Econometrics 710
Midterm Exam
March 21, 2002

1. The model is iid data, $i = 1, ..., n$,

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

Does the presence of heteroskedasticity invalidate the application of the non-parametric bootstrap? Explain briefly.

2. The model is

$$\begin{aligned} Y &= Z\beta + e \\ Z &= X\Gamma \\ E(e \mid X) &= 0 \end{aligned}$$

where X is $n \times k$, Γ is $k \times m$, $k > m$, β is $m \times 1$. Suppose that Γ is unknown, but it is estimated by $\hat{\Gamma}$ which satisfies

$$\hat{\Gamma} \to_p \Gamma.$$

Assume that Γ has full rank m and that $\beta = 0$ (Hint: both are important). Set $\hat{Z} = X\hat{\Gamma}$, and let $\hat{\beta} = \left(\hat{Z}'\hat{Z}\right)^{-1} \hat{Z}'Y$. Derive the asymptotic distribution of $\sqrt{n}\hat{\beta}$.

3. The model is iid data, $i = 1, ..., n$,

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

Let $\hat{\beta}$ be the OLS estimator of β, and let $\hat{V}_n$ be the White covariance matrix estimator of $V_n = Var(\hat{\beta})$. Suppose that in addition to the above model assumptions, it is true that

$$E\left(e_i^2 \mid x_i\right) = \sigma^2.$$

Under these conditions, find $E\left(\hat{V}_n \mid X\right)$.

Hint: First find $E\left(\hat{e}_i^2 \mid X\right)$, where $\hat{e}_i$ is the OLS residual. In particular, show that

$$E\left(\hat{e}_i^2 \mid X\right) = \sigma\lambda_i$$

where λ_i is specific function of x_i and $(X'X)^{-1}$.