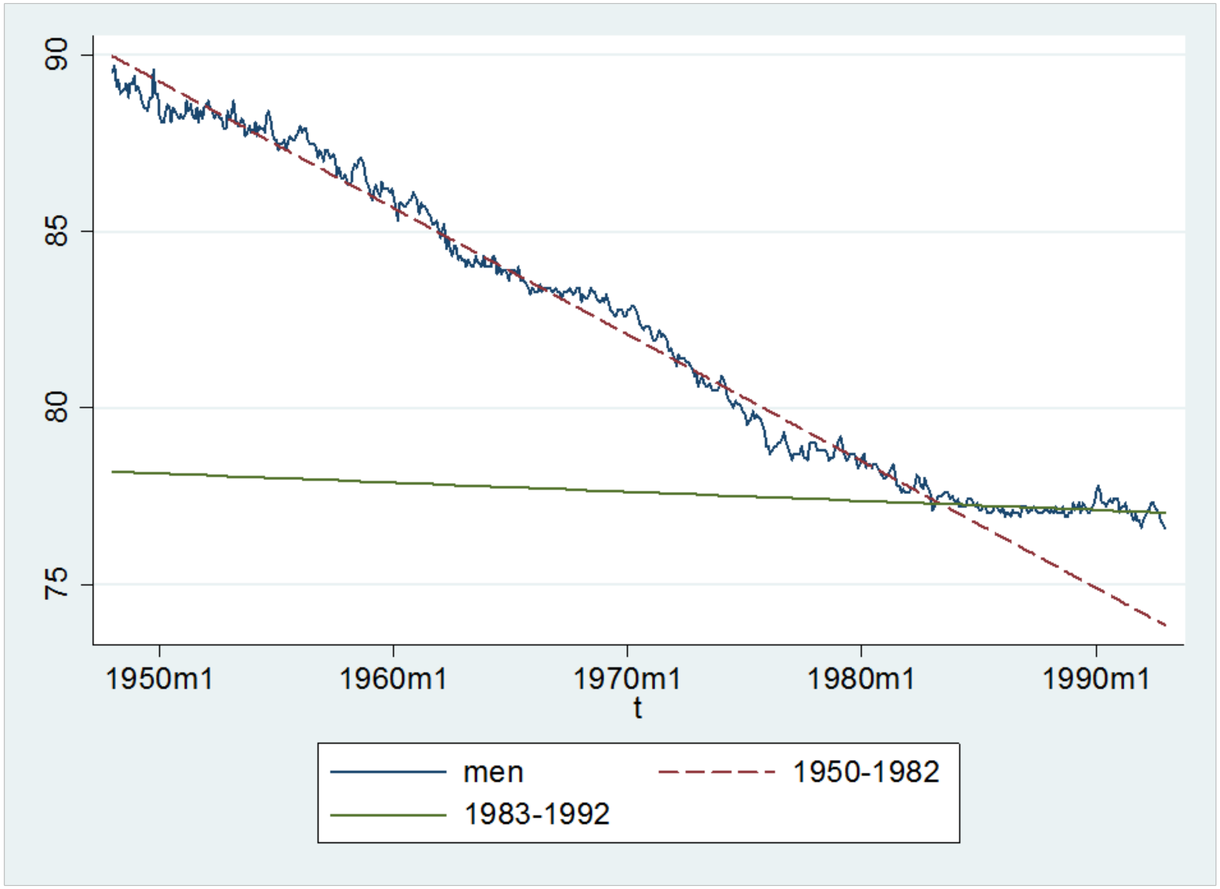
Changing Trends

- We have seen in some cases that it appears that the trend slope has changed at some point.
- This is a type of structural change, sometimes called a **changing trend** or **breaking trend**.
- We can model this using the interaction of dummy variables with the trend.

Labor Force Participation - Men



- Separate trends fit to 1950-1982 and 1983-1992

Sub-Sample Trend Lines

- If you fit a trend for observations before and after a breakdate τ, then for $t \leq \tau$

$$T_t = \beta_0 + \beta_1 Time_t$$

  and for $t > \tau$

$$T_t = \alpha_0 + \alpha_1 Time_t$$

- Notice that both the intercept and slope change

Estimation

- You can simply estimate on each sub-sample separately, and then forecast using the second set of estimates.
- Or, you can use dummy variable interactions.
- Define the dummy variable for observations after time τ

