Regression Models

- Bivariate data (y,x)
- Multivariate $(y,x_1,\ldots,x_k)$
- Suppose the conditional mean of *y* is a function of x

$$E\left(y_t \mid x_t\right) = \alpha + \beta x_t$$

- Then the regression function is the optimal forecast of *y* given *x*

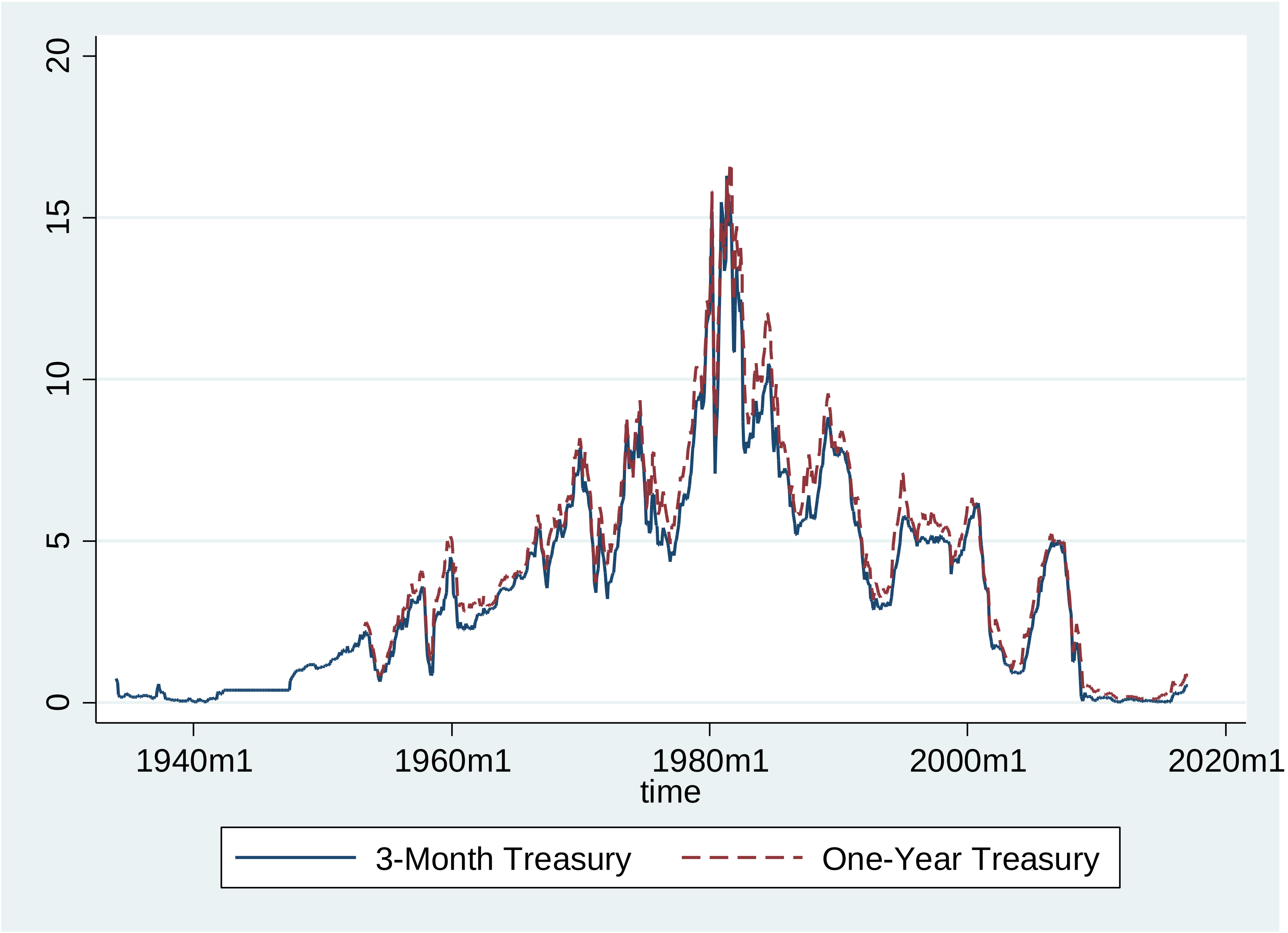
Regression

- Model

$$y_t = \alpha + \beta x_t + e_t$$

- Estimation: Least-Squares
- Example: Interest Rates
  - Monthly
  - Rates on 3-month and 1-year U.S. Treasury Bonds
  - 3-month bond series dates from 1934m1
  - 1-year bond series dates from 1953m4

