Stability

- Coefficients may change over time
  - Evolution of the economy
  - Policy changes

Time-Varying Parameters

$$y_t = \alpha_t + x_t \beta_t + e_t$$

- Coefficients depend on the time period
- If the coefficients vary randomly and are unpredictable, then they cannot be estimated
  - As there would be only one observation for each set of coefficients
  - We cannot estimate coefficients from just one observation!

Smoothly Time-Varying Parameters

$$y_t = \alpha_t + x_t \beta_t + e_t$$

- If the coefficients change gradually over time, then the coefficients are similar in adjacent time periods.
- We could try to estimate the coefficients for time period *t* by estimating the regression using observations [*t*- *w*/2,…, *t*+ *w*/2] where *w* is called the *window width*.
- *w* is the number of observations used for local estimation

Rolling Estimation

- This is called *rolling* estimation
- For a given window width w, you roll through the sample, using w observations for estimation.
- You advance one observation at a time and repeat
- Then you can plot the estimated coefficients against time

What to expect

- Rolling estimates will be a combination of true coefficients and sampling error
- The sampling error can be large
  - Fluctuations in the estimates can be just error
- If the true coefficients are trending
  - Expect the estimated coefficients to display trend plus noise
- If the true coefficients are constant
  - Expect the estimated coefficients to display random fluctuation and noise

Example: GDP Growth

```
. reg gdp L(1/3).gdp,r

Linear regression                               Number of obs =     248
                                                F(  3,   244) =   10.73
                                                Prob > F      =  0.0000
                                                R-squared     =  0.1527
                                                Root MSE      =   3.815
```

gdp	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdp						
L1.	.3412071	.0764232	4.46	0.000	.1906738	.4917405
L2.	.1327376	.0826814	1.61	0.110	-.0301228	.2955981
L3.	-.1293765	.0731709	-1.77	0.078	-.2735037	.0147508
_cons	2.193251	.412281	5.32	0.000	1.381167	3.005335

STATA **rolling** command

- STATA has a command for rolling estimation:

.rolling, window(100) clear: regress gdp L(1/3).gdp

- In this command:
  - **window(100)** sets the window width
    - w=100
    - The number of observations for estimation will be 100
  - **clear**
    - Clears out the data in memory
    - The data will be replaced by the rolling estimates
    - It is necessary

rolling command

.rolling, window(100) clear: regress gdp L(1/3).gdp

- The part after the ":"
  - **regress gdp L(1/3).gdp**
  - This is the command that STATA will implement using the rolling method
  - An AR(3) will be fit using 100 observations, rolling through the sample

Example

- GDP is quarterly 1947Q1 through 2009Q4
  - 251 observations
- Using w=100
  - The first estimation window is 1947Q2-1972Q1
  - The second is 1947Q3-1972Q2
  - There are 152 estimation windows
  - The final is 1985Q1-2009Q4

- STATA Execution:

```
. rolling, window(100) clear: regress gdp L(1/3).gdp
(running regress on estimation sample)

Rolling replications (152)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50
..................................................   100
..................................................   150
..
```

After Rolling Execution

- The original data have been cleared from memory
- STATA shows new variables
  - start
  - end
  - _stat_1
  - _stat_2
  - _stat_3
  - _b_cons
- *start* and *end* are starting/ending dates for each window
  - *start* runs from 1947Q2 to 1985Q1
  - *end* runs from to 1972Q1 2009Q4
- The others are the rolling estimates, AR and intercept

Time reset

- As the original data have been cleared, so has your time index.
- So the **tsline** command does not work until you reset the time
- You can set the time to be start or end
  - **.tsset start**
  - **.tsset end**
- Or, more elegantly, you can set the time to be the mid-point of the window
  - **.gen t=round((start+end)/2)**
  - **.format t %tq**
  - **.tsset t**
  - This time index runs from 1959Q4 through 1997Q3

