Econometrics 715

Problem Set 1, Fall 2017

Due: Wednesday, Oct 4

This problem set includes both numerical and theoretical exercises.

Numerical Exercises:

The file ps1.dat is a text file with 1000 rows and 4 columns. Each row is an independent observation. The four variables are $(y_i, x_{1i}, x_{2i}, x_{3i})$. We will be estimating the parameters of the following equation

$$y_i = \theta_1 x_{1i} + \theta_2 x_{2i} + \theta_3 \exp(\theta_4 x_{3i}) + \theta_5 + e_i$$

Each estimator takes the form

$$\hat{\theta} = \operatorname{argmin} S_n(\theta)$$

where

$$S_n(\theta) = \frac{1}{n} \sum_{i=1}^{n} m(y_i, x_i, \theta)$$

for a different functions $m(y, x, \theta)$.

Do your numerical optimzation in Matlab, R, or your package of choice. Use the standard optimation software, but don't use packages specific for the problems. In each case, report the estimates $\hat{\theta}$ obtained, and (briefly) the method used for optimization. Include your code.

1. Nonlinear-least-squares (NLLS).

$$m(y, x, \theta) = (y - \theta_1 x_1 - \theta_2 x_2 - \theta_3 \exp(\theta_4 x_3) - \theta_5)^2$$

2. Huber criterion

$$\begin{aligned} m(y, x, \theta) &= h(y - \theta_1 x_1 - \theta_2 x_2 - \theta_3 \exp(\theta_4 x_3) - \theta_5) \\ h(u) &= \begin{cases} \frac{1}{2}u^2 & |u| \leq 1 \\ \\ |u| - \frac{1}{2} & |u| > 1 \end{cases} \end{aligned}$$

3. L^4

$$m(y,x,\theta) = (y - \theta_1 x_1 - \theta_2 x_2 - \theta_3 \exp(\theta_4 x_3) - \theta_5)^4$$

4. student t likelihood

   For this application, the model is that the error e_i takes the form $e_i = \theta_6 u_i$ where u_i has a student t density with degree-of-freedom θ_7. Thus u has the density

$$f(u) = \frac{\Gamma\left(\frac{\theta_7+1}{2}\right)}{\sqrt{\theta_7 \pi}\,\Gamma\left(\frac{\theta_7}{2}\right)} \left(1 + \frac{u^2}{\theta_7}\right)^{-(\theta_7+1)/2}$$

   Derive the conditional density for y given x and the conditional log-likelihood function for y. Write out $m(y,x,\theta)$ (the negative log-density). The parameters are $\theta = (\theta_1, \theta_2, ..., \theta_7)$.

Theoretical Exercises:

1. Take the NLLS estimator given above. Assume that $E(e_i|x_i) = 0$, so the model is correctly specified. Provide a set of conditions such that the NLLS estimator is consistent for the true parameter value. You may assume that the parameter space Θ is compact. Your conditions should be stated in terms of the distributions (or moments) of the observations and parameter values. (That is, they should be relatively primitive, not high-level.)

2. Take the Huber estimator. Do not assume $E(e_i|x_i) = 0$. Try to make a characterization of the pseudo-true value θ_0 for this problem.

3. Continuing with the Huber estimator, provide a set of conditions such that the Huber estimator is consistent for the pseudo-true value.