Interest Rates on Treasury Bonds



Least-Squares, 3-month on 1-year rate

$$\hat{y}_t = -0.19 + 0.94 x_t$$

```
. reg t3month t1year, r

Linear regression                                      Number of obs     =        767
                                                       F(1, 765)         =   23107.44
                                                       Prob > F          =     0.0000
                                                       R-squared         =     0.9876
                                                       Root MSE          =     .34516
```

t3month	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year	.9353714	.0061533	152.01	0.000	.923292	.9474507
_cons	-.1897946	.0240209	-7.90	0.000	-.2369494	-.1426399

Forecast

$$y_{n+h} = \alpha + \beta x_{n+h} + e_{n+h}$$

$$\hat{y}_{n+h|n} = \alpha + \beta x_{n+h}$$

- A forecast of y_{n+h} requires x_{n+h}
- This is not typically feasible, as x_{n+h} is unknown at time n.

Regressor forecast

- Suppose we have a forecast for x
- Then

$$\hat{y}_{n+h|n} = \alpha + \beta \hat{x}_{n+h}$$

- For example, if

$$E\left(x_t \mid \Omega_{t-h}\right) = \gamma + \phi x_{t-h}$$

then

$$\hat{x}_{n+h|n} = \gamma + \phi x_n$$

1-year Treasury on Lagged Value

- Regress x_t on x_{t-12} (12-month ahead forecast)

```
. reg t1year L12.t1year, r

Linear regression                               Number of obs     =        755
                                                F(1, 753)         =    1387.89
                                                Prob > F          =     0.0000
                                                R-squared         =     0.7694
                                                Root MSE          =     1.5853
```

t1year	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year L12.	.8829714	.0237011	37.25	0.000	.8364432	.9294995
_cons	.5644619	.0987198	5.72	0.000	.3706632	.7582606

$$\hat{x}_t = 0.56 + 0.88 x_{t-12}$$

Interest Rate Forecast, h=12

- Estimates

$$\hat{y}_t = -0.19 + 0.94 x_t$$
$$\hat{x}_t = 0.56 + 0.88 x_{t-12}$$

- Current: $x_{2017M2} = 0.82\%$

$$\hat{x}_{2018M2} = 0.56 + 0.88 \times 0.82 = 1.28$$
$$\hat{y}_{2018M2} = -0.19 + 0.94 \times 1.28 = 1.01$$

Example

- Current 3-month interest rate = 0.52%
- Current 1-year interest rate = 0.82%
- 12-step-ahead point forecast for 3-month rate
  - Regression model: 1.01%
  - AR(1) Model (Direct Method): 0.84%

Regression Method (Direct)

- Combine

$$E\left(y_t \mid x_t\right) = \alpha + \varphi x_t$$

$$E\left(x_t \mid \Omega_{t-h}\right) = \gamma + \phi x_{t-h}$$

- We obtain

$$\begin{aligned} E\left(y_t \mid \Omega_{t-h}\right) &= \alpha + \varphi E\left(x_t \mid \Omega_{t-h}\right) \\ &= \alpha + \varphi\left(\gamma + \phi x_{t-h}\right) \\ &= \mu + \beta x_{t-h} \end{aligned}$$