Time reset example

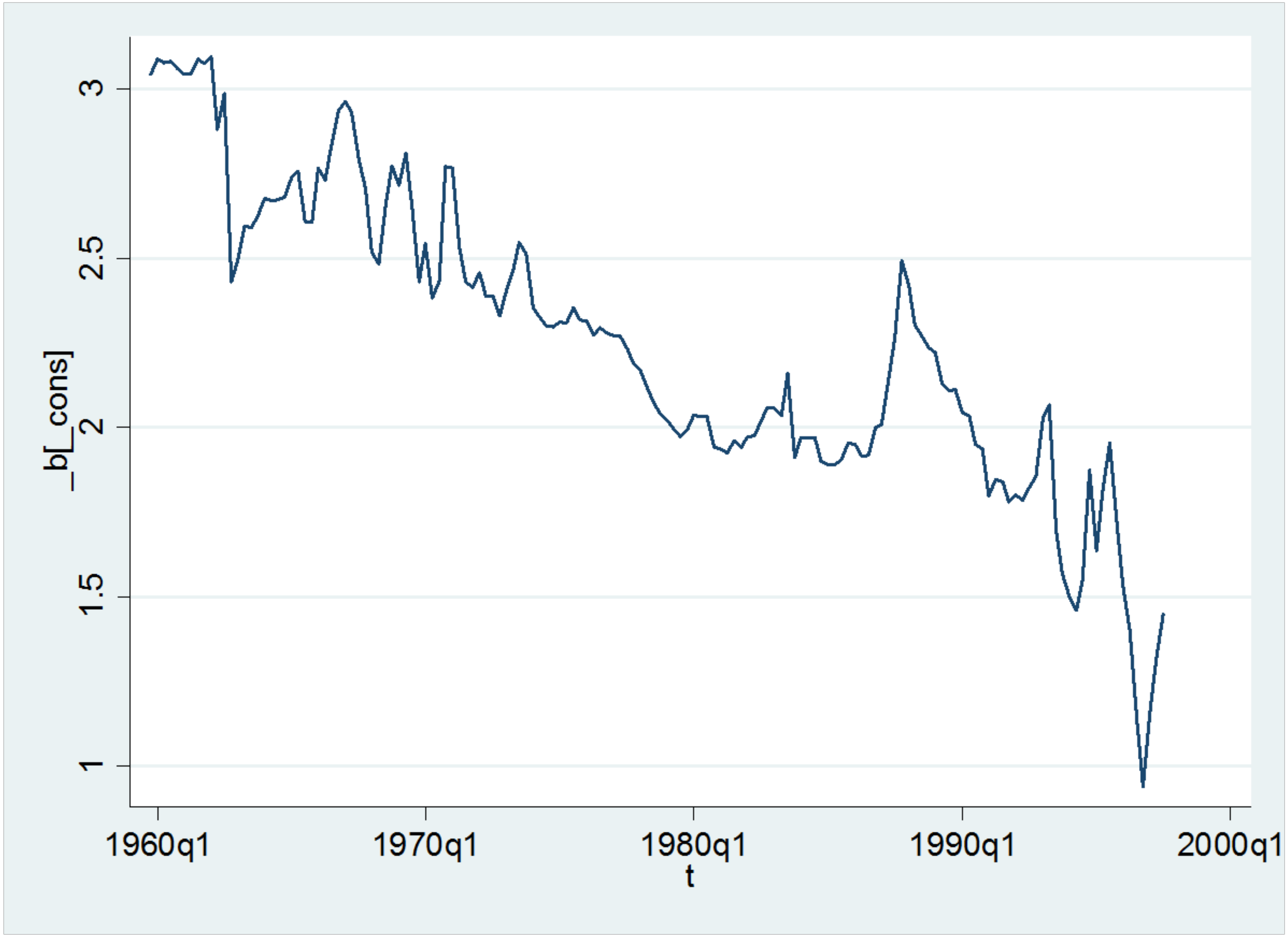
- Example

```
. gen t=round((start+end)/2)

. format t %tq

. tsset t
        time variable:  t, 1959q4 to 1997q3
                delta:  1 quarter

. tsline _b_cons
```

