Schedule

- Tuesday Feb 18: Problem Set 4 due
  - Problem Set Answer key will be posted
  - Return Problem Sets 1-3
- Thursday Feb 20: Midterm Exam
  - Humanities 1101, 1pm
  - No Reading Reflection
- Tuesday Feb 25: First Project Report Due
  - No Problem Set
- Thursday Feb 27: Reading Reflection Ch 4

First Midterm Exam

- Thursday Feb 20 in class
- Gonzalez-Rivera, Chapters 1-4, 10.1
- Review book, lectures, problem sets
- Calculators allowed
- Mix of conceptual, interpretive, and computational problems. No questions from Silver’s book.

First Project Report

- Due Tuesday Feb 25
- Requirements described on website
- Describe your variable(s)
  - Source, historical dates available
  - Which observations will you be forecasting out-of-sample?
  - Download the data, and present a time-series plot
- This is a preliminary proposal; you can change the series if desired

Estimation of Autocorrelations

- The autocorrelation is a function of moments

$$\rho(k) = \frac{\operatorname{cov}(Y_t, Y_{t-k})}{\operatorname{var}(Y_t)}$$

$$= \frac{\gamma(k)}{\gamma(0)}$$

$$\operatorname{cov}(Y_t, Y_{t-k}) = \gamma(k)$$

$$= E\left((Y_t - \mu)(Y_{t-k} - \mu)\right)$$

$$\mu = EY_t$$

- We estimate by replacing the population moments by sample moments

Estimation

- The population mean

$$\mu = EY_t$$

is estimated by the sample mean

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} Y_t$$

- The population covariance

$$\gamma(k) = E\left(\left(Y_t - \mu\right)\left(Y_{t-k} - \mu\right)\right)$$

is estimated by the sample covariance

$$\hat{\gamma}(k) = \frac{1}{T}\sum_{t=k+1}^{T}\left(Y_t - \hat{\mu}\right)\left(Y_{t-k} - \hat{\mu}\right)$$