$$d_t = 1(t \geq \tau)$$

Dummy Equation

$$
\begin{aligned}
T_t &= (\beta_0 + \beta_1 Time_t)1(t < \tau) + (\alpha_0 + \alpha_1 Time_t)1(t \geq \tau) \\
&= (\beta_0 + \beta_1 Time_t) + ((\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)Time_t)1(t \geq \tau) \\
&= \beta_0 + \beta_1 Time_t + \beta_2 d_t + \beta_3 Time_t d_t
\end{aligned}
$$

where

$$\beta_2 = \alpha_0 - \beta_0$$

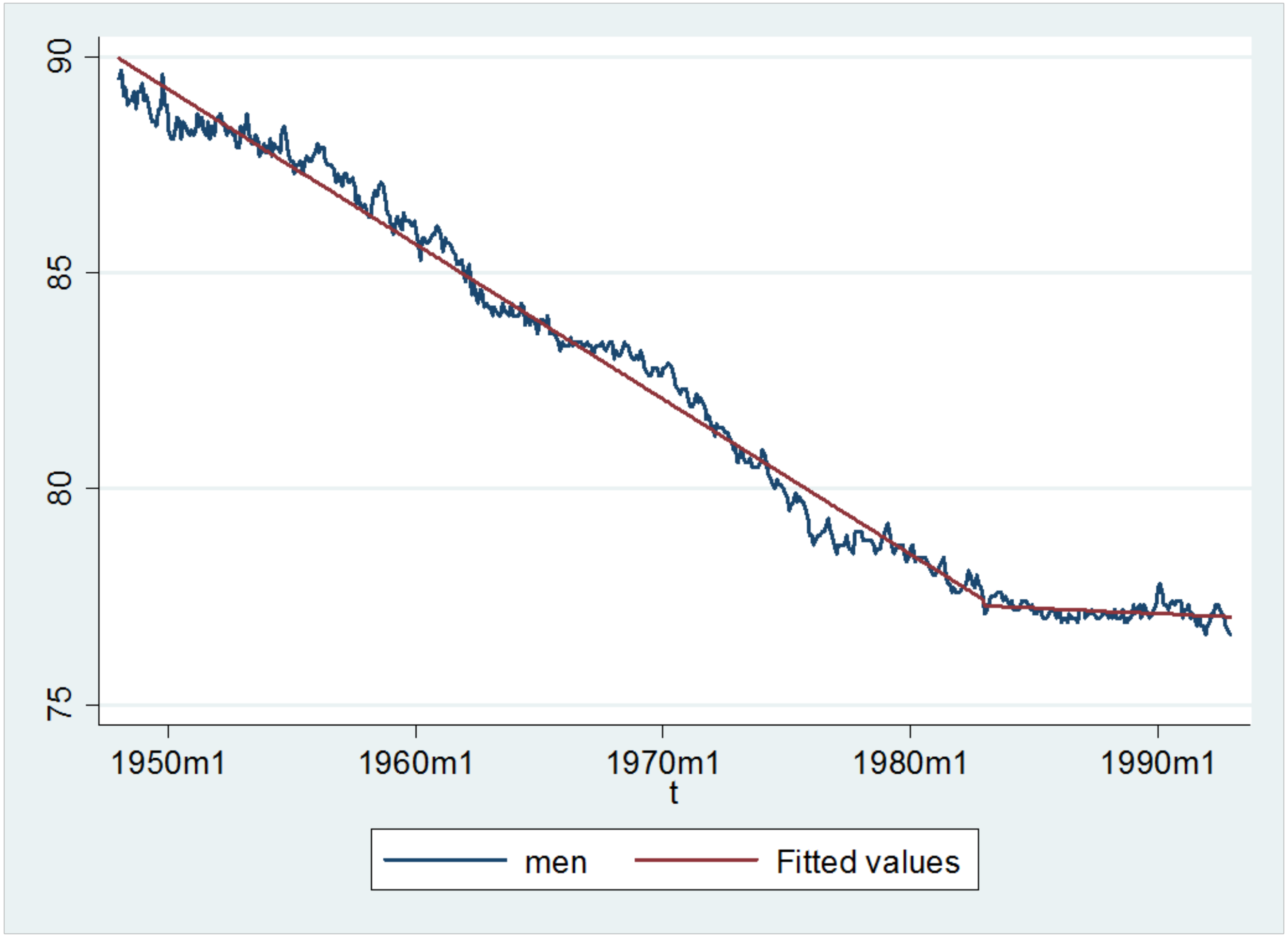
$$\beta_3 = \alpha_1 - \beta_1$$

- This is a linear regression, with regressors $Time_t$, d_t and $Time_t d_t$

Estimation

```
. generate d=(t>tm(1982m12))

. generate dt=d*t

. regress t d dt if (t<=tm(1992m12))

      Source |       SS       df       MS              Number of obs =     540
-------------+------------------------------           F(  2,   537) =  302.16
       Model |    6947990     2     3473995           Prob > F      =  0.0000
    Residual |    6173965   537  11497.1415           R-squared     =  0.5295
-------------+------------------------------           Adj R-squared =  0.5277
       Total |   13121955   539       24345           Root MSE      =  107.22

------------------------------------------------------------------------------
