulse responses at all horizons. As revealed by (41), these moving average components arise as intrinsic features of dynamic economic models with capital accumulation. Even with white noise structural errors, such models necessarily imply moving average terms.

Thus in general, the SVAR approach rules out compatibility with large classes of economic models not only through its auxiliary trend assumptions (as in the previous examples), but also through its a priori restrictions on short-run dynamics, as embodied in the use of VAR estimation of the reduced form. Like the stationarity assumptions, these restrictions on short-run dynamics are not given any theoretical interpretation or justification. The primary implication of these short-run restrictions is that they preclude viewing the dynamic responses of SVAR models as primarily determined by the data. Without this, the claim that these responses are intrinsic features of the data cannot be justified.

As with the previous examples, we know that this household production model above is 'false' because it is an abstraction. Nevertheless, it provides us with a coherent specification and interpretation of three essential features of the model: difference stationarity assumptions, long-run restrictions, and short-run dynamic specifications. As we have seen, each of these three aspects of model specification plays an important role in identification and interpretation. There is no reason to favor one particular type of assumption with a theoretical foundation, at the expense of the others. In contexts where we are unwilling to impose overidentifying restrictions, having some consistent, theoretical interpretation of *all* of the identifying restrictions seems highly desirable. The fact that the household production model in particular, and DGE models in general, provide a theoretical foundation for all three model features does not imply that those assumptions are more restrictive, or more likely to be false, than those imposed by SVAR methods. Rather it simply implies that the DGE-based assumptions rest upon a specific economic interpretation, whereas most SVAR-based assumptions do not.

Taken together these examples suggest that we cannot have much confidence in the robustness of the empirical findings from SVAR identification and estimation strategies. Instead, SVARs seem more useful as ways of interpreting the data from the perspective of particular, fully specified, economic models.

4. Identification failures

We have argued that all identifying restrictions, whether they be long run, short run, or orthogonality restrictions impose significant structure on a model, and consequently, should be firmly grounded in an economic model. Any such restriction implies the use of a corresponding instrumental variable (see Hausman and Taylor (1983)). Consequently, our examples above make clear the importance of deriving instrumental variables from economic theory. Seemingly innocuous identifying restrictions implicitly define instrumental variables which *fail* to be orthogonal to the true structural errors underlying the data. There is another aspect of these restrictions that should concern us. If the implied instruments turn out to be poorly correlated with their associated, endogenous, explanatory variables, we encounter an *identification failure* in the sense of Phillips (1989). Related ideas appear in recent papers by Pagan and Jung (1993), Staiger and Stock (1994), and Wang and Zivot (1996), on instrumental variables estimation with weak instruments.

4.1. Instrumental variable estimation

Identification failure in SVAR models is extensively discussed in a recent paper by Sarte (1994). The way to understand identification failure in this setting is to think of estimating our original structural VAR via instrumental variables. Since this is a simultaneous equation system the estimators can be given an instrumental variables interpretation. Consider the VAR in output and hours that we motivated on the basis of a two shock real business cycle model:

$$A(L)\begin{pmatrix} \Delta \ln h_t \\ \Delta \ln y_t \end{pmatrix} = \varepsilon_t. \tag{44}$$

If we do not impose the restriction that the preference shocks are temporary, the moving average form of the model is

$$\begin{pmatrix} \Delta \ln h_t \\ \Delta \ln y_t \end{pmatrix} = \begin{pmatrix} \phi_{h1}(L) & \phi_{h2}(L) \\ \phi_{y1}(L) & \phi_{y2}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \equiv C(L)\varepsilon_t. \tag{45}$$

The long-run responses are the elements of $C(1)$. Inverting these yields the elements of $A(1)$

$$\begin{pmatrix} \phi_{h1}(1) & \phi_{h2}(1) \\ \phi_{y1}(1) & \phi_{y2}(1) \end{pmatrix}^{-1} = \begin{pmatrix} \theta_{hh} & \theta_{hy} \\ \theta_{yh} & \theta_{yy} \end{pmatrix}. \tag{46}$$

Thus, the specification that preference shocks have no long-term effect on output amounts to specifying that $\phi_{y1}(1) = 0$, which in turn implies $\theta_{yh} = 0$. It is also important to see that pinning down θ_{yh} to any particular value serves to identify the system and thus the shocks in the way that we described in the earlier section. Sarte's analysis is focused on the identification restriction used by Shapiro and Watson (1988). They impose the restriction that $\theta_{hy} = 0$ which implies that technology shocks have no long-run effect on labor supply. This interpretation is consistent with our earlier interpretation of ε_1 as a preference or government spending shock. They also find, with this restriction, that technology shocks account for only a small fraction (about 35%) of the variation in output.

