

Wisconsin Unemployment Rate Forecast

- Recall, the Nov 2016 Wisconsin Unemployment Rate was 4.1%
- Wednesday, I forecasted the Dec 2016 rate
 - Point Forecast 4.0%
 - 80% Interval: (3.95%, 4.05%)

Data from Wisconsin DWD

- Google “Wisconsin Department of Workforce Development”
- Click “Newsroom”, “BLS Data”

Unemployment Release

State of Wisconsin Department of Workforce Development

Thursday, January 19, 2017

- Place of residence data: A preliminary seasonally adjusted unemployment rate of 4.0 percent in December 2016, down 0.1 percent from November.
- Place of work data: Based on preliminary data, the state added a statistically significant 28,700 total non-farm jobs and 28,900 private-sector jobs from December 2015 to December 2016, with a one-month gain of 900 private-sector jobs and significant one-month gains of 3,000 in manufacturing and 2,700 in Construction.

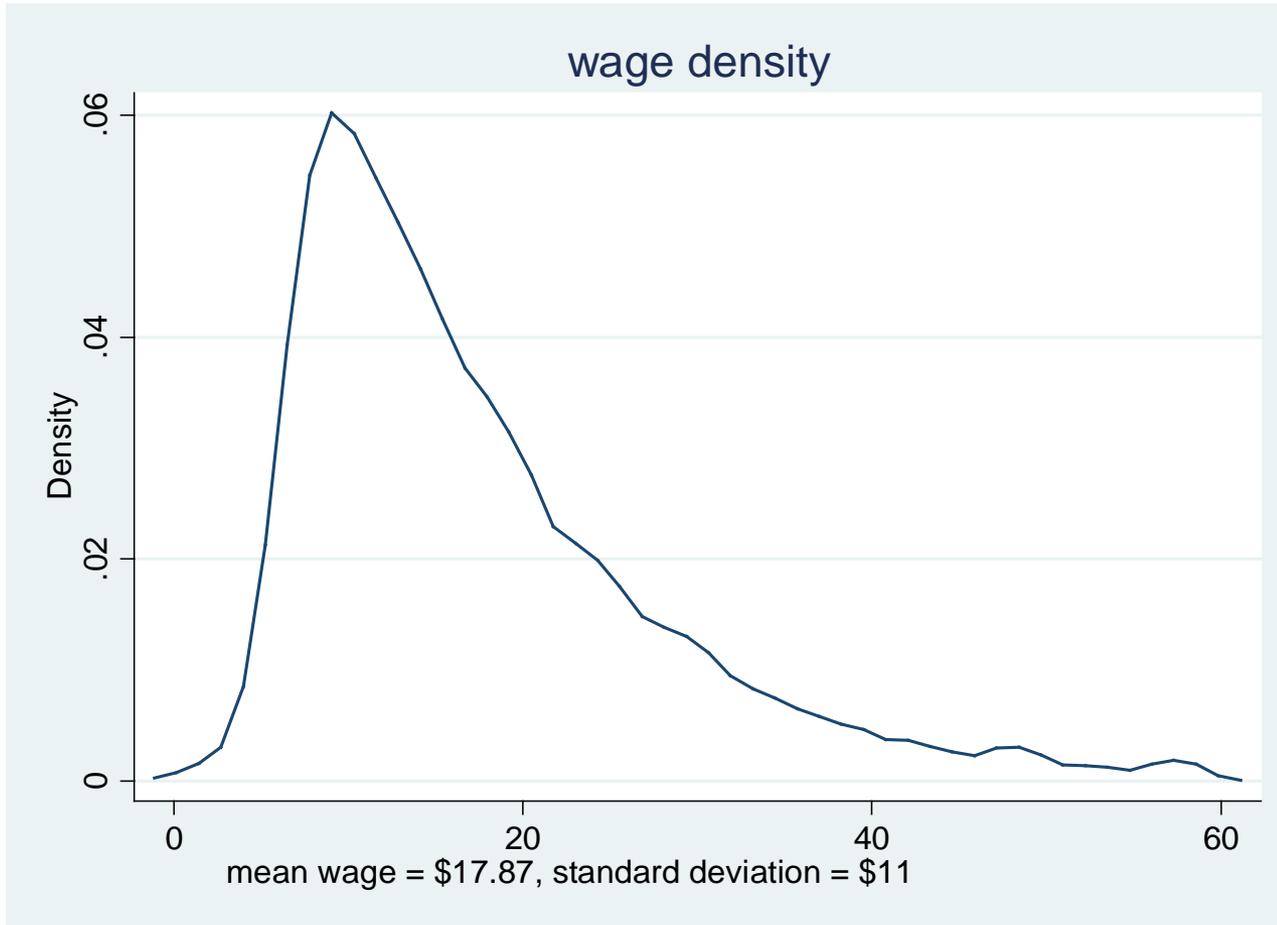
Forecasting

- A *forecast* is a guess about an unknown.
- An *economic forecast* is a forecast about an economic variable, event, outcome, or duration.

Let's Make a Forecast

- Suppose we take a random household in the United States.
- Let's forecast the wage (hourly) of the head of household.
- What is your forecast?
- Will your forecast be correct? Why?

Wage Density



Wage Distribution



Description Statistics in Stata

- Summarize, detail lists summary statistics
- Mean is 17.87, standard deviation 11.6

```
. summarize wage, detail
```

```
                wage
-----
```

	Percentiles	Smallest		
1%	4.375	.0166667		
5%	6.266667	.025		
10%	7.5	.025	Obs	18808
25%	10	.0307692	Sum of Wgt.	18808
50%	14.76333		Mean	17.87572
		Largest	Std. Dev.	11.59428
75%	22.45417	137.381		
90%	32.3	160	Variance	134.4272
95%	40.6	173	Skewness	2.321593
99%	57.7	231	Kurtosis	16.86886

Wage Forecast

- Wages have a distribution in the population.
- It is impossible to correctly forecast an individual's wage.
- The mean wage is \$17.87
- If we forecast “the wage will be \$17.87” it is close to impossible that a given person's wage will be exactly \$17.87.
- The most correct and accurate forecast is the entire distribution (or density).

Forecast Distribution

Percentiles

1%	\$ 4.37
5%	\$ 6.27
10%	\$ 7.50
25%	\$10.00
50%	\$14.76
75%	\$22.45
90%	\$32.30
95%	\$40.60
99%	\$57.70

Forecast Distribution

- Suppose we are trying to forecast an economic variable y
- For example, a random person's wage.
- y has a distribution $F(y)$ which is mathematically defined as

