

Econometrics 710
Final Exam
May 13, 2005

In all questions, the pair (y_i, x_i) are iid, and we observe a random sample $i = 1, \dots, n$ of observations from this distribution.

1. We want to estimate $\mu = Ey_i$, and we know that $Ex_i = 0$, where $x_i \in R$. Find an efficient GMM estimator for μ .
2. Take the regression model

$$\begin{aligned}y_i &= x_i' \beta + \varepsilon_i \\E(\varepsilon_i | x_i) &= 0 \\E(\varepsilon_i^2 | x_i) &= \sigma_i^2\end{aligned}$$

with $x_i \in R^k$. Assume that $P(\varepsilon_i = 0) = 0$. Consider the infeasible estimator

$$\tilde{\beta} = \left(\sum_{i=1}^n \varepsilon_i^{-2} x_i x_i' \right)^{-1} \left(\sum_{i=1}^n \varepsilon_i^{-2} x_i y_i \right).$$

This is a WLS estimator using the weights e_i^{-2} .

- (a) Find the asymptotic distribution of $\tilde{\beta}$
 - (b) Contrast your result with the asymptotic distribution of infeasible GLS.
3. Take the model

$$\begin{aligned}y_i &= x_{1i}' \beta_1 + x_{2i}' \beta_2 + e_i \\E(x_i e_i) &= 0\end{aligned}$$

with $x_{1i} \in R^k$ and $x_{2i} \in R^k$. Consider the hypothesis

$$\begin{aligned}H_0 &: \beta_1 = \beta_2 \\H_1 &: \beta_1 \neq \beta_2\end{aligned}$$

Describe a test of H_0 using the nonparametric bootstrap.

4. Consider the model

$$\begin{aligned}y_i &= z_i' \beta + e_i \\E(x_i e_i) &= 0 \\R' \beta &= 0\end{aligned}$$

The dimensions are $z_i \in R^k$, $x_i \in R^l$, $l > k$. The matrix R is $k \times q$, $1 \leq q < k$. Derive an efficient GMM estimator for β for this model.

5. Take the linear equation $Y = Z\beta + e$, and consider the following estimators of β .

- (a) $\hat{\beta}_1$. 2SLS using the instruments X_1
- (b) $\hat{\beta}_2$. 2SLS using the instruments X_2
- (c) $\tilde{\beta}$. GMM using the instruments $X = (X_1, X_2)$ and the weight matrix

$$W = \begin{pmatrix} (X_1'X_1)^{-1}\lambda & 0 \\ 0 & (X_2'X_2)^{-1}(1-\lambda) \end{pmatrix}$$

for $\lambda \in (0, 1)$

Find an expression for $\tilde{\beta}$ which shows that it is a specific weighted average of $\hat{\beta}_1$ and $\hat{\beta}_2$.