Seasonal + Cycle

- Consider a components model with seasonal and AR(1) cycle

$$y_t = S_t + C_t$$

$$C_t = \beta C_{t-1} + e_t$$

- The seasonal S_t is a set of seasonal dummies

$$S_t = \sum_{i=1}^{s} \alpha_i D_{it}$$

Transformation

$$y_t = S_t + C_t$$

$$C_t = \beta C_{t-1} + e_t$$

- Take the first equation and lag it once

$$y_{t-1} = S_{t-1} + C_{t-1}$$

- Multiply it by β

$$\beta y_{t-1} = \beta S_{t-1} + \beta C_{t-1}$$

- Then subtract it from the first equation to find

$$y_t = \beta y_{t-1} + S_t - \beta S_{t-1} + e_t$$

Seasonal Representation

- We find

$$y_t = \beta y_{t-1} + S_t - \beta S_{t-1} + e_t$$

- When the seasonal S_t is a set of seasonal dummies, one for each season, this equation suggests a regression on
  - y_{t-1}
  - Seasonal dummies
  - Lagged Seasonal dummies

Redundant

- But lagged seasonal dummies are redundant with the original seasonal dummies
- The set of lagged dummy variables are collinear with the current dummy variables
- Given that you know this month is February, there is no information in knowing that last month was January.
- The lagged dummies can be (should be) omitted

Seasonal + Cycle

- We have found that the regression model is

$$y_t = \sum_{i=1}^{s} \alpha_i D_{it} + \beta y_{t-1} + e_t$$

or

$$y_t = \alpha_0 + \sum_{i=1}^{s-1} \alpha_i D_{it} + \beta y_{t-1} + e_t$$

AR(p) Case

- If the cycle is an AR(p) we have

$$y_t = \sum_{i=1}^{s} \alpha_i D_{it} + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + e_t$$

- Estimate by least squares
- Linear Forecasting

Trend+Seasonal+Cycle

- A full model is

$$y_t = T_t + S_t + C_t$$

$$T_t = \mu_1 + \mu_2 t$$

$$S_t = \sum_{i=1}^{s} \alpha_i D_{it}$$

$$C_t = \beta_1 C_{t-1} + \cdots + \beta_p C_{t-p} + e_t$$

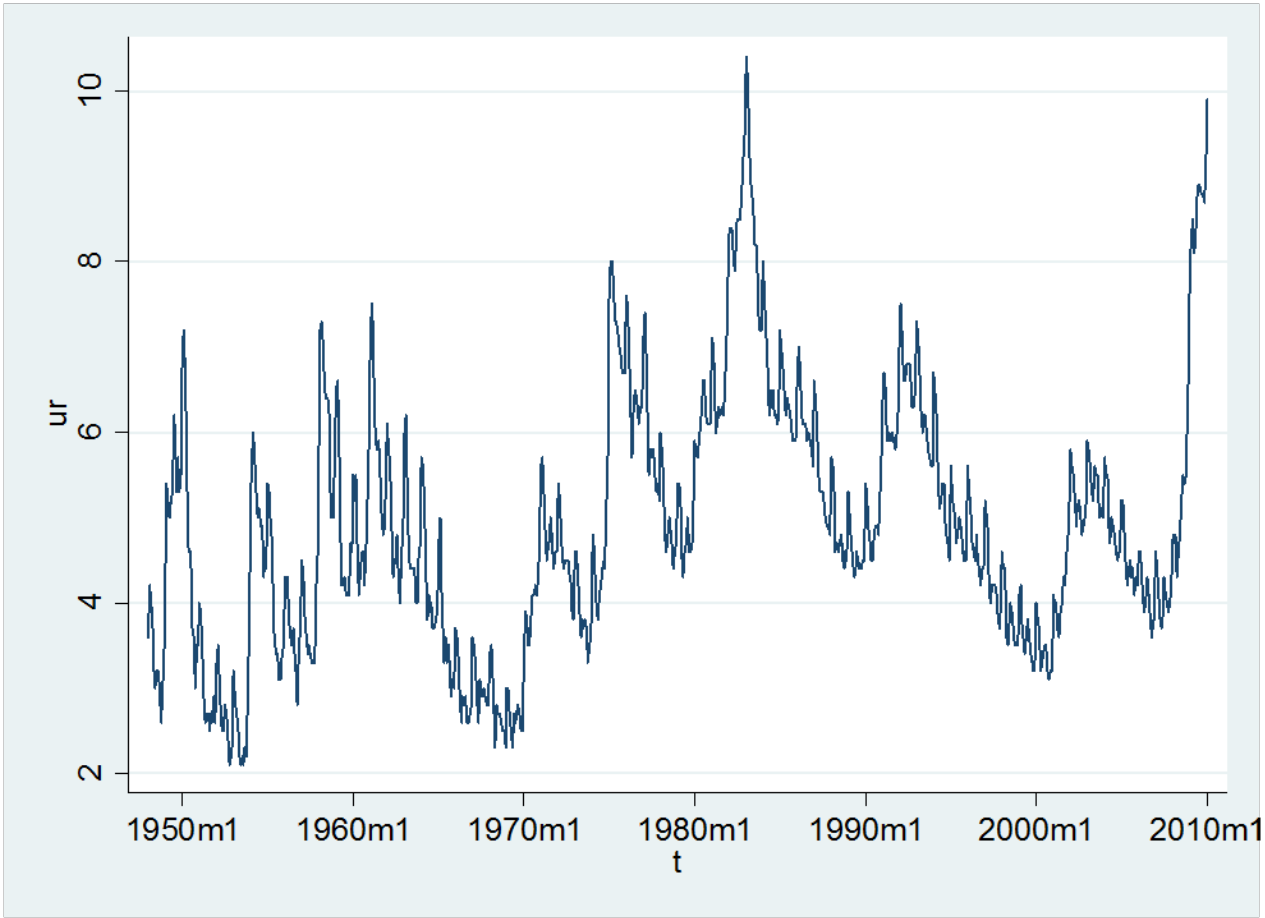
Regression Model

- The implied regression model is

$$y_t = \sum_{i=1}^{s} \alpha_i D_{it} + \gamma t + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + e_t$$

- This can be estimated by least-squares
- It is a complete forecasting model

Example: NSA Unemployment Rate



Regress on Dummies plus AR(12)

```
. reg ur b12.m L(1/12).ur
```

Source	SS	df	MS
Model	1609.97357	23	69.9988508
Residual	33.8239198	709	.047706516
Total	1643.79749	732	2.24562498

