Distribution of Least-Squares

- In classic regression, if the errors are iid normal, and independent of the regressors, then the least-squares estimates have an **exact** normal distribution, not just asymptotic
- This is not true in most time-series regressions.

Non-Classical Distributions

- Estimates in autoregressive models
  - Biased downwards
  - Skewed
  - Thick tails
- Especially
  - When autoregressive coefficients are large
  - Sample sizes are small
- These issues diminish in large samples

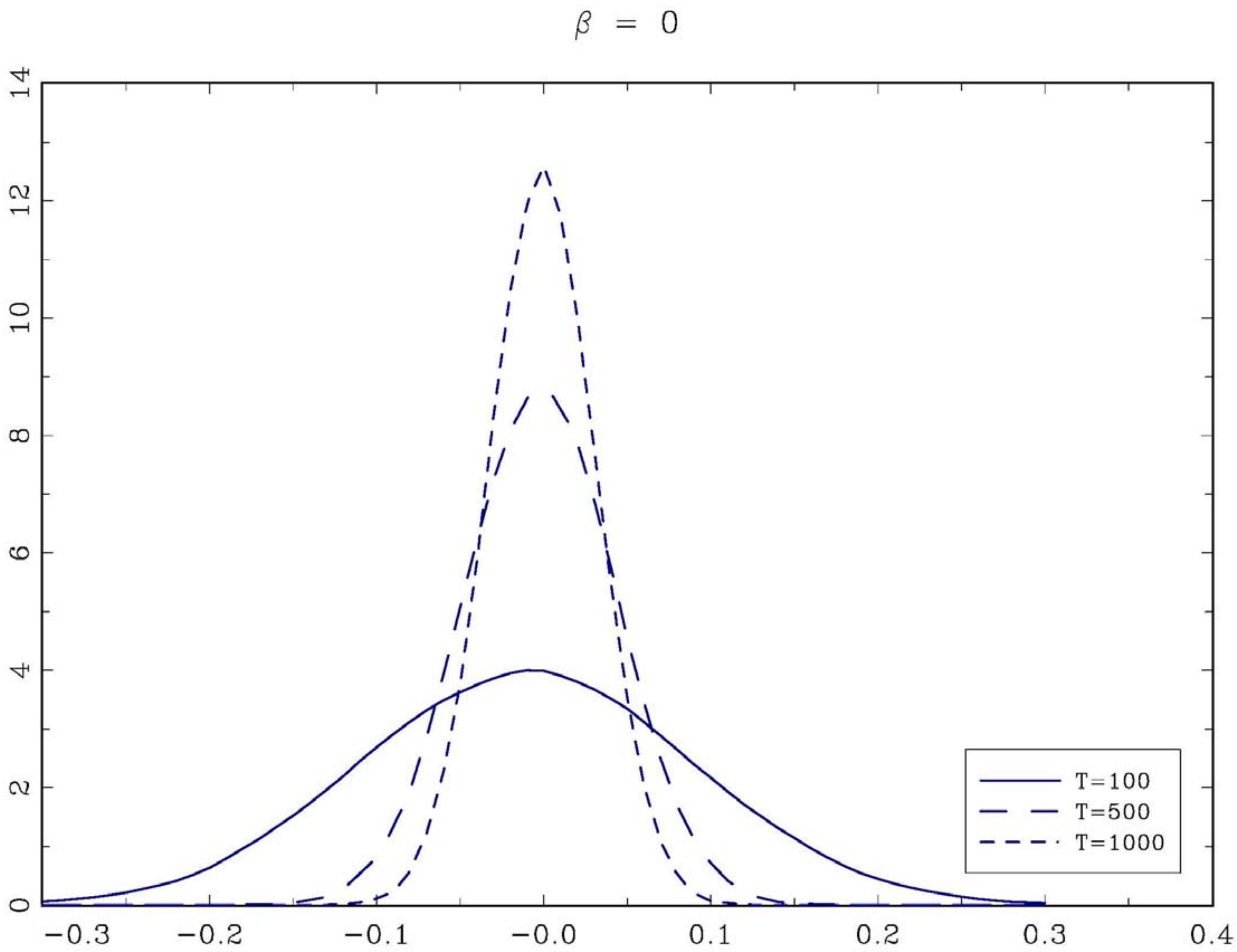
Example

- Take the AR(1) model with intercept

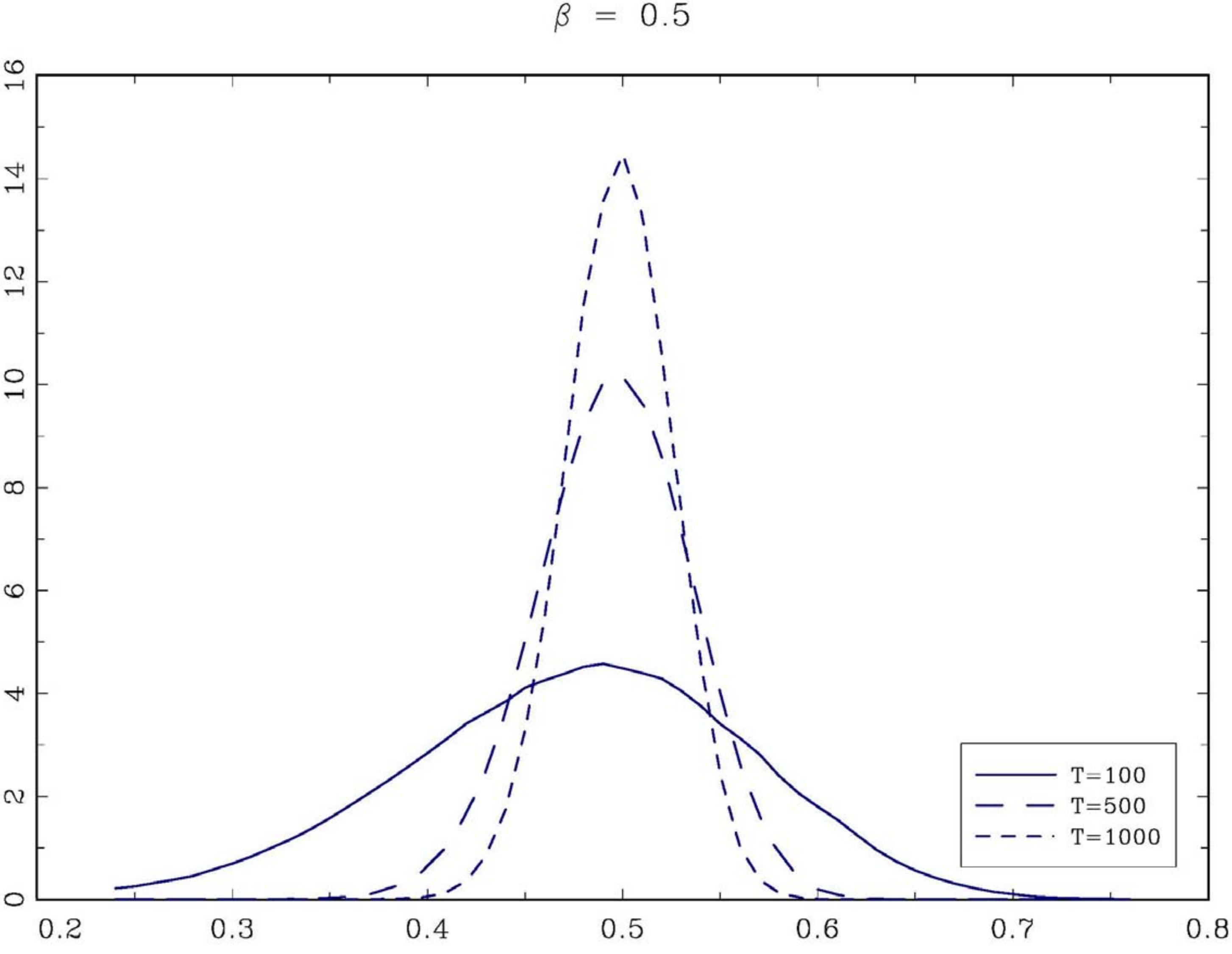
$$y_t = \alpha + \beta y_{t-1} + e_t$$

- e_t N(0,1)
- T=100, 500, 1000
- β=0.0, β=0.5, β=0.9,
- Numerically calculate distribution of least-squares estimate of β

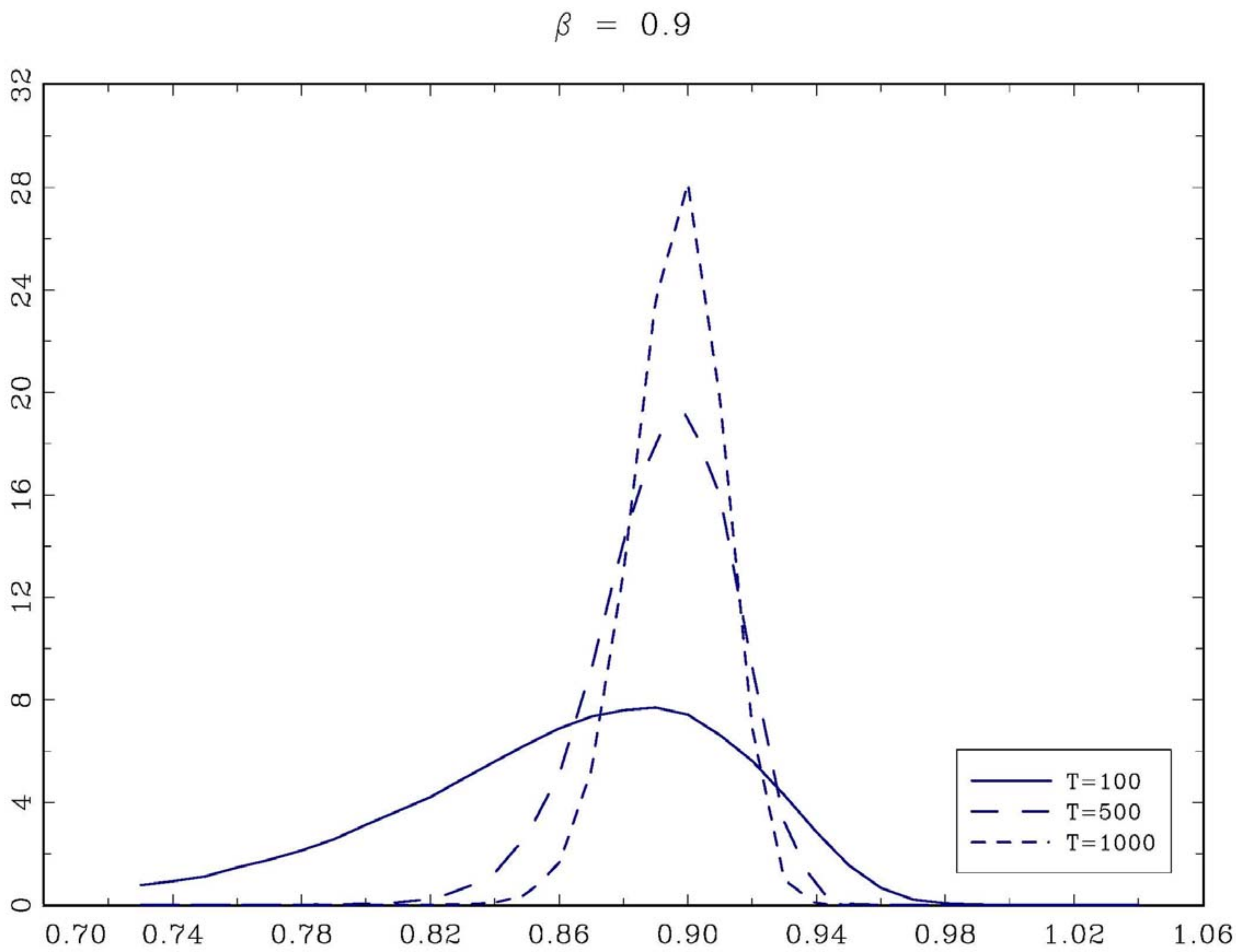
Distribution, β=0.0



Distribution, β=0.5



Distribution, β=0.9



Interpretation

- Estimates of autoregressive parameters are random
- Even if regression error is normal, the parameter estimates are not normally distributed
- Distributions are less normal when AR coefficient is large
- Distributions are more concentrated and normal when sample size is large

Asymptotic Standard Deviation

- The least-squares estimate is asymptotically (approximately) normally distributed
- In the simple model $y_t = \beta x_t + e_t$

  then

$$\hat{\beta} \overset{a}{\sim} N\left(\beta, \sigma_{\hat{\beta}}^2\right)$$

$$\sigma_{\hat{\beta}}^2 = \frac{1}{T} \frac{\operatorname{var}(x_t e_t)}{\left[\operatorname{var}(x_t)\right]^2}$$

- The standard deviation measures the precision of the estimate, but it is unknown.