To see the instrumental variables interpretation of the estimation problem rewrite (44) as

$$\begin{aligned} \Delta \ln h_t &= \sum_{j=0}^{k} \alpha_{hy,j} \Delta \ln y_{t-j} + \sum_{j=1}^{k} \alpha_{hh,j} \Delta \ln h_{t-j} + \varepsilon_{1t}, \\ \Delta \ln y_t &= \sum_{j=1}^{k} \alpha_{yy,j} \Delta \ln y_{t-j} + \sum_{j=0}^{k} \alpha_{yh,j} \Delta \ln h_{t-j} + \varepsilon_{2t}. \end{aligned} \tag{47}$$

Consistent estimation of these equations is not possible without further restrictions because of the presence of $\Delta \ln y_t$ on the right-hand side of the first equation and the presence of $\Delta \ln h_t$ on the right-hand side of the second equation. Suppose, however that we can impose the restriction

$$\sum_{j=0}^{k} \alpha_{hy,j} = \theta_{hy}, \tag{48}$$

where θ_{hy} could be zero, or anything else, so long as it is known *a priori*. This allows us to eliminate an exogenous variable from the first (hours) equation in (47), and hence to use it as an instrument for $\Delta \ln y_t$. We can now rewrite the first equation of (47) as:

$$\begin{aligned} \Delta \ln h_t - \theta_{hy} \Delta \ln y_{t-1} &= \alpha_{hy,0} \Delta^2 \ln y_t + \sum_{j=2}^{k} \alpha_{hy,j} (\Delta \ln y_{t-j} - \Delta \ln y_{t-1}) \\ &\quad + \sum_{j=1}^{k} \alpha_{hh,j} \Delta \ln h_{t-j} + \varepsilon_{1t}. \end{aligned} \tag{49}$$

This equation can be estimated consistently using $\Delta \ln y_{t-1}$ as an instrument for $\Delta^2 \ln y_t$. To identify the second (output) equation (47), Shapiro and Watson impose orthogonality between ε_1 and ε_2. Then the fitted residuals from the hours equation $\hat{\varepsilon}_1 = e_1$ can be used as an instrument for $\Delta \ln h_t$ in the output equation. The important thing to recognize is that these residuals are related to θ_{hy} so we denote them as $e_1(\theta_{hy})$.

Sarte (1994) shows that some identifying restrictions may not yield good instrumental variables. Thus, the 'strength' of the instrument should be examined. What is required for $e_1(\theta_{hy})$ to be a good instrument is that it be (a) uncorrelated with ε_{2t}, (which is guaranteed by the construction of the problem), and (b) highly correlated with the variable it is instrumenting for, in this case $\Delta \ln h_t$.

4.1.1. Weak instruments

Clearly, since $e_1(\theta_{hy})$ depends explicitly on the parameter used to achieve identification, its strength as an instrument can vary with the value of the identifying parameter. Sarte uses a two stage least squares interpretation of the estimator to show that, if

$$e_1(\theta_{hy})'\Delta \ln h_t = 0,$$

then the estimates may not be computed. This is an identification failure. Moreover, drawing on results in Phillips (1989) and Pagan and Jung (1993), Sarte shows that the coefficients of the remaining regressors begin to lose their conventional asymptotic properties as $\mathrm{E}[\varepsilon_{1t}(\theta_{hy})'\Delta \ln h_t] \to 0$. He argues that empirical evidence that identification failures may be a problem affecting the distribution of the remaining parameters is provided by looking at the contribution to R^2 from adding $\varepsilon_{1t}(\theta_{hy})$ to the set of instruments.