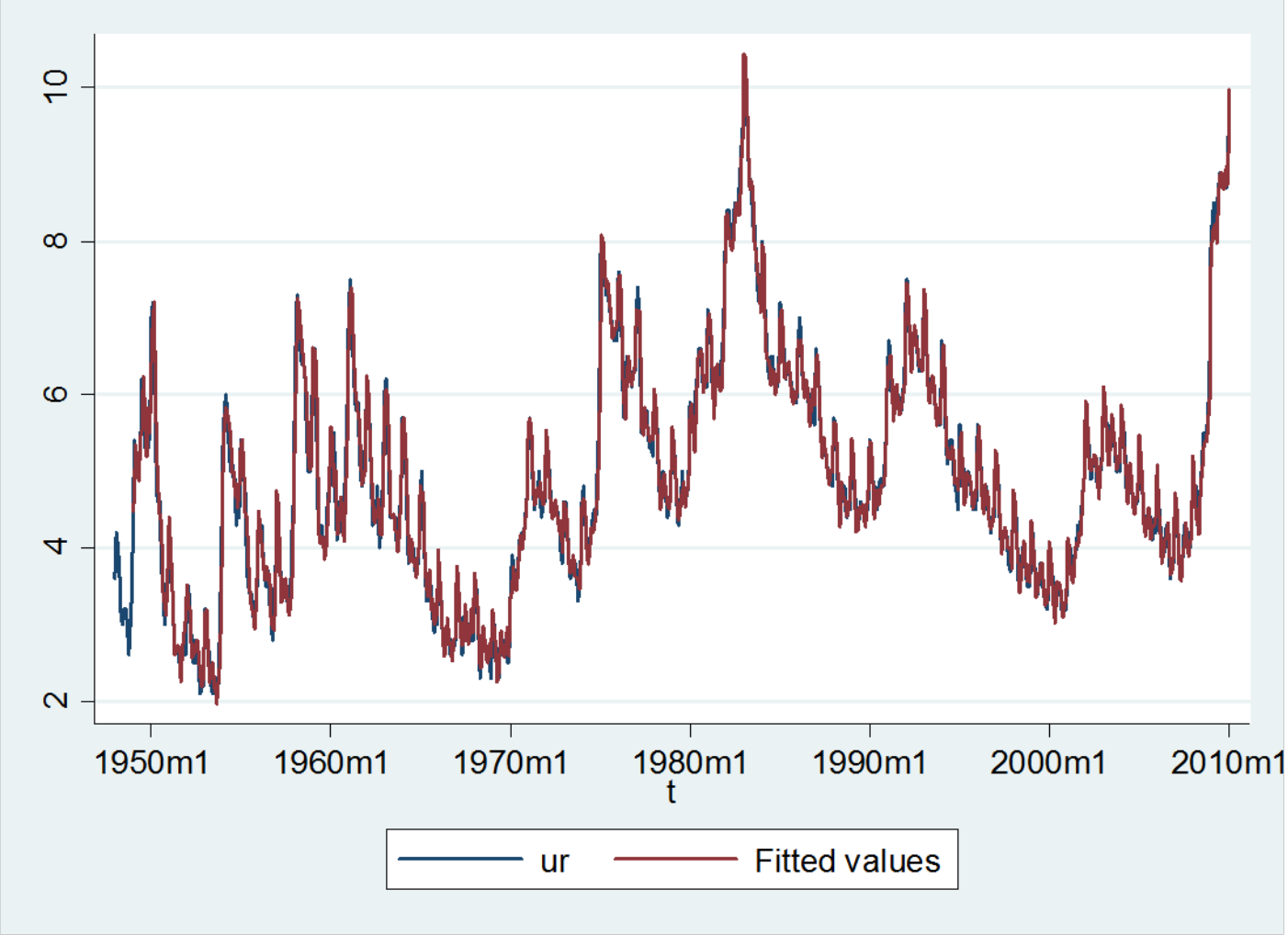
Number of obs	= 733
F(23, 709)	= 1467.28
Prob > F	= 0.0000
R-squared	= 0.9794
Adj R-squared	= 0.9788
Root MSE	= .21842

ur	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
m						
1	.8614153	.0577449	14.92	0.000	.7480438	.9747868
2	-.1534497	.0777527	-1.97	0.049	-.3061028	-.0007967
3	-.468174	.087469	-5.35	0.000	-.6399033	-.2964448
4	-.5474595	.0821548	-6.66	0.000	-.7087554	-.3861636
5	-.1594984	.0684777	-2.33	0.020	-.2939418	-.025055
6	.2175848	.0633983	3.43	0.001	.0931139	.3420557
7	.1527194	.0691765	2.21	0.028	.0169041	.2885346
8	-.0525233	.0835277	-0.63	0.530	-.2165145	.1114679
9	-.2668832	.089909	-2.97	0.003	-.4434029	-.0903635
10	-.1902467	.0812989	-2.34	0.020	-.3498622	-.0306312
11	.1247499	.0685836	1.82	0.069	-.0099012	.2594011