Standard Errors

- Estimates of the standard deviations are called **standard errors**, and are reported in regression output
- They are used to measure estimation precision.

Classical standard errors

A **classic standard error** is an estimate of the standard deviation from the formula

$$\sigma_{\hat{\beta}}^2 = \frac{1}{n}\frac{\mathrm{var}(e_t)}{\mathrm{var}(x_t)}$$

This formula is valid under conditional homoskedasticity

$$E(e_t \mid \Omega_{t-1}) = 0$$
$$E(e_t^2 \mid \Omega_{t-1}) = \sigma^2$$

Robust standard errors

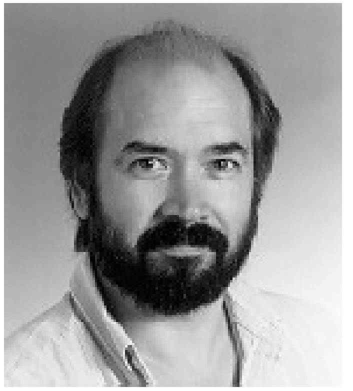
- "Robust" standard errors are estimates of

$$\sigma_\beta = \sqrt{\frac{1}{n} \frac{\mathrm{var}(x_t e_t)}{[\mathrm{var}(x_t)]^2}}$$

- These are the conventional standard errors for regression analysis
- Also known as "White" standard errors

Halbert White

- Professor Hal White, UCSD
- Leading contributor to econometric methods, especially time series analysis
- Introducted robust standard errors into econometrics (1980)
  - Most referenced paper in economics
- Founded Bates-White consulting firm, a leader in economic policy analysis



Have you seen robust standard errors?

- If you took an econometrics course other than 410, you may not be familiar with robust standard errors
- If you are currently taking 410, you won't cover robust standard errors until later in the course
  - Wooldridge uses the homoskedasticity assumption in the early part of his text
- Stock-Watson use robust standard errors throughout

Does the Choice Matter?

- Classic standard errors are for the assumption of conditional homoskedasticity

$$E\left(e_t^2 \mid \Omega_{t-1}\right) = \sigma^2$$

- This is **unforecastability in the variance**
  – This is not implied by conventional unforecastability
  – It may be a convenient approximation for macro data
  – It is a bad assumption (quite false) in financial data

Example: Stock Returns, AR(1)

r	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
r						
L1.	-.0163895	.0179105	-0.92	0.360	-.051507	.0187279
_cons	.0013626	.0003728	3.66	0.000	.0006317	.0020935

r	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
r						
L1.	-.0163895	.032377	-0.51	0.613	-.0798718	.0470927
_cons	.0013626	.0003878	3.51	0.000	.0006022	.002123

- The robust standard error on the AR(1) coefficient is almost twice as large as the conventional standard error

Computation

- In STATA, the default is conventional standard errors.
- They are automatically reported with the regress (reg) command
- For robust standard errors, use the "r" option

.reg y x, r

Example: Real GDP Growth

```
. reg gdp L(1/4).gdp

      Source |       SS       df       MS              Number of obs =     247
-------------+------------------------------           F(  4,   242) =   11.39
       Model |  662.232234     4  165.558059           Prob > F      =  0.0000
    Residual |  3518.78213   242   14.540422           R-squared     =  0.1584
-------------+------------------------------           Adj R-squared =  0.1445
       Total |  4181.01437   246  16.9959934           Root MSE      =  3.8132

------------------------------------------------------------------------------
         gdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |
         L1. |    .327656   .0640344     5.12   0.000     .2015202    .4537919
         L2. |   .1466135   .0670302     2.19   0.030     .0145764    .2786506
         L3. |  -.0980287    .066934    -1.46   0.144    -.2298764    .0338189
         L4. |  -.0889209   .0644466    -1.38   0.169    -.2158689     .038027
             |
       _cons |   2.378427    .389677     6.10   0.000     1.610836    3.146019
------------------------------------------------------------------------------
```

With Robust st. errors

```
. reg gdp L(1/4).gdp,r

Linear regression                                      Number of obs =     247
                                                       F(  4,   242) =    8.85
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1584
                                                       Root MSE      =  3.8132

------------------------------------------------------------------------------
             |               Robust
         gdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |
         L1. |    .327656    .076895     4.26   0.000     .1761871     .479125
         L2. |   .1466135   .0858808     1.71   0.089    -.0225558    .3157828
         L3. |  -.0980287   .0728951    -1.34   0.180    -.2416186    .0455611
         L4. |  -.0889209   .0790354    -1.13   0.262     -.244606    .0667641
             |
       _cons |   2.378427   .4731312     5.03   0.000     1.446447    3.310408
------------------------------------------------------------------------------
```

Robust st. errors

- With the “r” option

  .reg y x, r
- You get robust
  - Standard errors
  - t statistics and p-values
  - test statistics

Annoyance

- In STATA, with the "r" option, STATA omits sum of squared error table
  - Yet this can be useful
- So both commands may be useful

.reg y x

.reg y x, r

Interpretation of standard errors

- The standard errors measure precision of the estimate
  - Forecasts use *estimated* coefficients.
- Small standard errors mean the estimate is precise
  - Good for forecasting
- Large standard errors mean the estimate is not precise
  - Bad for forecasting
  - Inaccurate estimates leads to inaccurate forecasts

gdp	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdp						
L1.	.327656	.076895	4.26	0.000	.1761871	.479125
L2.	.1466135	.0858808	1.71	0.089	-.0225558	.3157828
L3.	-.0980287	.0728951	-1.34	0.180	-.2416186	.0455611
L4.	-.0889209	.0790354	-1.13	0.262	-.244606	.0667641
_cons	2.378427	.4731312	5.03	0.000	1.446447	3.310408

