

# Economic Policy and Equality of Opportunity

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## Abstract

We employ EOP definitions that have appeared in the philosophical literature on distributive justice, and apply it to a formal economic model which incorporates human capital investment and luck within and across generations. The human capital acquisition technology features multiple stages of parental investments during childhood, and on-the-job investments in adulthood. The benchmark model features an intergenerational borrowing constraint, education and low-income subsidies which are financed by a progressive income tax. The model is calibrated to the U.S. in 1990, on which we operationalize philosophical concepts by using the children's continuation values as outcomes, and the state space of the recursive decision problem as circumstances. This reveals that intergenerational investments should be accounted for when comparing measures of EOP. When conditioning on parental luck instead of outcomes, EOP looks much larger: luck is overwhelmed by human capital accumulation. In contrast, when looking at children's net wealth instead of lifetime earnings, EOP looks worse: intergenerational borrowing constraints are more pronounced in wealth than in earnings. In counterfactual experiments, we find that education subsidies increase utilitarian welfare and decrease overall inequality, but does little to promote EOP. This is for two reasons: when one takes the view that intergenerational efforts should be rewarded, there is little room for improvement in EOP to begin with. On the other hand, when one takes the view that intergenerational efforts should not be rewarded, much stronger redistribution is needed for the policies to have a quantitative impact.

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# 1. Introduction

We present a quantitative economic model of human capital investment within and across generations, with incomplete markets and government transfer programs. We then map the outcome distribution of the model into existing notions of equality that have appeared in the philosophical literature on distributive justice. Our analysis is novel in that we provide an operationalization of philosophical concepts in a quantitative economic model. The advantage of our model-based approach is that we can apply these notions to objects that are discussed in the abstract but for which no such data exists (unobservable genetic traits, effects from and to far away generations). A counterfactual policy analysis shows that egalitarian policies that target the equality of outcomes may have little impact on equality of opportunity (EOP).

Equality, or inequality, is at the heart of many branches of the social sciences. A major challenge in philosophy is to establish an ethically defensible notion of equality that a society should strive for. There is an implicit consensus that outcomes should not depend on circumstances that are beyond an individual's control, but individual efforts given such circumstances should be rewarded ([Arneson, 1989](#); [Roemer, 1993](#)). Put simply, opportunities should be equalized while responsibility should be rewarded. From this it also follows that equal efforts should be rewarded equally. This hardly solves the problem, since we then need to be able to identify what constitutes an opportunity that an individual is not responsible for, and have a notion of how to compare efforts from individuals with different circumstances. Even equipped with such definitions, obstacles abound when trying to implement such notions to existing data in practice. Some recent empirical attempts are by [Lefranc et al. \(2008, 2009\)](#), who operationalize successive developments since Roemer.

Mainstream economics has taken a different route. Despite some early discussions among economists ([Atkinson, 1970](#); [Sen, 1970](#); [Okun, 1975](#)), the main emphasis in traditional Neoclassical theory is on economic efficiency rather than equality. This is well manifested in an influential paper by [Lucas \(2003\)](#). Existing normative studies on economic inequality and redistribution either rely on Pareto improvements or assume as given a social welfare function. The latter is typically a Benthamian-type average utility which is maximized by a social planner. Depending on the assumed preferences, technology, and environment, the socially optimal outcome becomes a mixture of maximizing aggregate efficiency while minimizing disparate jumps in the individuals' consumption paths over time.

The emphasis on efficiency notwithstanding, economists have long been interested in measures of inequality such as Gini coefficients, Lorenz curves, and percentile income concentrations, and analyzing how these objects would change in response to different policies. In the recent literature the goal is more positive than normative, i.e., to compute the costs and benefits of potential policies at the disposal of a social planner according to her own objective function, not tell the planner what the objective should be. For this, quantitative economic research is able to incorporate degrees of heterogeneity that were previously unimaginable. However, to the best of our knowledge, no attempts have been made to apply egalitarian notions of EOP in quantitative research, since the welfare criterion is always utilitarian.

Our contribution is two-fold: for philosophers, we present a model-based approach which can serve as a test-bed for EOP notions that can be difficult to recover directly from the data. For economists, we illustrate that the "veil of ignorance" criterion far from reflect egalitarian concerns, as has been critiqued by [Roemer \(2002\)](#). We do not develop new notions of EOP, but rather define, for a given generation in the model, which variables should be included as circumstantial (for which an individual should not be rewarded; also referred to as or "types") vis a vis outcomes. We also show that decreasing the overall inequality of outcomes is not equivalent to EOP.