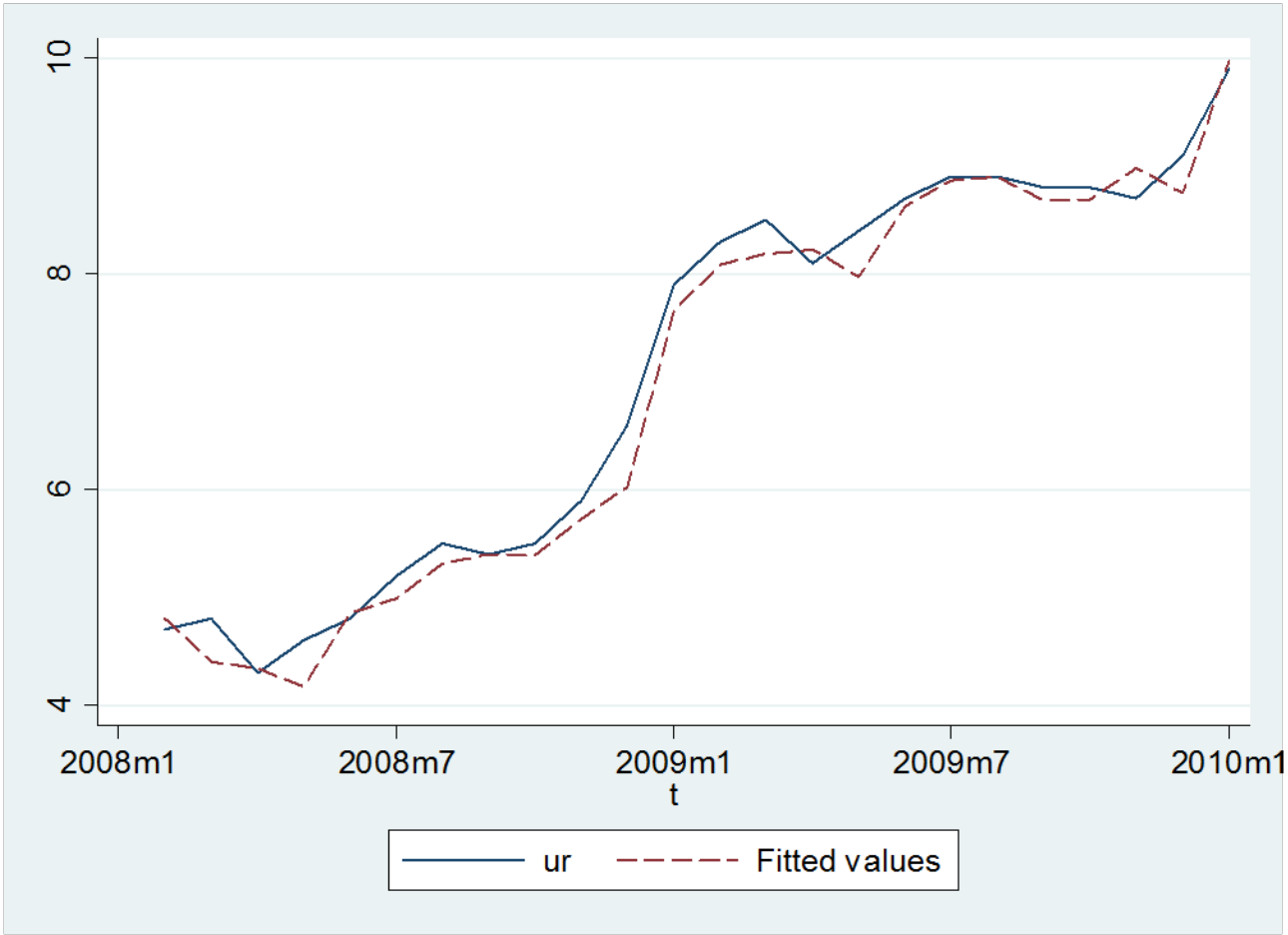
AR Coefficients

ur	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
L1.	1.119934	.0373309	30.00	0.000	1.046642	1.193227
L2.	.0911447	.0562169	1.62	0.105	-.0192268	.2015162
L3.	-.1434849	.0563492	-2.55	0.011	-.2541162	-.0328536
L4.	-.1024096	.0566111	-1.81	0.071	-.213555	.0087359
L5.	.0776225	.0566159	1.37	0.171	-.0335323	.1887774
L6.	-.1096277	.0566984	-1.93	0.054	-.2209446	.0016892
L7.	.0013858	.0565946	0.02	0.980	-.1097274	.1124989
L8.	.0828428	.0565074	1.47	0.143	-.0280991	.1937846
L9.	-.0202508	.0564409	-0.36	0.720	-.1310621	.0905605
L10.	-.0057293	.0563117	-0.10	0.919	-.116287	.1048284
L11.	.0887878	.0562091	1.58	0.115	-.0215683	.199144
L12.	-.1025115	.0373434	-2.75	0.006	-.1758284	-.0291946
_cons	.1530973	.059812	2.56	0.011	.0356676	.2705271

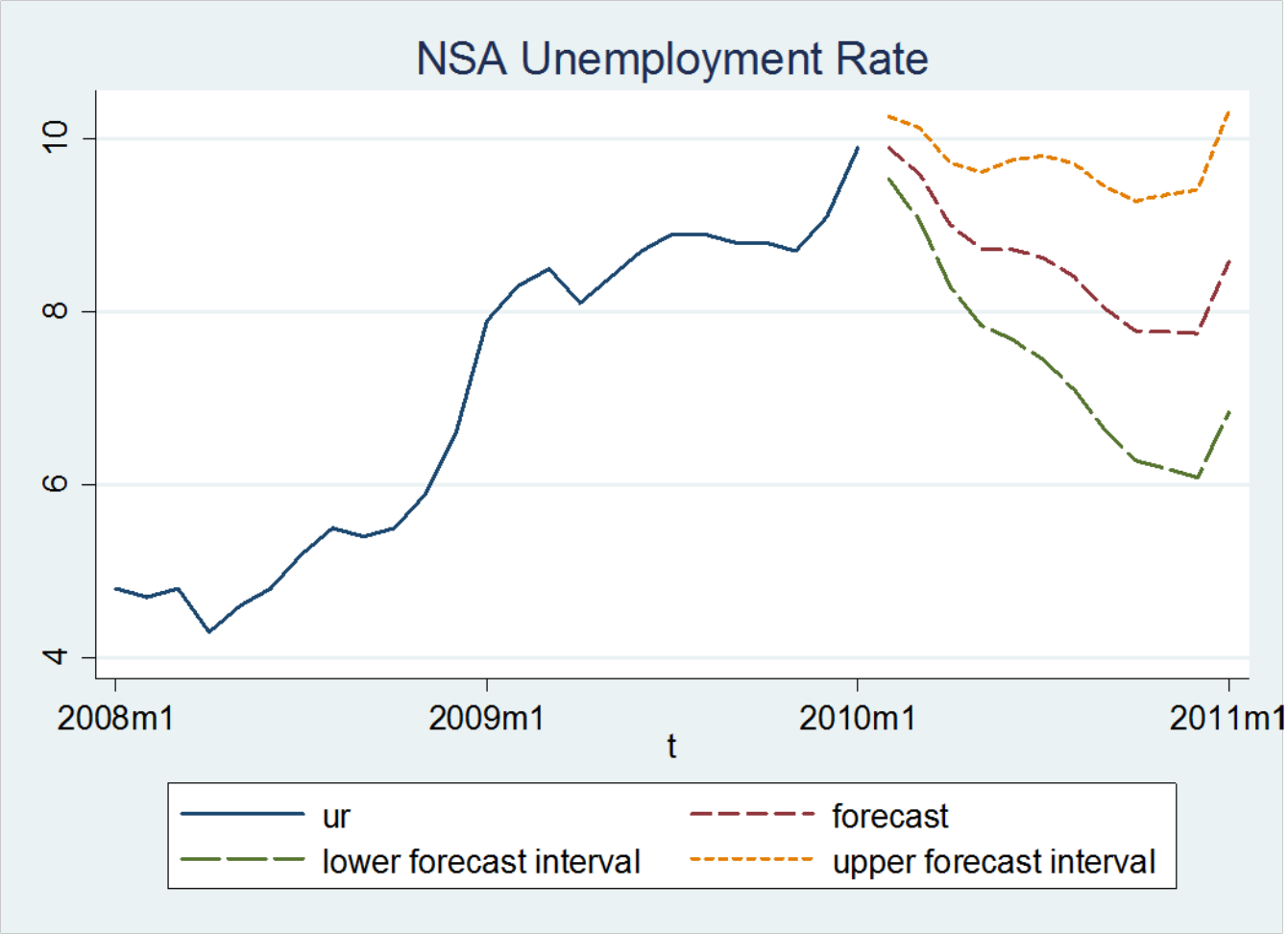
Fitted Values



Last 2 years


10
8
6
4
2008m1
2008m7
2009m1
2009m7
2010m1
t
ur
Fitted values

12-month forecsat



Forecasting with Seasonal Dummy

- To forecast in STATA with seasonal dummies, the dummy variables must be defined for the forecast period
- **After** you use the **tsappend** command, you create the month variable

  .gen m=month(dofm(t))

  or

  .replace m=month(dofm(t))
- Otherwise m will have missing values for the forecast period

Example: Retail Sales

- U.S. Census Bureau
  - Monthly Retail Sales
  - Not Seasonally Adjusted and Seasonally Adjusted
  - Sales listed by variety of categories
  - 1992-2009