Forecast Regression

- h-step-ahead

$$y_t = \mu + \beta x_{t-h} + e_t$$

- Forecast

$$y_{n+h|n} = \mu + \beta x_n$$

3-month rate on Lagged 1-year rate

```
. reg t3month L12.t1year, r

Linear regression                               Number of obs     =        755
                                                F(1, 753)         =    1064.42
                                                Prob > F          =     0.0000
                                                R-squared         =     0.7492
                                                Root MSE          =     1.5567
```

t3month	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year						
L12.	.8203938	.0251459	32.63	0.000	.7710294	.8697582
_cons	.3647291	.1022803	3.57	0.000	.1639407	.5655174

$$y_{n+h|n} = 0.36 + .82 x_n$$

$$\hat{y}_{n+h|n} = 0.36 + .82 \times 0.82 = 1.03$$

AR(q) Regressors

- Suppose x is an AR(q)

$$y_t = \alpha + \beta x_t + e_t$$

$$x_t = \gamma + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_q x_{t-q} + u_t$$

- Then a one-step forecasting equation for *y* is

$$y_t = \mu + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \cdots + \beta_q x_{t-q} + e_t$$

- And an h-step is

$$y_t = \mu + \beta_1 x_{t-h} + \beta_2 x_{t-h-1} + \cdots + \beta_q x_{t-h+1-q} + e_t$$

T-Bill example: AR(12) for 1-year rate

```
. reg t1year L(12/23).t1year, r

Linear regression                                      Number of obs     =        744
                                                       F(12, 731)        =     140.73
                                                       Prob > F          =     0.0000
                                                       R-squared         =     0.7739
                                                       Root MSE          =     1.5765
```

t1year	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year						
L12.	1.375194	.2810166	4.89	0.000	.823498	1.92689
L13.	-.7200031	.4123016	-1.75	0.081	-1.52944	.0894334
L14.	.4364246	.5145301	0.85	0.397	-.5737084	1.446558
L15.	-.2173301	.5456363	-0.40	0.691	-1.288531	.8538709
L16.	.2494866	.5387952	0.46	0.643	-.808284	1.307257
L17.	-.319791	.5098665	-0.63	0.531	-1.320768	.6811864
L18.	.3348807	.4845582	0.69	0.490	-.6164109	1.286172
L19.	.0684706	.4996681	0.14	0.891	-.9124851	1.049426
L20.	-.3466629	.4921321	-0.70	0.481	-1.312824	.6194979
L21.	-.0119003	.4554983	-0.03	0.979	-.9061412	.8823406
L22.	-.0637893	.3952368	-0.16	0.872	-.8397238	.7121453
L23.	.0826626	.2095718	0.39	0.693	-.3287718	.494097
_cons	.6621646	.1013239	6.54	0.000	.463244	.8610851

Regress 3-month on 12 lags of 12-month

```
. reg t3month L(12/23).t1year, r

Linear regression                               Number of obs     =        744
                                                F(12, 731)        =     117.81
                                                Prob > F          =     0.0000
                                                R-squared         =     0.7580
                                                Root MSE          =     1.5372
```

t3month	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year						
L12.	1.292683	.2894151	4.47	0.000	.7244992	1.860867
L13.	-.7221754	.4248946	-1.70	0.090	-1.556335	.1119839
L14.	.5122536	.5458989	0.94	0.348	-.559463	1.58397
L15.	-.1776701	.5981948	-0.30	0.767	-1.352055	.9967147
L16.	.1975543	.588286	0.34	0.737	-.9573773	1.352486
L17.	-.2590636	.5325977	-0.49	0.627	-1.304667	.78654
L18.	.3023064	.4852666	0.62	0.533	-.6503761	1.254989
L19.	.0544363	.4924534	0.11	0.912	-.9123555	1.021228
L20.	-.3141731	.4712961	-0.67	0.505	-1.239428	.6110823
L21.	-.0615082	.4250759	-0.14	0.885	-.8960234	.7730069
L22.	-.0813998	.3716856	-0.22	0.827	-.8110983	.6482988
L23.	.0562509	.2059851	0.27	0.785	-.348142	.4606438
_cons	.4889849	.1007357	4.85	0.000	.2912192	.6867507

