Econometrics 710

Final Exam

May 11, 1999

You have 150 minutes for the exam. Complete answers get full credit. Incomplete answers get partial credit.

Be careful to write full and complete answers where possible. At the same time, you should exercise judgement and only write down the important (and of course correct) points.

1. Take a linear regression model

$$Y = X\beta + e$$

where X is $n \times k$ and β is $k \times 1$. Assume $E(e \mid X) = 0$. Suppose the parameters β are known to satisfy the restrictions

$$\beta = Q\theta$$

where Q is $k \times m$ and θ is $m \times 1$, $m < k$. Q is known but θ is unknown.

(a) Is there a simple way to estimate θ by least squares? Find this estimator ($\tilde{\theta}$).

(b) Let $\hat{\beta}$ denote the OLS estimator for β. One can also estimate θ from $\hat{\beta}$ using the "minumum chi-square criterion." Specifically, for some positive definate $k \times k$ matrix W, define

$$C(\theta) = \left(\hat{\beta} - Q\theta\right)' W \left(\hat{\beta} - Q\theta\right)$$

and define $\hat{\theta}$ as the θ which minimizes $C(\theta)$:

$$\hat{\theta} = \text{Argmin } C(\theta).$$

Find this $\hat{\theta}$.

(c) Is $\hat{\theta}$ consistent for θ?

(d) Find the asymptotic distribution of $\hat{\theta}$.

(e) Is there a choice of W so that $\tilde{\theta} = \hat{\theta}$? [$\tilde{\theta}$ is defined in part (a)]

2. Take the linear model

$$Y = X\beta + e.$$

Let the OLS estimator for β be $\hat{\beta}$ and the OLS residual be $\hat{e} = Y - X\hat{\beta}$.

Let the 2SLS estimator for β using some instrument Z be $\tilde{\beta}$ and the 2SLS residual be $\tilde{e} = Y - X\tilde{\beta}$. If X is indeed endogeneous, will 2SLS "fit" better than OLS, in the sense that $\tilde{e}'\tilde{e} < \hat{e}'\hat{e}$, at least in large samples?

3. Consider the single equation model

$$y_i = x_i\beta + e_i,$$

where y_i and x_i are both real-valued (1×1). Let $\hat{\beta}$ denote the 2SLS estimator of β using as an instrument a dummy variable d_i (takes only the values 0 and 1). Find a simple expression for the 2SLS estimator in this context.

4. Suppose that y_t is generated by

$$y_t = \alpha y_{t-2} + e_t$$

with e_t iid $(0, \sigma^2)$ and $0 < \alpha < 1$.

(a) Is y_t stationary and ergodic?

(b) Find the asymptotic distribution of the OLS estimator $\hat{\alpha}$.

5. A latent variable y_i^* is generated by

$$y_i^* = x_i\beta + e_i$$

The distribution of e_i, conditional on x_i, is $N(0, \sigma_i^2)$, where $\sigma_i^2 = \gamma_0 + x_i^2\gamma_1$ with $\gamma_0 > 0$ and $\gamma_1 > 0$. The binary variable y_i equals 1 if $y_i^* \geq 0$, else $y_i = 0$. Find the log-likelihood function for the conditional distribution of y_i given x_i (the parameters are $\beta, \gamma_0, \gamma_1$).

6. The equation of interest is

$$\begin{aligned} y_i &= g(x_i, \beta) + e_i \\ E(e_i \mid z_i) &= 0. \end{aligned}$$

The observed data is (y_i, x_i, z_i). z_i is $k \times 1$ and β is $m \times 1$. Show how to construct the efficient GMM estimator for β.

7. In the linear model

$$y_i = x_i \beta + e_i$$

suppose $\sigma_i^2 = E\left(e_i^2 \mid x_i\right)$ is known. Show that the GLS estimator of β can be written as an instrumental variables estimator using some instrument z_i. (Find an expression for z_i.)

8. Take the linear model

$$\begin{aligned} y_i &= x_i \beta + e_i \\ E\left(e_i \mid x_i\right) &= 0. \end{aligned}$$

where x_i and β are 1×1.

(a) Show that $E\left(x_i e_i\right) = 0$ and $E\left(x_i^2 e_i\right) = 0$. Is $z_i = \left(x_i \quad x_i^2\right)$ a valid instrumental variable for estimation of β?

(b) Define the 2SLS estimator of β, using z_i as an instrument for x_i. How does this differ from OLS?

(c) Find the efficient GMM estimator of β, based on the moment condition

$$E\left(z_i\left(y_i - x_i \beta\right)\right) = 0.$$

Does this differ from 2SLS and/or OLS?