$$F(u) = P(y \leq u)$$

- Visually, we represent distributions through their density functions $f(y) = \frac{d}{dy} F(y)$

Forecast Distribution

- A complete forecast for y is its distribution $F(y)$ or density $f(y)$.
- Either $F(y)$ or $f(y)$ summarizes all that is known and unknown about the potential values for y .
- We call F the *forecast* or *predictive* distribution.
- Can you forecast a person's wage?
 - We cannot know with certainty the wage
 - We know (or can estimate) the distribution:
 - The range and likelihood of possible wages.

What might this matter?

- Suppose your company underwrites unemployment insurance which pay a person's wage y if they become unemployed.
- Suppose a random person loses their job.
- What is the cost to the company?
- We cannot know with certainty, but we may know the *distribution* of the potential costs.

Point Forecast

- In many applications, users want a single number.
- A *point forecast* \hat{y} is our best guess for the unknown object.
- A point forecast for a random person's wage could be the mean \$17.87 or the median \$14.76

Point Forecast

- A point forecast \hat{y} is a function of the predictive distribution F
- It can be viewed as a summary of F .
- Which function should be used?
- What is our best guess for y based on F ?
- It turns out that the answer depends on our *loss function* – how we measure the costs due to potential forecast error.

Forecast Error

- If we forecast a random variable y with a forecast f we say that the *forecast error* is

$$e = y - \hat{y}$$

- The forecast error is the difference between the actual and the forecast.
- For example, if we forecasted that an individual's wage would be \$18, but it turns out that it is \$24, then the error is $24 - 18 = 6$. If their wage was actually \$14, then the error would be $14 - 18 = -4$.

Forecast error

- So long as the variable y is random (not perfectly forecastable) then there will always be forecast error.
- This cannot be avoided.
- However, errors are costly.
- A user can assign costs to a forecast error.
- We call this the loss function

$$L(e) = \textit{Loss Function}$$

Loss Functions

- Common mathematical choices

- Quadratic Loss

$$L(e) = e^2$$

- Absolute Loss

$$L(e) = |e|$$

- Both are symmetric

- Treat positive and negative forecast errors symmetrically

- Quadratic loss penalizes large errors much more than small errors.
- Asymmetric Loss Functions also possible.

