

Econometrics 710
Final Exam, Spring 2009

1. The observed data is $\{y_i, x_i\} \in \mathbb{R} \times \mathbb{R}^k$, $k > 1$, $i = 1, \dots, n$. Take the model

$$\begin{aligned} y_i &= x_i' \beta + e_i \\ E(x_i e_i) &= 0 \\ \mu_3 &= E(e_i^3) \end{aligned}$$

- (a) Write down an estimator for μ_3
- (b) Explain how to use the Efron percentile method to construct a 90% confidence interval for μ_3 in this specific model.
2. An economist reports a set of parameter estimates, including the coefficient estimates $\hat{\beta}_1 = 1.0$, $\hat{\beta}_2 = 0.8$, and standard errors $s(\hat{\beta}_1) = 0.07$ and $s(\hat{\beta}_2) = 0.07$. The author writes “The estimates show that β_1 is larger than β_2 .”
- (a) Write down the formula for an asymptotic 95% confidence interval for $\theta = \beta_1 - \beta_2$, expressed as a function of $\hat{\beta}_1$, $\hat{\beta}_2$, $s(\hat{\beta}_1)$, $s(\hat{\beta}_2)$ and $\hat{\rho}$, where $\hat{\rho}$ is the estimated correlation between $\hat{\beta}_1$ and $\hat{\beta}_2$.
- (b) Can $\hat{\rho}$ be calculated from the reported information?
- (c) Is the author correct? Does the reported information support the author’s claim?
3. The observed data is $\{y_i, x_i, z_i\} \in \mathbb{R} \times \mathbb{R}^k \times \mathbb{R}^\ell$, $k > 1$ and $\ell > k > 1$, $i = 1, \dots, n$. The model is

$$\begin{aligned} y_i &= x_i' \beta + e_i \\ E(z_i e_i) &= 0 \end{aligned} \tag{1}$$

- (a) Given a weight matrix $W > 0$, write down the GMM estimator for $\hat{\beta}$.
- (b) Suppose model (1) is misspecified in that

$$\begin{aligned} e_i &= \delta n^{-1/2} + u_i \\ E(u_i | z_i) &= 0 \end{aligned} \tag{2}$$

with $\mu_z = E z_i \neq 0$ and $\delta \neq 0$.

Show that (2) implies (1) is false.

- (c) Express $\sqrt{n}(\hat{\beta} - \beta)$ as a function of W , n , δ , and the variables (x_i, z_i, u_i) .
- (d) Find the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta)$ under Assumption (2).
4. The observed data is $\{y_i, x_i, z_i\} \in \mathbb{R} \times \mathbb{R}^k \times \mathbb{R}^\ell$, $k > 1$ and $\ell > 1$, $i = 1, \dots, n$. An econometrician first estimates

$$y_i = x_i' \hat{\beta} + \hat{e}_i \tag{3}$$

by least squares. The econometrician next regresses the residual \hat{e}_i on z_i , which can be written as

$$\hat{e}_i = z_i' \tilde{\gamma} + \tilde{u}_i \tag{4}$$

- (a) Define the population γ being estimated in the (4).
- (b) Find the probability limit for $\tilde{\gamma}$.
- (c) Suppose the econometrician constructs a Wald statistic W_n for $H_0 : \gamma = 0$ from the regression (4), ignoring regression (3). Write down the formula for W_n .
- (d) Assuming $E(z_i x_i') = 0$, find the asymptotic distribution for W_n under $H_0 : \gamma = 0$.
- (e) If $E(z_i x_i') \neq 0$ will your answer to (d) change?