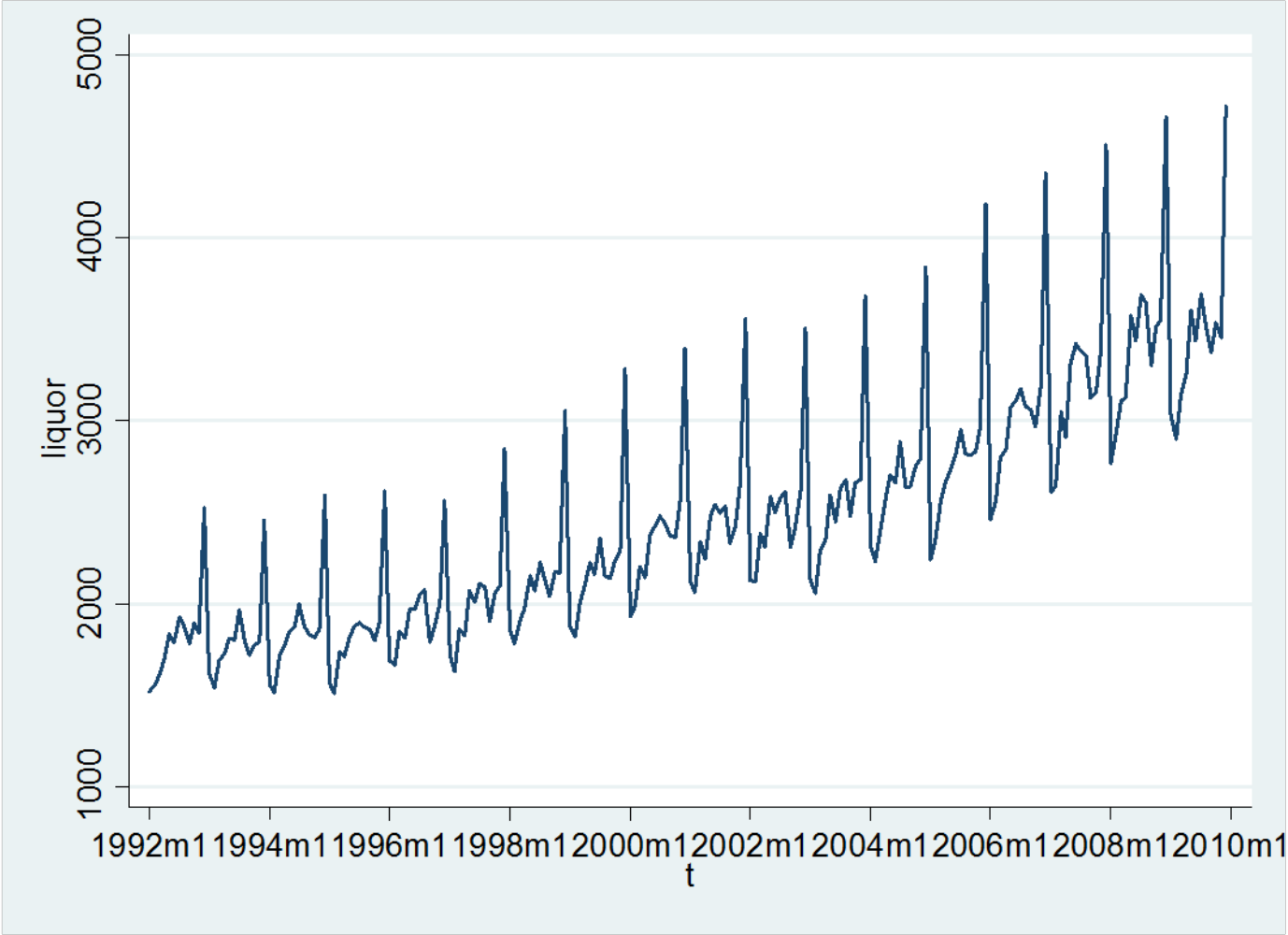
From Census Bureau Spreadsheet

	Jan. 2009	Feb. 2009	Mar. 2009	Apr. 2009	May 2009	Jun. 2009
NOT ADJUSTED						
Retail and food services sales, total	313,593	304,056	334,149	336,155	354,668	351,418
Retail sales and food services excl motor vehicle and parts	262,237	252,560	275,000	277,938	294,559	289,242
Retail sales, total	277,402	269,015	295,520	298,119	313,979	312,547
Retail sales, total (excl. motor vehicle and parts dealers)	226,046	217,519	236,371	239,902	253,870	250,371
GAFO(1)	83,323	82,918	87,477	87,248	93,069	88,330
Motor vehicle and parts dealers	51,356	51,496	59,149	58,217	60,109	62,176
Automobile and other motor vehicle dealers	45,385	45,544	52,498	51,558	53,528	55,239
Automobile dealers	41,790	41,632	46,834	45,373	46,601	48,405
New car dealers	36,032	35,304	40,575	39,492	40,941	42,530
Used car dealers	5,758	6,328	6,259	5,881	5,660	5,875
Automotive parts, acc., and tire stores	5,971	5,952	6,651	6,659	6,581	6,937
Furniture, home furn, electronics, and appliance stores	16,069	15,700	15,530	14,545	15,362	15,519
Furniture and home furnishings stores	7,428	7,219	7,601	7,293	7,689	7,728
Furniture stores	4,230	4,296	4,262	3,969	4,261	4,148
Home furnishings stores	3,198	2,923	3,339	3,324	3,428	3,580
Floor covering stores	1,381	1,335	1,448	1,475	1,462	1,673
All other home furnishings stores	1,698	1,480	1,762	1,725	1,851	1,790
Electronics and appliance stores	8,641	8,481	7,929	7,252	7,673	7,791
Appl.,TV, and other elect. stores	6,706	6,615	5,936	5,374	5,827	5,857
Household appliance stores	1,306	1,201	1,249	1,252	1,317	1,349
Radio, T.V., and other elect. stores	5,400	5,414	4,687	4,122	4,510	4,508
Computer and software stores	1,734	1,722	1,790	1,679	1,612	1,706

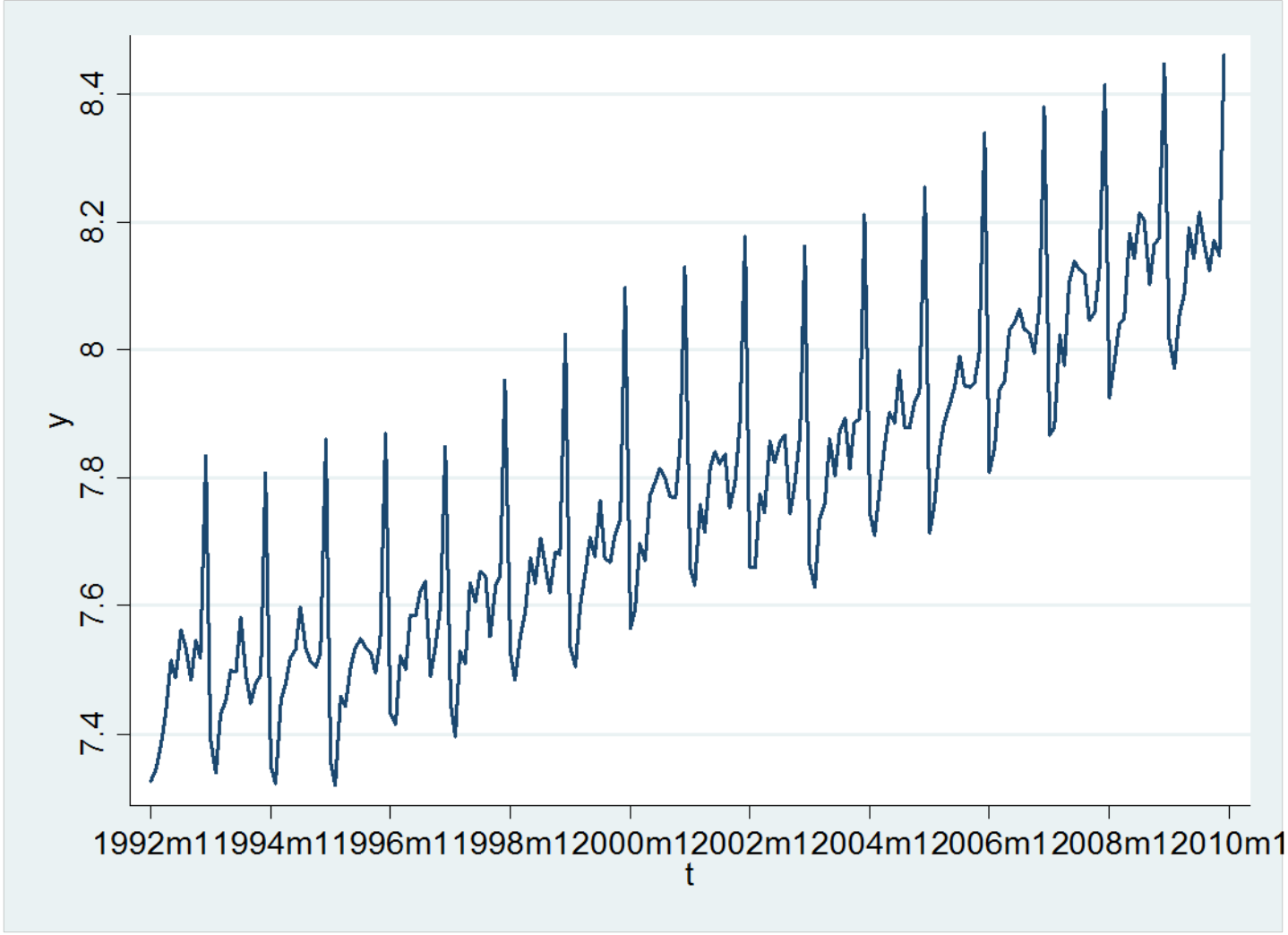
Liquor Sales

- Beer, wine and liquor
- Analyzed in Diebold for 1968-1993
- Our data is 1992-2009
- Not Seasonally Adjusted
- Big spike in December