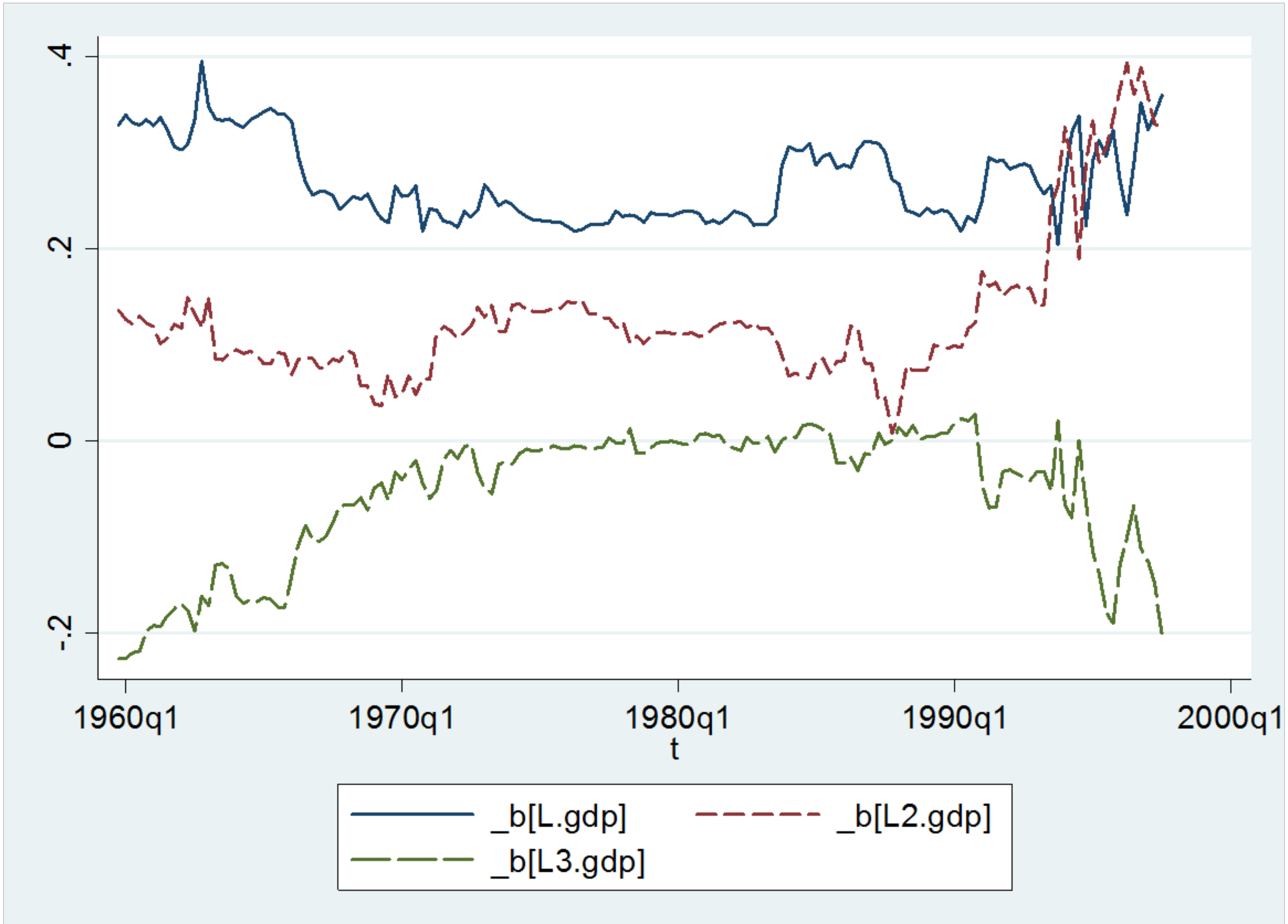
Plot Rolling Coefficients

- Now you can plot the estimated coefficients against time
  - You can use separate or joint plots
  - **.tsline _b_cons**
  - **.tsline _stat_1 _stat_2 _stat_3**

Rolling Intercept

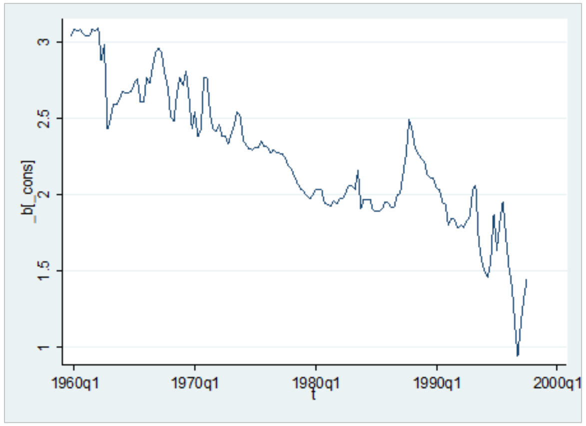


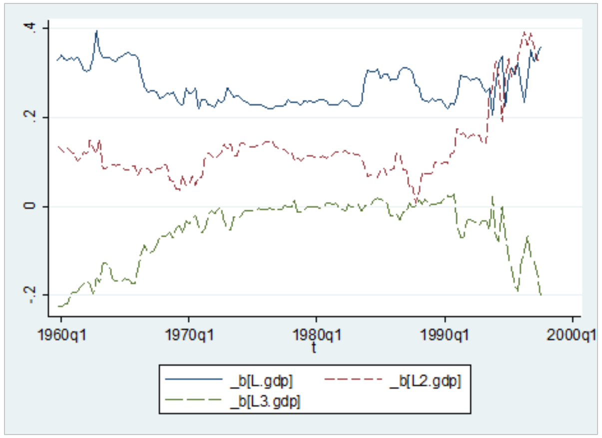
Rolling AR coefficients



Analysis

- The estimated intercept is decreasing gradually
- The AR(1) coef is quite stable
- The AR(2) coef starts increasing around 1990
- The AR(3) coef is 0 most of the period, but is negative from 1960-1973 and after 1995
- All of the graphs go a bit crazy over 1990-1997





Sequential (Recursive) Estimation

- As an alternative to rolling estimation, *sequential* or *recursive* estimation uses all the data up to the window width
  - First window: [1,w]
  - Second window: [1,w+1]
  - Final window: [1,T]
- With sequential estimation, *window* is the length of the first estimation window