Forecast for 3-month rate

- Predicted value for 2018M2 using 12 lags =1.22
- Predicted value using 3 lags=1.05

Distributed Lags

- This class of models is called *distributed lags*

$$y_t = \mu + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \cdots + \beta_q x_{t-q} + e_t$$
$$= \mu + B(L) x_{t-1} + e_t$$

- If we interpret the coefficients as the effect of *x* on *y*, we sometimes say
  - β_1 is the immediate impact
  - $\beta_1 + \ldots + \beta_n = B(1)$ is the long-run impact

Regressors and Dynamics

- We have seen AR forecasting models
- And now distributed lag model
- Add both together! ADL model.

$$A(L)y_t = \mu + B(L)x_{t-1} + e_t$$

or

$$y_t = \mu + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p}$$

$$+ \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q} + e_t$$

h-step

- Regress on lags of y and x, h periods back
- Estimate by least squares
- Forecast using estimated coefficients and final values

$$y_t = \mu + \alpha_1 y_{t-h} + \cdots + \alpha_p y_{t-h+1-p}$$
$$+ \beta_1 x_{t-h} + \cdots + \beta_q x_{t-h+1-q} + e_t$$

3-month t-bill forecast

t3month	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t3month						
L12.	.2200665	.5728128	0.38	0.701	-.9045189	1.344652
L13.	.3183035	.7708815	0.41	0.680	-1.195144	1.831751
L14.	-.3067659	.7271241	-0.42	0.673	-1.734306	1.120774
L15.	-.2821578	.7825353	-0.36	0.719	-1.818485	1.254169
L16.	.0164269	.7887997	0.02	0.983	-1.532199	1.565053
L17.	-.0574395	.7689248	-0.07	0.940	-1.567046	1.452167
L18.	.5585693	.7486907	0.75	0.456	-.9113118	2.02845
L19.	-.5694826	.6867805	-0.83	0.407	-1.917817	.7788521
L20.	-.2759316	.6119251	-0.45	0.652	-1.477305	.9254419
L21.	-.371824	.5475765	-0.68	0.497	-1.446864	.7032159
L22.	.0639512	.5345311	0.12	0.905	-.985477	1.113379
L23.	-.6263078	.3730493	-1.68	0.094	-1.358704	.1060883
t1year						
L12.	1.076166	.484711	2.22	0.027	.1245485	2.027784
L13.	-1.005522	.7235237	-1.39	0.165	-2.425993	.4149498
L14.	.7447113	.6913634	1.08	0.282	-.6126209	2.102044
L15.	.1073409	.7203744	0.15	0.882	-1.306948	1.521629
L16.	.2140472	.7638801	0.28	0.779	-1.285655	1.713749
L17.	-.191681	.7183557	-0.27	0.790	-1.602006	1.218644
L18.	-.2229479	.7122459	-0.31	0.754	-1.621278	1.175382
L19.	.5599726	.6842226	0.82	0.413	-.7833403	1.903286
L20.	-.0260269	.6364245	-0.04	0.967	-1.275499	1.223446
L21.	.300346	.5827375	0.52	0.606	-.8437243	1.444416
L22.	-.2333542	.540865	-0.43	0.666	-1.295218	.8285092
L23.	.703305	.3402191	2.07	0.039	.0353635	1.371246
_cons	.2473085	.0910063	2.72	0.007	.0686386	.4259783

Forecast for 3-month rate

- Using ADL model
- Predicted value for 2018M2=1.34

Model Selection

- The dynamic distributed lag model has p lags of *y* and q lags of *x*, a total of 1+p+q estimated coefficients
- Models (p and q) can be selected by calculating and minimizing the AIC

$$AIC = T \ln\left(\frac{SSR}{T}\right) + 2(p+q+1)$$

- Penalty is 2 times number of estimated coefficients

Predictive Causality

- The variable x affects a forecast for y if lagged values of x have true non-zero coefficients in the dynamic regression of y on lagged y's and lagged x's
- If one of the β's are non-zero