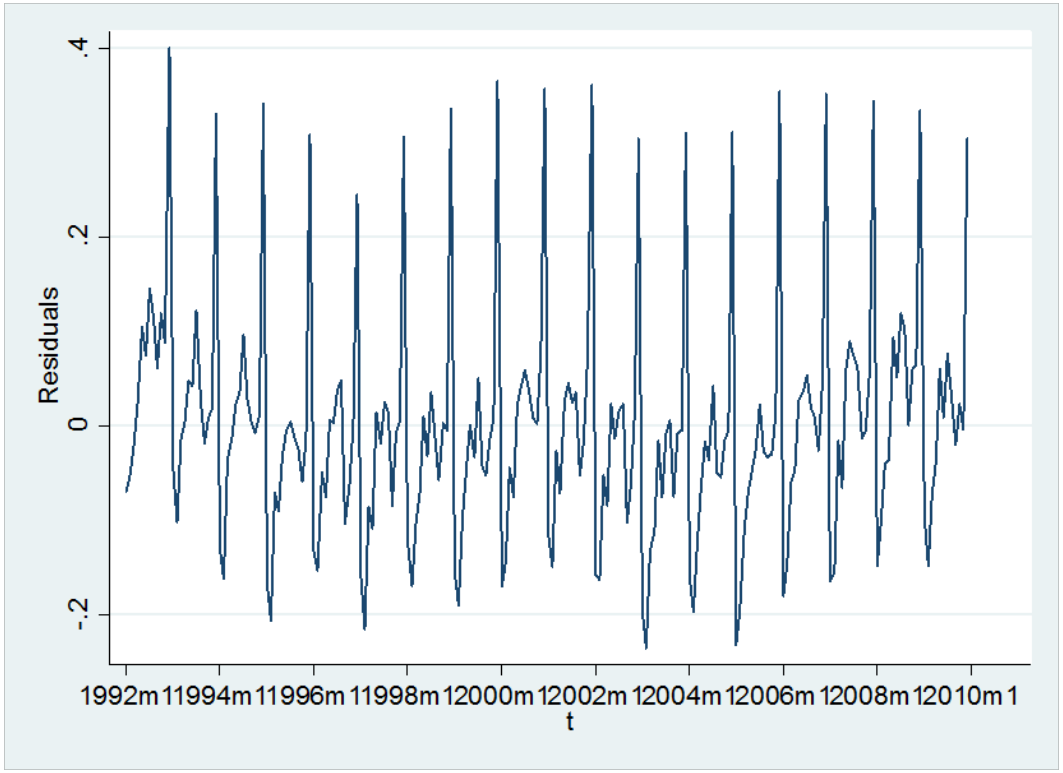
Liquor Sales (Millions of $)



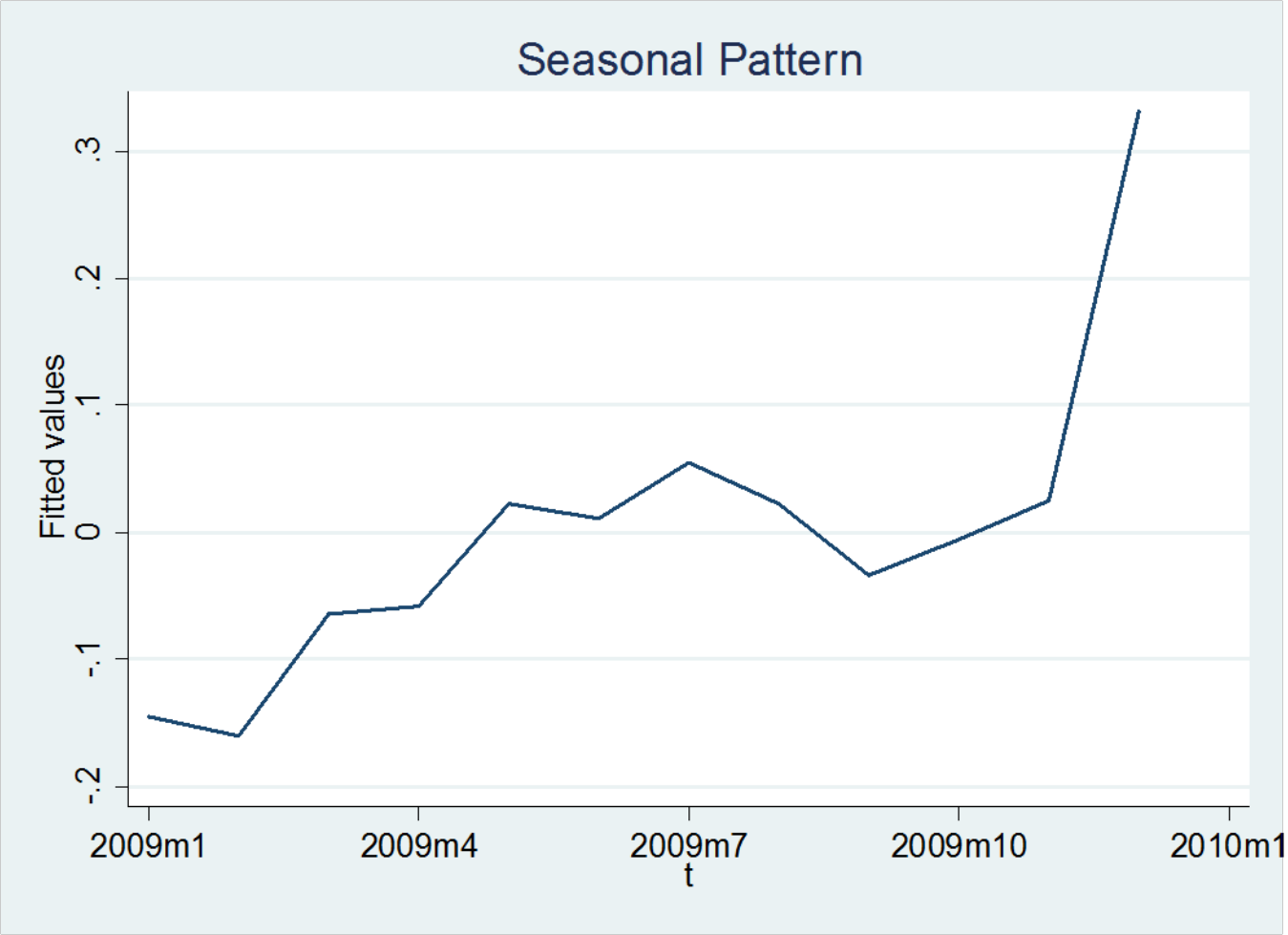
ln(Liquor Sales)



