Forecasting Economic Time Series

Economic forecasts rely on time series data – observations which are recorded sequentially over time.

Time Series Data

- A time series is written as y_t
- The index t denotes the time period.
- A time period may be a year, quarter, month, week, day, transaction, or any other time unit.
- We call this the data frequency.

Lags and Leads

- We will often talk about lags and leads
- The first lag of y_t is written $y_{t\text{-}1}$
  - It is the observation from the previous period
  - For example, the lag of November is October
- The second lag is $y_{t\text{-}2}$, the k'th lag is $y_{t\text{-k}}$
- The first lead of y_t is y_{t+1}
- The k'th lead is $y_{t+\text{k}}$

Time Series Samples

- A historical sample is a set of observations in contiguous time, written as

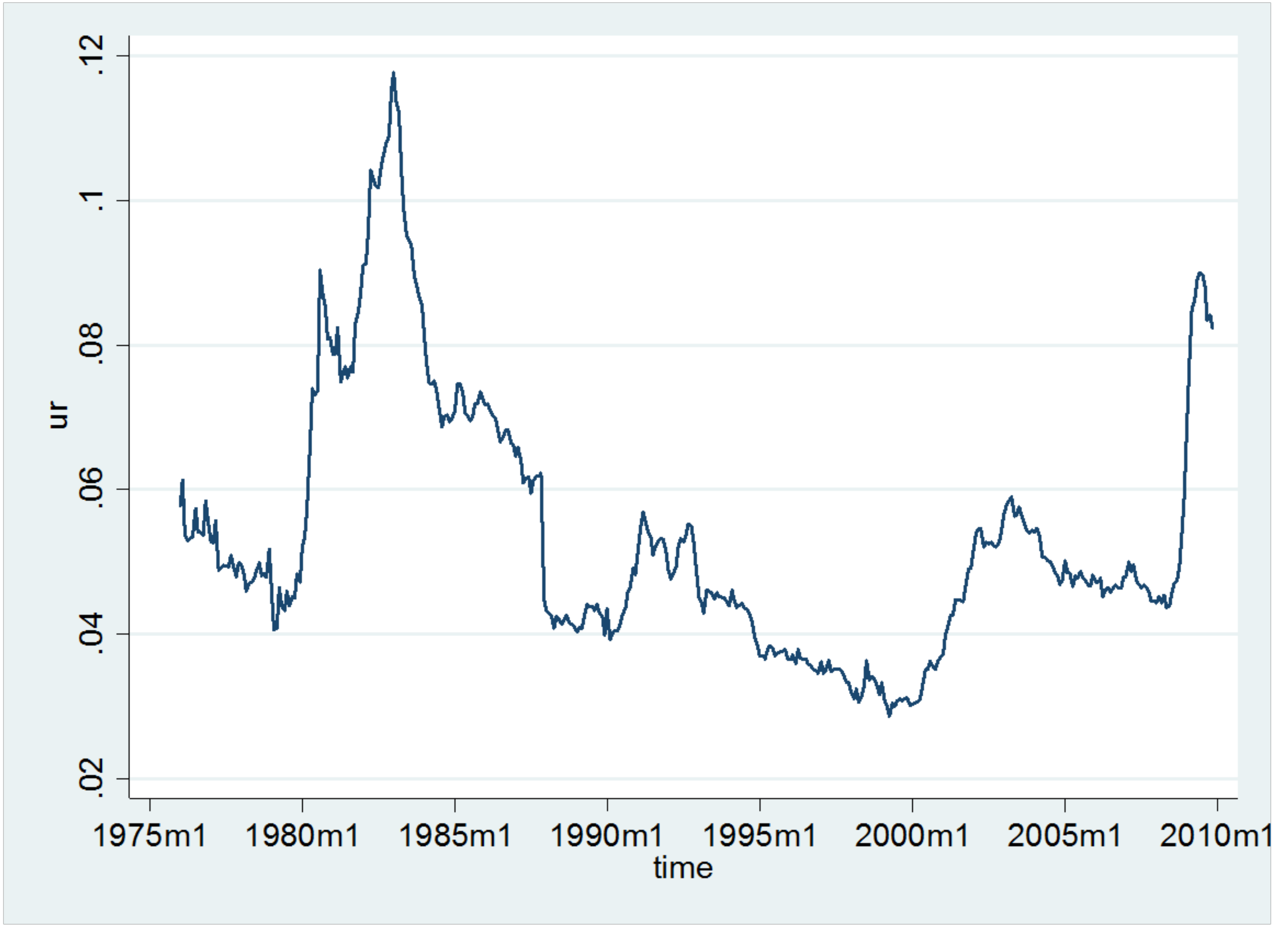
$$\{y_1, y_2, \ldots, y_T\}$$

- T is the number of observations in-sample.
- The number of observations does not equal the number of years, unless the frequency is annual

Examples

- Number of Unemployed
- Unemployment Rate
- GDP
- Real GDP
- Price Level
- Inflation Rate
- New Housing Starts

Wisconsin Unemployment Rate



Forecast Period

- In-sample observations: $\left\{ y_1, y_2, \ldots, y_T \right\}$
- Out-of-sample period: $\left\{ y_{T+1}, y_{T+2}, \ldots, y_{T+h} \right\}$
- h is called the forecast horizon

Forecast Notation

- We denoted $\hat{y}$ as the point forecast for y.
- This suggests $\hat{y}_{T+h}$ as the point forecast for y_{T+h}.
- But is not enough. While it is the forecast for the time series at time period $T+h$, it is not clear when the forecast is made.
  - At time period T
  - At time period $T+1$
  - At time period $T+h-1$

Notation

- We will use the notation

$$\hat{y}_{t+h|t}$$

to refer to the forecast of y_{t+h} made at time t.

- Thus $\hat{y}_{T+h|T}$, $\hat{y}_{T+h|T+1}$, $\hat{y}_{T+h|T+2}$, etc. are the sequence of forecasts of y_{T+h} made in time periods *T*, *T+1*, *T+2*, etc.

Notation

- Similarly, the forecast distribution and density for y_{t+h} made at time t will be written as

