Econometrics 710
Midterm Exam
March 4, 1999

1. Let Y be $n \times 1$, X be $n \times k$ (rank k), and $Z = XB$, where B is $k \times k$ with rank k. Let $(\hat{\beta}, \hat{e})$ denote the OLS coefficients and residuals from regression of y on X. Similarly, let $(\tilde{\beta}, \tilde{e})$ denote these from OLS regression of y on Z. Find the relationship between $\hat{\beta}$ and $\tilde{\beta}$, and the relationship between $\hat{e}$ and $\tilde{e}$.

2. Let Y be $n \times 1$, X be $n \times k$ (rank k). Suppose that $E(Y \mid X) = X\beta$. Define the *ridge regression* estimator
$$\hat{\beta} = \left(X'X + \lambda I_k\right)^{-1} (X'Y)$$
where $\lambda > 0$ is a fixed constant. Find $E\left(\hat{\beta} \mid X\right)$. Is $\hat{\beta}$ biased for β?

3. Of the random variables (Y^*, Y, X) only the pair (Y, X) are observed. (In this case, we say that Y^* is a *latent* variable.) Suppose $E(Y^* \mid X) = X\beta$ and $Y = Y^* + u$, where u is a measurement error satisfying $E(u \mid Y^*, X) = 0$. Let $\hat{\beta}$ denote the OLS coefficient from the regression of Y on X.

   (a) Find $E(Y \mid X)$.
   (b) Is $\hat{\beta}$ consistent for β as $n \to \infty$?
   (c) Find the asymptotic distribution of $\sqrt{n}\left(\hat{\beta} - \beta\right)$ as $n \to \infty$.

4. You run an OLS regression of the form $\hat{y} = \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$, where y=executive salaries on x_1 =sales and x_2=profits, across a sample of 102 firms. The results are

$$\underset{}{\hat{y} =} \underset{(.83)}{0.50}\, x_1 + \underset{(.83)}{0.40}\, x_2, \qquad X'X = \begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}, \qquad \hat{V} = \begin{pmatrix} .7 & -.5 \\ -.5 & .7 \end{pmatrix},$$

   (All variables are expressed as deviations about their means. The numbers in parenthesis are standard errors. $\hat{V}$ is the estimated covariance matrix for $\hat{\beta}$)

   (a) Someone suggests that the high collinearity between sales and profits has prevented precise estimation of the parameters. Does this seem reasonable, based on the evidence presented? (Hint: I am not expecting anything detailed here.)
   (b) Someone else suggests a method to eliminate this problem. First, regress profits on sales, and obtain the residuals x_2^*. Second, regress y on x_1 and x_2^* to estimate the salary function. Denote the results of the second step by $\tilde{y} = \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2$. Find an expression for x_2^*.
   (c) Calculate $\tilde{\beta}_1$ and $\tilde{\beta}_2$.
   (d) Calculate their conventional standard errors.
   (e) Evaluate this proposal as a device to eliminate (or reduce) collinearity.