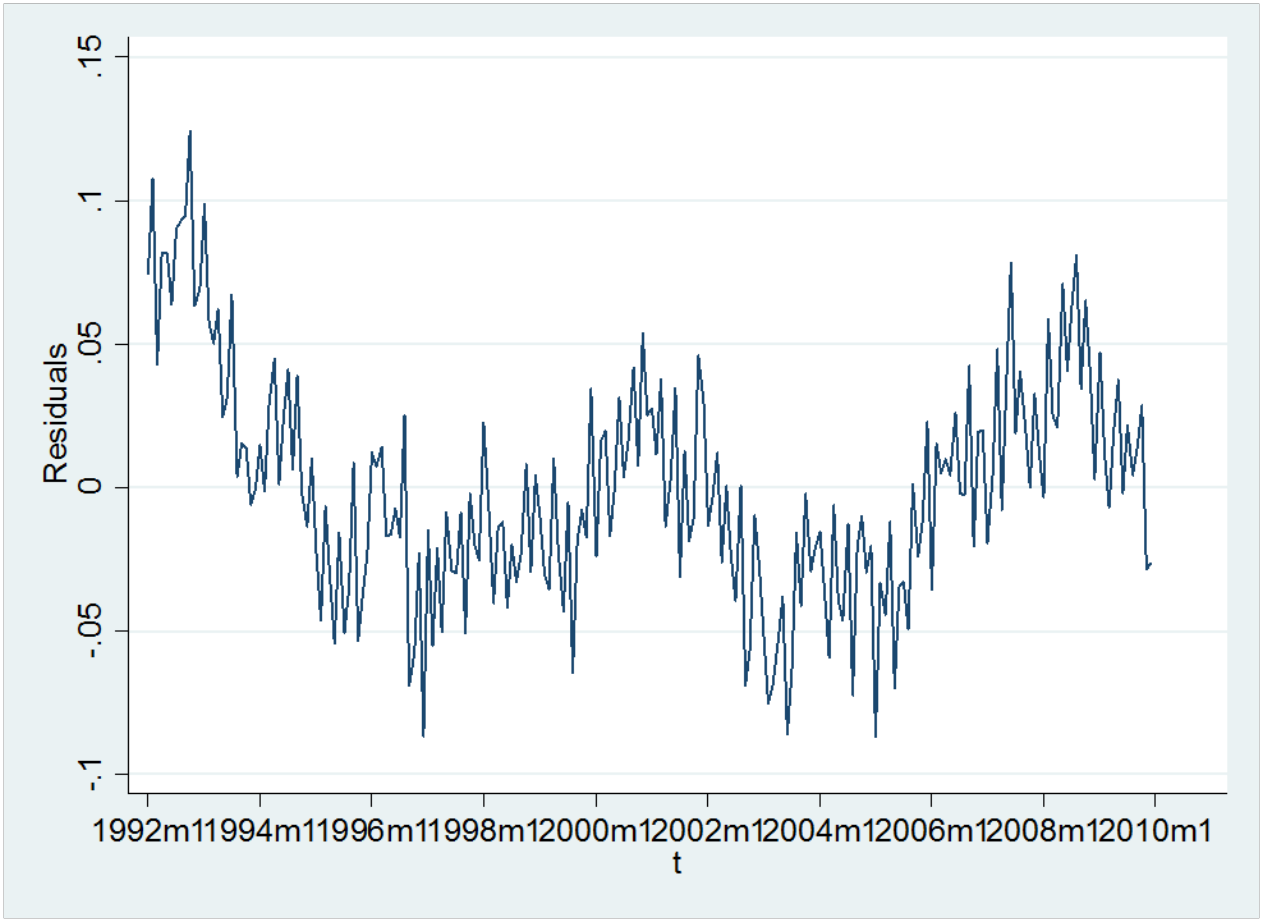
Residual from Linear Trend



Seasonal Dummy



Residuals after Seasonal Dummies



Full Estimation

```
. reg y t b12.m L(1/12).y
```

Source	SS	df	MS
Model	12.6032786	24	.52513661
Residual	.083727442	179	.000467751
Total	12.6870061	203	.062497567

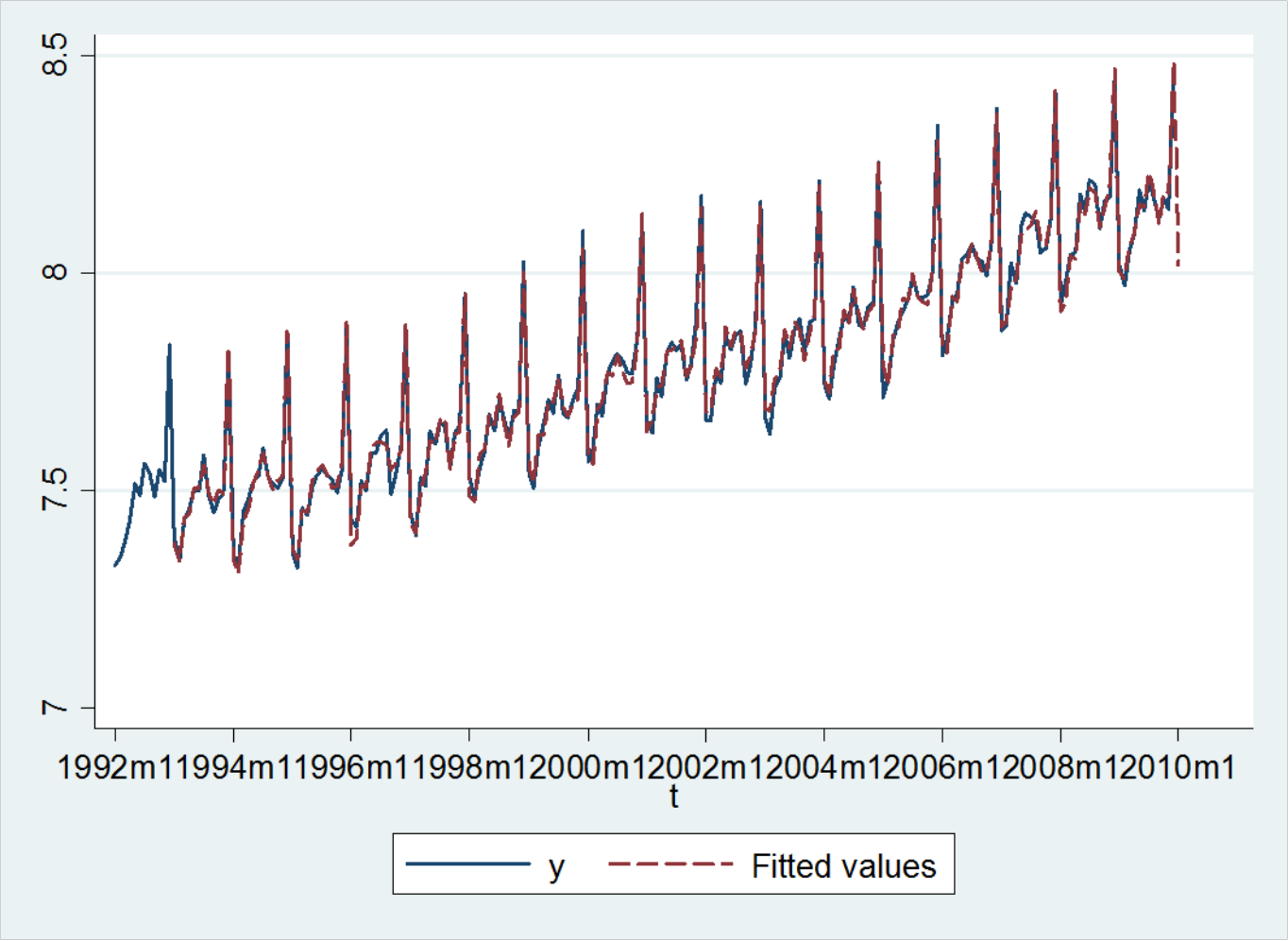
Number of obs	204
F(24, 179)	1122.68
Prob > F	0.0000
R-squared	0.9934
Adj R-squared	0.9925
Root MSE	.02163