This notion of identification failure also provides us with another tool for looking and the robustness of conclusions drawn from estimated SVARs. We can assess the fragility of conclusions by seeing how they vary with the values assigned to identifying parameters. Two examples will make this clear.

4.1.2. Hours and output

The first example is one we have been considering throughout, the bivariate SVAR between hours and output. Here, however, we follow the example in Sarte (1994) and use the long-run restrictions imposed by Shapiro and Watson (1988). Fig. 5 shows the correlation between the estimated instrument e_1 and $\Delta \ln h_t$ as the value of θ_{hy} is varied. It also shows the contribution to R^2 from the inclusion of the instrumented term. For this example we can see that the correlation is pretty high for $\theta_{hy} = 0$, but it rises steadily until it reaches a value of $\theta_{hy} = 0.65$. At this point the instruments seem the strongest. It appears that a clear identification failure occurs in this problem as the value of θ_{hy} gets close to 0.8.

Fig. 6 shows the impulse response functions for output and hours as a function of the identifying parameters. The striking thing about these pictures is that the estimated response of output to technology shocks depends crucially on the identification. Stated differently, the qualitative conclusions about the importance of productivity shocks for output fluctuations at the business cycle frequencies are not at all robust to changes in the identifying restrictions. This is further verification of the findings in section three.

4.1.3. Money and output

The second example to be considered is a structural VAR between money and output. Such relationships have been extensively analyzed by a number of authors, including, Fisher and Seater (1993) and King and Watson (1992,1993). These authors impose different identifying restrictions on bivariate money–output VARs and examine the estimated impulse responses. There are many contentious issues in the analysis of these types of VARs, including the appropriate choice

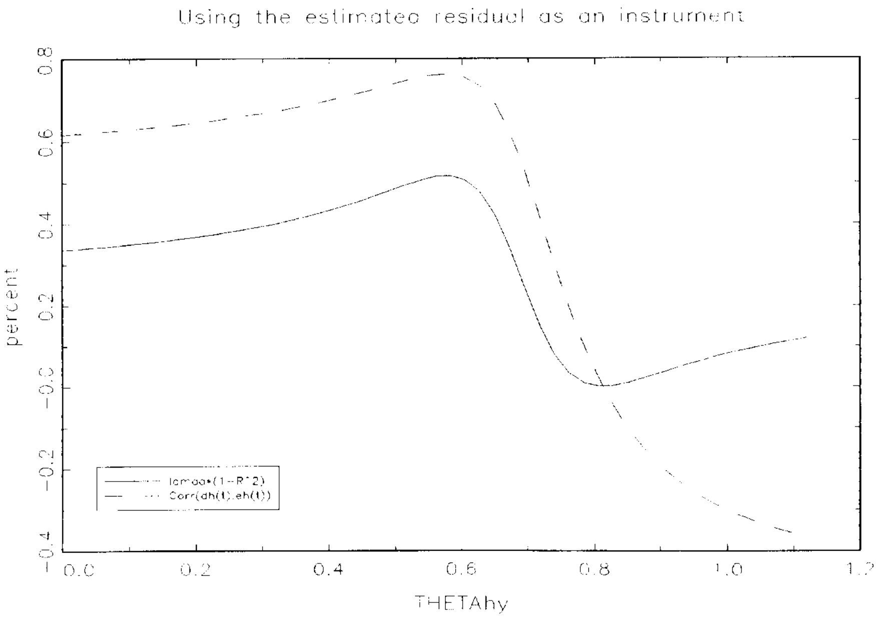

Fig. 5. Variation in instrument relevance with long-run hours effect on output.

of monetary aggregate, the role of aggregation and the role of common trends. We sidestep all of these issues and focus again on the idea of robustness of the empirical conclusions.

A common strategy in the estimation of bi-variate money–output VARs is to impose an identifying restriction that sets the long-run response of output to monetary shocks to be zero. This is indeed a property of some monetary models so it is a restriction that can be motivated by strong theory. Cooley and Hansen (1995) consider a series of monetary models all of which have the property that money is not neutral. Here we adopt a framework that is broadly consistent with monetary business cycle models and look for evidence of identification failures or fragility in the implied response of output to monetary shocks for various values of the identifying restriction.