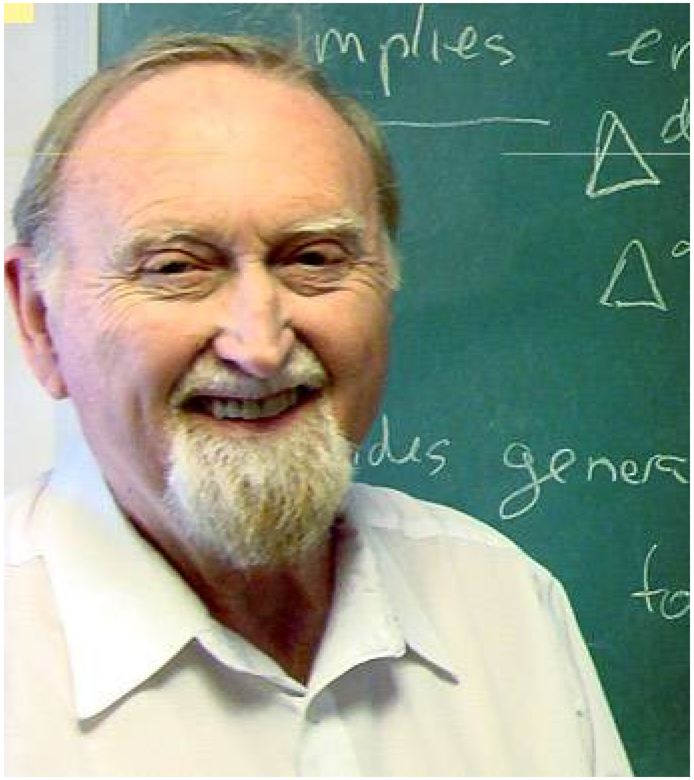
$$y_t = \mu + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p}$$
$$+ \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q} + e_t$$

Predictive Causality

- In this case, we say that “x causes y”
  - It does not mean causality in a mechanical sense
  - Only that x “predictively causes” y
  - True causality could actually be the reverse
- In economics, “predictive causality” is frequently called “Granger causality”

Clive Granger

- UCSD econometrician
  - 1934-2009
  - Winner of 2003 Nobel Prize
  - Greatest time-series econometrician of all time
- Many accomplishments
  - Granger causality
  - Spurious regression
  - Cointegration



Non-Causality

- Hypothesis:
  - x does not predictively (Granger) cause y
- Test
  - Reject hypothesis of non-causality if joint test of all lags on x are zero
  - F test using robust r option

$$y_t = \mu + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p}$$
$$+ \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q} + e_t$$

STATA Command

- **.reg t3month L(1/12).t3month L(1/12).t1year, r**
- **.testparm L(1/12).t1year**

3 month on 1 year Rate

```
. reg t3month L(1/12).t3month L(1/12).t1year, r

Linear regression                               Number of obs     =        755
                                                F(24, 730)        =    1340.06
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9878
                                                Root MSE          =      .3486
```

t3month	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t3month						
L1.	.9007304	.1635996	5.51	0.000	.5795485	1.221912
L2.	.0351072	.2452419	0.14	0.886	-.4463563	.5165706
L3.	-.0256843	.2307462	-0.11	0.911	-.4786895	.427321
L4.	-.0203787	.2036107	-0.10	0.920	-.4201111	.3793536
L5.	-.0234755	.2001105	-0.12	0.907	-.4163362	.3693851
L6.	-.2897435	.1765702	-1.64	0.101	-.6363894	.0569024
L7.	.5790217	.1615369	3.58	0.000	.2618893	.896154
L8.	-.2872972	.1844941	-1.56	0.120	-.6494996	.0749052
L9.	.130556	.1844517	0.71	0.479	-.231563	.492675
L10.	-.2429758	.2086496	-1.16	0.245	-.6526007	.1666491
L11.	-.0124844	.2136691	-0.06	0.953	-.4319636	.4069948
L12.	.0936189	.1309328	0.72	0.475	-.1634308	.3506685

