1. Let $Y$ be $n \times 1$, $X$ be $n \times k$, and $X^* = XC$ where $C$ is $k \times k$ and full-rank. Let $\hat{\beta}$ be the LS estimator from the regression of $Y$ on $X$, and let $\hat{V}$ be the estimate of its asymptotic covariance matrix. Let $\hat{\beta}^*$ and $\hat{V}^*$ be those from the regression of $Y$ on $X^*$. Derive an expression for $\hat{V}^*$ as a function of $\hat{V}$.

2. You have a random sample from the model

$$y_i = x_i \beta_1 + x_i^2 \beta_2 + e_i$$

$$E(e_i \mid x_i) = 0$$

where $y_i$ is wages (dollars per hour) and $x_i$ is age. Describe how you would test the hypothesis that the expected wage for a 40-year-old worker is $20 an hour. You do not need to derive the theory behind your procedure.

3. Take the standard model

$$y_i = x_i' \beta + e_i$$

$$E(x_i e_i) = 0$$

For a positive function $w(x)$, let $w_i = w(x_i)$. Consider the estimator

$$\tilde{\beta} = \left( \sum_{i=1}^{n} w_i x_i x_i' \right)^{-1} \left( \sum_{i=1}^{n} w_i x_i y_i \right).$$

Find the probability limit (as $n \to \infty$) of $\tilde{\beta}$. Is $\tilde{\beta}$ consistent for $\beta$? If not, under what assumption is $\tilde{\beta}$ consistent for $\beta$?

4. Take the model

$$Y = X \beta + \epsilon$$

$$E(\epsilon \mid X) = 0$$

$$E(\epsilon \epsilon' \mid X) = D$$

Assume for simplicity that $D$ is known. Consider the OLS and GLS estimators $\hat{\beta} = (X'X)^{-1} (X'Y)$ and $\tilde{\beta} = (X'D^{-1}X)^{-1} (X'D^{-1}Y)$. Compute the (conditional) covariance between $\hat{\beta}$ and $\tilde{\beta}$:

$$E \left( (\hat{\beta} - \beta) (\tilde{\beta} - \beta)' \mid X \right)$$

Find the (conditional) covariance matrix for $\hat{\beta} - \tilde{\beta}$:

$$E \left( (\hat{\beta} - \tilde{\beta}) (\hat{\beta} - \tilde{\beta})' \mid X \right)$$