Econometrics 710
Midterm Exam
March 9, 2006

1. Let Y be $n \times 1$, X be $n \times k$, and $X^* = XC$ where C is $k \times k$ and full-rank. Let $\hat{\beta}$ be the LS estimator from the regression of Y on X, and let $\hat{V}$ be the estimate of its asymptotic covariance matrix. Let $\hat{\beta}^*$ and $\hat{V}^*$ be those from the regression of Y on X^*. Derive an expression for $\hat{V}^*$ as a function of $\hat{V}$.

2. You have a random sample from the model

$$\begin{aligned} y_i &= x_i\beta_1 + x_i^2\beta_2 + e_i \\ E\left(e_i \mid x_i\right) &= 0 \end{aligned}$$

where y_i is wages (dollars per hour) and x_i is age. Describe how you would test the hypothesis that the expected wage for a 40-year-old worker is \$20 an hour. You do not need to derive the theory behind your procedure.

3. Take the standard model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E\left(x_i e_i\right) &= 0 \end{aligned}$$

For a positive function $w(x)$, let $w_i = w(x_i)$. Consider the estimator

$$\tilde{\beta} = \left(\sum_{i=1}^n w_i x_i x_i'\right)^{-1}\left(\sum_{i=1}^n w_i x_i y_i\right).$$

Find the probability limit (as $n \to \infty$) of $\tilde{\beta}$. Is $\tilde{\beta}$ consistent for β? If not, under what assumption is $\tilde{\beta}$ consistent for β?

4. Take the model

$$\begin{aligned} Y &= X\beta + e \\ E\left(e \mid X\right) &= 0 \\ E\left(ee' \mid X\right) &= D \end{aligned}$$

Assume for simplicity that D is known. Consider the OLS and GLS estimators $\hat{\beta} = \left(X'X\right)^{-1}\left(X'Y\right)$ and $\tilde{\beta} = \left(X'D^{-1}X\right)^{-1}\left(X'D^{-1}Y\right)$. Compute the (conditional) covariance between $\hat{\beta}$ and $\tilde{\beta}$:

$$E\left(\left(\hat{\beta} - \beta\right)\left(\tilde{\beta} - \beta\right)' \mid X\right)$$

. Find the (conditional) covariance matrix for $\hat{\beta} - \tilde{\beta}$:

$$E\left(\left(\hat{\beta} - \tilde{\beta}\right)\left(\hat{\beta} - \tilde{\beta}\right)' \mid X\right)$$