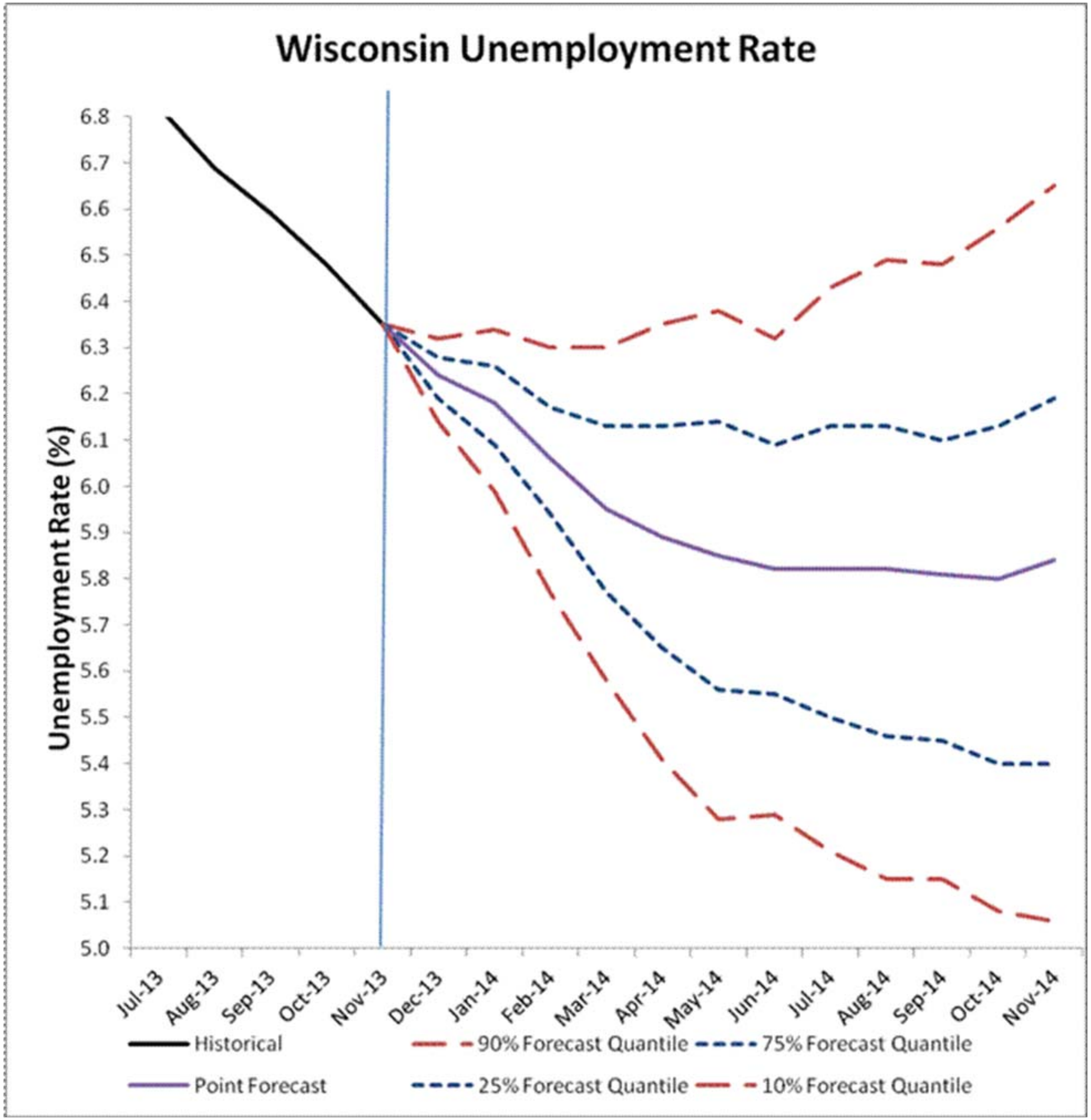
$$F_{t+h|t}(y)$$

and

$$f_{t+h|t}(y)$$

Extrapolative Forecasts and Fan Charts

- At time T, make a sequence of forecasts for time periods T+1, T+2, T+3,…, T+h
- Point forecasts: $\hat{y}_{T+1|\mathrm{T}}$, $\hat{y}_{T+2|\mathrm{T}}$, …, $\hat{y}_{T+h|\mathrm{T}}$
- Add interval forecasts
- Plot over forecast horizon
- This is called a *fan chart*.
- The intervals tend to fan out with the forecast horizon.



Wisconsin Unemployment Rate
Unemployment Rate (%)
6.8
6.7
6.6
6.5
6.4
6.3
6.2
6.1
6.0
5.9
5.8
5.7
5.6
5.5
5.4
5.3
5.2
5.1
5.0
Jul-13
Aug-13
Sep-13
Oct-13
Nov-13
Dec-13
Jan-14
Feb-14
Mar-14
Apr-14
May-14
Jun-14
Jul-14
Aug-14
Sep-14