y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
t	.0005763	.0001903	3.03	0.003	.0002007	.0009519
m						
1	-.4414455	.0475001	-9.29	0.000	-.5351778	-.3477132
2	-.4495358	.0524949	-8.56	0.000	-.5531243	-.3459472
3	-.32453	.0590456	-5.50	0.000	-.441045	-.2080149
4	-.1865844	.0477627	-3.91	0.000	-.2808349	-.0923339
5	-.2373487	.0430689	-5.51	0.000	-.3223367	-.1523606
6	-.1554224	.0503231	-3.09	0.002	-.2547253	-.0561194
7	-.1289456	.050234	-2.57	0.011	-.2280725	-.0298187
8	-.2270499	.0500036	-4.54	0.000	-.3257222	-.1283775
9	-.4130694	.0607582	-6.80	0.000	-.5329639	-.293175
10	-.2073198	.0597305	-3.47	0.001	-.3251863	-.0894532
11	-.2796951	.0484164	-5.78	0.000	-.3752355	-.1841547

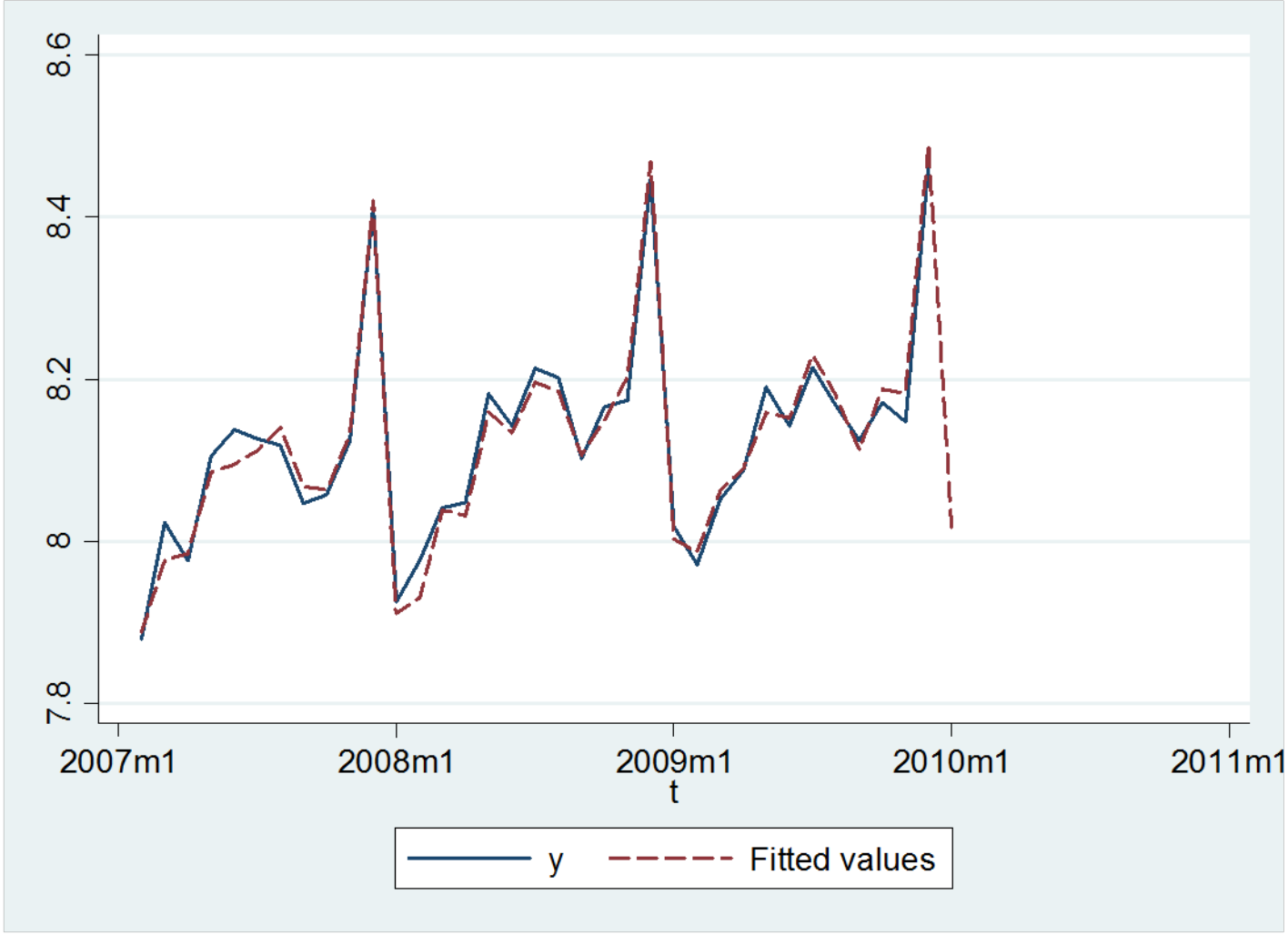
AR Coefficients

y						
L1.	.225336	.0741289	3.04	0.003	.079057	.3716149
L2.	.2636816	.075335	3.50	0.001	.1150227	.4123405
L3.	.3490587	.0749059	4.66	0.000	.2012465	.496871
L4.	-.0818078	.0778903	-1.05	0.295	-.2355091	.0718935
L5.	.2386739	.0769151	3.10	0.002	.0868969	.3904509
L6.	-.0707958	.0780874	-0.91	0.366	-.2248862	.0832946
L7.	-.1447524	.0781988	-1.85	0.066	-.2990625	.0095577
L8.	-.1762223	.0774924	-2.27	0.024	-.3291385	-.0233061
L9.	.2302193	.078365	2.94	0.004	.0755813	.3848574
L10.	-.2566324	.0753442	-3.41	0.001	-.4053095	-.1079553
L11.	.101947	.0733289	1.39	0.166	-.0427534	.2466473
L12.	.1686073	.0720459	2.34	0.020	.0264388	.3107759
_cons	1.164893	.3237073	3.60	0.000	.5261194	1.803666

