Problem Set #5
Spring 2014

1. Rewrite the following expressions without using the lag operator
   (a) $(1-\rho L)y_t = \varepsilon_t$
   (b) $(1+.2L-.8L^2)y_t = \varepsilon_t$
   (c) $y_t = (1-\theta L)\varepsilon_t$
   (d) $y_t = (1-.3L+.5L^2)\varepsilon_t$
2. Rewrite the following expressions in lag operator form
   (a) $y_t = .8y_{t-1} + \varepsilon_t$
   (b) $y_t = .2y_{t-1} + .3y_{t-2} - .7y_{t-5} + \varepsilon_t$
   (c) $y_t = \varepsilon_t - .7\varepsilon_{t-1}$
   (d) $y_t = \varepsilon_t + .3\varepsilon_{t-4}$
3. Consider the following MA(1) process

$$y_t = 2.3 - 0.95\varepsilon_{t-1} + \varepsilon_t$$

   with ε_t iid $N(0,1)$.
   (a) What is the optimal forecast for time periods $T+h$, $h=1,2,3$. Write your answer as a function of $y_1,...,y_T$ and/or $\varepsilon_1,...,\varepsilon_T$
   (b) Now suppose that $\varepsilon_T = 0.4$ and $\varepsilon_{T-1} = -1.2$. Re-answer part (a)
4. The stata file "gdp2013.dta" contains quarterly observations on the real growth rates of the components of U.S. gdp.

   Pick two of the series. Graph their autocorrelation functions. Comment and discuss.
5. The file "housingstarts.dta" has data on housing starts. Graph the autocorrelation function for the "south" and "midwest" regions. Comment and discuss.
6. The file "s&p.dta" has stock return dat, include transaction volume. In an earlier exercise you estimated a trend model for the natural log of volume. Re-estimate that model, but this time on the full sample. That is, take the log of the volume and regress it on a time trend. Take the residuals of that regression. (The command is **predict e, residuals** after estimating the regression). Graph a time-series plot of the residuals. Graph the autocorrelation function of the residuals. Comment and discuss.
7. For this problem you will create simulated data. See the description of simulation in STATA at the end of this problem set.

   Simulate each of processes with $T = 100$ observations, with ε_t iid $N(0,1)$. For each series, graph the autocorrelation function, and estimate a MA(2) model. Commend and discuss. For estimation, use the command (where y is the name of the series you created)

   .arima y arima(0,0,2)
   (a) $y_t = \varepsilon_t$ (white noise)
   (b) $y_t = \varepsilon_t + 0.8\varepsilon_{t-1}$
   (c) $y_t = \varepsilon_t - 0.6\varepsilon_{t-1} + 0.4\varepsilon_{t-2}$
   (d) $y_t = .5y_{t-1} + \varepsilon_t$

Simulation in STATA

Starting with a blank STATA session, you can set the number of observations and declare the file to be time-series as follows. If you want to create a file with 100 observations, use the commands

```
.set obs 100
.gen t=_n
.tsset t
```

To simulate a Gaussian white noise process use the command

```
.gen e=rnormal()
```

This generates (creates) a variable "e" (or any name you decide) filled with random normal N(0,1) innovations

To create a moving average process $y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$, use the command

```
.gen y=e+0.3*L.e
```

This takes the variable "e" previously defined, creates the lagged value, and creates the moving average with the coefficient 0.3

To create an autoregression, you need a bit more creativity because of the recursion. To create the process $x_t = 0.5x_{t-1} + \varepsilon_t$ with $x_1 = 0$

```
.gen x=0
.replace x=0.5*L.x+e if t>1
```

The first command sets the variable $x_t = 0$. We only use this for the first observation, thus $x_1 = 0$. (You can relace 0 with whatever starting value is appropriate.) Then the second command replaces the x_t by the recursion $x_t = 0.5x_{t-1} + e_t$, for observations $t > 1$, where the e variable is previously defined, and the time index t was previously defined. The command is **replace** rather than **gen** since the variable x already exists, and **gen** will not write over an existing variable.