Many Loss Functions are (Approximately) Quadratic

- Consider a monopolist selling a product Q at a price P , with linear demand and zero cost.
- The monopolist sets price P and then sells Q .
- The demand equation is $Q=2a-P$
- The profit function is $\pi(P)=2aP-P^2$
- The optimal price is $P^*=a$, optimal profit $\pi^*=\pi(P^*)=a^2$.
- Let \hat{a} be a forecast of a with error $e=a-\hat{a}$.
- The monopolist sets $P=\hat{a}$
- The Loss is $L=\pi^*-\pi(\hat{a})=a^2-2a\hat{a}+\hat{a}^2=(a-\hat{a})^2=e^2$
- This is quadratic loss.

Risk

- The *Risk* of a forecast is its expected loss.
- Mathematically,

$$R(\hat{y}) = \mathbb{E} L(e) = \mathbb{E} L(y - \hat{y})$$

- For quadratic loss

$$R(\hat{y}) = \mathbb{E}(y - \hat{y})^2$$

- For absolute loss

$$R(\hat{y}) = \mathbb{E}|y - \hat{y}|$$

Optimal Point Forecast

- The optimal (best) point forecast is the function \hat{y} of the predictive distribution F which minimizes the risk (minimizes the expected loss).
- In the quadratic case

$$\begin{aligned}R(\hat{y}) &= \text{E}(y - \hat{y})^2 \\ &= \text{E} y^2 - 2\hat{y} \text{E} y + \hat{y}^2\end{aligned}$$

which is a quadratic in \hat{y}

Optimal Point Forecast – Quadratic Loss

- The \hat{y} which minimizes the Risk is found by differentiation

$$0 = -2E y + 2\hat{y}$$

- Which has the solution

$$\hat{y} = E y$$

- The optimal point forecast is the mean of the predictive distribution

Optimal Forecast Under Quadratic Loss is the Mean

- The optimal point forecast under quadratic loss is the mean.
- For example, to forecast the wage of a random person, our optimal point forecast is \$17.87

Optimal Prediction Under Absolute Loss is the Median

- The risk of a forecast is

$$R(\hat{y}) = E|y - \hat{y}|$$

- This is minimized by the median
- The optimal forecast under absolute loss is

$$\hat{y} = \textit{Median}(y)$$

- For example, to forecast the wage of a random person, the optimal point forecast is \$14.76

Choice of Loss Function

- We have learned that the optimal point forecast depends upon the loss function
- The mean Ey minimizes the expected squared error
- The median minimizes the expected absolute error.
- Other loss functions lead to different solutions.
- In most cases, we do not have an explicit loss function. So we take the simplest approach and use the mean, which is equivalent to squared loss.
- However, in a real-world application, you might be able to articulate the explicit loss due to forecasting error. In this case, it would be best to use the loss function explicitly, leading to specialized estimators and forecasts.

Summary so Far

- The optimal (best) point forecast of a random variable y from a distribution $F(y)$, when our loss function is quadratic, is the mean Ey of the distribution
- Our forecast of a random person's wage is the mean wage, \$17.87

Interval Forecast

- We have said that a complete forecast for the unknown wage y is its density f , but for simplicity users often want a point forecast \hat{y}
- An intermediate solution is to report a **forecast interval** $C=[\hat{y}_L, \hat{y}_U]$.
- A forecast interval is similar to a confidence interval in statistics.
- The goal is for the unknown wage y to lie in the forecast interval with a pre-specified probability.

Forecast Interval

- An $x\%$ forecast interval C satisfies

$$P(y \in C) = x$$

- Common choices for x include
 - $x = .90$ (90%)
 - $x = .80$ (80%)
 - $x = .50$ (50%)
- 50% intervals have the simple property that they contain the unknown y with even odds.

Use Forecast Intervals!

