Econometrics 710
Midterm Exam
March 11, 2014

1. Consider an iid sample $\{y_i, x_i\}$ $i = 1, ..., n$ where $x_i$ is $k \times 1$. Assume the linear conditional expectation model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

Assume that $n^{-1}X'X = I_k$ (orthogonal regressors). Consider the OLS estimator $\widehat{\beta}$ for $\beta$.

   (a) Find $V_{\widehat{\beta}} = \text{var}(\widehat{\beta})$

   (b) In general, are $\widehat{\beta}_j$ and $\widehat{\beta}_\ell$ for $j \neq \ell$ correlated or uncorrelated?

   (c) Find a sufficient condition so that $\widehat{\beta}_j$ and $\widehat{\beta}_\ell$ for $j \neq \ell$ are uncorrelated.

2. Consider an iid sample $\{y_i, x_i\}$ $i = 1, ..., n$ where $y_i$ and $x_i$ are scalar. Consider the reverse projection model

$$\begin{aligned} x_i &= y_i\gamma + u_i \\ E(y_i u_i) &= 0 \end{aligned}$$

and define the parameter of interest as $\theta = 1/\gamma$

   (a) Propose an estimator $\widehat{\gamma}$ of $\gamma$. (You do not need to appeal to an efficiency justification.)

   (b) Propose an estimator $\widehat{\theta}$ of $\theta$. (You do not need to appeal to an efficiency justification.)

   (c) Find the asymptotic distribution of $\widehat{\theta}$.

   (d) Find an asymptotic standard error for $\widehat{\theta}$.

3. Suppose you have two independent samples

$$y_{1i} = x_{1i}'\beta_1 + e_{1i}$$

and

$$y_{2i} = x_{2i}'\beta_2 + e_{2i}$$

both of sample size $n$, and both $x_{1i}$ and $x_{2i}$ are $k \times 1$. You estimate $\beta_1$ and $\beta_2$ by OLS, $\widehat{\beta}_1$ and $\widehat{\beta}_2$, say, with asymptotic covariance matrix estimators $\widehat{V}_{\beta_1}$ and $\widehat{V}_{\beta_2}$ (which are consistent for the asymptotic covariance matrices $V_{\beta_1}$ and $V_{\beta_2}$). Consider efficient minimimum distance estimation under the restriction $\beta_1 = \beta_2$.

   (a) Find the estimator $\widetilde{\beta}$ of $\beta = \beta_1 = \beta_2$

   (b) Find the asymptotic distribution of $\widetilde{\beta}$.

   (c) Extra and Very Optional: (Only attempt if you have time.) How would you approach the problem if the sample sizes are different, say $n_1$ and $n_2$?