Integration

• Orders of Integration Terminology
  – A series with a unit root (a random walk) is said to be **integrated of order one**, or \( I(1) \)
  – A stationary series without a trend is said to be **integrated of order 0**, or \( I(0) \)
  – An \( I(1) \) series is differenced once to be \( I(0) \)
  – In general, we say that a series is \( I(d) \) if its \( d \)'th difference is stationary.
Integrated of order $d$

• A series is $I(d)$ if

$$(1-L)^d y_t = z_t$$

is stationary and without trend.

• Examples

  – $I(0)$: \( y_t = z_t \)
  – $I(1)$: \( (1-L)y_t = z_t \)
  – $I(2)$: \( (1-L)^2 y_t = z_t \)

• Possible $I(2)$ series are price levels and money supply
Fractional Integration

• **Advanced side note!**
• We said a series is $I(d)$ if
  \[(1 - L)^d y_t = z_t\]
• We did not require $d$ to be an integer
• We say that $y$ is **fractionally integrated** if $0 < d < 1$ or $-1 < d < 0$
• A fractionally integrated series is in between $I(0)$ and $I(1)$
• Strong dependence, slow autocorrelation decay
• Popular model for asset return volatility.
Fractional Differencing

• The fractional differencing operator is an infinite series

\[(1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k\]

\[= \sum_{k=0}^{\infty} \frac{\prod_{a=0}^{k-1} (d - a)}{k!} (-B)^k\]

\[= 1 - dB + \frac{d(d-1)}{2} B^2 - \ldots .\]
Co-Integration

- We say that two series are co-integrated if a linear combination has a lower level of integration.
- If $y$ and $x$ are each $I(1)$, yet $z = y - \theta x$ is $I(0)$.
- Example: Term Structure
  - We saw before that $t_{3\text{month}}$ appears to have a unit root.
  - But the spread $t_{10\text{year}} - t_{3\text{month}}$ was stationary.
  - $t_{3}$ and $t_{10\text{year}}$ are co-integrated!
Common Co-Integration Relations

• Interest Rates of different maturities
• Stock prices and dividends
  – (Campbell and Shiller)
• Aggregate consumption and income
  – (Campbell and Shiller)
• Aggregate output, consumption, and investment
  – King, Plosser, Stock and Watson
Cointegrating Equation

• We said that $y$ and $x$ are cointegrated if

\[ z_t = y_t - \theta x_t \]

is stationary

• This is called the cointegrating equation

• $\theta$ is the cointegrating coefficient

• In some cases, $\theta$ is known from theory
  – often $\theta=1$
Great Ratios

• If the aggregate variables $Y$ and $X$ are proportional in the long run, then

$$Z_t = \frac{Y_t}{X_t}$$

is stationary.

• Then

$$\log(Z_t) = \log(Y_t) - \log(X_t)$$

and

$$z_t = y_t - x_t$$

where $y = \log(Y)$ and $x = \log(X)$

• In this case, the logs $y$ and $x$ are cointegrated with coefficient 1.
Equilibrium Error

• The difference $z_t = y_t - \theta x_t$

is sometimes called the equilibrium error, as it measures the deviation of $y$ and $x$ from the long-term cointegrating relationship.
Simulated Example
Scatter plot

Variables stay close to cointegration line
Granger Representation Theory

• If $y$ and $x$ are I(1) and cointegrated, then the optimal regression for $y$ takes the form

$$
\Delta y_t = \mu + \gamma z_{t-1} + \alpha_1 \Delta y_{t-1} + \cdots + \alpha_p \Delta y_{t-p} + \beta_1 \Delta x_{t-1} + \cdots + \beta_q \Delta x_{t-q} + e_t
$$

$$
z_{t-1} = y_{t-1} - \theta x_{t-1}
$$

• A dynamic regression in first differences, plus the error correction term $z$. 
Answer to spurious regression

• The reaction to spurious regression was:
  – If the series are I(1), then do regressions in differences

• Cointegration says:
  – Add the error correction $z$!