- Forecast intervals are simple, yet less commonly reported.
- [In policy work, they are often calculated and used by the analysts, but not discussed in public discussion.]
- A point forecast by itself does not communicate the uncertainty in the forecast
- A forecast interval is easier to interpret than the entire distribution

Percentiles

- A percentile (of a distribution) is the value below which a given percentage of the distribution is found
- For example, in the wage data set, 99% of wages are below \$57. So the 99th percentile is \$57.
- 10% of wages are at or below \$6.26. So the 10th percentile is \$6.26

Quantiles

- Another name for percentiles is *quantiles*
- The difference is that we refer to quantiles in terms of fractions (numbers between 0 and 1) rather than percentages.
- Example: The 0.99 quantile of the wage distribution is \$57. The 0.10 quantile is \$6.26
- Definition: The α 'th quantile of y is the number q_α which satisfies
$$\alpha = F(q_\alpha)$$
- They are found by inverting the distribution function $F(y)$

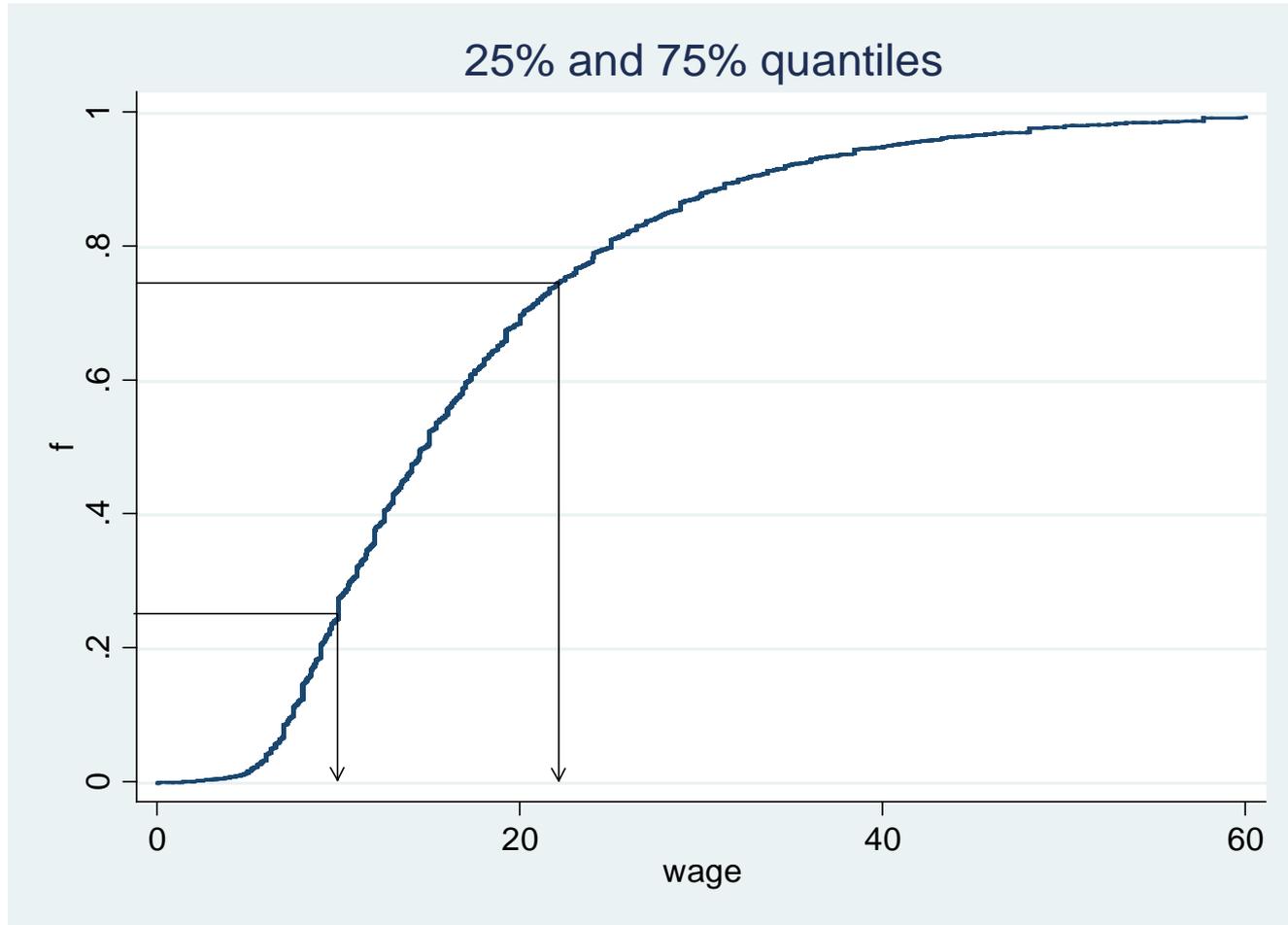
Quantiles and Forecast Intervals

- An $x\%$ forecast interval $C=[\hat{y}_L, \hat{y}_U]$ should satisfy

$$P(y \in C) = x$$

- This holds if $P(\hat{y}_L < x < \hat{y}_U) = P(x < \hat{y}_U) - P(x < \hat{y}_L) = x$
- This means that the endpoints $[\hat{y}_L, \hat{y}_U]$ are quantiles of the distribution F of y .
- For a $x\%$ interval, you need the $x/2$ and $1-x/2$ quantiles
