Econometrics 710
Final Exam, Spring 2017

Write complete answers. Be specific about estimators and covariance matrix estimators.

1. Consider the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i|z_i) &= 0 \end{aligned}$$

with y_i scalar and x_i and z_i each a k vector. You have a random sample $(y_i, x_i, z_i : i = 1, ..., n)$.

(a) Write the IV estimator $\widehat{\beta}$ for β

(b) Suppose that x_i is exogeneous in the sense that $E(e_i|z_i, x_i) = 0$. Is $\widehat{\beta}$ unbiased for β?

(c) Continuing to assume that x_i is exogeneous, find the variance matrix for $\widehat{\beta}$, $\text{var}\left(\widehat{\beta}|X, Z\right)$.

2. Consider the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ x_i &= \Gamma' z_i + u_i \\ E(z_i e_i) &= 0 \\ E\left(z_i u_i'\right) &= 0 \end{aligned}$$

with y_i scalar and x_i and z_i each a k vector. You have a random sample $(y_i, x_i, z_i : i = 1, ..., n)$. Take the control function equation

$$\begin{aligned} e_i &= u_i'\gamma + \varepsilon_i \\ E(u_i \varepsilon_i) &= 0 \end{aligned}$$

and assume for simplicity that u_i is observed. Inserting into the structural equation we find

$$y_i = x_i'\beta + u_i'\gamma + \varepsilon_i \tag{1}$$

The control function estimator $(\widehat{\beta}, \widehat{\gamma})$ is OLS estimation of (1).

(a) Show that $E(x_i \varepsilon_i) = 0$ (algebraically)

(b) Derive the asymptotic distribution of $(\widehat{\beta}, \widehat{\gamma})$.

3. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E\left(z_i e_i\right) &= 0 \end{aligned}$$

y_i scalar, x_i a k vector and z_i an ℓ vector, $\ell \geq k$. Assume iid observations. Consider the statistic

$$\begin{aligned} J_n(\beta) &= n\overline{m}_n(\beta)' W \overline{m}_n(\beta) \\ \overline{m}_n(\beta) &= \frac{1}{n}\sum_{i=1}^{n} z_i\left(y_i - x_i'\beta\right) \end{aligned}$$

for some weight matrix $W > 0$.

(a) Take the hypothesis

$$H_0 : \beta = \beta_0$$

Derive the asymptotic distribution of $J_n(\beta_0)$ under H_0 as $n \to \infty$.

(b) What choice for W yields a known asymptotic distribution in part a? (Be specific about degrees of freedom.)

(c) Write down an appropriate estimator $\widehat{W}$ for W which takes advantage of H_0. (You do not need to demonstrate consistency or unbiasedness.)

(d) Describe an asymptotic test of H_0 against $H_1 : \beta \neq \beta_0$ based on this statistic.

(e) Use the result in part (d) to construct a confidence region for β. What can you say about the form of this region? For example, does the confidence region take the form of an ellipse, similar to conventional confidence regions?

(f) Describe a bootstrap test of H_0 against $H_1 : \beta \neq \beta_0$ based on the statistic $J_n(\beta_0)$.
Hint: The key is to find an appropriate bootstrap estimate of the distribution of $J_n(\beta_0)$.