           t |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           d |      -65.5    95.4503    -0.69   0.493    -253.0018    122.0018
          dt |          1   .2825718     3.54   0.000     .4449184    1.555082
       _cons |       65.5   5.232031    12.52   0.000     55.22224    75.77776
------------------------------------------------------------------------------

. predict yp
(option xb assumed; fitted values)
```

Fitted



90
85
80
75
1950m1
1960m1
1970m1
1980m1
1990m1
t
men
Fitted values

Discontinuity

- One problem with this method is that the estimated trend function can be discontinuous
  - At the breakdate τ there might be a jump in the trend function
  - This might not be sensible
  - We may wish to impose continuity
- In the model, this requires

$$\beta_0 + \beta_1 \tau = \alpha_0 + \alpha_1 \tau$$

or

$$\beta_2 + \beta_3 \tau = 0$$

Continuous Break

- You can impose a continuous trend by using a technique known as a **spline**

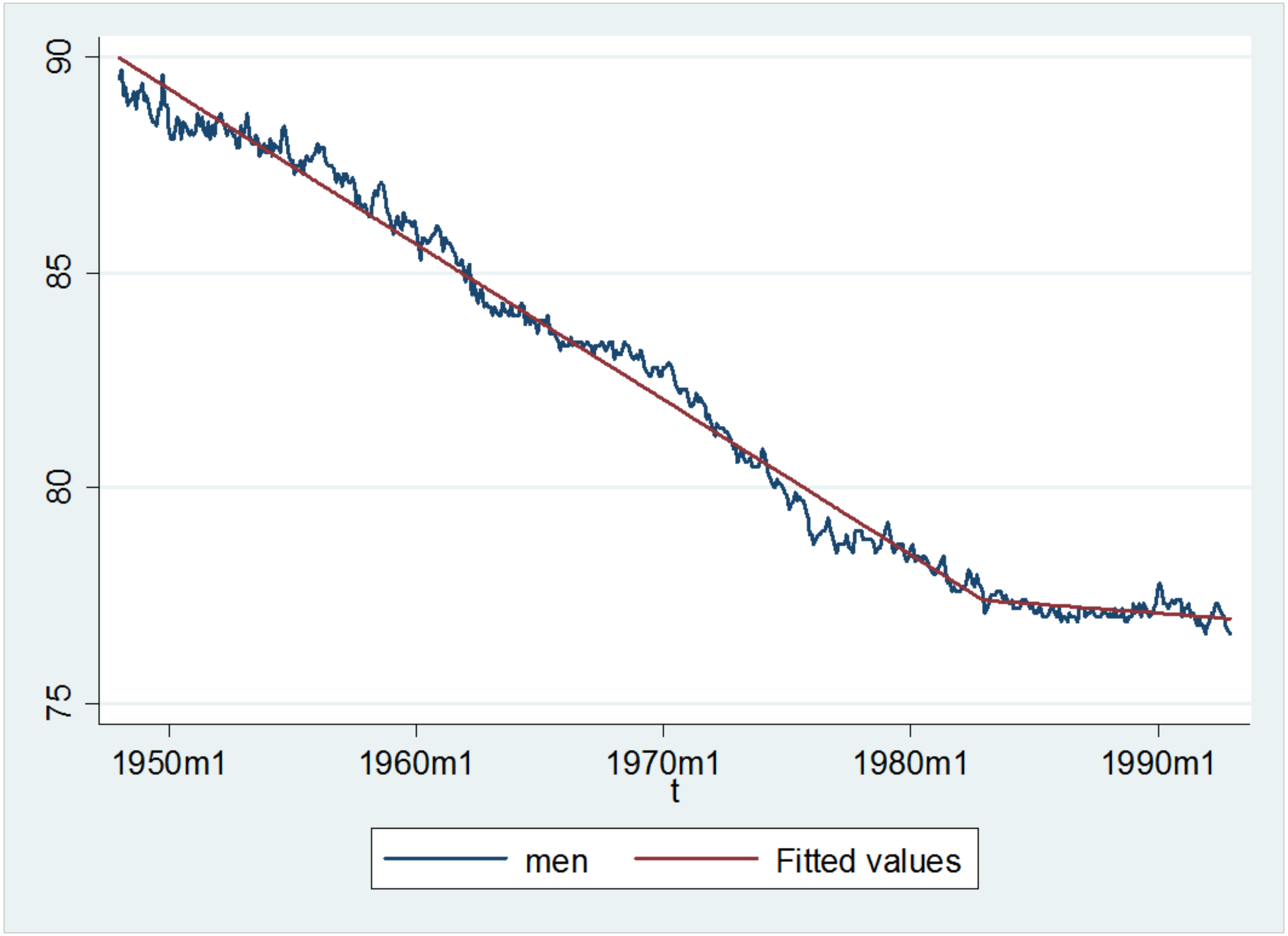
$$T_t = \beta_0 + \beta_1 Time_t + \beta_2 (Time_t - \tau) 1(t \geq \tau)$$
$$= \beta_0 + \beta_1 Time_t + \beta_2 Time_t^*$$

