

Econometrics 710
Midterm Exam
March 11, 2008

1. Take the model

$$\begin{aligned}y_i &= x'_{1i}\beta_1 + x'_{2i}\beta_2 + e_i \\E(e_i | x_i) &= 0 \\E(e_i^2 | x_i) &= \sigma^2\end{aligned}$$

where $x_i = (x_{1i}, x_{2i})$, with x_{1i} $k_1 \times 1$ and x_{2i} $k_2 \times 1$. Consider the short regression

$$y_i = x'_{1i}\hat{\beta}_1 + \hat{e}_i$$

and define the error variance estimator

$$s^2 = \frac{1}{n - k_1} \sum_{i=1}^n \hat{e}_i^2.$$

- (a) Find $E(s^2 | X)$
- (b) Find the probability limit of s^2 as $n \rightarrow \infty$.

2. Take the model

$$\begin{aligned}y_i &= x_i\beta + e_i \\E(x_i e_i) &= 0\end{aligned}$$

with x_i scalar and $E x_i^2 = 1$. Let V be the asymptotic variance of the least-squares estimator of β , and let V^0 be the “homoskedastic” form of the asymptotic variance.

- (a) Find V and V^0 . (You do not need to re-derive the asymptotic distribution.)
- (b) Find the relationship between V , V^0 , and $C = cov(x_i^2, e_i^2)$.

3. Take the model

$$\begin{aligned}y_i &= x'_i\beta + e_i \\E(e_i | x_i) &= 0 \\E(e_i^2 | x_i) &= \sigma_i^2 = z'_i\gamma\end{aligned}$$

where z_i is a (vector) function of x_i . The sample is $i = 1, \dots, n$ with iid observations. For simplicity, assume that $z'_i\gamma > 0$ for all z_i . Suppose you are interested in forecasting y_{n+1} given $x_{n+1} = x$ and $z_{n+1} = z$ for some out-of-sample observation $n + 1$. Describe how you would construct a point forecast and a forecast interval for y_{n+1} .

4. Take the model

$$\begin{aligned}y_i &= x_i' \beta + e_i \\E(e_i | x_i) &= 0 \\z_i &= (x_i' \beta) \gamma + u_i \\E(u_i | x_i) &= 0\end{aligned}$$

Your goal is to estimate γ . (Note that γ is real-valued.) You use a two-step estimator:

- (a) Estimate $\hat{\beta}$ by least-squares of y_i on x_i
- (b) Estimate $\hat{\gamma}$ by least-squares of z_i on $x_i' \hat{\beta}$

Show that $\hat{\gamma}$ is consistent for γ .

Extra Credit (**only** if you have time): Find the asymptotic distribution of $\hat{\gamma}$ when $\gamma = 0$ /