Lags on 1-year Rate

t1year						
L1.	.5882765	.1654434	3.56	0.000	.2634748	.9130781
L2.	-.7452902	.2445862	-3.05	0.002	-1.225466	-.265114
L3.	.3041305	.2410746	1.26	0.208	-.1691518	.7774128
L4.	-.1373067	.2449349	-0.56	0.575	-.6181674	.3435541
L5.	.2761729	.1987497	1.39	0.165	-.1140162	.6663621
L6.	-.0738674	.1648722	-0.45	0.654	-.3975476	.2498128
L7.	-.4169761	.1623806	-2.57	0.010	-.7357647	-.0981874
L8.	.3873454	.2052571	1.89	0.060	-.0156194	.7903101
L9.	-.0714845	.2071423	-0.35	0.730	-.4781503	.3351812
L10.	.090841	.2023076	0.45	0.654	-.306333	.4880151
L11.	.0949785	.169495	0.56	0.575	-.2377773	.4277343
L12.	-.1527069	.0970236	-1.57	0.116	-.3431854	.0377717
_cons	.0102915	.0237535	0.43	0.665	-.0363418	.0569249

Causality Test

```
. testparm L(1/12).t1year

 ( 1)  L.t1year = 0
 ( 2)  L2.t1year = 0
 ( 3)  L3.t1year = 0
 ( 4)  L4.t1year = 0
 ( 5)  L5.t1year = 0
 ( 6)  L6.t1year = 0
 ( 7)  L7.t1year = 0
 ( 8)  L8.t1year = 0
 ( 9)  L9.t1year = 0
 (10)  L10.t1year = 0
 (11)  L11.t1year = 0
 (12)  L12.t1year = 0

       F( 12,   730) =    2.93
            Prob > F =    0.0006
```

- P-value is near zero
- Reject hypothesis of non-causality
- Infer that 1-year Treasury rate helps predict 3-month rate
- Long rates help to predict short rates

Reverse: 1-year on 3-month

- Do short rates help to forecast long rates?
- Regress 1-year rate on lagged values, and lags of 3-month rate

1-year rate on 3-month rate

```
. reg t1year L(1/12).t1year L(1/12).t3month, r

Linear regression                               Number of obs     =        755
                                                F(24, 730)        =    1519.69
                                                Prob > F          =     0.0000
                                                R-squared         =     0.9895
                                                Root MSE          =     .34391
```

t1year	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
t1year						
L1.	1.651651	.1410236	11.71	0.000	1.374791	1.928512
L2.	-1.084735	.245776	-4.41	0.000	-1.567247	-.6022224
L3.	.5124624	.2279814	2.25	0.025	.0648851	.9600397
L4.	-.2411744	.1998496	-1.21	0.228	-.6335229	.151174
L5.	.32389	.1764972	1.84	0.067	-.0226126	.6703926
L6.	-.147861	.1555466	-0.95	0.342	-.453233	.157511
L7.	-.2843866	.1616608	-1.76	0.079	-.6017622	.032989
L8.	.282631	.183567	1.54	0.124	-.0777511	.6430132
L9.	-.0364799	.1906547	-0.19	0.848	-.4107767	.3378169
L10.	.0532745	.1738142	0.31	0.759	-.2879609	.3945099
L11.	.2585591	.1651651	1.57	0.118	-.0656961	.5828143
L12.	-.2764837	.1008495	-2.74	0.006	-.4744734	-.0784939