Let the law of motion for technology shocks be given by

$$z_t = z_{t-1} + \rho_z(L)\varepsilon_t^z, \tag{50}$$

and the law of motion for monetary shocks be given by

$$u_t = u_{t-1} + \rho_m(L)\varepsilon_t^m. \tag{51}$$

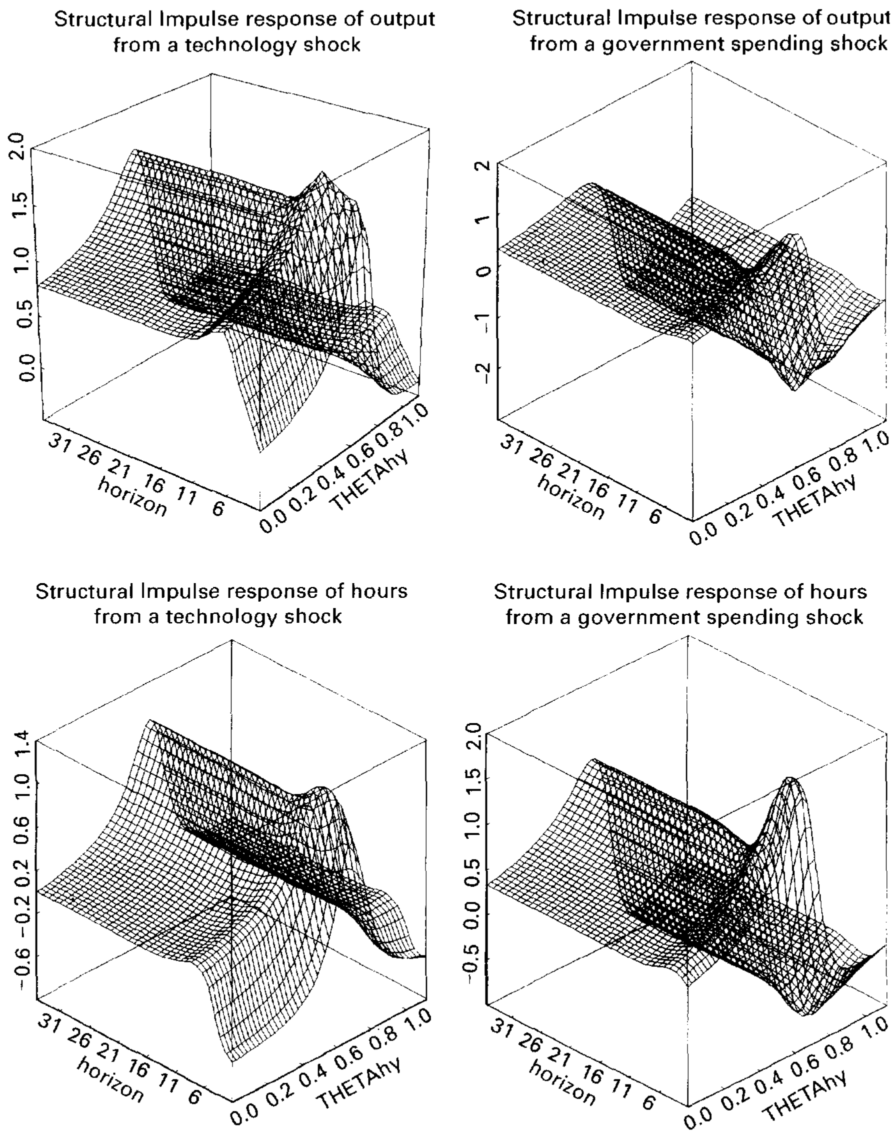

Fig. 6. Impulse responses for hours–output VAR.

