Problem Set #7
Spring 2017

1. You are a consulting to MedTrax, a large pharmaceutical company, which released a new ulcer drug 3 months ago and is concerned about recovering research and development costs. Accordingly, MedTrax has approached you for drug sales projections at 1- through 12-month-ahead horizons, which it will use to guide potential sales force realignments. In briefing you, MedTrax indicated that it expects your long-run forecasts (e.g., 12-month-ahead) to be just as accurate as your short-horizon forecasts (e.g., 1-month-ahead). Explain to MedTrax (briefly) why that is not likely to be the case, even if you do the best forecasting job possible.

2. Use the iteration rule to forecast the AR process

$$y_t = \beta y_{t-1} + \varepsilon_t$$

Assume that all parameters are known.

(a) Show that the optimal forecasts are

$$\begin{aligned} y_{T+1|T} &= \beta y_T \\ y_{T+2|T} &= \beta^2 y_T \\ &\cdots \\ y_{T+h|T} &= \beta^h y_T \end{aligned}$$

(b) Show that the corresponding forecast errors are

$$\begin{aligned} u_{T+1|T} &= y_{T+1} - y_{T+1|T} = \varepsilon_{T+1} \\ u_{T+2|T} &= y_{T+2} - y_{T+2|T} = \varepsilon_{T+2} + \beta\varepsilon_{T+1} \\ &\cdots \\ u_{T+h|T} &= y_{T+h} - y_{T+h|T} = \varepsilon_{T+h} + \beta\varepsilon_{T+h-1} + \cdots + \beta^{h-1}\varepsilon_{T+1} \end{aligned}$$

(c) Show that the forecast error variances are

$$\begin{aligned} \sigma_1^2 &= \sigma^2 \\ \sigma_2^2 &= \sigma^2\left(1+\beta^2\right) \\ &\cdots \\ \sigma_h^2 &= \sigma^2\left(1+\beta^2+\cdots+\beta^{2h-2}\right) = \sigma^2\sum_{i=0}^{h-1}\beta^{2i} \end{aligned}$$

(d) Show that the limiting forecast error variance is

$$\lim_{h\to\infty}\sigma_h^2 = \frac{\sigma^2}{1-\beta^2}$$

the unconditional variance of the AR(1) process.

3. Take a stationary AR(p) process

$$y_t = \alpha + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + e_t$$

Calculate the mean $E(y_t) = \mu$.

4. Take the AR(1) model

$$y_t = \alpha + \beta y_{t-1} + e_t$$

where the errors e_t are iid white noise $N(0,1)$

(a) Set $\alpha = 1$ and $\beta = 0.25$

i. Calculate the mean $\mu = E(y_t)$.

ii. Simulate a series of length $T = 240$. Set the initial value $y_1 = \mu$ to equal the unconditional mean (from part (a)). (This is similar to problem 4 from the last problem set). Create a time-series plot of your series.

iii. Estimate a AR(1) model. Are your coefficient estimates close to the true values?

(b) Repeat with $\alpha = 10$, $\beta = 0.9$

(c) Repeat with $\alpha = 0$, $\beta = -.5$.

5. In the STATA file "realgdpgrowth.dta" the series *pdi* contains quarterly U.S. seasonally adjusted aggregate investment growth rates, from the BEA. Estimate an AR(4) model for this series. Which lags appear to be most important? Generate a point and 90% interval one-step-ahead forecast for 2017q1.

6. In the same file the series *exports* is aggregate U.S. exports (growth rates). Estimate an AR(4) model for this series. Which lags appear to be important? Do you notice anything interesting about the coefficients on the relevant lags? Generate a point and 90% interval one-step-ahead forecast for 2017q1.

7. In the same file the series *pdi_residential* is aggregate residential investment growth rates. Using an AR(4) model, generate point and interval forecasts for 2017q1 through 2017q4 using the direct method. Create a plot of the forecasts and intervals.