Oct-14
Nov-14
Historical
Point Forecast
90% Forecast Quantile
25% Forecast Quantile
75% Forecast Quantile
10% Forecast Quantile

Extrapolative Forecasts

	Point Forecast	50% Interval Forecast	80% Interval Forecast
2013:12	6.2%	(6.2%, 6.3%)	(6.1%, 6.3%)
2014: 1	6.2%	(6.1%, 6.3%)	(6.0%, 6.3%)
2014: 2	6.1%	(5.9%, 6.2%)	(5.8%, 6.3%)
2014: 3	6.0%	(5.8%, 6.1%)	(5.6%, 6.3%)
2014: 4	5.9%	(5.7%, 6.1%)	(5.4%, 6.4%)
2014: 5	5.9%	(5.6%, 6.1%)	(5.3%, 6.4%)
2014: 6	5.8%	(5.6%, 6.1%)	(5.3%, 6.3%)
2014: 7	5.8%	(5.5%, 6.1%)	(5.2%, 6.4%)
2014: 8	5.8%	(5.5%, 6.1%)	(5.2%, 6.5%)
2014: 9	5.8%	(5.5%, 6.1%)	(5.2%, 6.5%)
2014:10	5.8%	(5.4%, 6.1%)	(5.1%, 6.6%)
2014:11	5.8%	(5.4%, 6.2%)	(5.1%, 6.7%)

