Econometrics 710
Midterm Exam
March 16, 2017

1. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \end{aligned}$$

with parameter of interest $\theta = R'\beta$ with R $k \times 1$. Let $\widehat{\beta}$ be the least-squares estimate and $\widehat{V}_{\widehat{\beta}}$ its variance estimate.

   (a) Write down $\widehat{C}$, the 95% asymptotic confidence interval for θ, in terms of $\widehat{\beta}$, $\widehat{V}_{\widehat{\beta}}$, R, and $z = 1.96$ (the 97.5% quantile of $N(0,1)$).

   (b) Show that the decision "Reject H_0 if $\theta_0 \notin \widehat{C}$" is an asymptotic 5% test of $H_0 : \theta = \theta_0$.

2. Consider the least-squares regression estimates

$$y_i = x_{1i}\widehat{\beta}_1 + x_{2i}\widehat{\beta}_2 + \widehat{e}_i$$

and the "one regressor at a time" regression estimates

$$y_i = \widetilde{\beta}_1 x_{1i} + \widetilde{e}_{1i} \qquad y_i = \widetilde{\beta}_2 x_{2i} + \widetilde{e}_{2i}$$

Under what condition does $\widetilde{\beta}_1 = \widehat{\beta}_1$ and $\widetilde{\beta}_2 = \widehat{\beta}_2$?

3. Take a regression model with i.i.d. observations (y_i, x_i) and scalar x_i

$$y_i = x_i\beta + e_i$$

$$E(e_i \mid x_i) = 0$$

The parameter of interest is $\theta = \beta^2$. Consider the OLS estimates $\widehat{\beta}$ and $\widehat{\theta} = \widehat{\beta}^2$

   (a) Find $E(\widehat{\theta}|X)$ using our knowledge of $E(\widehat{\beta}|X)$ and $V_{\widehat{\beta}} = \text{var}(\widehat{\beta}|X)$. Is $\widehat{\theta}$ biased for θ?

   (b) Suggest an (approximate) biased-corrected estimator $\widehat{\theta}^*$ using an estimate $\widehat{V}_{\widehat{\beta}}$ for $V_{\widehat{\beta}}$.

   (c) For $\widehat{\theta}^*$ to be potentially unbiased, which estimate of $V_{\widehat{\beta}}$ is most appropriate?
   Under which conditions is $\widehat{\theta}^*$ unbiased?

4. Take a regression model with i.i.d. observations (y_i, x_i) and scalar x_i

$$y_i = x_i\beta + e_i$$

$$E(e_i \mid x_i) = 0$$

$$\Omega = E\left(x_i^2 e_i^2\right)$$

Let $\widehat{\beta}$ be the OLS estimate of β with residuals $\widehat{e}_i = y_i - x_i\widehat{\beta}$. Consider the estimates of Ω

$$\begin{aligned} \widetilde{\Omega} &= \frac{1}{n}\sum_{i=1}^n x_i^2 e_i^2 \\ \widehat{\Omega} &= \frac{1}{n}\sum_{i=1}^n x_i^2 \widehat{e}_i^2 \end{aligned}$$

   (a) Find the asymptotic distribution of $\sqrt{n}\left(\widetilde{\Omega} - \Omega\right)$ as $n \to \infty$.

   (b) Find the asymptotic distribution of $\sqrt{n}\left(\widehat{\Omega} - \Omega\right)$ as $n \to \infty$.

   (c) How do you use the regression assumption $E(e_i \mid x_i) = 0$ in your answer to (b)?