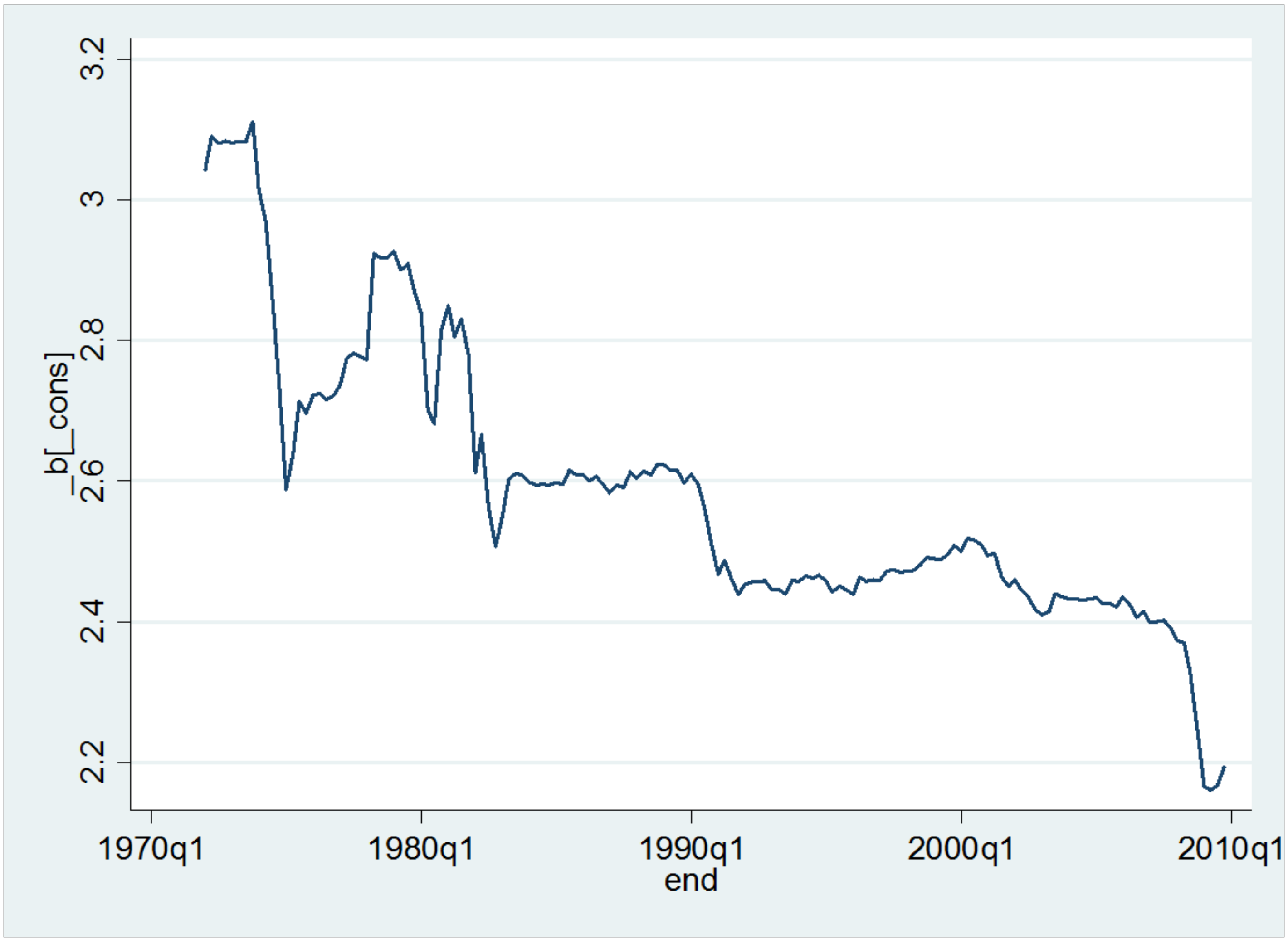
Recursive Estimation

- STATA command is similar, but adds **recursive** after comma

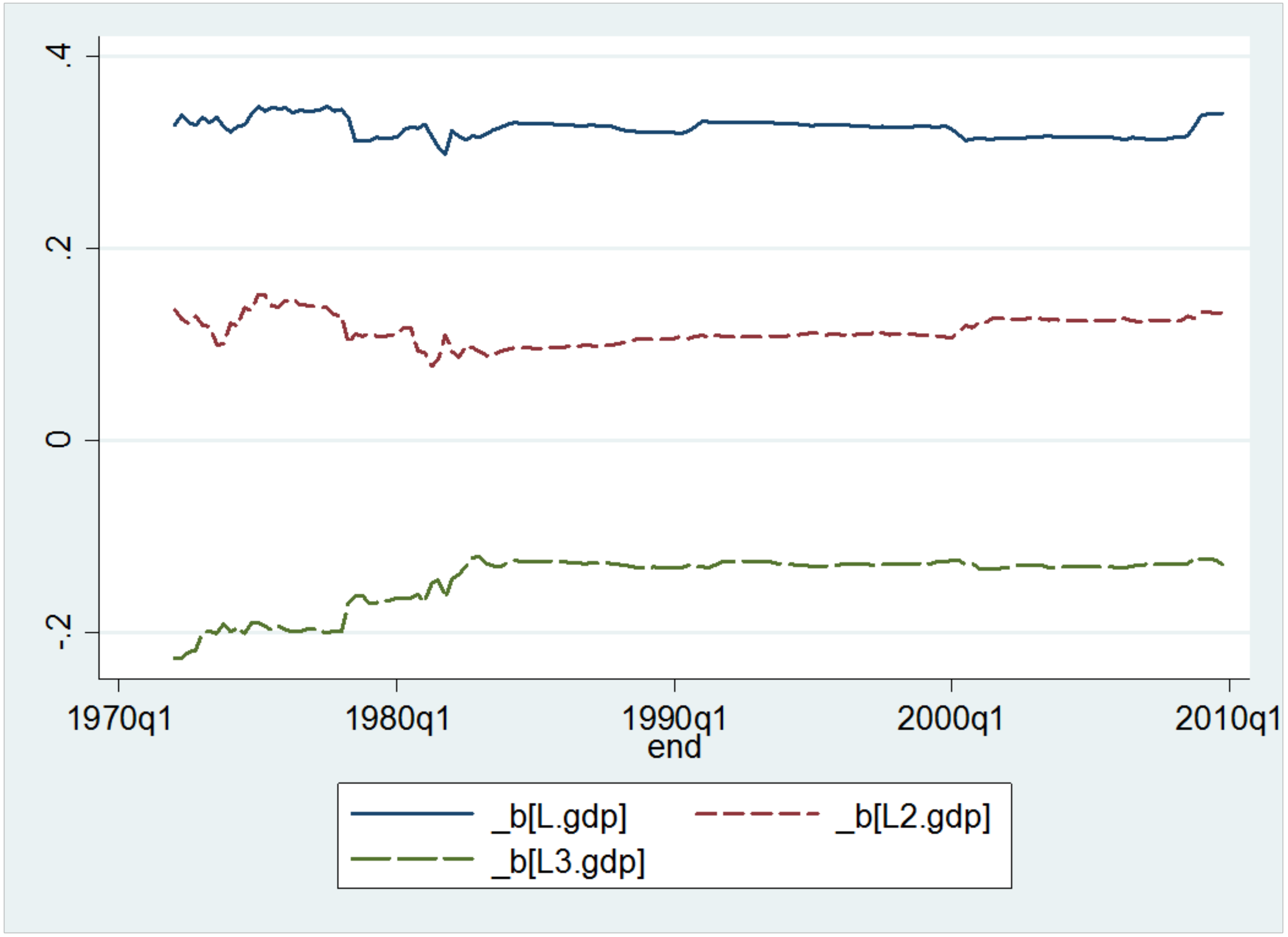
.rolling, recursive window(100) clear: regress gdp L(1/3).gdp

- STATA clears data set, replaces with *start*, *end*, and recursive coefficient estimates *_b_cons*, *_stat_1*, etc.
- Use *end* for time variable
  - **.tsset end**