Information Set

- To forecast y_{t+h} at time t we use relevant information.
- Most information is values of other economic variables.
- Those observed up to time t.
- This includes previous values of the variable y_t .
- It can also include other relevant variables.
- All this information is the *Information Set*, written as Ω_t .
- For example $\Omega_t = \{y_1, y_2, y_3, \ldots, y_t\}$ is the set of previous values.
- $\Omega_t = \{y_1, x_1, y_2, x_2, y_3, x_3, \ldots, y_t, x_t\}$ includes another variable x_t

Conditional Mean Forecasts

- Under squared error loss, the optimal *unconditional* point forecast is the mean Ey

$$\mathrm{E}(y) = \int y f(y) dy$$

- Under squared error loss, the optimal *conditional* point forecast is the *conditional mean*

$$\mathrm{E}(y \mid \Omega_t) = \int y f(y \mid \Omega_t) dy$$

where $f(y|\Omega_t)$ is the conditional density of y given Ω_t

Conditional Mean

- The unconditional mean E(*y*) is the average value of *y* in the entire population
- The conditional mean E(*y*|*x*) given a set of variables *x* is the average value of *y* for the subpopulation with the variables *x*.
- The conditional mean E(*y*| Ω_t) given an information set is the average value of *y* in the subhistory with the previous history Ω_t.

Conditional Mean

- What is E(y| Ω_t) ?
- When Ω_t is discrete, then E(y| Ω_t) is simply the mean in the subpopulation.
- For example, the average wage among white male college graduates.
- When Ω_t large and/or is continuously distributed, this definition does not work.

Conditional Mean

- Let $f(y, \Omega_t)$ denote the joint density of (y, Ω_t). Then the conditional density of y given Ω_t is

$$f\left(y \mid \Omega_t\right) = \frac{f\left(y, \Omega_t\right)}{f\left(\Omega_t\right)}$$

- It is the distribution of y holding Ω_t fixed.
- It is a slice of the joint density
- Then the conditional mean is

$$\mathrm{E}\left(y \mid \Omega_t\right) = \int y f\left(y \mid \Omega_t\right) dy$$

Forecast Intervals

- Forecast intervals are constructed from the conditional distribution $F(y|\,\Omega_t)$.
- The endpoints are the conditional quantiles.
- Definition: The α'th conditional quantile of y given Ω_t is the number $q_\alpha(\Omega_t)$ which satisfies

$$\alpha = F\left(q_\alpha(\Omega_\mathrm{t}) \,|\, \Omega_\mathrm{t}\right)$$

- It looks more complicated, but it is identical with the case with no Ω_t.

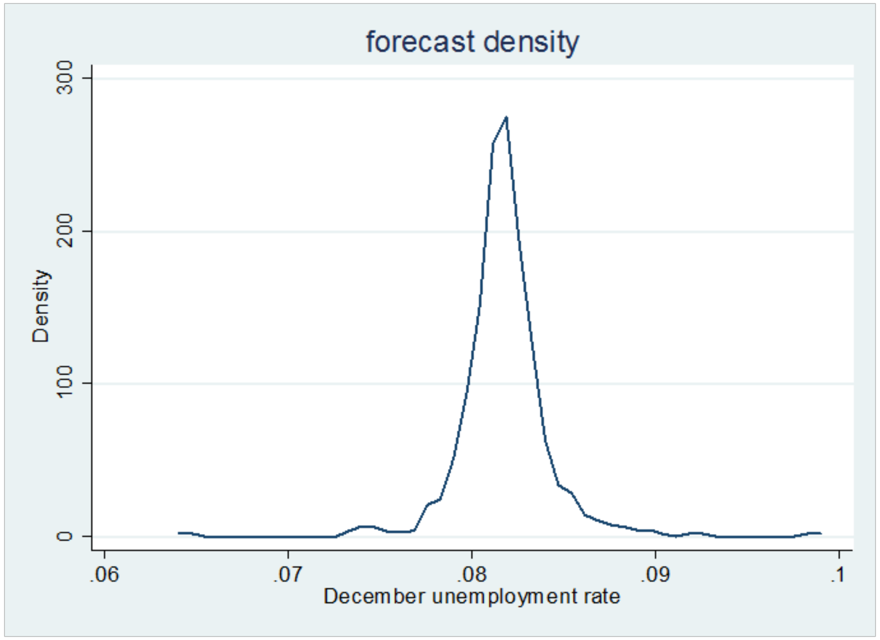
When does Conditioning Help?