Estimation

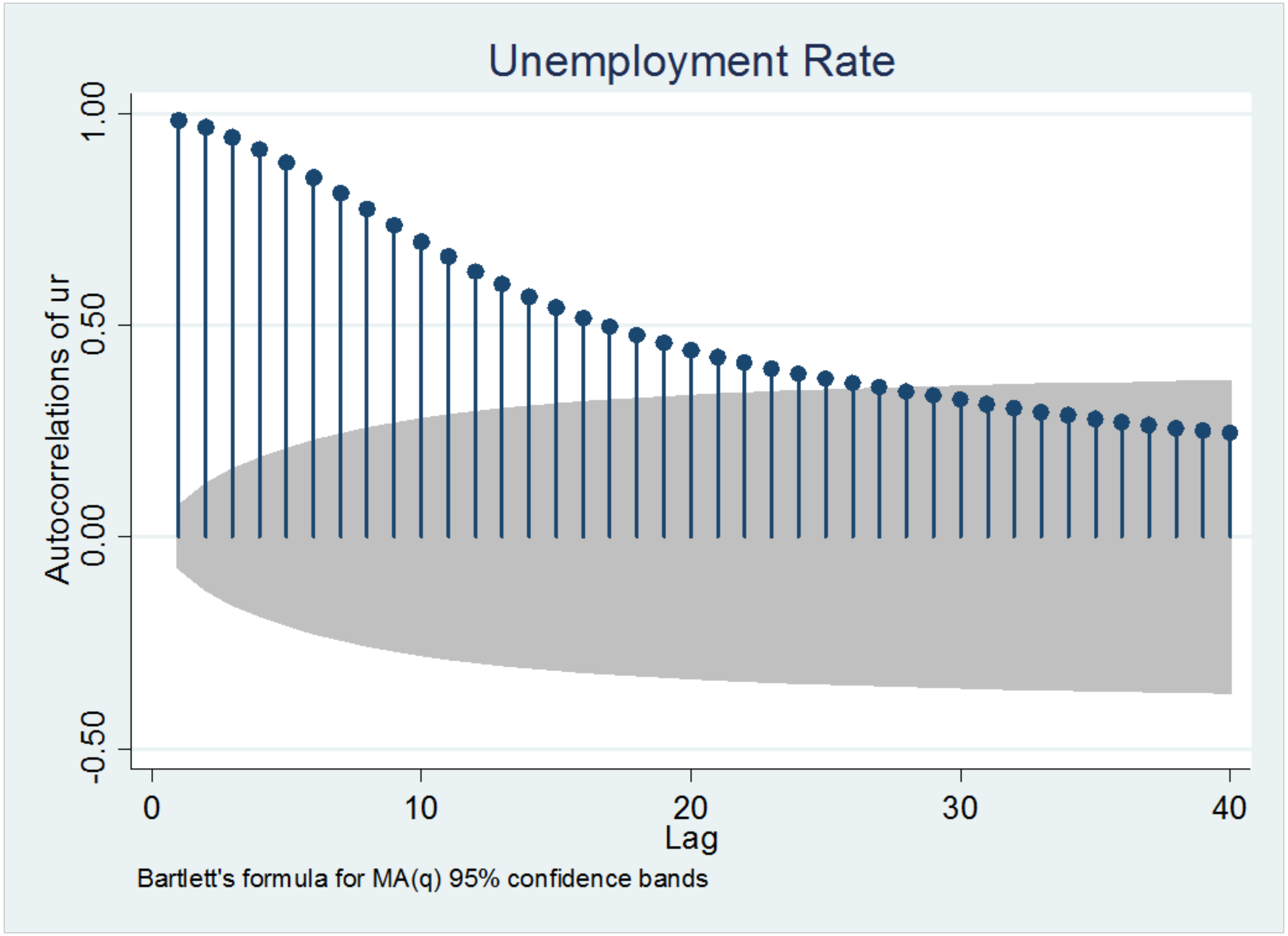
- The population autocorrelation

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)}$$

is estimated by the ratio of sample autocovariances

$$\hat{\rho}(k) = \frac{\hat{\gamma}(k)}{\hat{\gamma}(0)}$$

Autocorrelation Plot



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ac ur, title("Unemployment Rate")

Sampling Uncertainty

- The sample autocorrelations are estimates of the population autocorrelations, and are thus random.
- Just because the estimated autocorrelation is positive does not mean that the true autocorrelation is positive. The estimate contains sampling error.

Confidence Bands – White Noise Case

- If Y_t is independent white noise, then

$$\mathrm{var}\left(\hat{\rho}(k)\right) \approx \frac{1}{T}$$

  which means that the standard error is $1/T^{1/2}$
- Thus the sample values will lie in the region $[-2/T^{1/2}, 2/T^{1/2}]$ with 95% probability.
- Consequently, a common measure of uncertainty for sample autocorrelations is to plot $\pm 2/T^{1/2}$ confidence bands about zero.
- The interpretation is that if the sample autocorrelation is within the bands, it is not statistically different from zero.

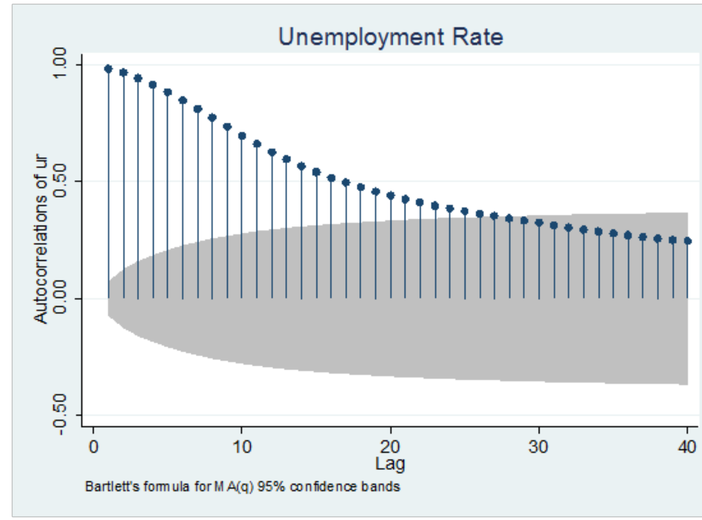
Confidence Bands – Correlated Case

- The $\pm 2/T^{1/2}$ confidence bands are only valid if the true process is white noise
- In general, the confidence bands depend upon the actual autocorrelation.
- Bartlett worked out an approximation based on a moving average model

Bartlett Confidence Bands

- Suppose that the autocorrelations up to order k-1 are the estimated values, but the remaining autocorrelations

  (order k and above) are zero.
- The Bartlett confidence band is the 95% sampling interval for the estimated k'th autocorrelation
- STATA displays the Bartlett bands as the shaded region
- The interpretation is that if the estimated autocorrelation falls outside the shaded region, it is statistically different than zero.