The decision rules (the reduced form) for this economy will have the general form

$$\begin{aligned} \Delta \ln m_t &= \phi_{ym}(L)\varepsilon_t^z + \phi_{mm}(L)\varepsilon_t^m + \text{temporary shocks}, \\ \Delta \ln y_t &= \phi_{yy}(L)\varepsilon_t^z + \phi_{ym}(L)\varepsilon_t + \text{temporary shocks}. \end{aligned} \tag{52}$$

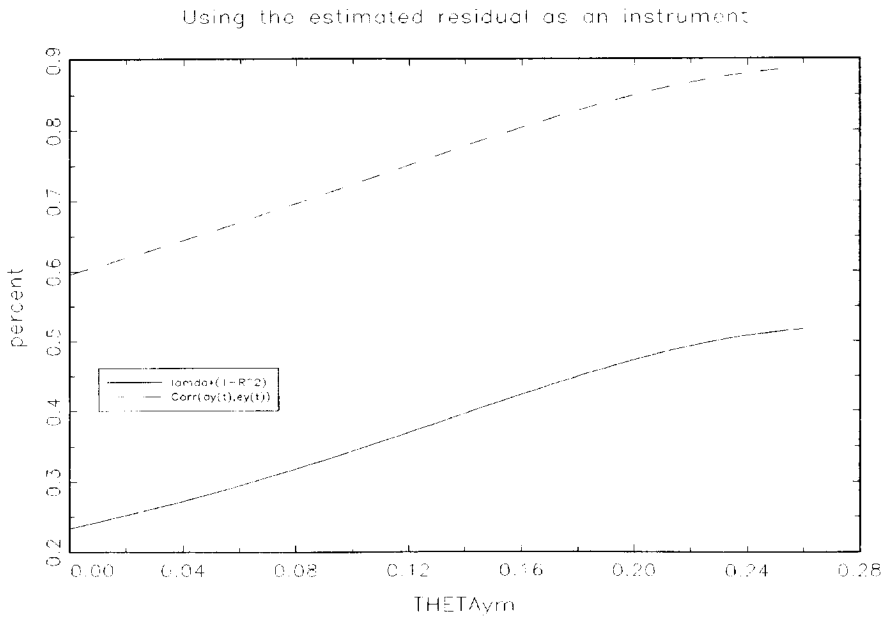

Fig. 7. Variation in instrument relevance with long-run money effect on output.

For convenience we assume all temporary shocks are zero and rewrite the system as

$$\begin{pmatrix} \Delta \ln m_t \\ \Delta \ln y_t \end{pmatrix} = \begin{pmatrix} \phi_{mm}(L) & \phi_{ym}(L) \\ \phi_{ym}(L) & \phi_{yy}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^z \end{pmatrix}. \tag{53}$$

As is customary we will assume that ε_t^m and ε_t^z are independent of one another although this identifying restriction is *not* a property of any particular model. It is an assumption made about the forcing processes. Consider again the inverse of the matrix of long-run multipliers given by the representation

$$\begin{pmatrix} \theta_{mm} & \theta_{my} \\ \theta_{ym} & \theta_{yy} \end{pmatrix} \begin{pmatrix} \phi_{mm}(1) & \phi_{my}(1) \\ \phi_{ym}(1) & \phi_{yy}(1) \end{pmatrix}^{-1}. \tag{54}$$

The restriction $\theta_{ym} = 0$ implies that money is neutral in the long-run. Once again, using the instrumental variable interpretation we can vary the parameter used to achieve identification and look at the strength of the instrument and look for evidence of identification failure.

Fig. 7 shows the correlation of the estimated residual and $\Delta \ln y_t$, the variable for which it is an instrument, for various values of the identifying parameter.[9]

[9] The corresponding values of θ_{ym} were determined by solving for the reduced form VARs implied by the decision rules in the economies studied by Cooley and Hansen (1995). These imply long-run responses for output and money growth given the calibration in those papers.

As we can see, the strength of the instrument increases as the long-run neutrality restriction is relaxed in favor of models where money shocks have a more significant long-run effect on output. There is no obvious evidence of identification failure here, since the R^2 and the correlation are well behaved for all values of θ_{ym}. Thus although the identification restriction may lead to different conclusions about the long-term effect of money on output, the instruments are well behaved.

Fig. 8 shows the family of impulse response functions that emerge as we vary the identifying restriction. The impulse response functions for output appear to be well behaved. What is most remarkable is that pattern of the responses is largely invariant to the identifying restrictions. The hump-shaped response of output to technology shocks is largely invariant across identification regimes. The shape of the response of output to monetary shocks is similarly robust. An examination of the confidence bands for the impulse response functions at the extremes suggests that most of the variation in output is accounted for by variation in productivity while most of the variation in money is accounted for by monetary shocks. The one empirical finding that does appear fragile here is the liquidity effect of technology shocks. These impulse responses vary from positive to negative across the range of identifying values of θ_{ym}.