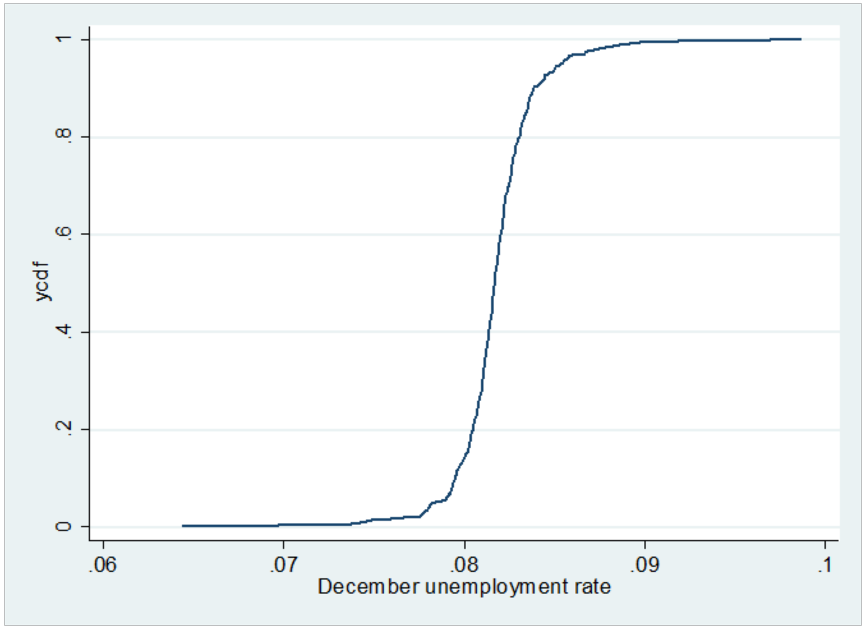
- It helps if y and Ω_t are *dependent* or *correlated*.
- If y and Ω_t are independent, then $f(y|\ \Omega_t)=f(y)$ and $E(y|\ \Omega_t)=E(y)$.
  – There is no gain from conditioning.
- To optimally forecast unknown y we want to use observable variables Ω_t which are highly correlated with y.

Summary – Conditional Forecasts

- Information improves forecasts
- Conditioning on relevant variables reduces the risk (expected loss) of forecasts
- Point and interval forecasts are functions of the conditioning variables

Forecast Density and Distribution





Actual Forecasting

- Even if the variables in the information set Ω_t are known, the conditional mean function $E(y_{t+h} \mid \Omega_t)$ is unknown
  - The functional form is unknown
  - The parameters of the function are unknown
- Thus to make an actual forecast, we need to:
  - Create an approximate model for $E(y_{t+h} \mid \Omega_t)$
  - Estimate the model parameters from data.

Time-Series Components

- Recall that the optimal point forecast of a series y_{t+h} is its conditional mean.
- It is useful to decompose this mean into components

$$\mathrm{E}\left(y_{t+h} \mid \Omega_t\right) = T_t + S_t + C_t$$

  - T_t = Trend
  - S_t = Seasonal
  - C_t = Cycle

Components

- Trend
  - Very long term (decades)
  - Smooth
- Seasonal
  - Patterns which repeat annually
  - May be constant or variable
- Cycle
  - Business cycle
  - Correlation over 2-7 years
- It is useful to consider the components separately

