Econometrics 710
Final Exam, Spring 2011

The exam consists of one question, broken in several parts.

The model is

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + e_i \tag{1}$$

$$E(\mathbf{z}_i e_i) = 0 \tag{2}$$

The dimensions are: $\mathbf{x}_i$, $\mathbf{z}_i$, and $\boldsymbol{\beta}$ are $k \times 1$, $k > 1$, and y_i and e_i are 1×1. Let

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_{xx} & \boldsymbol{Q}_{xz} \\ \boldsymbol{Q}_{zx} & \boldsymbol{Q}_{zz} \end{bmatrix} = \begin{bmatrix} E(\mathbf{x}_i\mathbf{x}_i') & E(\mathbf{x}_i\mathbf{z}_i') \\ E(\mathbf{z}_i\mathbf{x}_i') & E(\mathbf{z}_i\mathbf{z}_i') \end{bmatrix}$$

Assume both $\boldsymbol{Q}_{xx}$ and $\boldsymbol{Q}_{xz}$ have full rank k.

Let $\widehat{\boldsymbol{\beta}}$ be the least-squares estimate obtained by regressing y_i on $\mathbf{x}_i$, and let $\widetilde{\boldsymbol{\beta}}$ be the 2SLS estimator obtained by estimation of (1) using the instrument $\mathbf{z}_i$.

1. Find
$$\delta = \operatorname*{plim}_{n\to\infty} \left(\widehat{\boldsymbol{\beta}} - \widetilde{\boldsymbol{\beta}}\right)$$
2. Suppose that in addition to (1) and (2),
$$E(\mathbf{x}_i e_i) = 0 \tag{3}$$
Quite simply, what does this condition mean? What is δ under this assumption?
3. Write the difference $\widehat{\boldsymbol{\beta}} - \widetilde{\boldsymbol{\beta}}$ as a function of sample moments of $\mathbf{x}_i$, $\mathbf{z}_i$, and e_i.
4. Under (1)-(3), find the asymptotic distribution of
$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \begin{pmatrix} \mathbf{x}_i \\ \mathbf{z}_i \end{pmatrix} e_i$$
as $n \to \infty$.
5. Under (1)-(3) find the asymptotic distribution of $\sqrt{n}\left(\widehat{\boldsymbol{\beta}} - \widetilde{\boldsymbol{\beta}}\right)$ as $n \to \infty$.
6. Suppose that
$$E\left(e_i^2 | \mathbf{x}_i, \mathbf{z}_i\right) = \sigma^2 \tag{4}$$
How does the asymptotic variance from question 5 simplify under (4)?
7. Propose an estimator of the asymptotic variance under (4).
8. Propose a test statistic for (3) under (4) and find its asymptotic distribution under the assumption that $\boldsymbol{Q} > 0$
9. Describe how to use this statistic to test the hypothesis that $\mathbf{x}_i$ is exogenous.
10. Extra Credit. Show where $\boldsymbol{Q} > 0$ is used in the answer to question 8.