where

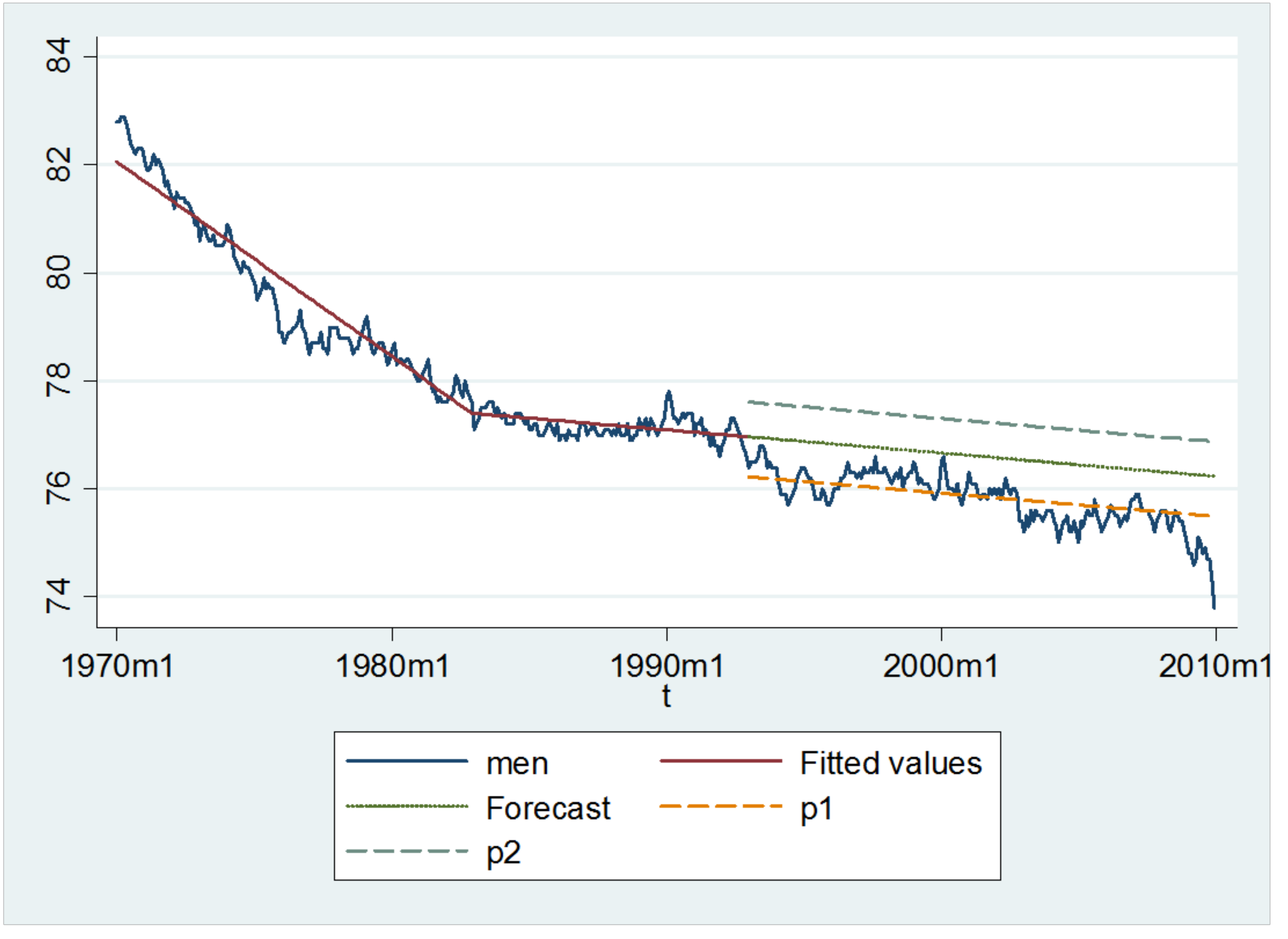
$$Time_t^* = (Time_t - \tau) 1(t \geq \tau)$$