One finding that seems consistent across these many experiments whether they are money–output or output–hours VARs is the response of output to technology shocks. This is quite consistent with the representation of the decision rules that one would derive from what we have called strong theory. The decision rules show the evolution of each variable as function of realizations of the state variables. In system like that the evolution of output would be expected to be invariant to the variables it is paired with in a bi-variate representation. These results seem consistent with that view.

5. Concluding comments

Structural vector autoregressions employ a mixture of theoretical and atheoretical restrictions for identifying models. The plausibility of these atheoretical restrictions may be addressed via pretesting, but they are uninterpretable without a fully articulated economic model. Structural VARs offer a simple empirical methodology for finding the contributions of various shocks to the fluctuations in output. They also offer a means of describing a richer set of stylized facts that students of business cycles can try to incorporate in explicit theories of the cycle. Very simple structural VARs and dynamic general equilibrium analysis have reached very different conclusions about the relative importance of technology and demand shocks for the fluctuations in output at business cycle frequencies. In general the SVAR methodology ascribes a much larger role for demand or transitory shocks than do identification strategies based on richer theory. An

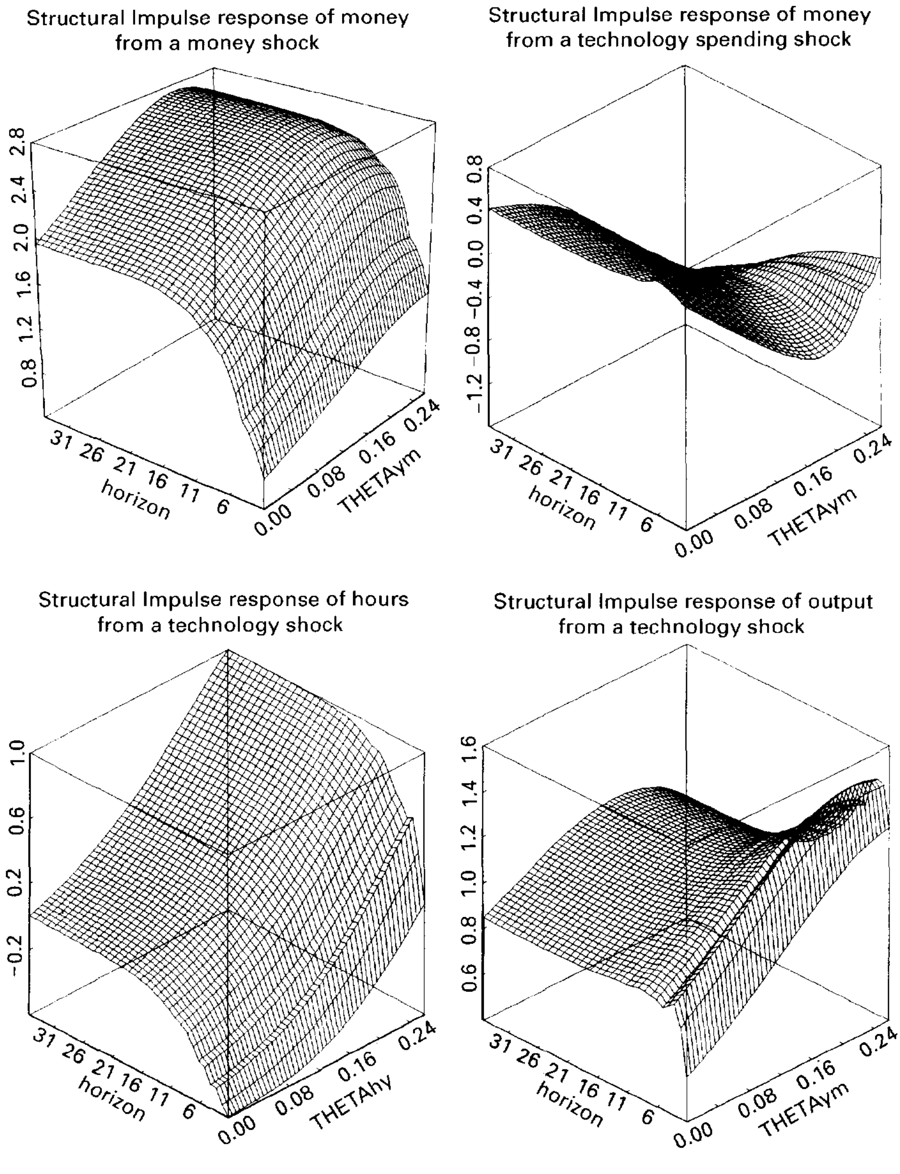

Fig. 8. Impulse responses for money–output VARs.

exception to this is the more recent work by Leeper and Sims (1994) that has looked at a class of structural VARs that are more elaborate and can be derived from explicit theoretical models. The findings of this paper suggest that conclusions about the importance of technology and other shocks based on simple

SVARs are certainly not invariant to the identifying assumptions and may not be very reliable as vehicles for identifying the relative importance of shocks. At the same time certain empirical regularities in the data are revealed by SVARs and appear robust. The hump shaped response of output to technology shocks, for example, seems quite robust across bi-variate models and identifying restrictions.