Lags on 3-month rate

t3month						
L1.	-.1775868	.1527564	-1.16	0.245	-.477481	.1223074
L2.	.3959462	.2542087	1.56	0.120	-.1031212	.8950135
L3.	-.2825128	.222148	-1.27	0.204	-.7186379	.1536124
L4.	.1435659	.1702035	0.84	0.399	-.1905808	.4777125
L5.	-.0526498	.1698017	-0.31	0.757	-.3860077	.2807082
L6.	-.3013846	.1533138	-1.97	0.050	-.6023731	-.0003961
L7.	.5642528	.1572944	3.59	0.000	.2554495	.8730561
L8.	-.2042855	.165599	-1.23	0.218	-.5293927	.1208216
L9.	-.0413791	.1719692	-0.24	0.810	-.3789923	.2962342
L10.	-.1109756	.1785176	-0.62	0.534	-.4614448	.2394936
L11.	-.1324585	.1892281	-0.70	0.484	-.5039548	.2390377
L12.	.1779926	.1224971	1.45	0.147	-.0624961	.4184812
_cons	.0392638	.0229089	1.71	0.087	-.0057114	.0842391

Causality Test

```
. testparm L(1/12).t3month

 ( 1)  L.t3month = 0
 ( 2)  L2.t3month = 0
 ( 3)  L3.t3month = 0
 ( 4)  L4.t3month = 0
 ( 5)  L5.t3month = 0
 ( 6)  L6.t3month = 0
 ( 7)  L7.t3month = 0
 ( 8)  L8.t3month = 0
 ( 9)  L9.t3month = 0
 (10)  L10.t3month = 0
 (11)  L11.t3month = 0
 (12)  L12.t3month = 0

       F( 12,   730) =    1.73
            Prob > F =    0.0567
```

- P-value is nearly significant
- Not clear if we reject hypothesis of non-causality
- Unclear if 3-month Treasury rate helps predict 1-year rate
  - If short rates help to predict long rates

Term Structure Theory

- This is not surprising, given the theory of the **term structure of interest rates**
- Helpful to review interest rate theory

Bonds

- A bond with face value $1000 is a promise to pay $1000 at a specific date in the future
  - If that date is 3 months from today, it is a 3-month bonds
  - If that date is 12 months from today, it is a 12-month bond
- Rate: If a 3-month $1000 bond sells for $980, the interest percentage for the 3-month period is 100*20/980=2.04%, or 8.16% annual rate

Term Structure

- Suppose an investor has a 2-period horizon
  - They can purchase a 2-period bond
  - Or a sequence of one-period bonds
- Competitive equilibrium sets the prices of the bonds so they have equal expected returns.
  - The average expected one-period returns equal the two-period return
  - The two-period return is an expectation of future short rates

$$Long_t = \frac{Short_t + E\left(Short_{t+1} \mid \Omega_t\right)}{2}$$

Term Structure Regression

- This implies

$$E\left(Short_{t+1} \mid \Omega_t\right) = 2Long_t - Short_t$$

- Thus a predictive regression for short-term interest rates is a function of lagged long-term interest rates
- Long-term interest rates help forecast short term rates because long-term rates are themselves market forecast of future short rates
  - High long-term rates mean that investors expect short rates to rise in the future

Causality

- The theory of the term structure predicts that long-term rates will help predict short-term rates
- It does not predict the reverse
- This is consistent with our hypothesis tests
  - 1-year rate predicted 3-month rate
  - Unclear if 3-month predicts 12-year.

Selection of Causal Variables

- Even if we don't reject non-causality of y by x, we still might want to include x in forecast regression
  - Testing is not a good selection method
  - AIC is a better for selection

Example 1: Forecasting 3-month rate

- Model

$$y_t = \mu + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q} + e_t$$

- *Y* = 3-month interest rate
- *X* = 1-year interest rate
- AIC selection for
  - Number of AR lags *p* = {0, 1, 6, 12}
  - Number of regressor lags *q* = { 0, 1, 6, 12}
- Together this is 4x4=16 models
- As the largest number of lags is 12, to compute AIC & BIC, we estimate each model on the common restricted sample 1954m4-2017m2
- All models have the same 755 observations