Interpretation of t-statistics

- "t" is the coefficient estimate divided by the standard error.
- It is used to *test* if the coefficient is zero
  - "P>|t|" is the p-value of the t-statistic
  - If p<.05 you "reject" the hypothesis of a zero coefficient
- Hypothesis tests are useful for assessing economic theories
  - But are less useful for picking good forecasting models

gdp	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdp						
L1.	.327656	.076895	4.26	0.000	.1761871	.479125
L2.	.1466135	.0858808	1.71	0.089	-.0225558	.3157828
L3.	-.0980287	.0728951	-1.34	0.180	-.2416186	.0455611
L4.	-.0889209	.0790354	-1.13	0.262	-.244606	.0667641
_cons	2.378427	.4731312	5.03	0.000	1.446447	3.310408

Interpretation of Confidence Interval

- The 95% interval is the coefficient estimate plus and minus 1.96 times the standard error
- Helps gauge possible values for the true coefficient
- Useful tool

gdp	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdp						
L1.	.327656	.076895	4.26	0.000	.1761871	.479125
L2.	.1466135	.0858808	1.71	0.089	-.0225558	.3157828
L3.	-.0980287	.0728951	-1.34	0.180	-.2416186	.0455611
L4.	-.0889209	.0790354	-1.13	0.262	-.244606	.0667641
_cons	2.378427	.4731312	5.03	0.000	1.446447	3.310408

Summary

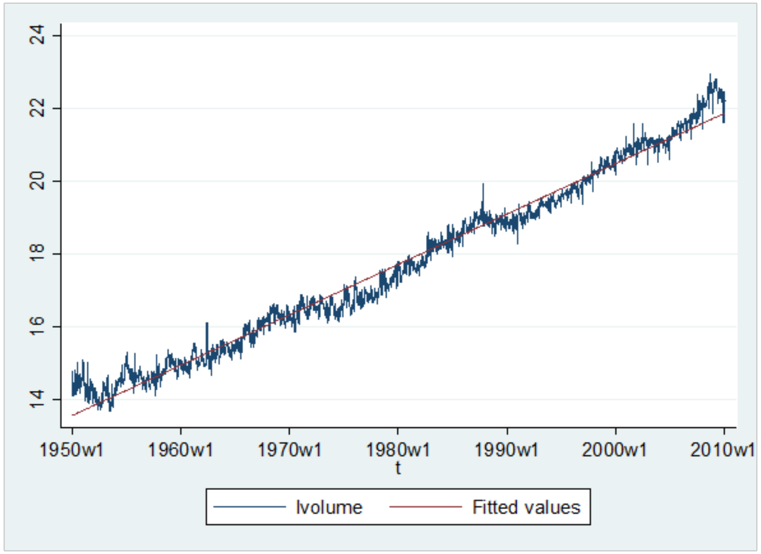
- In one-step-ahead forecast regressions with unforecastable errors
  - Robust standard errors generally appropriate
  - Classical standard errors appropriate under conditional homoskedasticity