  - This sets the time index to the end period used for estimation

Recursive Intercept

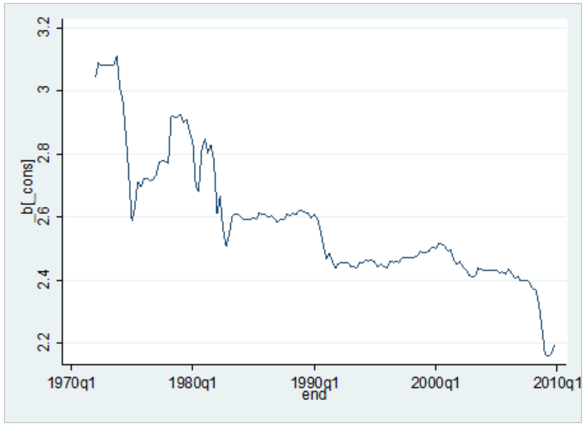


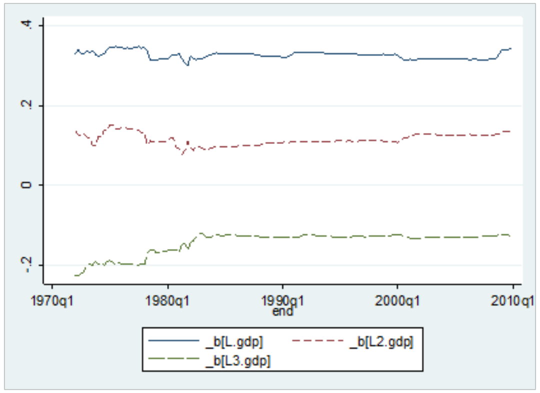
Recursive AR coefficients



Analysis

- The recursive intercept fluctuates, but decreases
  - Drops around 1984, and 1990
- The recursive AR(1) and AR(2) coefs are very stable
- The recursive AR(3) coef increases, and then becomes stable after 1984.





