Econometrics 710
Midterm Exam
March 17, 2005

1. Let the variable y_i be generated by $y_i = x_i^2 + \varepsilon_i$ where ε_i is independent of x_i, $E\varepsilon_i = 0$ and $E\varepsilon_i^2 = \sigma^2$. Suppose that $Ex_i = 0$, and let $\mu_2 = Ex_i^2$, $\mu_3 = Ex_i^3$ and $\mu_4 = Ex_i^4$. Using a random sample from (y_i, x_i), suppose you estimate (by OLS) a linear equation $y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{e}_i$. What are $\hat{\beta}_0$ and $\hat{\beta}_1$ estimating? Find expressions for β_0 and β_1 in terms of the moments of ε_i and x_i

2. Take the model $y_i = x_i'\beta + e_i$ with $E(x_i e_i) = 0$. Suppose you have two independent random samples of observations (y_i, x_i) of size n_1 and n_2. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ denote the least-squares estimate of β on each sample. Let $\tilde{\beta} = \left(\hat{\beta}_1 + \hat{\beta}_2\right)/2$ denote the average of the two estimates. Let $\hat{\beta}$ denote the least-squares estimate on the combined sample. Which is more efficient, $\tilde{\beta}$ or $\hat{\beta}$? When are they asymptotically equivalent?

3. Take the homoskedastic linear regrssion

$$\begin{aligned} y_i &= x_{1i}'\beta_1 + x_{2i}'\beta_2 + \varepsilon_i \\ E\left(\varepsilon_i \mid x_{1i}, x_{2i}\right) &= 0 \\ E\left(\varepsilon_i^2 \mid x_{1i}, x_{2i}\right) &= \sigma^2 \end{aligned}$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} Ex_{1i}x_{1i}' & Ex_{1i}x_{2i}' \\ Ex_{2i}x_{1i}' & Ex_{2i}x_{2i}' \end{bmatrix}.$$

   Consider two estimators of β_1, based on the long regression

$$y_i = x_{1i}'\hat{\beta}_1 + x_{2i}'\hat{\beta}_2 + \hat{e}_i$$

   and short regression

$$y_i = x_{1i}'\tilde{\beta}_1 + \tilde{u}_i$$

   using a sample of size n. Assume that $\beta_2 \neq 0$.

   Assume that $M_{21} = 0$. Find expressions for the (asymptotic) variance of $\hat{\beta}_1$ and $\tilde{\beta}_1$. Which is more efficient?

4. This is a continuation of question 3, but now assume that $M_{21} \neq 0$.

   Hint: Parts (b), (c) and (d) are challenging.

   (a) Find the (asymptotic) bias of $\tilde{\beta}_1$
   (b) Find the (asymptotic) variance $\tilde{\beta}_1$
   (c) Construct an expression for the mean-squared error of $\tilde{\beta}_1$
   (d) Contrast the expression in (c) with the asymptotic MSE of $\hat{\beta}_1$.