Clearly, we are not the first to apply philosophical concepts to economics, but to the best of our knowledge all previous attempts have been purely empirical. [Lefranc et al. \(2008\)](#) posits a philosophical definition of EOP owing to [Roemer \(1993\)](#) and extends it to an empirically testable definition which incorporates "luck," which is also important for our analysis. Applying this to French data from 1979-2000, they conclude that there is a significant degree of inequality based on observables that can be categorized as circumstantial, while "luck" seems to be rewarded equally conditional on circumstances. Since one cannot hope to be able to equalize all returns to luck, such an outcome is ethically desirable.

We go in the opposite direction. After presenting a quantitative economic model calibrated to the 1990 U.S., we explain how we would measure the degree of EOP from the model, employing existing definitions. The model incorporates multiple stages of human capital investment during childhood, a college decision, and on-the-job investments in adulthood. This emphasis on human capital acquisition is in line with recent studies that point to human capital as a major source of inequality ([Huggett et al., 2011](#)), which is especially sensitive to investments at earlier ages ([Cunha et al., 2010](#); [Corak, 2013](#)). The benchmark model also includes education and lump-sum subsidies financed by a progressive income tax, and a PAYGO social security system. Finally, in contrast to

empirical studies in which luck is the residual outcome that cannot be explained by observables (in an OLS sense), our model features two sources of fundamental luck: genetic luck and market luck. The effect of such luck cannot be directly measured in the data, as it affects individual decisions that translate non-linearly into (market) outcomes.

Because of the recursive representation of most quantitative macroeconomic models, circumstantial variables can be mapped into the states of the value function, or previous choices that led to those states. In our model, the education, human capital, and wealth of the parent correspond to "types" that can be observed in the data, while the child's (lifetime) earnings and wealth correspond to outcomes. EOP is measured by comparing the conditional distribution of children outcomes by type. With full EOP, these distributions should overlap.

Despite advances in quantitative research, it is difficult to incorporate all abstract notions of "circumstances" or "effort" into a single model.<sup>1</sup> We are not an exception, and we assume that all such objects are assumed to be the same across all individuals in the economy.<sup>2</sup> Instead we emphasize that our model, which builds on standard economic theories of education and economic outcomes, can address at least two concerns that are hard to implement in pure empirical studies in practice:

1. how to compare inequality across generations,
2. ex ante determined ability vis a vis ex post accumulated human capital.

The point is that when analyzing egalitarian policies, it is important to focus not only on how the distribution of parental circumstances affect the next generation, but how parental *efforts* affect the next generation. One of the questions we raise in this paper, which is central to economics and philosophy, is thus: if a well-educated parent affords better education for her child, should this be viewed as better circumstances for the child, which should be equalized away, or the effort of the grandparent, which should be rewarded ([Arneson, 1998](#))?

While we do not give an answer to this question, we argue that different type-outcome pairs should be considered when measuring EOP depending on which stance is taken. Specifically, if one takes the stance that parental efforts should be rewarded, we should look at children's earnings conditioned on parental luck or ability. If one takes the stance that they should not be

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<sup>1</sup>For example, most economic models allow little heterogeneity in preferences, technology, and/or belief formation.

<sup>2</sup>Assuming all individuals share the same belief about the state of the world is not as heroic as it may sound, at least for long-run outcomes, which we focus on.

rewarded, we should look at children's net wealth when the children are of an age that all investments in the *grandchildren* have been completed. We corroborate our argument by showing that such choice of pairs correctly reflect the distribution of continuation utilities, which is available in our structural model but unobserved in real data.

Following existing notions of EOP, we first analyze the conditional distributions of the next generation's lifetime earnings by the lifetime earnings of the parent. After arguing that such variables themselves, however, are the outcome *previous* generations (grandparents, great-grandparents, etc.) we then analyze the conditional distributions that are purely circumstantial in the model: the luck of the parents. The results are striking: when conditioning on parental luck instead of lifetime earnings, EOP is almost achieved, in the sense that the conditional distributions converge toward each other. The culprit is our multi-layered human capital acquisition technology—luck is overwhelmed by intergenerational human capital investments.

However, it is not necessarily ethical to award a future generation for accumulated efforts from far away generations. But if we were to ignore the fact that some circumstantial variables at least partially reflect the effort of previous generations, then the current generation should not be punished for what previous generations are not rewarded for (investment in children) either. Then the appropriate outcome variable we should look at is not children's earnings but the wealth level of the child at middle-age, less transfers to his own offspring. Surprisingly, we find that despite a smaller overall level of inequality in net wealth compared to lifetime earnings, EOP is smaller in the sense that the conditional distributions become farther from each other. This is mainly due to the lowest type: because of the intergenerational borrowing constraint (non-negative bequest constraint), poor households would persistently want to borrow from their children but cannot. This is also manifested as a wider variation in the conditional distributions of lower-types.