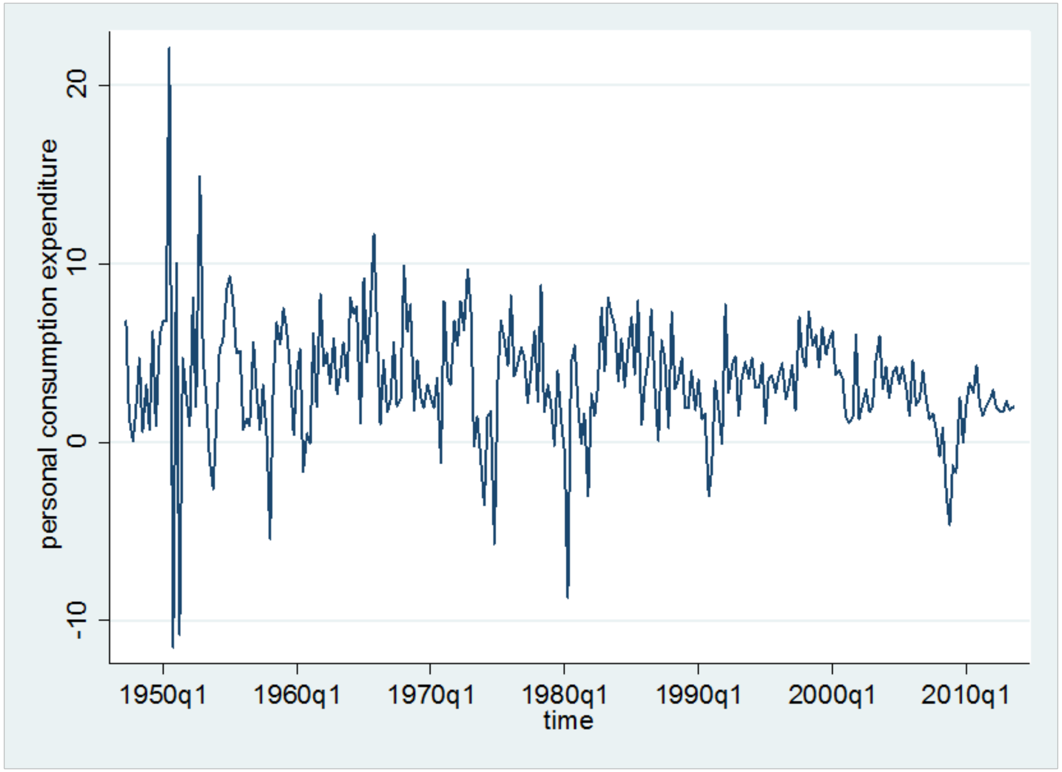
Mean Forecasting

- The simplest forecasting model has no trend, seasonal or cycle, only a constant mean

$$\mathrm{E}\left(y_{t+h} \mid \Omega_t\right) = \beta_0$$

- This might seem overly simple, but can be appropriate for a random **stationary** time-series
  - A series not growing or changing over time
  - Many series reported as percentage changes
- In this model, the optimal point forecast for y_{t+h} is $E(y_{t+h} \mid \Omega_t) = \beta_0$.
- An actual forecast is an estimate of β_0.

U.S. Real Personal Consumption (Quarterly) Percentage Change from Previous Period



Estimation

- If $E(y_{t+h} \mid \Omega_t) = \beta_0$ then the optimal forecast is the mean $\beta_0 = E(y_{t+h})$
- The estimate of β_0 is the sample mean

$$b_0 = \frac{1}{T}\sum_{t=1}^{T} y_{t+h}$$

- This is the estimate of the optimal point forecast when $E(y_{t+h} \mid \Omega_t) = \beta_0$
- b_0 is also the least-squares estimate in an intercept-only model

Estimation

- In STATA, use the **regress** command
- See *STATA Handout* on website
- Sample mean is estimated "constant"

```
. use gdp2013

. regress pce

      Source |       SS       df       MS              Number of obs =     266
-------------+------------------------------           F(  0,   265) =    0.00
       Model |           0     0           .           Prob > F      =       .
    Residual |  3088.35013   265  11.6541514           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared =  0.0000
       Total |  3088.35013   265  11.6541514           Root MSE      =  3.4138

------------------------------------------------------------------------------
         pce |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   3.408647   .2093146    16.28   0.000     2.996515    3.820778
------------------------------------------------------------------------------
```