- For example, for a 50% forecast interval, you want to know the 0.25 and 0.75 quantiles, or equivalently the 25% and 75% percentiles.
- They are found by inverting the distribution function

Quantiles of Wage Distribution



Implication for Forecast Intervals

- We need to find specified quantiles (percentiles) of the forecast distribution

Monotonicity Rule

- Suppose y is a random variable with α 'th quantile $q_\alpha(y)$
- Let $t=g(y)$ be an increasing transformation of y
 - Example 1: $t=a+by$
 - Example 2: $t=\ln(y)$
 - Example 3: $t=\exp(y)$
- Then the α 'th quantile of t is $g(q_\alpha)$
 - Example 1: $q_\alpha(t)=a+bq_\alpha(y)$
 - Example 2: $q_\alpha(t)=\ln(q_\alpha(y))$
 - Example 3: $q_\alpha(t)=\exp(q_\alpha(y))$

Forecast Interval Transformations

- Suppose an $x\%$ forecast interval for y is $[\hat{y}_L, \hat{y}_U]$
- Then an $x\%$ forecast interval for $t=g(y)$ is

$$[g(\hat{y}_L), g(\hat{y}_U)]$$

- Example 1: $t=a+by$
 - $[a+b\hat{y}_L, a+b\hat{y}_U]$
- Example 2: $t=\ln(y)$
 - $[\ln(\hat{y}_L), \ln(\hat{y}_U)]$
- Example 3: $t=\exp(y)$
 - $[\exp(\hat{y}_L), \exp(\hat{y}_U)]$

Normal Rule

- If the variable y is normally distributed $N(0,1)$
 - The point forecast for y is 0
 - An $\alpha\%$ forecast interval for y is $[-z_{\alpha/2}, z_{\alpha/2}]$ where $z_{\alpha/2}$ are quantiles from the normal distribution table.
 - For a 95% interval, $z_{.025}=1.96$,
 - For a 90% interval, $z_{.05}=1.645$,
 - For a 80% interval, $z_{.10}=1.282$,
 - For a 50% interval, $z_{.25}=0.675$

