Econometrics 710
Midterm Exam
March 22, 2012

1. Take the linear model with restrictions

$$\begin{aligned} y_i &= \boldsymbol{x}_i'\boldsymbol{\beta} + e_i \\ \mathbb{E}(\boldsymbol{x}_i e_i) &= \boldsymbol{0} \\ \boldsymbol{R}'\boldsymbol{\beta} &= \boldsymbol{c} \end{aligned}$$

with n observations. Consider three estimators for $\boldsymbol{\beta}$

- $\widehat{\boldsymbol{\beta}}$, the unconstrained least squares estimator
- $\widetilde{\boldsymbol{\beta}}$, the constrained least squares estimator
- $\overline{\boldsymbol{\beta}}$, the constrained efficient minimum distance estimator

For each estimator, define the residuals $\widehat{e}_i = y_i - \boldsymbol{x}_i'\widehat{\boldsymbol{\beta}}$, $\widetilde{e}_i = y_i - \boldsymbol{x}_i'\widetilde{\boldsymbol{\beta}}$, $\overline{e}_i = y_i - \boldsymbol{x}_i'\overline{\boldsymbol{\beta}}$, and variance estimators $\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n \widehat{e}_i^2$, $\widetilde{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n \widetilde{e}_i^2$, and $\overline{\sigma}^2 = \frac{1}{n}\sum_{i=1}^n \overline{e}_i^2$.

(a) As $\overline{\boldsymbol{\beta}}$ is the most efficient and $\widehat{\boldsymbol{\beta}}$ the least, do you expect that $\overline{\sigma}^2 < \widetilde{\sigma}^2 < \widehat{\sigma}^2$, in large samples?

(b) Consider the statistic

$$T_n = \hat{\sigma}^{-2} \sum_{i=1}^n (\widehat{e}_i - \widetilde{e}_i)^2$$

Find the asymptotic distribution for T_n when $\boldsymbol{R}'\boldsymbol{\beta} = \boldsymbol{c}$ is true.

(c) Does the result of the previous question simplify when the error e_i is homoskedastic?

2. Take the linear model

$$\begin{aligned} y_i &= x_{1i}\beta_1 + x_{2i}\beta_2 + e_i \\ \mathbb{E}(\boldsymbol{x}_i e_i) &= \boldsymbol{0} \end{aligned}$$

with n observations. Consider the restriction

$$\frac{\beta_1}{\beta_2} = 2 \tag{1}$$

(a) Find an explicit expression for the constrained least-squares (CLS) estimator $\widetilde{\boldsymbol{\beta}} = (\widetilde{\beta}_1, \widetilde{\beta}_2)$ of $\beta = (\beta_1, \beta_2)$ under (1). Your answer should be specific to the restriction (1), it should not be a generic formula for an abstract general restriction.

(b) Derive the asymptotic distribution of $\widetilde{\beta}_1$ under the assumption that (1) is a true restriction

3. Suppose that for a pair of observables (y_i, x_i) with $x_i > 0$ that an economic model implies

$$\mathbb{E}(y_i \mid x_i) = (\gamma + \theta x_i)^{1/2}. \tag{2}$$

A friend suggests that (given an iid sample) you estimate γ and θ by the linear regression of y_i^2 on x_i, that is, to estimate the equation

$$y_i^2 = \alpha + \beta x_i + e_i. \tag{3}$$

(a) Investigate your friend's suggestion. Define $u_i = y_i - (\gamma + \theta x_i)^{1/2}$. Show that $\mathbb{E}(u_i \mid x_i) = 0$ is implied by (2).

(b) Use $y_i = (\gamma + \theta x_i)^{1/2} + u_i$ to calculate $\mathbb{E}(y_i^2 \mid x_i)$. What does this tell you about the implied equation (3)?

(c) Can you recover either γ and/or θ from estimation of (3)? Are additional assumptions required?

(d) Is this a reasonable suggestion?