Summary

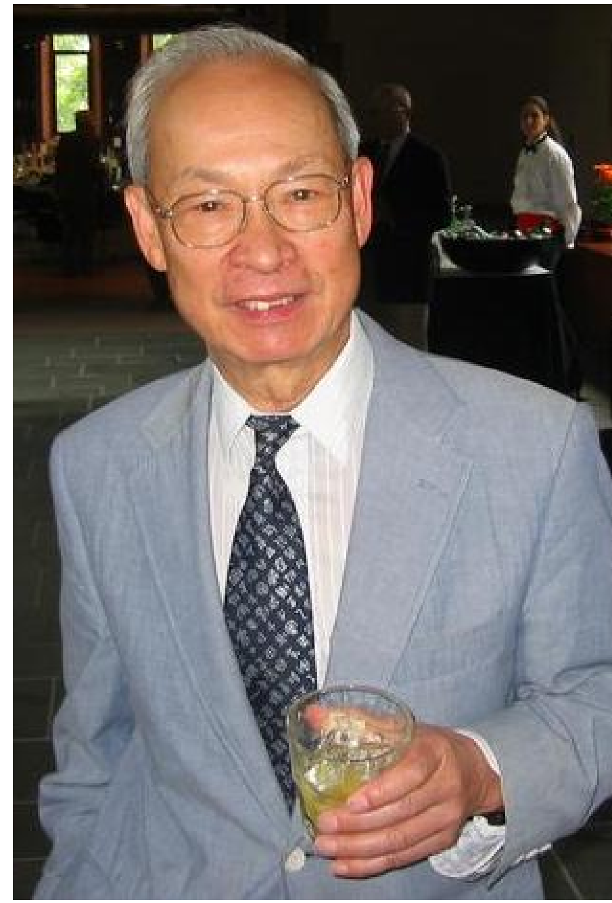
- Use rolling and recursive estimation to investigate stability of estimated coefficients
- Look for patterns and evidence of change
- Try to identify potential *breakdates*
- In GDP example, possible dates:
  - 1970, 1984, 1990

Testing for Breaks

- Did the coefficients change at some breakdate t^*?
- We can test if the coefficients before and after t^* are the same, or if they changed
- Simple to implement as an F test using dummy variables
- Known as a *Chow test*

Gregory Chow

- Professor Gregory Chow of Princeton University (emeritus)
- Proposed the “Chow Test” for structural change in a famous paper in 1960



Dummy Variable

- For a given breakdate t^*
- Define a dummy variable d
  - $d=1$ if $t>t^*$
- Include d and interactions d^*x to test for changes

Model with Breaks

- Original Model

$$y_t = \alpha + x_t \beta + e_t$$

- Model with break

$$y_t = \alpha + x_t \beta + \delta d_t + \gamma d_t x_t + e_t$$

- Interpreting the coefficients
  - δ=change in intercept
  - γ=change in slope

Chow Test

$$y_t = \alpha + x_t\beta + \delta d_t + \gamma d_t x_t + e_t$$

- The model has constant parameters if δ=γ=0
- Hypothesis test:
  - H_0: δ=0 and γ=0
- Implement as an *F* test after estimation
- If prob>.05, you do not reject the hypothesis of stable coefficients

Example: GDP

```
. gen d=(t>tq(1974q1))

. gen x1=d*L.gdp
(1 missing value generated)

. gen x2=d*L2.gdp
(2 missing values generated)

. gen x3=d*L3.gdp
(3 missing values generated)
```

```
. reg gdp L(1/3).gdp d x1 x2 x3,r

Linear regression                               Number of obs =     248
                                                F(  7,   240) =    6.21
                                                Prob > F      =  0.0000
                                                R-squared     =  0.1662
                                                Root MSE      =  3.8158

------------------------------------------------------------------------------
             |               Robust
         gdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |
         L1. |   .3220439    .111703     2.88   0.004     .1020005    .5420873
         L2. |   .1225762   .1097826     1.12   0.265    -.0936842    .3388366
         L3. |  -.1994019   .1035398    -1.93   0.055    -.4033648    .0045609
             |
           d |  -1.321552   .9254528    -1.43   0.155    -3.144599    .5014957
          x1 |   .0108441   .1466639     0.07   0.941    -.2780688    .2997569
          x2 |   .0100876   .1606384     0.06   0.950    -.3063537    .3265289
          x3 |   .1489167   .1495265     1.00   0.320    -.1456352    .4434686
       _cons |   3.014221   .8146597     3.70   0.000     1.409424    4.619017
------------------------------------------------------------------------------
```