• The difference is critical
  – The variable $z$ measures if $y$ is high or low relative to $x$
  – The error-correction variable $z$ pushes $y$ back towards the cointegration relationship
Origin of Cointegration

• British econometricians
  – Davidson, Hendry, Srba and Yeo (1978)
  – Suggested $ln(C_t) - ln(Y_t)$ was a valuable predictor of consumption growth $\Delta ln(C_t)$
  – This puzzled Clive Granger, as he knew that the variables were I(1), so should not be in a regression
Theory of Cointegration

• This led Clive Granger to develop the theory of cointegration and the Granger Representation Theorem
• The most influential statement was a co-authored paper with Robert Engle (1987)
• Granger and Engle shared the Nobel Prize in economics in 2003
Cointegration Development

- Much of the statistical theory was developed by Peter Phillips and his students at Yale
- A multivariate statistical method was developed by the statistician Soren Johansen (U Copenhagen)
- Some jointly with the economist Katarina Juselius (Copenhagen)
- Their methods are programmed in STATA as VECM (vector error-correction models)
Example: Term Structure

- Regress change in 3-month T-bill on lagged spread, lagged changes in 3-month and 10-year
- Positive error correction coefficient
- Short rate increases when long rate exceeds short

```
. reg d.t3 L.spread L(1/12).d.t3 L(1/12).d.t10year, r
```

Linear regression

| D.t3month | Coef.    | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|-----------|----------|-----------|-------|------|---------------------|
| spread L1.| 0.0274396| 0.0172321 | 1.59  | 0.112| -0.0063928 to 0.0612719 |

Number of obs = 730
F( 25, 704) = 4.41
Prob > F = 0.0000
R-squared = 0.2989
Root MSE = 0.36334
Regression for Long Rate

- Long Rate decreases when long rate exceeds short

```
reg d.t10year L.spread L(1/12).d.t3 L(1/12).d.t10year, r
```

Linear regression

| Coef.  | Std. Err. | t     | P>|t|   | [95% Conf. Interval] |
|--------|-----------|-------|-------|----------------------|
| spread | -0.0238377| 0.0107132 | -2.23 | 0.026 | [-0.0448714, -0.002804] |

Number of obs = 730
F(25, 704) = 5.31
Prob > F = 0.0000
R-squared = 0.2455
Root MSE = 0.24224
Unknown Cointegrating Coefficient

• If the cointegrating coefficient is unknown, it can be estimated
• Simplest estimator
  – Least squares of $y$ on $x$
  – Consistent (Stock, 1987), but inefficient
  – Standard errors meaningless

```
reg t3 t10year
```

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 743</th>
<th>F( 1, 741) = 4122.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5876.76913</td>
<td>1</td>
<td>5876.76913</td>
<td>Prob &gt; F = 0.0000</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>1056.41246</td>
<td>741</td>
<td>1.42565784</td>
<td>R-squared = 0.8476</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>6933.18159</td>
<td>742</td>
<td>9.3439105</td>
<td>Adj R-squared = 0.8474</td>
<td>Root MSE = 1.194</td>
</tr>
</tbody>
</table>

| t3month | Coef.    | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|---------|----------|-----------|-------|-----|---------------------|
| t10year | 1.00771  | 0.0156955 | 64.20 | 0.000 | .9768974 - 1.038523 |
| _cons  | -1.530229| 0.1043782 | -14.66| 0.000 | -1.735141 - 1.325317 |
Dynamic OLS

• Stock and Watson (1994) proposed a simple efficient estimator called dynamic OLS (DOLS)
• Regress $y$ on $x$ and leads and lags of $Dx$
• Use Newey-West standard errors
  – Lag $M = .75^{*}T^{1/3}$
• `newey t3 t120 L(-12/12).d.t120, lag(6)`
Interest Rate Cointegration

The estimated cointegrating coefficient is 0.99, almost exactly the expected value of 1.0!

The confidence interval contains our expected value of 1.

