Econometrics 710
Midterm Exam
March 10, 2011

1. Take the linear model

$$\begin{aligned} y_i &= \boldsymbol{x}_i'\boldsymbol{\beta} + e_i \\ E(e_i \mid \boldsymbol{x}_i) &= 0 \\ E\left(e_i^2 \mid \boldsymbol{x}_i\right) &= \sigma^2(\boldsymbol{x}_i) \end{aligned}$$

Consider two approximations to the conditional variance $\sigma^2(\boldsymbol{x})$

$$\boldsymbol{\gamma}_1 = \operatorname{argmin} E\left(\sigma^2(\boldsymbol{x}_i) - \boldsymbol{x}_i'\boldsymbol{\gamma}\right)^2$$

$$\boldsymbol{\gamma}_2 = \operatorname{argmin} E\left(e_i^2 - \boldsymbol{x}_i'\boldsymbol{\gamma}\right)^2$$

Show that either $\boldsymbol{\gamma}_1 = \boldsymbol{\gamma}_2$ or derive their difference.

2. Consider the short and long projections

$$y_i = x_i\gamma_1 + e_i$$

$$y_i = x_i\beta_1 + x_i^2\beta_2 + u_i$$

(a) Under what condition does $\gamma_1 = \beta_1$?

(b) Now suppose the long projection is

$$y_i = x_i\theta_1 + x_i^3\theta_2 + v_i$$

Is there a similar condition under which $\gamma_1 = \theta_1$?

3. Take the linear model

$$\begin{aligned} y_i &= x_i\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

with n observations and x_i is scalar (real-valued). Consider the estimator

$$\hat{\beta} = \frac{\sum_{i=1}^n x_i^3 y_i}{\sum_{i=1}^n x_i^4}$$

Find the asymptotic distribution of $\sqrt{n}\left(\hat{\beta} - \beta\right)$ as $n \to \infty$

4. Take the linear model

$$\begin{aligned} y_i &= \alpha + \boldsymbol{x}_i'\boldsymbol{\beta} + e_i \\ E(e_i) &= 0 \\ E(\boldsymbol{x}_i e_i) &= \mathbf{0} \end{aligned}$$

with n observations. Consider the restriction

$$\boldsymbol{\beta} = \mathbf{0} \tag{1}$$

(a) Find the constrained least-squares (CLS) estimator of α under (1).

(b) Find an expression for the efficient minimum distance estimator of α under (1).