Moving Average Processes

- Gonzalez-Rivera, Chapter 6
- These models are linear functions of stochastic errors

Innovations

- Time-series models are constructed as linear functions of fundamental forecasting errors e_t, also called **innovations** or **shocks**
- These basic building blocks satisfy
  - $Ee_t = 0$
  - $\text{var}(e_t) = Ee_t^2 = \sigma^2$
  - Serially uncorrelated
  - These errors e_t are called **white noise**
- In general, if you see an error e_t, it should be interpreted as white noise. We will write
  - e_t is WN(0, σ^2)

Unforecastable Innovations

- White noise processes are linearly unforecastable
- A stronger condition is unforecastable.
- The innovations e_t are **unforecastable** if
  - $E(e_t | \Omega_{t-1}) = 0$
  - This means the best forecast is zero
- For some purposes, we will assume the errors are unforecastable

MA(1) Process

- The **first-order moving average** process, or **MA(1)** process, is

$$y_t = e_t + \theta e_{t-1}$$

  where e_t is WN(0, σ^2)
- The MA coefficient θ controls the degree of serial correlation. It may be positive or negative.
- The innovations e_t impact y_t over two periods
  - An contemporaneous (same period) impact
  - A one-period delayed impact

Mean of MA(1)

- The unconditional mean of y_t is

$$\begin{aligned} E(y_t) &= E(e_t + \theta e_{t-1}) \\ &= E(e_t) + \theta E(e_{t-1}) \\ &= 0 \end{aligned}$$

Variance of MA(1)

- The unconditional variance of y_t is

$$
\begin{aligned}
&\operatorname{var}(y_t) = \operatorname{var}(e_t + \theta e_{t-1}) \\
&= \operatorname{var}(e_t) + \operatorname{var}(\theta e_{t-1}) + 2\operatorname{cov}(e_t, \theta e_{t-1}) \\
&= \sigma^2 + \theta^2 \sigma^2 + 0 \\
&= \left(1 + \theta^2\right)\sigma^2
\end{aligned}
$$

- This is a function of both the innovation variance σ^2 and the MA coefficient θ.

Conditional Mean of MA(1)

- If the error is unforecastable $E(e_t|\Omega_{t-1}) = 0$ then the conditional mean of y_t is

$$\begin{aligned} E(y_t \mid \Omega_{t-1}) &= E(e_t + \theta e_{t-1} \mid \Omega_{t-1}) \\ &= E(e_t \mid \Omega_{t-1}) + \theta E(e_{t-1} \mid \Omega_{t-1}) \\ &= \theta e_{t-1} \end{aligned}$$

- This is the best forecast of y_t .
- The optimal forecast error is

$$\begin{aligned} y_t - E(y_t \mid \Omega_{t-1}) &= \left(e_t + \theta e_{t-1}\right) - \theta e_{t-1} \\ &= e_t \end{aligned}$$

Conditional Variance of MA(1)

- The conditional variance of y_t is

$$
\begin{aligned}
\operatorname{var}(y_t \mid \Omega_{t-1}) &= \operatorname{var}(y_t - E(y_t \mid \Omega_{t-1}) \mid \Omega_{t-1}) \\
&= \operatorname{var}(e_t \mid \Omega_{t-1}) \\
&= \sigma^2
\end{aligned}
$$

- The conditional variance, the forecast variance, and the innovation variance are all the same thing

Autocovariance of MA(1)

- The first autocovariance is

$$
\begin{aligned}
\gamma(1) &= E(y_t y_{t-1}) \\
&= E\big((e_t + \theta e_{t-1})(e_{t-1} + \theta e_{t-2})\big) \\
&= E(e_t e_{t-1}) + \theta E(e_{t-1}^2) + \theta E(e_t e_{t-2}) + \theta^2 E(e_{t-1} e_{t-2}) \\
&= 0 + \theta E(e_{t-1}^2) + 0 + 0 \\
&= \theta \sigma^2
\end{aligned}
$$

Autocovariance of MA(1)

- The autocovariance for *k*>1 are

$$\begin{aligned}
\gamma(k) &= E(y_t y_{t-k}) \\
&= E\left((e_t + \theta e_{t-1})(e_{t-k} + \theta e_{t-k-1})\right) \\
&= E(e_t e_{t-k}) + \theta E(e_{t-1} e_{t-k}) + \theta E(e_t e_{t-k-1}) + \theta^2 E(e_{t-1} e_{t-k-1}) \\
&= 0 + 0 + 0 + 0 \\
&= 0
\end{aligned}$$

- Thus the autocovariance function is zero for k>1

Autocorrelations of MA(1)

- Since

$$\gamma(0) = \mathrm{var}\left(y_t\right) = \left(1+\theta^2\right)\sigma^2$$
$$\gamma(1) = \theta\sigma^2$$
$$\gamma(k) = 0, k \geq 2$$

then

$$\rho(1) = \frac{\theta\sigma^2}{\left(1+\theta^2\right)\sigma^2} = \frac{\theta}{1+\theta^2}$$
$$\rho(k) = 0, k \geq 2$$

- The autocorrelation function of an MA(1) is zero after the first lag.

First Autocorrelation

- The first autocorrelation has the same sign as θ

$$\rho(1) = \frac{\theta}{1+\theta^2}$$

- As θ ranges from -1 to 1, ρ(1) ranges from - ½ to ½

$$y_t = e_t + \theta e_{t-1}$$

- Θ<0 : negative autocorration