So in this case using the value 1 is recommended.
Estimated Cointegrating Coefficient

• Otherwise, the regression can use the estimated equilibrium error

\[ z_{t-1} = y_{t-1} - \hat{\theta}x_{t-1} \]
Johansen VECM Method

• Alternatively, you can estimate the full VECM

• `vec t3month  t10year, trend(constant) lags(12)`

• This estimates a Vector Error Correction model with the variables t3month and t10year, including a constant, and 12 lags of the variables

• This estimates equations for both variables, plus the cointegrating coefficient
Cointegrating Estimate

| beta     | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|----------|--------|-----------|-------|------|---------------------|
| _ce1     |        |           |       |      |                     |
| t3month  | 1      | .         | .     | .    | .                   |
| t10year  | -.9998363 | .0695128 | -14.38 | 0.000 | -1.136079 | -.8635938 |
| _cons    | 1.49518 | .         | .     | .    | .                   |

- The estimate is .99, similar to DOLS (.99)
- The DOLS method is simpler, but many econometricians prefer the VECM estimate.
Evaluating Forecasts

• Are our forecasts good?
• How do we know?
• How do we assess a historical forecast?
• How do we compare competing forecasts?
Properties of Forecasts

• What are the properties of a good forecast?
• We start by examining optimal forecasts.
Linear Representation

- The Wold representation for $y$, $h$ steps out, is
  \[ y_{n+h} = \mu + e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots \]

- The $h$-step-ahead optimal forecast is
  \[ y_{n+h|n} = \mu + b_h e_n + b_{h+1} e_{n-1} + b_{h+2} e_{n-2} + \cdots \]

- The $h$-step-ahead optimal forecast error is
  \[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]
Optimal Forecast is Unbiased

• The forecast error is

\[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]

• It has expectation

\[ E( e_{n+h|n} ) = 0 \]

• And thus the optimal forecast is unbiased
One-Step Errors are White Noise

• The one-step forecast error is

\[ e_{n+1|n} = e_{n+h} \]

• Which is unforecastable white noise

• Thus the optimal one-step-ahead forecast error is white noise and unforecastable
h-step-ahead errors are MA(h-1)

• The h-step forecast error is

\[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]

• This is a MA(h-1)

• Thus optimal h-step-ahead forecast errors are correlated, but at most a MA(h-1)
Forecast Variance

- The h-step forecast error is
  \[ e_{n+h|n} = e_{n+h} + b_1 e_{n+h-1} + b_2 e_{n+h-2} + \cdots + b_{h-1} e_{n+1} \]
- Its variance is the forecast variance, and is
  \[ \text{var}(e_{n+h|n}) = \left(1 + b_1^2 + b_2^2 + \cdots + b_{h-1}^2\right) \sigma^2 \]
- This is increasing in the forecast horizon \( h \)
- The variance of optimal forecasts increases with the forecast horizon
Unforecastable Errors

• The forecast errors should be unforecastable from all information available at the time of the forecast
• Not even the optimal forecast
• The coefficients should be zero in the regression

\[ e_{n+h|n} = \alpha + \beta y_{n+h|n} + \varepsilon_{n+h} \]
\[ \alpha = 0, \beta = 0 \]
Formal Comparison

• Since

\[ e_{n+h|n} = y_{n+h} - y_{n+h|n} \]

this implies

\[ y_{n+h} = \alpha + \beta y_{n+h|n} + e_{n+h|n} \]

\[ \alpha = 0, \beta = 1 \]

• The regression of the actual value on the ex-ante forecast should have a zero intercept and a coefficient of 1
Mincer-Zarnowitz Regression

• This is called a “Mincer-Zarnowitz” regression, proposed in a paper
  – “The evaluation of economic forecasts”
• Jacob Mincer (1922-2006)
  – Father of modern labor economics
• Victor Zarnowitz (1919-2009)
  – Leading figure in business cycle dating
Mincer-Zarnowitz Test

• Estimate the simple regression

\[ y_{n+h} = \alpha + \beta y_{n+h|n} + e_{n+h|h} \]

• Test the joint hypothesis

\[ \alpha = 0, \beta = 1 \]

• If the coefficients are different, it indicates systematic bias in the historical forecasts
Summary:
Properties of Optimal Forecasts

- Unbiased
- 1-step-ahead errors are white noise
- h-step-ahead errors are at most MA(h-1)
- Variance of h-step-ahead error is increasing in h
- Forecast errors should be unforecastable
Forecasting Average Growth