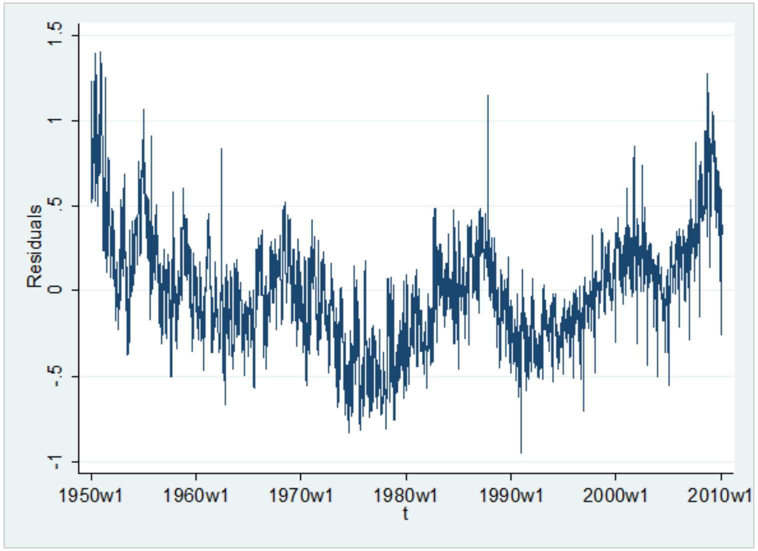
Regression with Correlated Errors

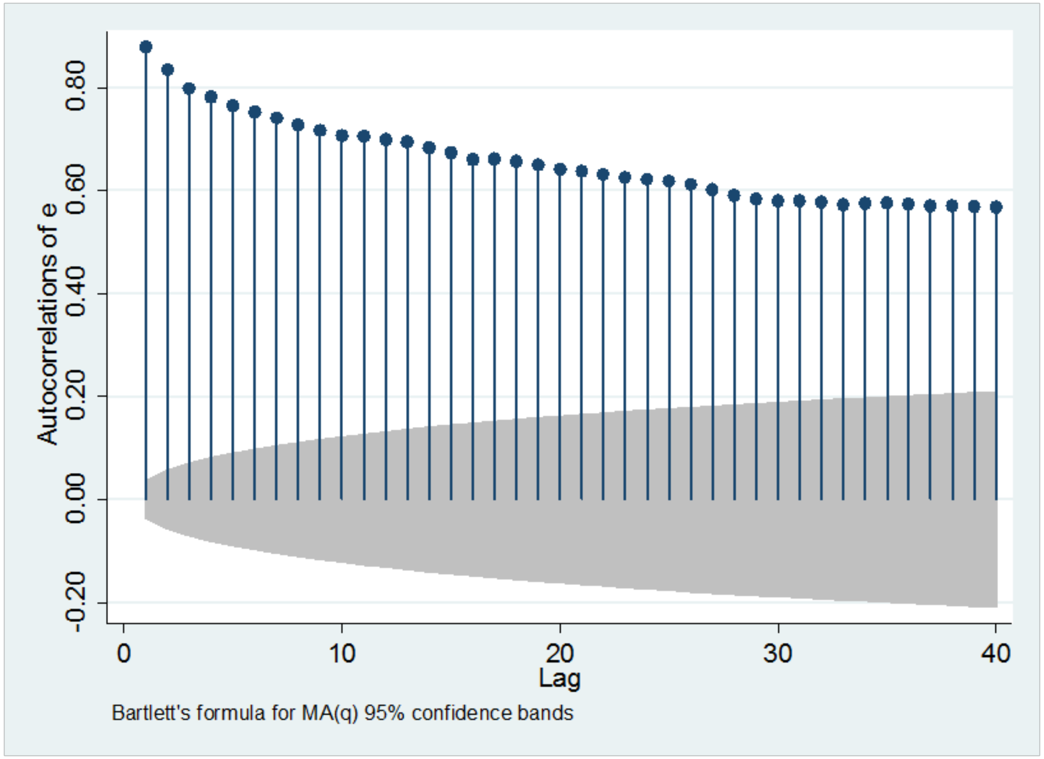
$$y_t = \alpha + \beta x_t + e_t$$

- In some regression models, the errors are correlated
  - Pure Trend Models
  - Pure Seasonality Models
- In these models the errors can be correlated
- Classical and robust standard errors are not appropriate

Example: Stock Volume







Least-Squares Variance Formula

Recall for $v_t = x_t e_t$

$$\operatorname{var}\left(\hat{\beta}\right) \overset{a}{\sim} \frac{\operatorname{var}\left(\sum_{t=1}^{T} v_t\right)}{\left[T \operatorname{var}(x_t)\right]^2}$$

When the v are uncorrelated

$$\operatorname{var}\left(\sum_{t=1}^{T} v_t\right) = \sum_{t=1}^{T} \operatorname{var}(v_t) = T \operatorname{var}(v_t)$$

$$\operatorname{var}\left(\hat{\beta}\right) \overset{a}{\sim} \frac{\operatorname{var}(v_t)}{T\left[\operatorname{var}(x_t)\right]^2}$$

General Formula

Define

$$f_T = \frac{\operatorname{var}\left(\sum_{t=1}^{T} v_t\right)}{T \operatorname{var}(v_t)}$$

When the v are uncorrelated f_T=1, otherwise not.

Then

$$\operatorname{var}\left(\hat{\beta}\right) \overset{a}{\sim} \frac{\operatorname{var}(x_t e_t)}{T\left[\operatorname{var}(x_t)\right]^2} f_T$$

Adjustment Factor

- The asymptotic variance of least-squares is the conventional, multiplied by an adjustment factor for the serial correlation

$$\operatorname{var}\left(\hat{\beta}\right) \overset{a}{\sim} \frac{\operatorname{var}\left(x_t e_t\right)}{T\left[\operatorname{var}\left(x_t\right)\right]^2} f_T$$

Autocovariance of v

- We want a useful formula for

$$f_T = \frac{\operatorname{var}\left(\sum_{t=1}^{T} v_t\right)}{T \operatorname{var}(v_t)}$$

- Since $E(v_t)=0$, then

$$E\left(v_t^2\right) = \operatorname{var}(v_t)$$

$$E\left(v_t v_j\right) = \operatorname{cov}\left(v_t v_j\right) = \gamma(t-j)$$

the autocovariance of v_t

Variance of sum of correlated v

$$\operatorname{var}\left(\sum_{t=1}^{T} v_t\right) = E\left(\sum_{t=1}^{T} v_t\right)^2$$

$$= E\left(\sum_{t=1}^{T} v_t \sum_{j=1}^{T} v_j\right)$$

$$= \sum_{t=1}^{T} \sum_{j=1}^{T} E\left(v_t v_j\right)$$

$$= \sum_{t=1}^{T} \sum_{j=1}^{T} \gamma\left(t-j\right)$$

Adjustment Factor

$$f_T = \frac{\operatorname{var}\left(\sum_{t=1}^{T} v_t\right)}{T \operatorname{var}(v_t)} = \frac{1}{T}\sum_{t=1}^{T}\ \sum_{j=1}^{T}\rho(t-j)$$

- Where the ρ(t-j) are the autocorrelations of v_t

- This double sum is the sum of all the elements in the matrix

$$\begin{bmatrix} \rho(0) & \rho(1) & \rho(2) & \cdots & \rho(T-1) \\ \rho(1) & \rho(0) & \rho(1) & \cdots & \rho(T-2) \\ \rho(2) & \rho(1) & \rho(0) & \cdots & \rho(T-3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho(T-1) & \rho(T-2) & \rho(T-3) & \cdots & \rho(0) \end{bmatrix}$$

- There are
  - *T* of the ρ(0)
  - 2*(T-1)* of the ρ(1)
  - 2*(T-2)* of the ρ(2)
  - …

$$T + \sum_{j=1}^{T-1} 2(T-j)\rho(j)$$

Adjustment Factor

- Dividing by T

$$f_T = \frac{1}{T}\sum_{t=1}^{T}\ \sum_{j=1}^{T}\rho(t-j)$$

$$= 1 + \sum_{j=1}^{T-1} 2\left(\frac{T-j}{T}\right)\rho(j)$$

- If T is large

$$f_T \rightarrow 1 + 2\sum_{j=1}^{\infty}\rho(j) = f$$

Summary: Least-Squares Variance

- When the errors are correlated

$$\operatorname{var}\left(\hat{\beta}\right) \overset{a}{\sim} \frac{\operatorname{var}\left(x_t e_t\right)}{T\left[\operatorname{var}\left(x_t\right)\right]^2} f$$

$$f = 1 + 2\sum_{j=1}^{\infty} \rho\left(j\right)$$

- The conventional formula is multiplied by an adjustment for autocorrelation

HAC Estimation

- Estimation of f
  - For variances and standard errors under autocorrelation
- Called heteroskedasticity and autocorrelation consistent (HAC) variance estimation
- Multiply conventional variance estimates by estimates of f

HAC Estimation

- The adjustment is

$$f = 1 + 2\sum_{j=1}^{\infty} \rho(j)$$

where ρ(j) are the autocorrelations of $v_t = x_t e_t$

- Estimate ρ(j) by sample autocorrelations using least-squares residuals
- But in a sample of length T we cannot estimate all autocorrelations well