- The variable $Time_t^*$ is 0 before the breakdate, and is a smoothly increasing trend afterwards.

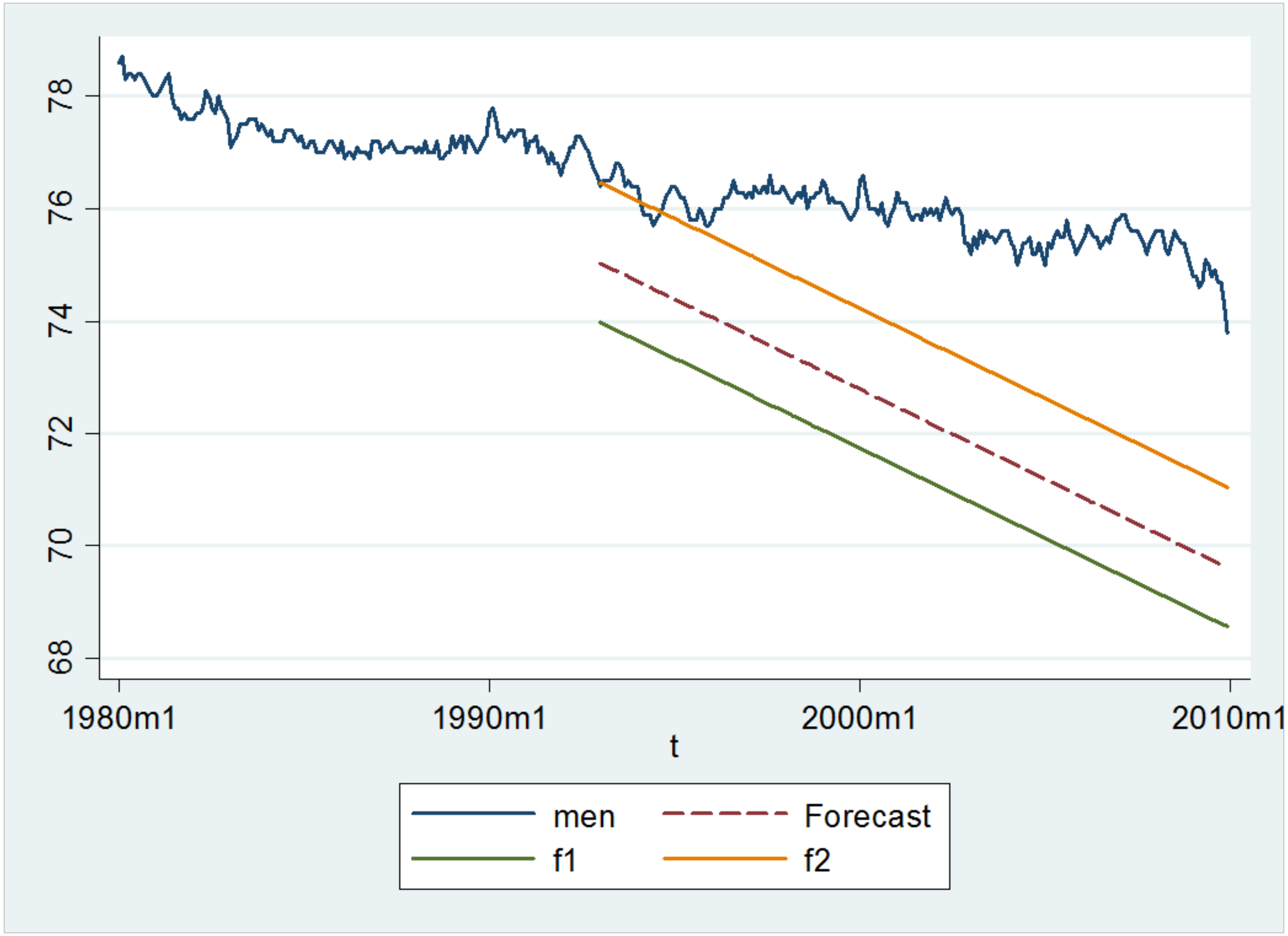
Fitted Continuous Trend



Continuous Trend Forecast



Contrast with Linear Trend Forecast

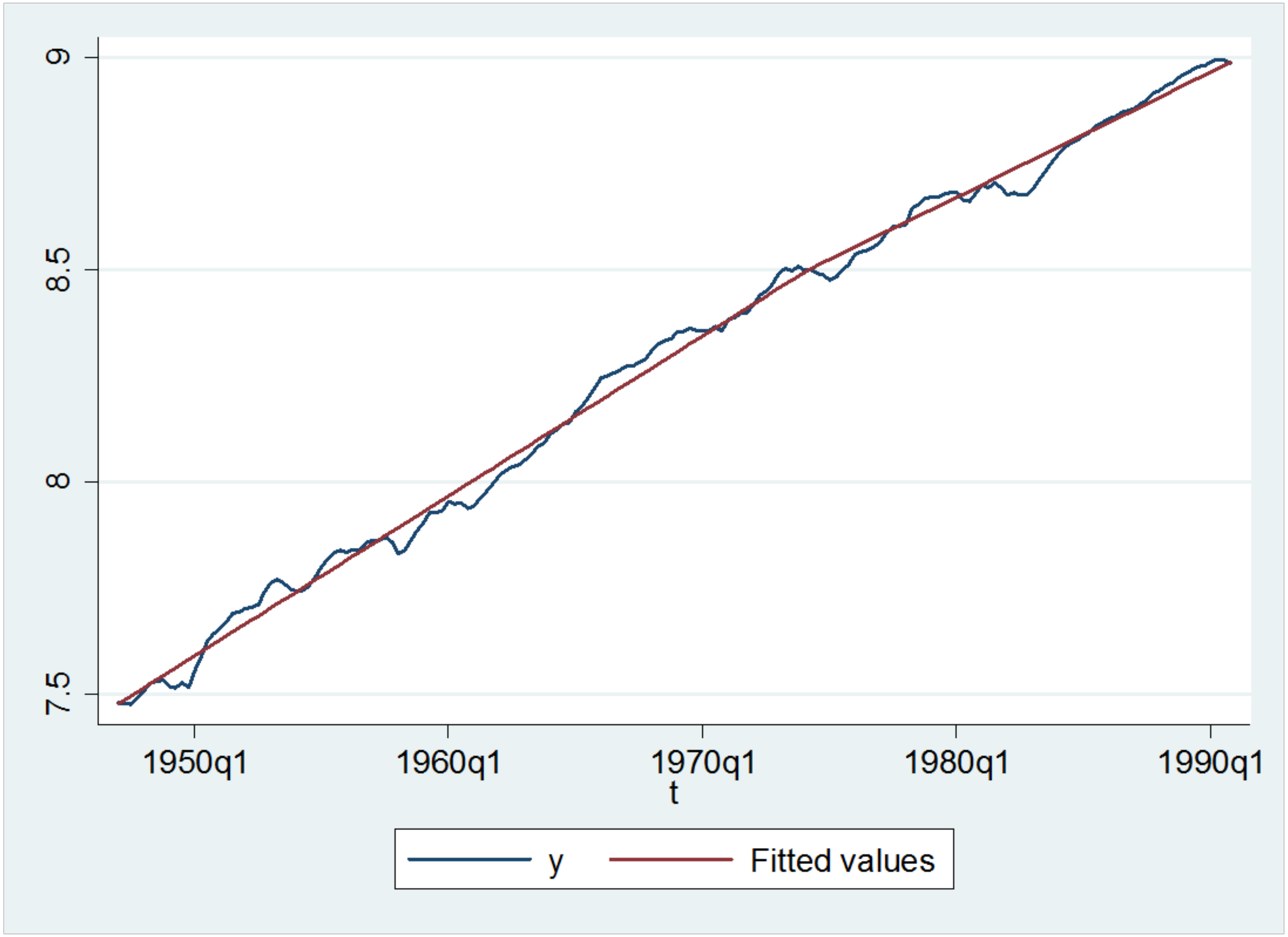


Real GDP

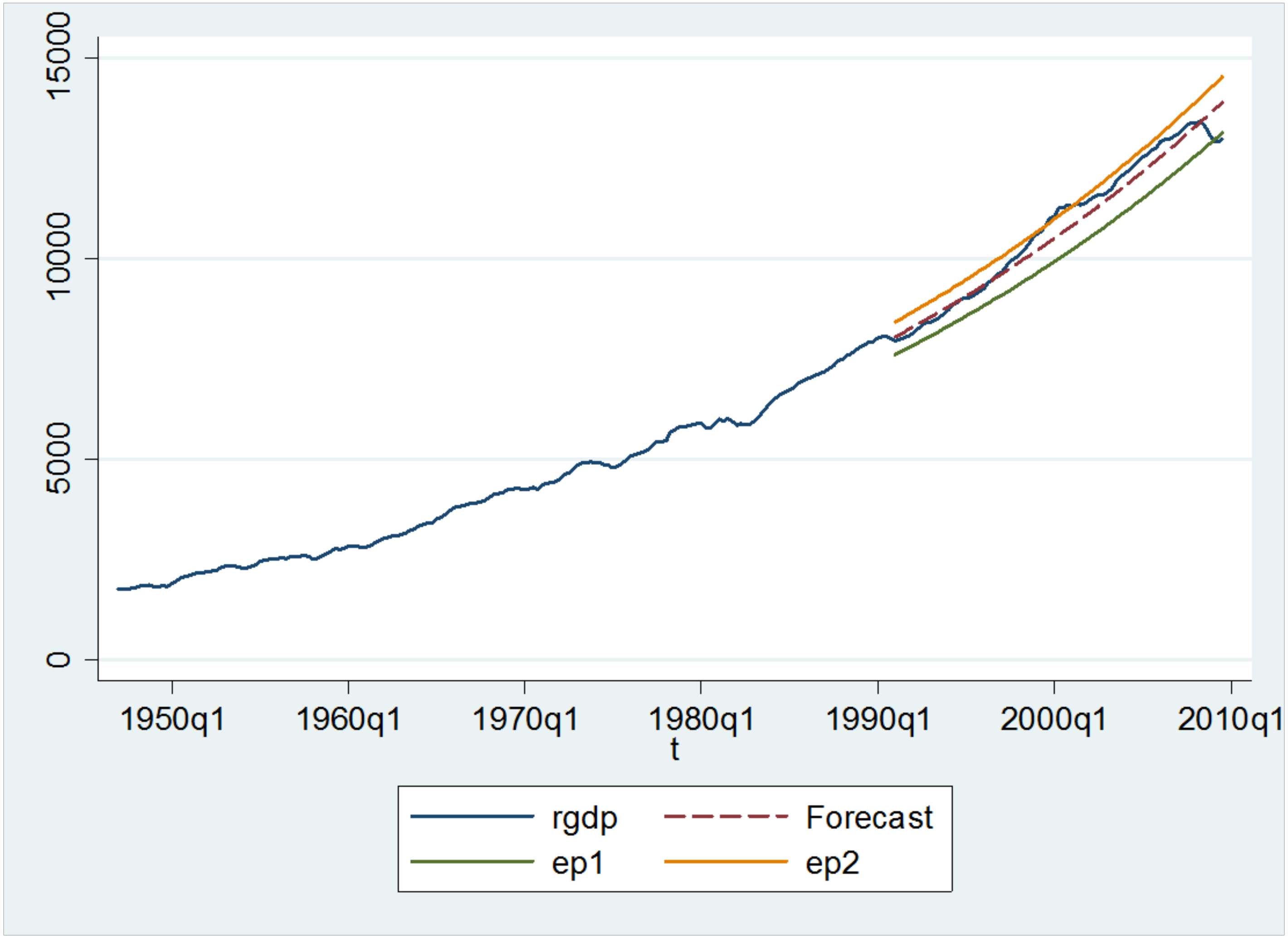
```
. generate tstar=(t-tq(1974q1))*(t>=tq(1974q1))

. regress y t tstar if t<=tq(1990q4)

      Source |       SS       df       MS              Number of obs =     176
-------------+------------------------------           F(  2,   173) =19367.20
       Model |  34.7825634     2  17.3912817           Prob > F      =  0.0000
    Residual |  .155349843   173  .000897976           R-squared     =  0.9956
-------------+------------------------------           Adj R-squared =  0.9955
       Total |  34.9379132   175  .199645218           Root MSE      =  .02997

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |   .0094132   .0000775   121.43   0.000     .0092602    .0095662
       tstar |  -.0020582   .0001934   -10.64   0.000    -.0024398   -.0016766
       _cons |    7.96762   .0027667  2879.80   0.000     7.962159    7.973081
------------------------------------------------------------------------------
```