Chow test

```
. test d x1 x2 x3

 ( 1)  d = 0
 ( 2)  x1 = 0
 ( 3)  x2 = 0
 ( 4)  x3 = 0

       F(  4,   240) =    0.72
            Prob > F =    0.5797
```

- The p-value is larger than 0.05
- It is not significant
- We do not reject hypothesis of constant coefficients

Fishing for a Breakdate

- An important trouble with the Chow test is that it assumes that the breakdate is known – before looking at the data
- But we selected the breakdate by examining rolling and recursive estimates
- This means that are **too likely** to find misleading “evidence” of non-constant coefficients

Fishing

- We could consider picking multiple possible breakdates $t^*=[t_1, t_2, \ldots, t_M]$
- For each breakdate t^*, we could estimate the regression and compute the Chow statistic $F(t^*)$
- Fishing for a breakdate is similar to searching for a big (significant) Chow statistic.

The Quandt Likelihod Ratio (QLR) Statistic

(also called the "sup-Wald" statistic)

The QLR statistic = the maximal Chow statistics

- Let $F(\tau)$ = the Chow test statistic testing the hypothesis of no break at date τ.
- The *QLR* test statistic is the ***maximum*** of all the Chow *F*-statistics, over a range of τ, $\tau_0 \leq \tau \leq \tau_1$:

$$QLR = \max[F(\tau_0), F(\tau_0+1), \ldots, F(\tau_1-1), F(\tau_1)]$$

- A conventional choice for τ_0 and τ_1 are the inner 70% of the sample (exclude the first and last 15%.

Richard Quandt

- Professor Richard Quandt (1930-)
  - Princeton University
  - Estimation of breakdate (Quandt, 1958)
  - QLR test (Quandt, 1960)

QLR Critical Values

$$QLR = \max[F(\tau_0), F(\tau_0+1), \ldots, F(\tau_1-1), F(\tau_1)]$$

- Should you use the usual critical values?
- The large-sample null distribution of $F(\tau)$ for a given (fixed, not estimated) τ is $F_{q,\infty}$
- But if you get to compute two Chow tests and choose the biggest one, the critical value must be larger than the critical value for a single Chow test.
- If you compute very many Chow test statistics – for example, all dates in the central 70% of the sample – the critical value must be larger still!

- ***Get this***: in large samples, *QLR* has the distribution,

$$\max_{a \le s \le 1-a} \left(\frac{1}{q} \sum_{i=1}^{q} \frac{B_i(s)^2}{s(1-s)} \right),$$

where $\{B_i\}$, $i = 1,\ldots,n$, are independent continuous-time "Brownian Bridges" on $0 \le s \le 1$ (a Brownian Bridge is a Brownian motion deviated from its mean), and where $a = .15$ (exclude first and last 15% of the sample)

- Critical values are tabulated in SW Table 14.6…

TABLE 14.6 Critical Values of the QLR Statistic with 15% Trimming

Number of Restrictions (q)	10%	5%	1%
1	7.12	8.68	12.16
2	5.00	5.86	7.78
3	4.09	4.71	6.02
4	3.59	4.09	5.12
5	3.26	3.66	4.53
6	3.02	3.37	4.12
7	2.84	3.15	3.82
8	2.69	2.98	3.57
9	2.58	2.84	3.38
10	2.48	2.71	3.23

Note that these critical values are larger than the $F_{q,\infty}$ critical values – for example, $F_{1,\infty}$ 5% critical value is 3.84.

QLR Theory

- Distribution theory for the QLR statistic
- Developed by
  - Professor Donald Andrews (Yale)

Has the postwar U.S. Phillips Curve been stable?

Consider a model of ΔInf_t given $Unemp_t$ – the empirical backwards-looking Phillips curve, estimated over (1962 – 2004):

$$\Delta Inf_t = \underset{(.44)}{1.30} - \underset{(.08)}{.42}\Delta Inf_{t-1} - \underset{(.09)}{.37}\Delta Inf_{t-2} + \underset{(.08)}{.06}\Delta Inf_{t-3} - \underset{(.08)}{.04}\Delta Inf_{t-4}$$