AIC & BIC comparisons

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
model00	755	-1926.476	-1926.476	1	3854.952	3859.578
model10	755	-1926.476	-413.1019	2	830.2038	839.4573
model60	755	-1926.476	-348.7713	7	711.5425	743.9295
model120	755	-1926.476	-311.9893	13	649.9786	710.1259
model01	755	-1926.476	-555.3144	2	1114.629	1123.882
model11	755	-1926.476	-406.2515	3	818.503	832.3832
model61	755	-1926.476	-339.8814	8	695.7627	732.7765
model121	755	-1926.476	-304.1252	14	636.2505	701.0245
model06	755	-1926.476	-504.6276	7	1023.255	1055.642
model16	755	-1926.476	-316.9731	8	649.9462	686.96
model66	755	-1926.476	-314.2659	13	654.5318	714.6791
model126	755	-1926.476	-276.4706	19	590.9413	678.8489
model012	755	-1926.476	-485.8565	13	997.7131	1057.86
model112	755	-1926.476	-281.7844	14	591.5687	656.3428
model612	755	-1926.476	-280.0678	19	598.1356	686.0433
model1212	755	-1926.476	-262.9498	25	575.8997	691.5676

- “modelpq” means p AR lags and q regressor lags
- Smallest AIC (575) for p=12 and q=12
- Smallest BIC (656) for p=1 and q=12

AIC in Table format

	q=0	q=1	q=6	q=12
p=0	3855	1115	1023	998
p=1	830	819	650	592
p=6	712	696	655	598
p=12	650	636	591	576*

- Large improvement in AIC by including at least one AR lag
- Large improvement in AIC by including regressors
- Best forecasting model (among those considered) is p=12, q=12

Example 2: Forecasting 1-year rate

- Model

$$y_t = \mu + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p}$$
$$+ \beta_1 x_{t-1} + \cdots + \beta_q x_{t-q} + e_t$$

- *Y* = 1-year interest rate
- *X* = 3-month interest rate
- AIC selection for
  – Number of AR lags *p* = {0, 1, 6, 12}
  – Number of regressor lags *q* = { 0, 1, 6, 12}

AIC & BIC comparisons

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
tmodel00	755	-1971.927	-1971.927	1	3945.854	3950.481
tmodel10	755	-1971.927	-630.3739	2	1264.748	1274.001
tmodel60	755	-1971.927	-597.8546	7	1209.709	1242.096
tmodel120	755	-1971.927	-569.9831	13	1165.966	1226.114
tmodel01	755	-1971.927	-400.0361	2	804.0723	813.3257
tmodel11	755	-1971.927	-399.7569	3	805.5138	819.394
tmodel61	755	-1971.927	-354.9752	8	725.9504	762.9642
tmodel121	755	-1971.927	-318.1507	14	664.3014	729.0755
tmodel06	755	-1971.927	-318.1547	7	650.3094	682.6965
tmodel16	755	-1971.927	-318.1443	8	652.2886	689.3023
tmodel66	755	-1971.927	-308.7783	13	643.5566	703.7039
tmodel126	755	-1971.927	-268.979	19	575.958	663.8657
tmodel012	755	-1971.927	-279.5303	13	585.0607	645.208
tmodel112	755	-1971.927	-279.5297	14	587.0594	651.8335
tmodel612	755	-1971.927	-272.7685	19	583.537	671.4446
tmodel1212	755	-1971.927	-252.709	25	555.4179	671.0859

- “modelqp” means q regressor lags and p AR lags (reverse from previous table)
- Smallest AIC (555) for p=12 and q=12
- Smallest BIC (645) for p=12 and q=0

AIC in Table format

	q=0	q=1	q=6	q=12
p=0	3946	1265	1210	1166
p=1	804	806	726	664
p=6	650	652	644	576
p=12	585	587	584	555*

- Large improvement in AIC by including AR lags
- Given AR(12) no improvement in AIC from a few regressors, only with q=12
- Recall that we failed to reject Granger non-causality of 1-year rate by 3-month rate
- AIC suggests: Best forecasting model (among those considered) is p=12, q=12

Assignments

- Diebold, Chapter 15
- Problem Set #8
  - Due Tuesday (4/4)
- Read Chapter 9 from *The Signal and the Noise*
  - Reading Reflection
  - Thursday (4/6)