• When we are forecasting future growth, we may be interested in total future growth out to \( h \) periods
• For example, the growth rate of GDP during 2015
• This is the average of the growth rates during the four quarters 2015q1, ..., 2015q4
Average Growth

- If $y_t$ is the growth rate in period $t$, then the average future $h$-step growth is

$$y_{n+1:n+h} = \frac{y_{n+1} + \cdots + y_{n+h}}{h}$$

- The forecast of the average growth is

$$y_{n+1:n+h|n} = \frac{y_{n+1|n} + \cdots + y_{n+h|n}}{h}$$

- What are its properties?
Average Forecast Error

• The error of the average forecast is

\[ e_{n+1:n+h|n} = y_{n+1:n+h|n} - y_{n+1:n+h} \]

\[ = \frac{y_{n+1|n} + \cdots + y_{n+h|n}}{h} - \frac{y_{n+1} + \cdots + y_{n+h}}{h} \]

\[ = \frac{(y_{n+1|n} - y_{n+1}) + \cdots + (y_{n+h|n} - y_{n+h})}{h} \]

\[ = \frac{e_{n+1|n} + \cdots + e_{n+h|n}}{h} \]

• Which is the average of the 1-step through h-step errors
Average Forecast Error Variance

• Since the average forecast error is the average of forecast errors, it has a smaller variance than the h-step variance

\[ \text{var}(e_{n+1:n+h|n}) = \text{var} \left( \frac{e_{n+1|n} + \cdots + e_{n+h|n}}{h} \right) \leq \text{var}(e_{n+h|n}) \]

• So multi-period growth rate forecasts will have smaller variance than h-step ahead growth forecasts
  – The forecasted average growth rate for 2015 has a smaller variance than the forecasted growth rate for 2015q4
Evaluating Forecasts

• Suppose we have a sequence of real forecasts
• Perhaps they are our own forecasts
• How can we evaluate the forecasts?
Measures of Forecast Performance

- Form the historical sequence of forecasts and actual values.
- Construct the forecast error as the difference
Example

• CBO’s Economic Forecasting Record: 2009 Update

• Economic forecasts made by
  – Congressional budget office (CBO)
  – U.S. Administration
  – Private forecasters
    • Blue Chip average
  – CBO regularly assesses their forecasts
CBO Comparison

- Real Output
- Nominal Output
- Inflation
- 3-month T-Bill rate
- 10-year Treasury note rate
- Difference between CPI and GDP inflation
- Both 2-year and 5-year forecasts
Table 4.
Comparison of CBO’s, Blue Chip’s, and the Administration’s Forecasts of Two-Year Average Growth Rates for Nominal Output

(Percent, by calendar year)

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>CBO</th>
<th>Blue Chip</th>
<th>Administration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast</td>
<td>Error</td>
<td>Forecast</td>
<td>Error</td>
</tr>
<tr>
<td>GNP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1976-1977</td>
<td>11.5</td>
<td>13.1</td>
<td>1.7</td>
<td>*</td>
</tr>
<tr>
<td>1977-1978</td>
<td>12.1</td>
<td>10.8</td>
<td>-1.3</td>
<td>*</td>
</tr>
<tr>
<td>1978-1979</td>
<td>12.5</td>
<td>10.9</td>
<td>-1.6</td>
<td>*</td>
</tr>
<tr>
<td>1979-1980</td>
<td>10.4</td>
<td>11.0</td>
<td>0.5</td>
<td>*</td>
</tr>
<tr>
<td>1980-1981</td>
<td>10.4</td>
<td>9.7</td>
<td>-0.7</td>
<td>*</td>
</tr>
<tr>
<td>1981-1982</td>
<td>8.0</td>
<td>12.1</td>
<td>4.1</td>
<td>*</td>
</tr>
<tr>
<td>1982-1983</td>
<td>6.3</td>
<td>9.7</td>
<td>3.4</td>
<td>9.5</td>
</tr>
<tr>
<td>1983-1984</td>
<td>9.8</td>
<td>8.2</td>
<td>-1.6</td>
<td>9.0</td>
</tr>
<tr>
<td>1984-1985</td>
<td>9.0</td>
<td>9.9</td>
<td>0.9</td>
<td>9.6</td>
</tr>
<tr>
<td>1985-1986</td>
<td>6.2</td>
<td>7.6</td>
<td>1.3</td>
<td>7.4</td>
</tr>
<tr>
<td>1986-1987</td>
<td>5.8</td>
<td>7.1</td>
<td>1.3</td>
<td>6.7</td>
</tr>
<tr>
<td>1987-1988</td>
<td>7.0</td>
<td>6.5</td>
<td>-0.5</td>
<td>6.4</td>
</tr>
<tr>
<td>1988-1989</td>
<td>7.6</td>
<td>6.3</td>
<td>-1.3</td>
<td>6.1</td>
</tr>
<tr>
<td>1989-1990</td>
<td>6.7</td>
<td>6.8</td>
<td>0.1</td>
<td>6.6</td>
</tr>
<tr>
<td>1990-1991</td>
<td>4.6</td>
<td>6.1</td>
<td>1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>1991-1992</td>
<td>4.4</td>
<td>5.7</td>
<td>1.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>
Comparison