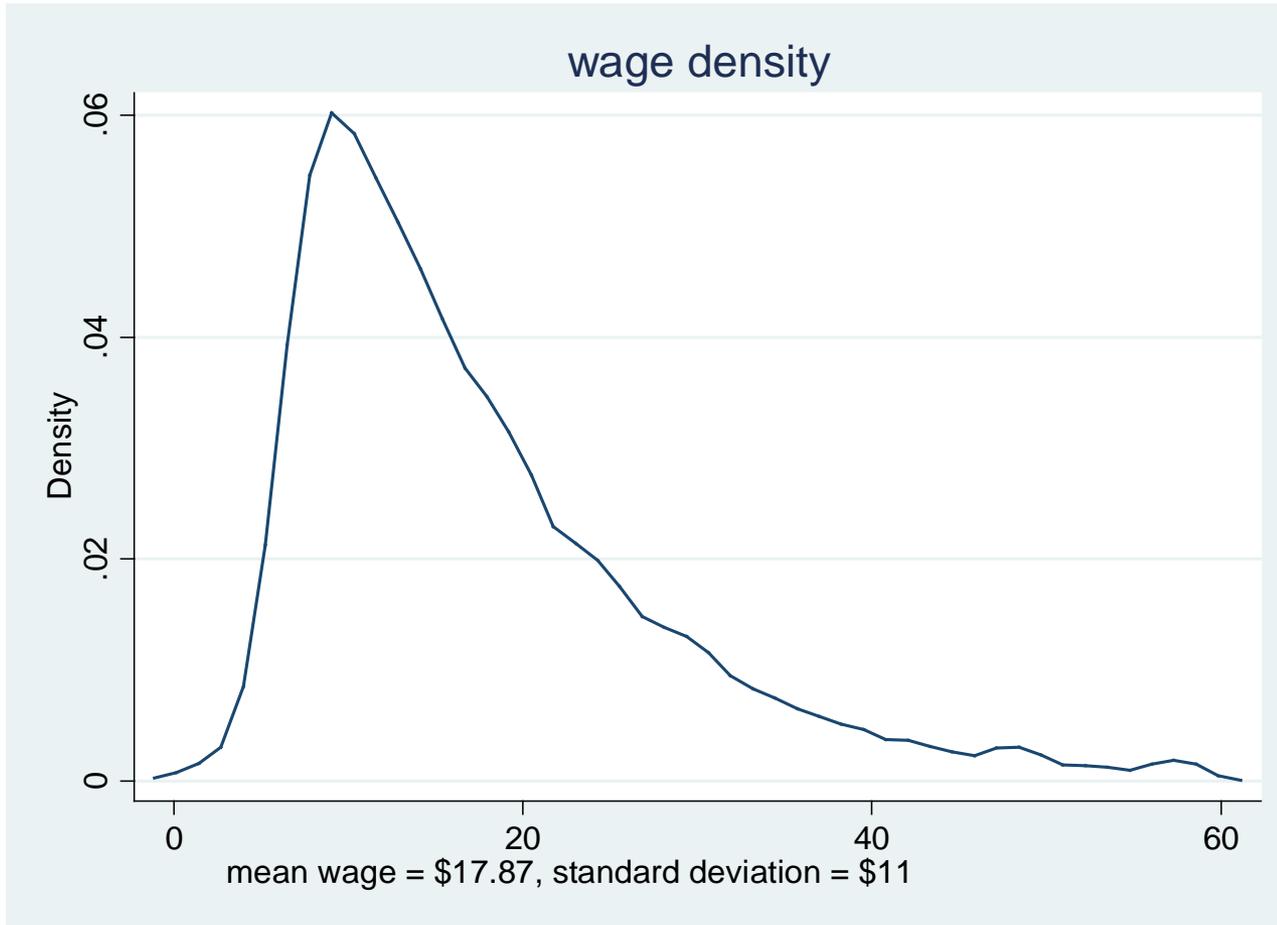
Normal Rule

- Suppose the variable y is normally distributed $N(\mu, \sigma^2)$
- The optimal (mean-square) point forecast is the mean μ
- The linear function $t=(y-\mu)/\sigma$ is distributed $N(0,1)$
 - Forecast intervals for t are $[-z_{\alpha/2}, z_{\alpha/2}]$
 - Forecast intervals for $y=\mu+\sigma t$ are
 - $[\mu-\sigma z_{\alpha/2}, \mu+\sigma z_{\alpha/2}] = \mu \pm \sigma z_{\alpha/2}$
- All you need to know is the mean and standard deviation

Example: Wage

- For the U.S. wage distribution
 - $\mu=17.9$
 - $\sigma=11.6$
- A 90% forecast interval is then
 - $17.9 \pm 11.6 * 1.645$
 - $[-\$1.18, \$36.98]$
- The lower bound is negative!
 - This is because the wage distribution is not well approximated by the normal

