Econometrics 710
Final Exam
May 13, 2004

1. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(x_i e_i) &= 0 \end{aligned}$$

The parameter of interest is $\theta = \beta_1 \beta_2$, where β_1 and β_2 are the first and second elements of β. Show how to construct a confidence interval for θ using the following three methods.

   (a) Asymptotic Theory
   (b) Percentile Bootstrap
   (c) Equal-Tailed Percentile-t Bootstrap.

   (Your answer should be specific, not general.)

2. Take the model

$$\begin{aligned} y_i &= x_i'\beta + e_i \\ E(e_i \mid x_i) &= 0 \end{aligned}$$

Let $\hat{\beta}$ denote the OLS estimator for β based on an available sample.

   (a) Suppose that the i'th observation is in the sample only if $x_{1i} > 0$, where x_{1i} is an element of x_i.
      i. Is $\hat{\beta}$ consistent for β?
      ii. If not, can you obtain an expression for its probability limit?
      (For this, you may assume that e_i is independent of x_i and $N(0, \sigma^2)$.)
   (b) Suppose that the i'th observation is in the sample only if $y_i > 0$.
      i. Is $\hat{\beta}$ consistent for β?
      ii. If not, can you obtain an expression for its probability limit?
      (For this, you may assume that e_i is independent of x_i and $N(0, \sigma^2)$.)

3. Let Y_i be iid, $\mu = EY_i > 0$, and $\theta = \mu^{-1}$. Let $\hat{\mu} = \overline{Y}_n$ be the sample mean and $\hat{\theta} = \hat{\mu}^{-1}$.

   (a) Is $\hat{\theta}$ unbiased for θ?
   (b) If $\hat{\theta}$ is biased, can you determine the direction of the bias $E\hat{\theta} - \theta$ (up or down)?
   (c) Obtain an approximation to the bias using a second-order Taylor series.
   (d) Could the nonparametric bootstrap be used to estimate the bias? If so, explain how.

4. Consider the just-identified model

$$
\begin{aligned}
y_i &= z_{1i}'\beta_1 + z_{2i}'\beta_2 + e_i \\
E(x_i e_i) &= 0
\end{aligned}
$$

where $z_i = (z_{1i}' \; z_{2i}')'$ and x_i are $k \times 1$. The hypothesis of interest is

$$H_0 : \beta_1 = 0$$

Three econometricians are called to advise on how to test H_0.

- Econometrician 1 proposes testing H_0 by a Wald statistic.
- Econometrician 2 suggests testing H_0 by the GMM Distance Statistic.
- Econometrician 3 suggests testing H_0 using the test of overidentifying restrictions.

You are asked to settle this dispute. Explain the advantages and/or disadvantages of the different procedures, in this specific context.

5. The model is

$$
\begin{aligned}
y_i &= z_i'\beta + e_i \\
E(x_i e_i) &= 0
\end{aligned}
$$

An economist wants to obtain the 2SLS estimates and standard errors for β. He uses the following steps

(a) Regresses z_i on x_i, obtains the predicted values $\hat{z}_i$.

(b) Regresses y_i on $\hat{z}_i$, obtains the coefficient estimate $\hat{\beta}$ and standard error $s(\hat{\beta})$ from this regression.

Is this correct? Does this produce the 2SLS estimates and standard errors?

6. Let T_n be a test statistic such that under H_0, $T_n \rightarrow_d \chi_3^2$. Since $P(\chi_3^2 > 7.815) = .05$, an asymptotic 5% tets of H_0 rejects when $T_n > 7.815$. An econometrician is interested in the Type I error of this test when $n = 100$ and the data structure is well specified. She performs the following Monte Carlo experiment.

(a) $B = 200$ samples of size $n = 100$ are generated from a distribution satisfying H_0.

(b) On each sample, the test statistic T_{nb} is calculated.

(c) She calculates $\hat{p} = \frac{1}{B}\sum_{b=1}^{B} 1(T_{nb} > 7.815) = 0.070$

(d) The econometrician concludes that the test T_n is oversized in this context – it rejects too frequently under H_0.

Is her conclusion correct, incorrect, or incomplete? Be specific in your answer.