Unweighted HAC Estimator

- For some **truncation parameter** m,

$$\hat{f} = 1 + 2\sum_{j=1}^{m} \hat{\rho}(j)$$

- Original proposal
  - L. Hansen, Hodrick (1978)
  - Hal White (1982)
- Difficiencies
  - This estimator is not smooth in the truncation parameter
  - The sample estimate can be negative

Lars Hansen

- Professor Lars Hansen, U Chicago
- Invented Generalized Method of Moments, the leading estimation method for applied econometrics
- Introduced unweighted HAC estimator for multi-step regression models
- On short list for Nobel in economics

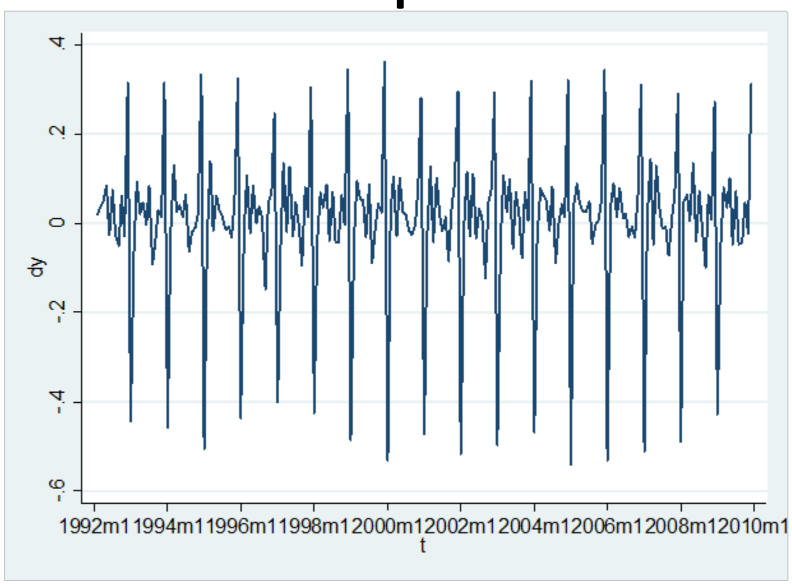
Example of Negative Estimate

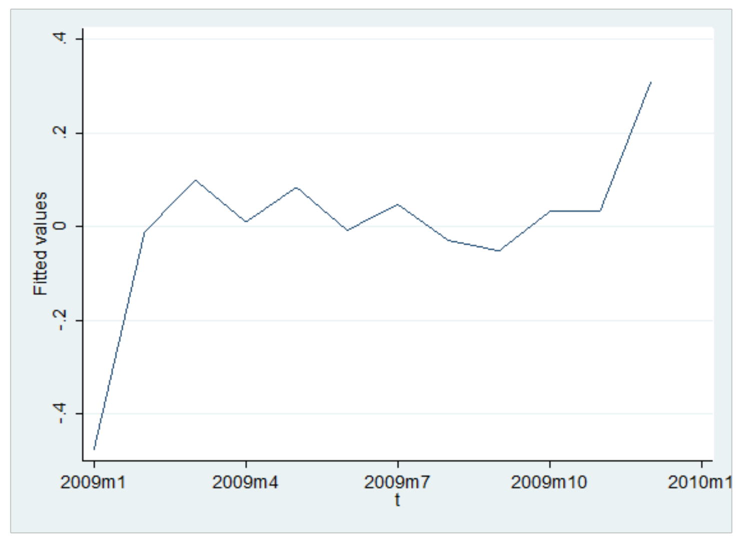
- Take m=1
- Then $\hat{f} = 1 + 2\hat{\rho}(1) < 0$

  if estimated ρ(1)<-1/2

Example: Liquor Sales

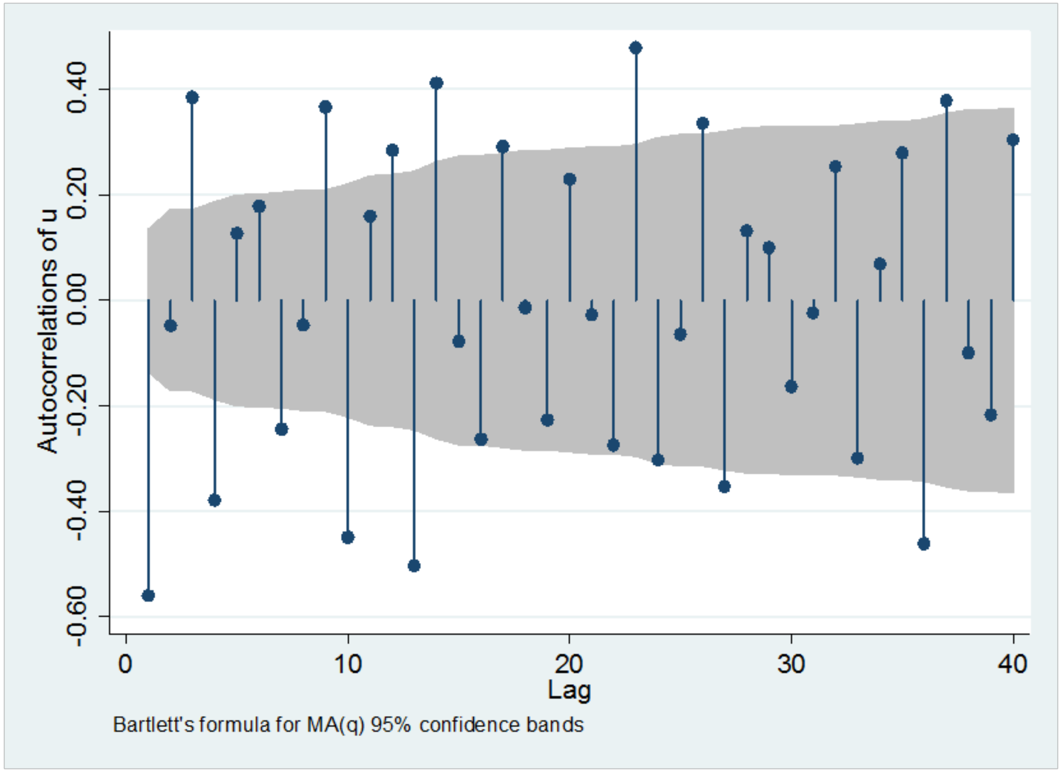
- Transform to growth rates
- Monthly change in log liquor sales
- Regress on Seasonal Dummies only to obtain seasonal pattern