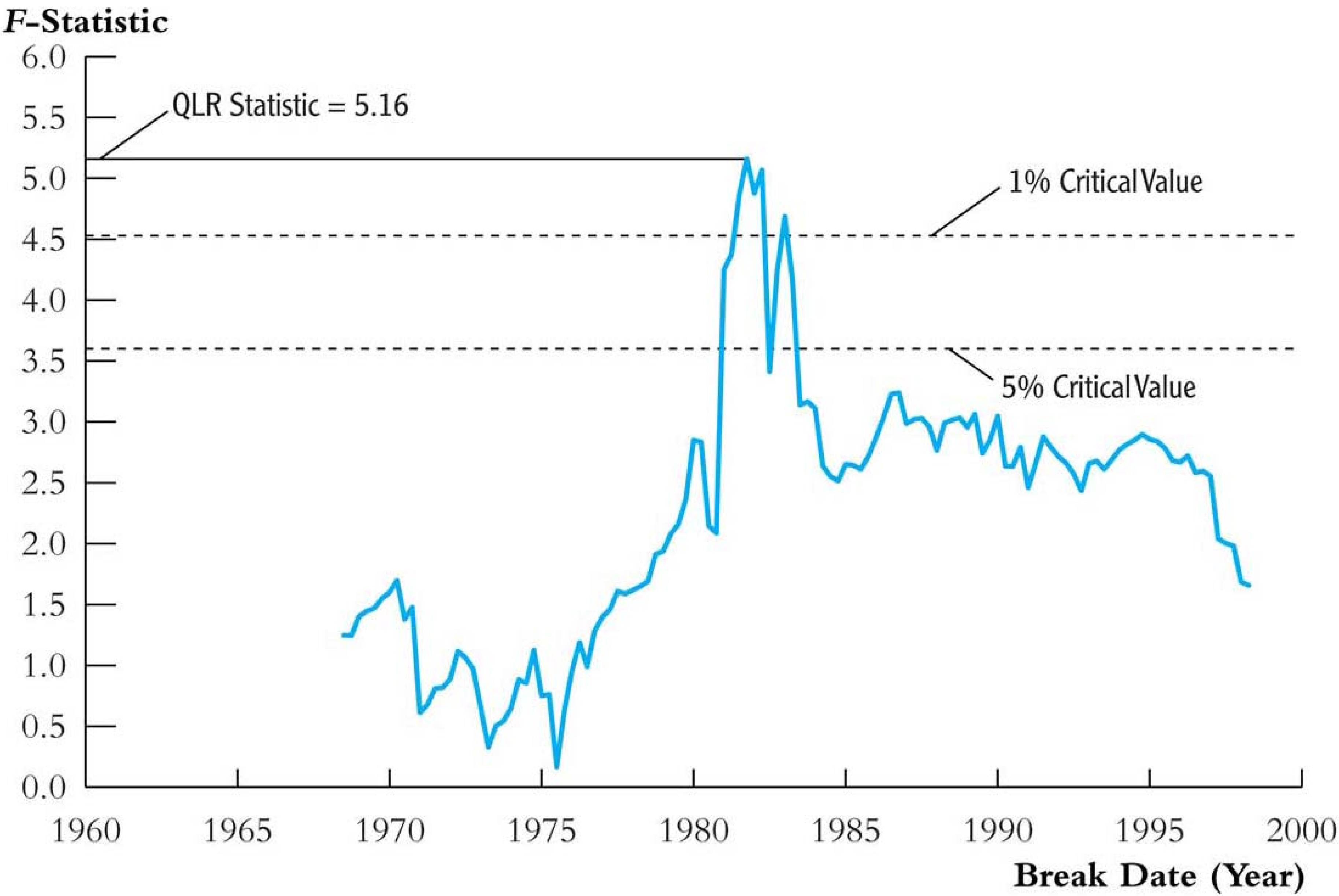
$$- \underset{(.46)}{2.64}Unem_{t-1} + \underset{(.86)}{3.04}Unem_{t-2} - \underset{(.89)}{0.38}Unem_{t-3} + \underset{(.45)}{.25}Unemp_{t-4}$$

Has this model been stable over the full period 1962-2004?

QLR tests of stability of the Phillips curve.

dependent variable: ΔInf_t
regressors: intercept, $\Delta Inf_{t-1},\ldots, \Delta Inf_{t-4}$,
$Unemp_{t-1},\ldots, Unemp_{t-4}$

- test for constancy of intercept only (other coefficients are assumed constant): $QLR = 2.865$ ($q = 1$).
  - 10% critical value = 7.12 $\Rightarrow$ don't reject at 10% level
- test for constancy of intercept and coefficients on $Unemp_t,\ldots, Unemp_{t-3}$ (coefficients on $\Delta Inf_{t-1},\ldots, \Delta Inf_{t-4}$ are constant): $QLR = 5.158$ ($q = 5$)
  - 1% critical value = 4.53 $\Rightarrow$ reject at 1% level
  - Break date estimate: maximal F occurs in 1981:IV
- Conclude that there is a break in the inflation – unemployment relation, with estimated date of 1981:IV


F-Statistic
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
QLR Statistic = 5.16
1% Critical Value
5% Critical Value
1960
1965
1970
1975
1980
1985
1990
1995
2000
Break Date (Year)

Implementation

- It is difficult to compute QLR without using some programming.
- But it is well approximated by
  - Examining rolling and recursive estimates for possible breaks
  - Computing Chow test at potential breakdates.
- Don’t use STATA’s p-value!
- Use Table 14.6 from SW (or earlier slide).