Appendix

In this appendix, we construct the error correction representation of the household production economy discussed in Section 3.3 above. We also show how to represent capital in the model in terms of hours and output. Let $X_t = (\ln h_t, \ln y_t, \ln k_t)'$. The common stochastic trends [10] of the system are apparent if we rewrite (38)–(40) as

$$\begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{13}\beta_{31} \\ \beta_{21} & \beta_{23}\beta_{31} \\ 0 & \beta_{31} \end{pmatrix} \begin{pmatrix} z_{1t} \\ \tilde{z}_{1t} \end{pmatrix} + \begin{pmatrix} \beta_{12} & \beta_{13}\beta_{32} \\ \beta_{22} & \beta_{23}\beta_{32} \\ 0 & \beta_{32} \end{pmatrix} \begin{pmatrix} z_{2t} \\ \tilde{z}_{2t} \end{pmatrix}. \tag{A.1}$$

The cointegrating vector can be expressed as:

$$\gamma = [-1,\ \beta_{22}^{-1}\beta_{12},\ (\beta_{13} - \beta_{22}^{-1}\beta_{12}\beta_{23})]'.$$

Next, let $U_t = (z_{1t}, \Delta z_{2t})'$. After some algebra, we can express the system above in an error-correction form as [11]

$$\Delta X_t = A\Delta X_{t-1} + a\gamma' X_{t-1} + D(L)U_t, \tag{A.2}$$

where $D(L)U_t = D_0 U_t + D_1 U_{t-1} + D_2 U_{t-2}$, A is (3×3), and a is (3×1). This notation is meant to indicate that the error correction model (ECM) considered here has VMA(2) components in U_t, as well as VAR(1) components. Then from (31) and (34)

$$U_t = \begin{bmatrix} \varphi_1(L) & 0 \\ 0 & \varphi_2(L) \end{bmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix},$$

so that the order of the VMA component is two greater than the maximum lag polynomial order of $\varphi_1(L)$ and $\varphi_2(L)$.

To see how capital is determined, let $W_t = \Delta X_t - A\Delta X_{t-1} - a\gamma' X_{t-1}$. Then isolate the capital component via

$$W_t = \begin{pmatrix} \tilde{W}_t \\ w_{3t} \end{pmatrix} = D(L)U_t = \begin{bmatrix} D_{12}(L) \\ d_3(L)' \end{bmatrix} U_t.$$

[10] See Stock and Watson (1988).

[11] See Engle and Granger (1987) and Davidson et al. (1978).

This allows us to express U_t as

$$U_t = D_{12}^{-1}(L)\tilde{W}_t,$$

and

$$w_{3t} = d_3(L)' D_{12}^{-1}(L)\tilde{W}_t.$$

This implies that we can express $x_{3t} = \ln k_t$ completely in terms of contemporaneous and lagged values of ΔX_t, and in terms of lags of the cointegrating residual $\gamma' X_t$.

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