Fitted Values

- Fitted values are the sample mean

$$\hat{y}_t = b_0$$

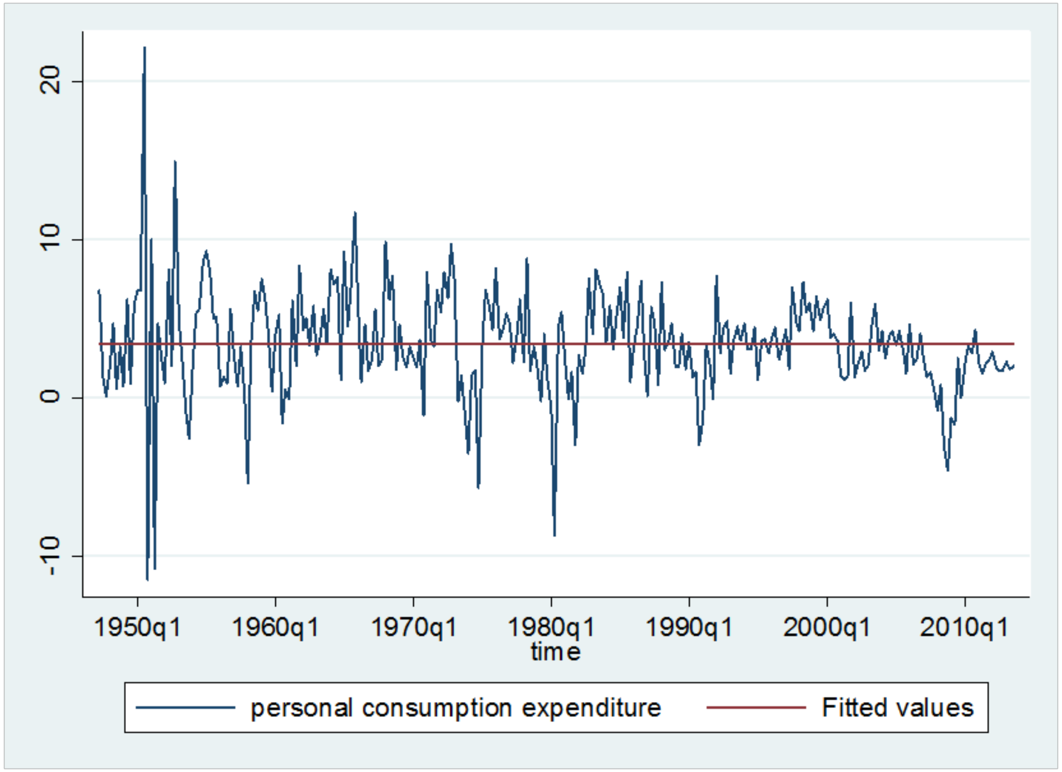
- In STATA use the **predict** command

```
. predict yp
(option xb assumed; fitted values)
```

- This creates a variable "yp" of fitted values

Plot actual against fitted

```
. tsline pce yp
```



Out-of-Sample

- Point forecasts are the sample mean

$$\hat{y}_{T+h} = b_0$$

- In STATA, use **tsappend** to expand sample, and **predict** to generate point forecasts.