- Break in 1974q1

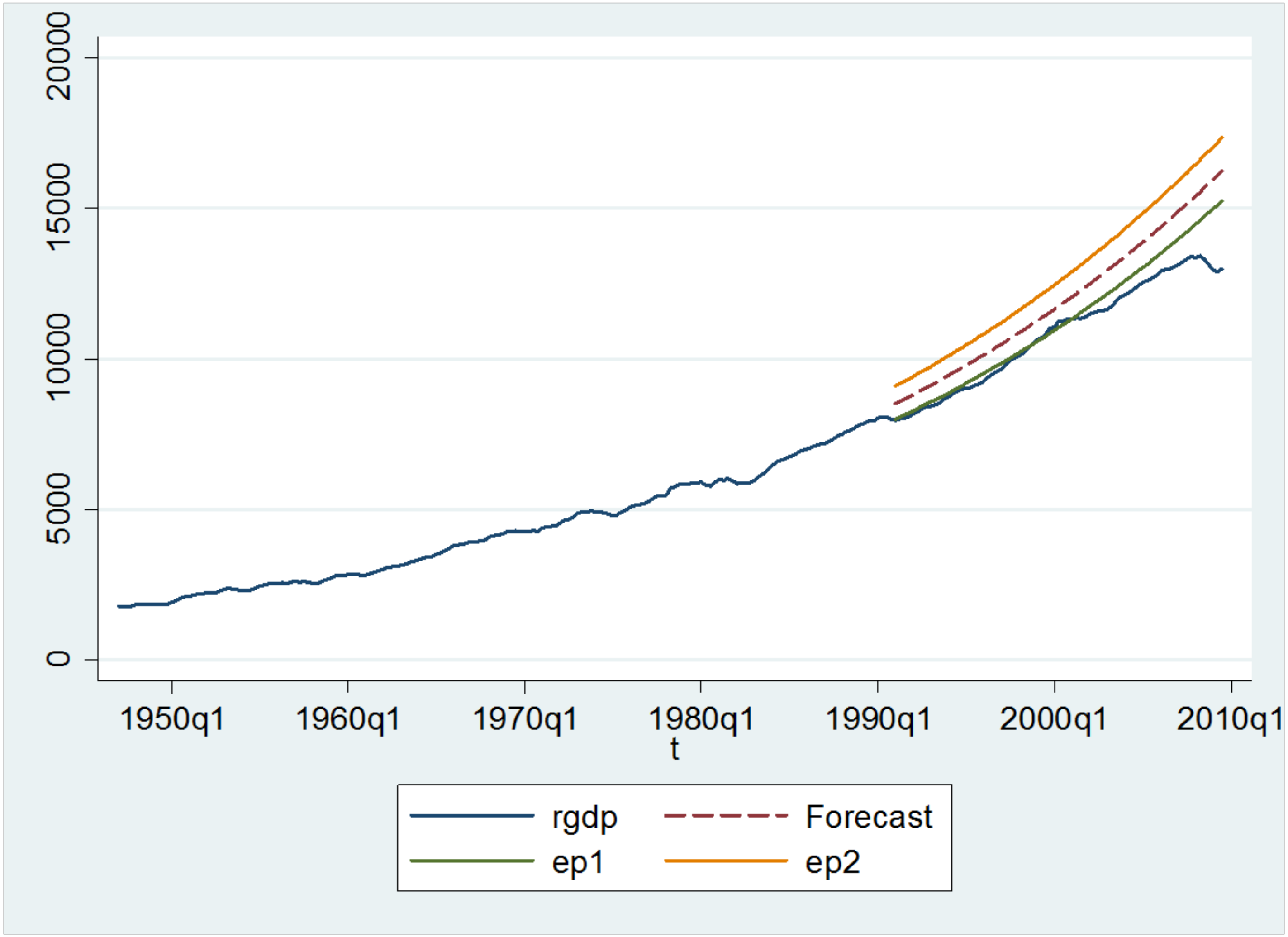
Real GDP - fitted



Forecast – Breaking Trend Model



Contrast – Forecast from Linear Trend



How to pick Breaks/Breakdates

- With caution, and skeptically
- Always have plenty of data (at least 10 years) after the breakdate
- Look for economic explanations
- Formally, the breakdate can be selected by minimizing the sum of squared errors