1. Consider an iid sample \(\{y_i, x_i\}_{i=1}^n\) where \(x_i\) is \(k \times 1\). Assume the linear conditional expectation model

\[
y_i = x_i' \beta + e_i, \quad E(e_i | x_i) = 0
\]

Assume that \(n^{-1}X'X = I_k\) (orthogonal regressors). Consider the OLS estimator \(\hat{\beta}\) for \(\beta\).

(a) Find \(\text{var}(\hat{\beta})\)

(b) In general, are \(\hat{\beta}_j\) and \(\hat{\beta}_\ell\) for \(j \neq \ell\) correlated or uncorrelated?

(c) Find a sufficient condition so that \(\hat{\beta}_j\) and \(\hat{\beta}_\ell\) for \(j \neq \ell\) are uncorrelated.

2. Consider an iid sample \(\{y_i, x_i\}_{i=1}^n\) where \(y_i\) and \(x_i\) are scalar. Consider the reverse projection model

\[
x_i = y_i + u_i, \quad E(y_i, u_i) = 0
\]

and define the parameter of interest as \(\theta = 1/\gamma\)

(a) Propose an estimator \(\hat{\gamma}\) of \(\gamma\). (You do not need to appeal to an efficiency justification.)

(b) Propose an estimator \(\hat{\theta}\) of \(\theta\). (You do not need to appeal to an efficiency justification.)

(c) Find the asymptotic distribution of \(\hat{\theta}\).

(d) Find an asymptotic standard error for \(\hat{\theta}\).

3. Suppose you have two independent samples

\[
y_{1i} = x_{1i}' \beta_1 + e_{1i}
\]

and

\[
y_{2i} = x_{2i}' \beta_2 + e_{2i}
\]

both of sample size \(n\), and both \(x_{1i}\) and \(x_{2i}\) are \(k \times 1\). You estimate \(\beta_1\) and \(\beta_2\) by OLS, \(\hat{\beta}_1\) and \(\hat{\beta}_2\), say, with asymptotic covariance matrix estimators \(\hat{\Sigma}_1\) and \(\hat{\Sigma}_2\) (which are consistent for the asymptotic covariance matrices \(V_{\beta_1}\) and \(V_{\beta_2}\)). Consider efficient minimum distance estimation under the restriction \(\beta_1 = \beta_2\).

(a) Find the estimator \(\hat{\beta}\) of \(\beta = \beta_1 = \beta_2\)

(b) Find the asymptotic distribution of \(\hat{\beta}\).

(c) Extra and Very Optional: (Only attempt if you have time.) How would you approach the problem if the sample sizes are different, say \(n_1\) and \(n_2\)?