Wage Density

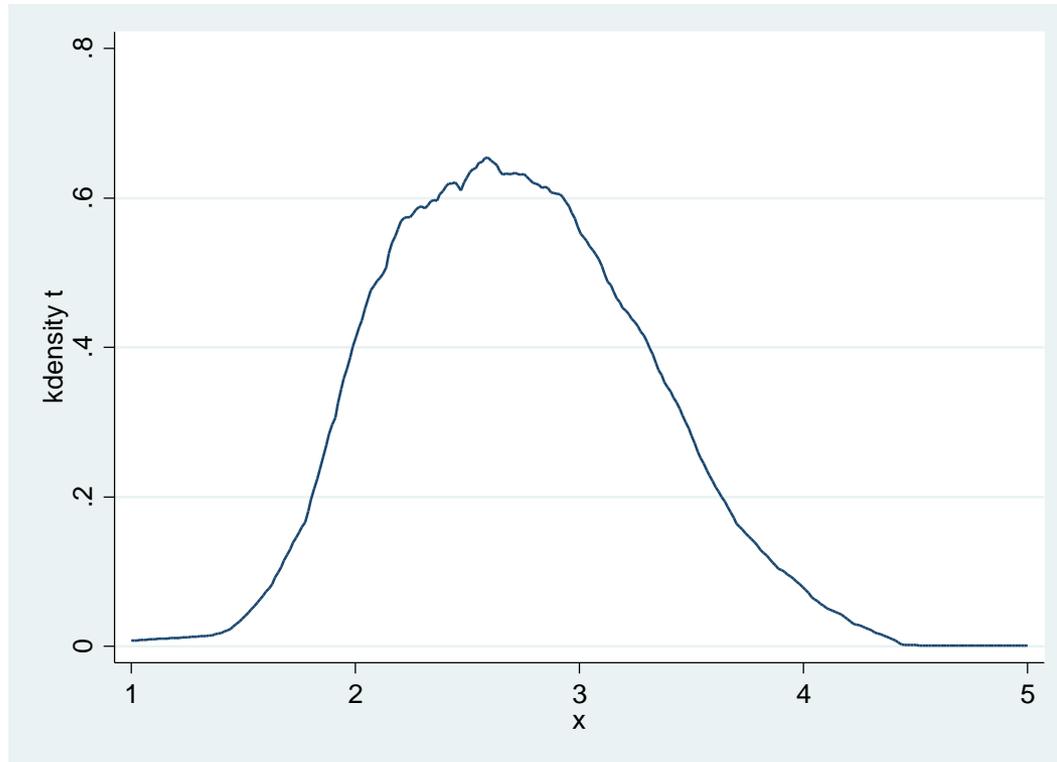


Log Normality

- Some economic variables, such as wages, are quite non-normal, but are close to “log normal”
- Def: y is **log normal** if $\ln(y)$ is distributed $N(\mu, \sigma^2)$
- Can visually check by creating a variable $t = \ln(y)$, and plotting its density
- [Note: This only works if y is strictly positive]
 - `gen t = ln(y)`
 - `twoway kdensity t, range(1 5)`

Log Wage

- $\text{Log}(\text{wage})$ is closer to normal than wages
- Mean = 2.7
- St Dev = 0.6



Log Transformation Forecast Intervals

- 90% forecast interval for log wages
 - $2.7 \pm 0.6 * 1.645 = [1.7, 3.7]$
- By monotonicity, a 90% forecast interval for $wages = \exp(\ln(wage))$
 - $[\exp(1.7), \exp(3.7)] = [\$5.47, \$40.45]$
- This is more reasonable than the $[-\$1.18, \$36.98]$ interval found by applying normality.

Empirical Quantile Intervals

- The desired $x\%$ forecast interval is from the $x/2$ to $1-x/2$ quantile of y 's distribution
- We can use the *empirical quantiles (or percentiles)* estimate these endpoint
- For a 90% interval, this is [\$6.27, \$40.60]

```
. summarize wage, detail
```

wage				
	Percentiles	Smallest		
1%	4.375	.0166667		
5%	6.266667	.025		
10%	7.5	.025	Obs	18808
25%	10	.0307692	Sum of Wgt.	18808
50%	14.76333		Mean	17.87572
		Largest	Std. Dev.	11.59428
75%	22.45417	137.381		
90%	32.3	160	Variance	134.4272
95%	40.6	173	Skewness	2.321593
99%	57.7	231	Kurtosis	16.86886

Summary – Unconditional Forecasts

- A complete forecast of a random variable y is the distribution F or density f of the variable.
- A point forecast is a single number \hat{y} to summarize the distribution.
 - The optimal choice depends on the loss function.
 - When loss is quadratic, the optimal point forecast is the mean.
- Forecast intervals are quantiles of the forecast distribution, and convey useful information about the uncertain in y .
 - They can be computed by the normal rule, using a log transformation, or using empirical quantiles

Assignments for the week

- Read Chapters 1-2 from Diebold
- Problem Set # 1
 - Due Tuesday (1/24)
- Read Chapter 1 from *The Signal and the Noise*
 - Reading Reflection
 - Due Thursday (1/26)
 - We will have a discussion that day in class