Autocorrelation of Residual

- The first autocorrelation is less than -1/2



Weighted HAC Estimator

$$\hat{f} = 1 + 2\sum_{j=1}^{m}\left(\frac{m-j}{m}\right)\hat{\rho}(j)$$

- Called Newey-West variance estimator
  - Whitney Newey, Ken West (1987)
- This weighted estimator is always positive
- Smoothly changes in truncation parameter *m*

Whitney Newey and Ken West

- Professor Whitney Newey, MIT
  - Leading econometric theorist
- Professor Ken West, Wisconsin
  - Macroeconomist, econometrician
  - Forecast evaluation and comparison
- Joint paper in 1987
  - Weighted HAC estimator
  - One of the most referenced papers in econometrics

Computation

- In STATA, replace **regress** command with **newey** command

  .newey y x, lag(m)

- You supply the truncation parameter “m”
- Similar to regression with robust standard errors
- These are identical

  .newey y x, lag(0)

  .reg y x, r

Example: Liquor Sales

```
. reg dy b12.m,r

Linear regression                               Number of obs =      215
                                                F( 11,   203) =   423.80
                                                Prob > F      =   0.0000
                                                R-squared     =   0.9613
                                                Root MSE      =    .0347
```

dy	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
m						
1	-.788371	.0120765	-65.28	0.000	-.8121825	-.7645595
2	-.3218705	.0105478	-30.52	0.000	-.3426677	-.3010733
3	-.2103181	.0094619	-22.23	0.000	-.2289744	-.1916619
4	-.3002915	.010514	-28.56	0.000	-.3210222	-.2795607
5	-.2258118	.0100036	-22.57	0.000	-.245536	-.2060876
6	-.3185358	.0096047	-33.16	0.000	-.3374735	-.2995981
7	-.2618824	.0100737	-26.00	0.000	-.2817449	-.2420198
8	-.3392591	.0107775	-31.48	0.000	-.3605093	-.3180088
9	-.3624475	.0123023	-29.46	0.000	-.3867042	-.3381907
10	-.2782956	.010299	-27.02	0.000	-.2986023	-.257989
11	-.2761872	.0108553	-25.44	0.000	-.2975908	-.2547835
_cons	.3099733	.0065735	47.16	0.000	.2970122	.3229343

With Newey-West standard errors

```
. newey dy b12.m, lag(12)
```

Regression with Newey-West standard errors
maximum lag: 12

Number of obs = 215
F(11, 203) = 908.34
Prob > F = 0.0000

dy	Coef.	Newey-West Std. Err.	t	P>|t|	[95% Conf.	Interval]
m						
1	-.788371	.0149943	-52.58	0.000	-.8179356	-.7588064
2	-.3218705	.0093479	-34.43	0.000	-.3403018	-.3034391
3	-.2103181	.0100234	-20.98	0.000	-.2300816	-.1905547
4	-.3002915	.0087418	-34.35	0.000	-.3175278	-.2830551
5	-.2258118	.0128307	-17.60	0.000	-.2511104	-.2005132
6	-.3185358	.0087245	-36.51	0.000	-.335738	-.3013336
7	-.2618824	.0090442	-28.96	0.000	-.279715	-.2440498
8	-.3392591	.0134996	-25.13	0.000	-.3658765	-.3126416
9	-.3624475	.0075171	-48.22	0.000	-.377269	-.3476259
10	-.2782956	.0116472	-23.89	0.000	-.3012606	-.2553307
11	-.2761872	.0126533	-21.83	0.000	-.3011359	-.2512384
_cons	.3099733	.0066381	46.70	0.000	.2968848	.3230618

Truncation Parameter

- *m* should be large when autocorrelation is large
- Sophistical data-dependent methods to pick m have been developed, but are not in STATA
- Stock-Watson default (explanatory x's)

$$m = 0.75T^{1/3}$$

- Trend/Seasonal default

$$m = 1.4T^{1/3}$$

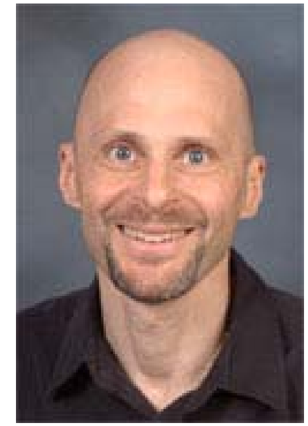
Derivation of Defaults

- Due to Andrews (1991)
- The optimal m minimizes the mean-squared error of the estimate of f
- When v_t is an AR(1) with coefficient ρ, Andrews found the optimal m is

$$m = CT^{1/3}$$

$$C = \left(\frac{6\rho^2}{\left(1-\rho^2\right)^2} \right)^{1/3}$$

Donald Andrews



- Professor Donald Andrews, Yale
- Leading econometric theorist
- Contributions to time-series
  - Optimal selection of truncation parameter
  - Tests for structural change

Default Values

$$m = CT^{1/3}$$

$$C = \left(\frac{6\rho^2}{\left(1-\rho^2\right)^2} \right)^{1/3}$$

- Stock-Watson
  - If both x_t and e_t are AR(1) with coef ½, then $v_t = x_t e_t$ has AR(1) coefficient ρ=.25. Plug this in, and C=.75
- Trend-Seasonal
  - If x_t is trend and/or seasonal and e_t are AR(1) with coef ½, then $v_t = x_t e_t$ has AR(1) coefficient ρ=.5. Plug this in, and C=1.4