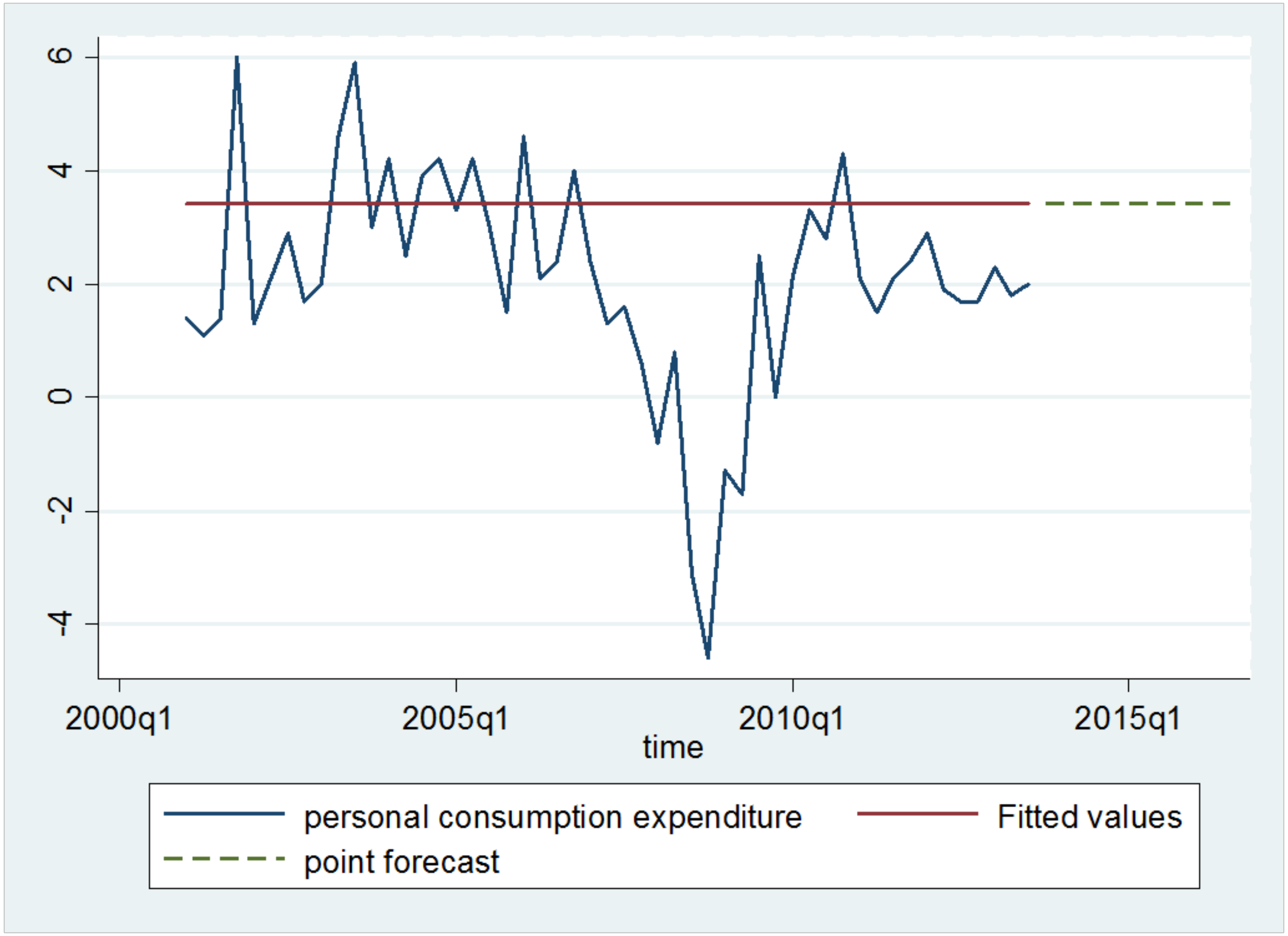
```
. tsappend, add(12)

. predict p if time>tq(2013q3)
(option xb assumed; fitted values)
(266 missing values generated)

. label variable p "point forecast"

. tsline pce yp p if time>tq(2000q4)
```

Out-of-Sample



Forecast Errors

- The forecast error e_t is the difference between the realized value and the conditional mean.

$$e_t = y_{t+h} - \mathrm{E}\left(y_{t+h} \mid \Omega_t\right)$$

or equivalently

$$y_{t+h} = \mathrm{E}\left(y_{t+h} \mid \Omega_t\right) + e_t$$

- We call e_t the forecast error.

Residuals

- The residuals are the in-sample fitted errors.
- The difference between the realized value and the in-sample forecast.

$$\begin{aligned}\hat{e}_t &= y_{t+h} - \hat{y}_{t+h} \\ &= y_{t+h} - b_0\end{aligned}$$

- In general, it is useful to plot the residuals against time, to see if any time series pattern remains.

Calculate and Plot Residuals

```
. predict e, residuals
(12 missing values generated)

. tsline e
```