• By showing the actual, forecasts, and forecast errors side-by-side, we can informally see which forecast performs better
Formal Comparison

• The forecasts can be compared by estimating the **bias** and **risk** (expected loss) of the forecasts

• They are estimated from $R$ forecast errors:
  
  – Bias, Mean Absolute Error, Root Mean Squared Error

  \[
  Bias = \frac{1}{R} \sum_{n=1}^{R} e_{n+h|n} \\
  MAE = \frac{1}{R} \sum_{n=1}^{R} |e_{n+h|n}| \\
  RMSE = \left( \frac{1}{R} \sum_{n=1}^{R} e_{n+h|n}^2 \right)^{1/2}
  \]
# CBO Comparison

Table 1. Summary Measures of Performance for Two-Year Average Forecasts

<table>
<thead>
<tr>
<th></th>
<th>CBO</th>
<th>Blue Chip(^a)</th>
<th>Administration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth Rate for Real Output (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>1.2</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>Growth Rate for Nominal Output (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>1.0</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>1.2</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Inflation in the Consumer Price Index (1982-2007)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Table 2.

Summary Measures of Performance for Five-Year Average Projections

(Percentage points)

<table>
<thead>
<tr>
<th></th>
<th>CBO</th>
<th>Blue Chip&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Administration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth Rate for Real Output (1979-2004)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.6</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>0.9</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td><strong>Growth Rate for Nominal Output (1982-2004)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>0.3</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>0.9</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Difference Between Inflation in the CPI and the GDP Price Index (1983-2004)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean error</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Root-mean-square error</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Data Revision

• A major difficulty with forecast evaluation is that for many series, there are serious data revisions
• The data used for forecasting, and the series published today, are different
• The series forecasted, and the series reported today, are different
• Price series, and real series based on price levels, are rebased every few years
• These rebasing are not scale transformations, because the construction of real output is done at a disaggregate level, and then aggregated.
## Real Output

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Meese-Rogoff Puzzle

• The most influential paper using the method of forecast model comparison is
  – “Empirical exchange rate models of the seventies”
  – Richard Meese and Kenneth Rogoff
  – *Journal of International Economics, 1983*
Meese-Rogoff

- Ken Rogoff (currently Harvard)
  - Recent book
  - *This Time is Different: Eight Centuries of Financial Folly*

- Dick Meese (formerly Berkeley, now Barclay Global Investors)
  - 1978 UW Ph.D.
  - Economics Dept Advisory Board
Meese-Rogoff paper

- They compare the RMSE and bias of 1-month, 6-month and 12-month forecasts of a set of exchange rates, using structural models.
- They compare the performance of the economic models with the performance of a random walk.
- They found the random walk beat the economic models.
- Very influential paper.
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Summary

• Evaluation of forecasts achieve by comparing the bias, MAE and RMSE of forecast errors

• Most influential paper is Meese-Rogoff, because they showed that naïve random walk model has lower forecast risk than structural economic models

• This established a challenge for economic modeling and forecasting.
  – Can we beat simple naïve models?!