Liquor Sales again

```
. dis 1.4*e(N)^(1/3)
8.387017

. newey dy b12.m, lag(8)

Regression with Newey-West standard errors          Number of obs  =       215
maximum lag: 8                                      F( 11,   203)  =    736.19
                                                    Prob > F       =    0.0000

------------------------------------------------------------------------------
             |             Newey-West
          dy |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           m |
           1 |   -.788371   .0146673   -53.75   0.000    -.8172907   -.7594513
           2 |  -.3218705   .0089781   -35.85   0.000    -.3395727   -.3041682
           3 |  -.2103181   .0097191   -21.64   0.000    -.2294815   -.1911548
           4 |  -.3002915   .0097151   -30.91   0.000     -.319447    -.281136
           5 |  -.2258118   .0116748   -19.34   0.000    -.2488312   -.2027924
           6 |  -.3185358   .0089588   -35.56   0.000    -.3362001   -.3008715
           7 |  -.2618824     .00916   -28.59   0.000    -.2799433   -.2438214
           8 |  -.3392591   .0126319   -26.86   0.000    -.3641655   -.3143526
           9 |  -.3624475   .0091312   -39.69   0.000    -.3804516   -.3444434
          10 |  -.2782956   .0106888   -26.04   0.000    -.2993709   -.2572204
          11 |  -.2761872   .0126343   -21.86   0.000    -.3010984   -.2512759
             |
       _cons |   .3099733   .0065735    47.16   0.000     .2970122    .3229343
------------------------------------------------------------------------------
```

Example: Men's Labor Force Participation Rate, Trend Model

```
. reg m t

      Source |       SS       df       MS              Number of obs =     744
-------------+------------------------------           F(  1,   742) = 9554.38
       Model |  14659.2499     1  14659.2499           Prob > F      =  0.0000
    Residual |   1138.4477   742  1.53429609           R-squared     =  0.9279
-------------+------------------------------           Adj R-squared =  0.9278
       Total |  15797.6976   743  21.2620426           Root MSE      =  1.2387

------------------------------------------------------------------------------
         men |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |  -.0206675   .0002114   -97.75   0.000    -.0210826   -.0202524
       _cons |   85.18169   .0661519  1287.67   0.000     85.05182    85.31156
------------------------------------------------------------------------------

. dis 1.4*e(N)^(1/3)
12.685834

. newey men t, lag(13)

Regression with Newey-West standard errors          Number of obs  =       744
maximum lag: 13                                     F(  1,   742)  =    692.69
                                                    Prob > F       =    0.0000

------------------------------------------------------------------------------
             |             Newey-West
         men |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |  -.0206675   .0007853   -26.32   0.000    -.0222091   -.0191259
       _cons |   85.18169   .2168636   392.79   0.000     84.75595    85.60743
```

Summary

- In one-step-ahead forecast regressions
- If the errors are serially uncorrelated
  - Use Robust standard errors
    - reg with r option
- If the errors are correlated
  - Use Newey-West standard errors
    - newey y x, lag(m)
  - In pure trend or seasonality models
    - Set $m=1.4T^{1/3}$
  - In dynamic regression
    - Set $m=.75T^{1/3}$

h-step-ahead forecasts

- In the AR(1) Model

$$y_t = \alpha + \beta y_{t-1} + e_t$$

- The optimal h-step forecasting regression takes the form

$$y_t = \alpha + \beta^h y_{t-h} + u_t$$

$$u_t = e_t + \beta e_{t-1} + \beta^2 e_{t-2} + \cdots + \beta^{h-1} e_{t-h+1}$$

- The error u_t is a correlated MA(h-1)
  - Unless β=0

h-step-ahead models

- In any h-step model

$$y_t = \alpha + \beta y_{t-h} + u_t$$

  the variable $v_t = y_{t-h} e_t$ is generally serially correlated
- Generally MA(h-1)
- Correct adjustment term

$$f = 1 + 2\sum_{j=1}^{h-1} \rho(j)$$

Newey-West Standard Errors

- Standard errors can be estimated using the Newey-West method
- Truncation parameter set to forecast horizon
  - m=h

$$\hat{f} = 1 + 2\sum_{j=1}^{h-1}\left(\frac{h-j}{h}\right)\hat{\rho}(j)$$

Example: Unemployment Rate

- 12-month-ahead forecast with 4 AR lags
  - Robust standard errors:

```
. reg ur L(12/15).ur,r

Linear regression                               Number of obs =      730
                                                F(  4,   725) =   139.36
                                                Prob > F      =   0.0000
                                                R-squared     =   0.4955
                                                Root MSE      =   1.1088
```

ur	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
ur						
L12.	1.686434	.2920485	5.77	0.000	1.113072	2.259795
L13.	-.0698989	.3908098	-0.18	0.858	-.837153	.6973552
L14.	-.5401552	.3461042	-1.56	0.119	-1.219641	.1393309
L15.	-.4100512	.2538791	-1.62	0.107	-.9084772	.0883747
_cons	1.94875	.1705347	11.43	0.000	1.613949	2.28355

Example: Unemployment Rate

- Newey-West standard errors:
- Standard errors on lag 13 and 14 decrease by half
- Standard error on constant more than doubles

```
. newey ur L(12/15).ur, lag(12)

Regression with Newey-West standard errors          Number of obs  =       730
maximum lag: 12                                     F(  4,    725) =     21.00
                                                    Prob > F       =    0.0000
```

ur	Coef.	Newey-West Std. Err.	t	P>|t|	[95% Conf.	Interval]
ur						
L12.	1.686434	.273372	6.17	0.000	1.149738	2.223129
L13.	-.0698989	.1564772	-0.45	0.655	-.3771014	.2373036
L14.	-.5401552	.1378278	-3.92	0.000	-.8107445	-.2695658
L15.	-.4100512	.246517	-1.66	0.097	-.8940236	.0739212
_cons	1.94875	.4550687	4.28	0.000	1.05534	2.842159

newey and forecasting

- **predict** works after **newey** command
- **e(rmse)** does not work, only after **regress** or **reg**
  - rmse not computed or reported
- Use **newey** to assess model and examine coefficients
- Use **reg** to compute out-of-sample forecast intervals

Summary

- In one-step-ahead forecast regressions
  - If the errors are serially uncorrelated, use r option
  - If the errors are correlated
    - Use **newey** for standard errors
      - In pure trend or seasonality models set $m=1.4T^{1/3}$
      - In dynamic regression set $m=.75T^{1/3}n$
    - Use **reg** and **e(rmse)** for forecast intervals
- In h-step-ahead forecast regressions
  - Use **newey** with m=h for standard errors
  - Use **reg** and **e(rmse)** for forecast intervals