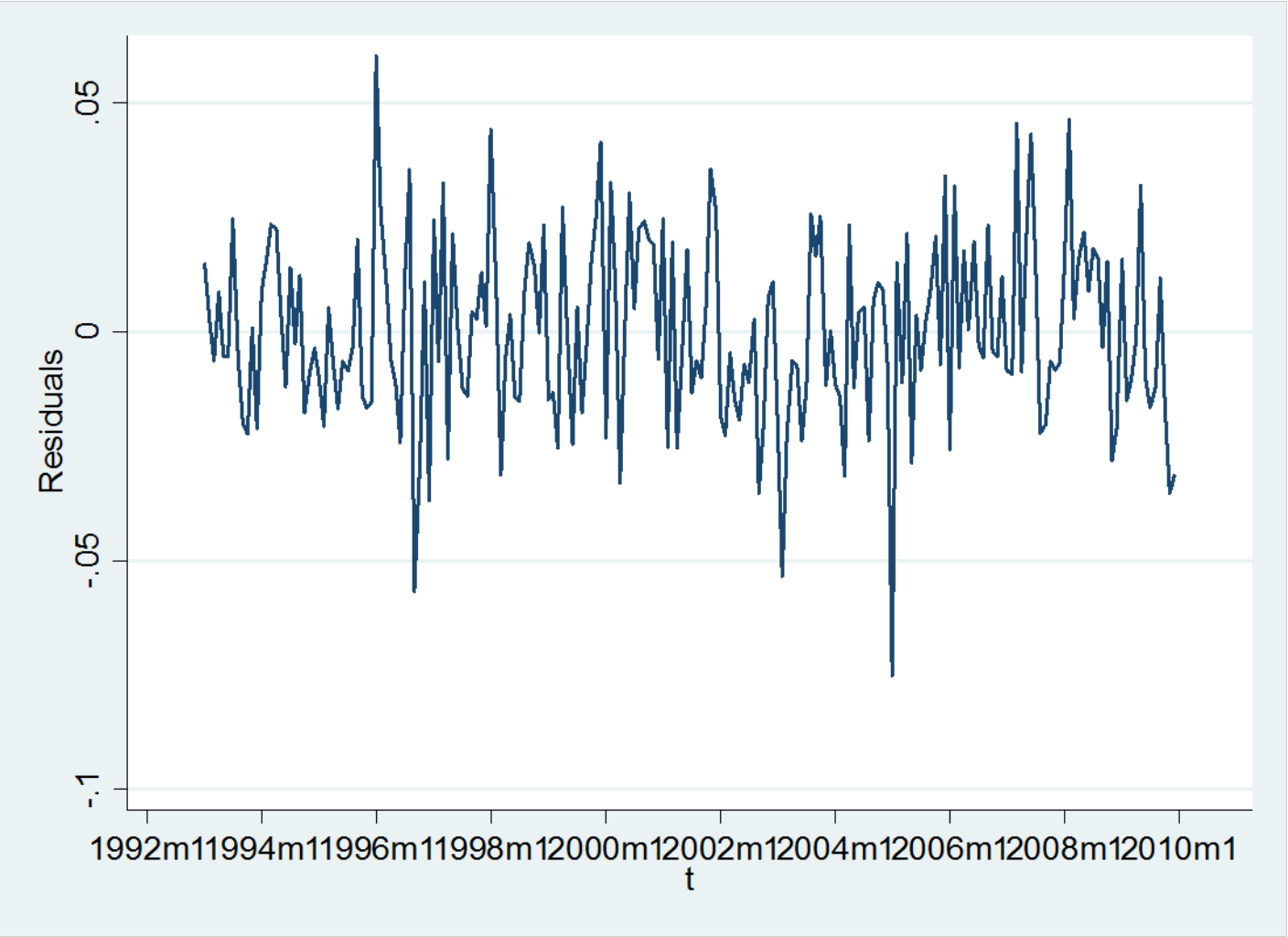
Fitted Values



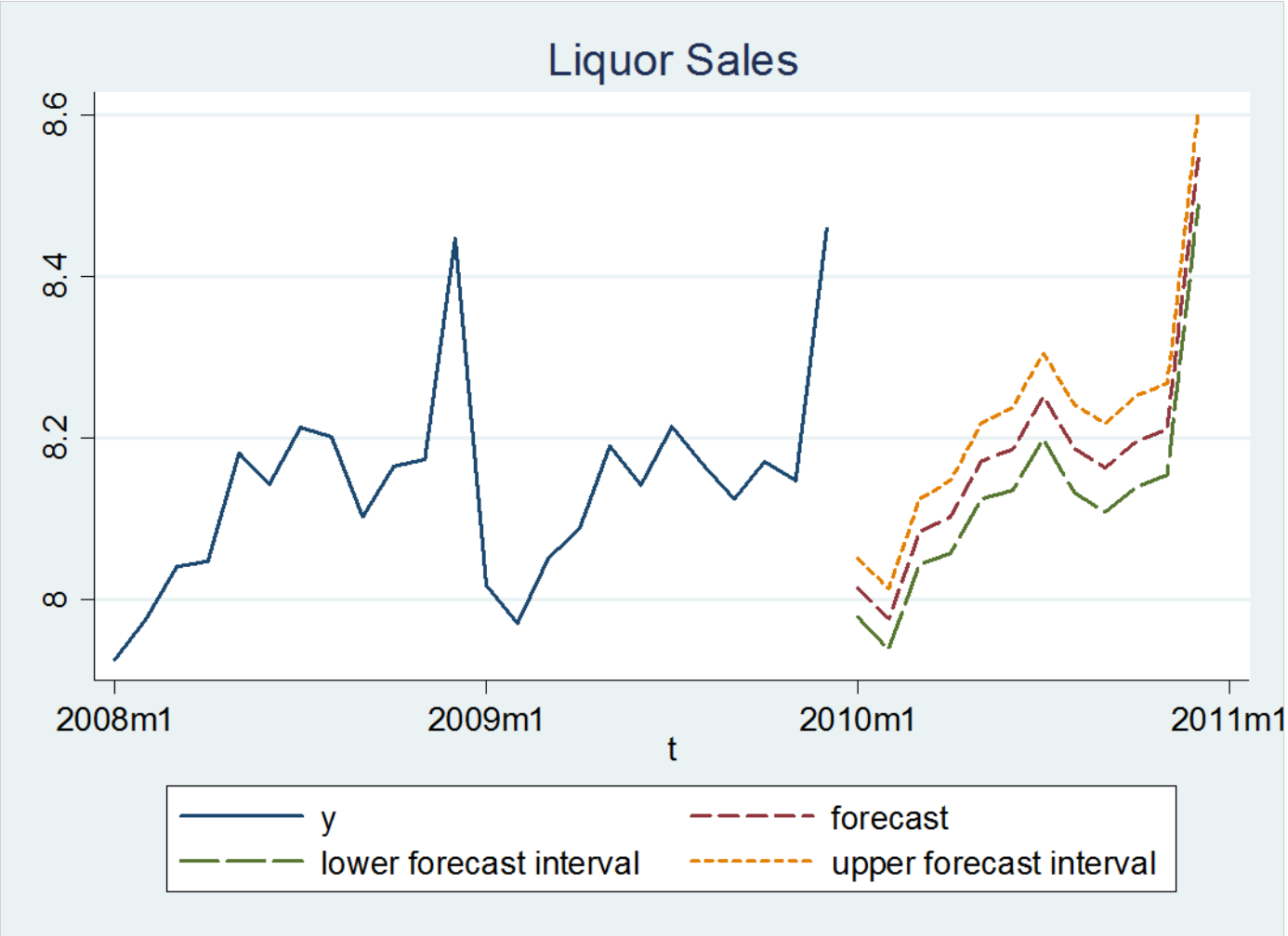
Last 3 years


8.6
8.4
8.2
8
7.8
2007m1
2008m1
2009m1
2010m1
2011m1
t
y
Fitted values

Residuals


Residuals
.05
0
-.05
-.1
1992m1
1994m1
1996m1
1998m1
2000m1
2002m1
2004m1
2006m1
2008m1
2010m1
t

12-Month Forecast



12-month forecast

- The big jump down in the forecast for January 2010 is because of the seasonal dummy effect