Since individuals in our model are fully rational, such efforts across generations occur because they derive utility from them. So from the model's perspective, regardless of whether one views circumstantial variables as the outcome of efforts from the previous generations, what should be compared are conditional *continuation utilities*, since it takes into account both the (utility) benefits and costs of having richer offspring. This has less to do with the expected utility notions emphasized by [Vallentyne \(2002\)](#) and others and more about dynamics: the point is that the continuation value of subsequent generations must be taken into account, not the expected utilities at a single point in time. The distributions when we condition on either the parent's lifetime earnings or abilities closely mirror the two cases in the preceding paragraphs. Thus, it confirms our emphasis

that intergenerational investments should be accounted for when comparing measures of EOP. Empirically, it calls for the need to estimate either genetic abilities or the net wealth of individuals in order for EOP to reflect utilities rather than dollar outcomes.

We then analyze how EOP would change in response to a change in a government policy by contrasting the observed, outcome distribution to the unobserved, structural distribution. Our model-based approach allows a joint analysis of different measures of EOP and utilitarian welfare, which can guide a policy-maker who faces both efficiency and egalitarian concerns. To weigh such concerns, a cardinal measure is needed, and we advocate the usage of the Theil index. The Theil index maps nicely into both philosophical and economic concepts. First, because it is readily decomposable, it is suitable for measuring compensation between and within types, which has been the major platform of redistributive justice debates. Second, it can be used as an input into the social welfare function of a planner who is concerned with both equity and efficiency in which the planner's weight on equity can be represented by an exogenous parameter.

Since we keep the model and parameter values constant, we cannot deconstruct how much of inequality stems from the transmission technology (human capital acquisition) vis a vis the distribution of parental types, as in [Lefranc et al. \(2012\)](#). The advantage, however, is the framework serves as a testbed for "controlled experiments": we can attribute all the shifts in the parental distribution and resulting measures of EOP to the exogenously changing policy variable. To the extent that the transmission technology is affected by the distribution of types (through policy and equilibrium effects), we find such an experiment appealing. The results are striking: while education subsidies have large, positive effects on utilitarian welfare and equality of outcomes, it does little to promote EOP. This is for two reasons: when one takes the view that intergenerational efforts should be rewarded, there is little room for improvement in EOP to begin with. On the other hand when one takes the view that intergenerational efforts should not be rewarded, much stronger redistribution is needed for the policy to have a quantitative impact.

Of course, all our results depend on the values of the deep parameters of the model, and even more on the posited model itself. Given that the individual elements of our model are widely accepted and that the model economy replicates the data very closely, though, our message should not be ignored. However, even if one is to reject our model, it should be clear that results relying on simple observables may not properly measure the EOP that abstract notions were meant to capture, not only because of the lack of data but also because the true data-generating process is non-linear. The main message is that regardless of one's notion of what constitutes parental

circumstances that should be compensated for, intergenerational efforts should be controlled for in the data; and that at least in the U.S., much stronger redistribution is called for to achieve EOP if one is to compensate for intergenerational investments (or the lack thereof). A utilitarian criterion may not sufficiently capture such egalitarian concerns [Arneson \(1999\)](#); [Roemer \(2002\)](#).

The rest of the paper is organized as follows. Sections 2-3 lays out our model and numerical calibration, which borrows heavily from [Lee and Seshadri \(2014\)](#). In Section 4, we spell out how we apply existing EOP notions to our model. Section 5 presents the outcome of the application and our counterfactual policy analysis. Section 6 concludes.

## 2. Model

In [Lee and Seshadri \(2014\)](#) we combine a fairly standard life-cycle model of human capital accumulation à la [Ben-Porath \(1967\)](#) with an intergenerational transmission mechanism à la [Becker and Tomes \(1986\)](#). Individuals differ in their genetic ability and human capital, but they share identical preferences and face the same technologies. Ability is transmitted across generations, and pre-labor market initial conditions are determined by endogenous parental investments. Bequests occur after the parent completes child investments. Parents' anticipation of the life-cycle decisions their children make as grown-ups also affect child investments.

We cast our model in an overlapping generations framework with infinitely-lived dynasties who are altruistic. So in contrast to many models in which parental states are fixed exogenously, and cross-sectional inequality among children is pre-determined once they become adults, current states are decided by all previous members of a dynasty, while current decisions are affected by all subsequent members of a dynasty. The life-cycle and intergenerational structure, along with multiple stages of human capital investment, complicate notions of equality: parent's own human capital accumulation over the life-cycle happens simultaneously with child investments but prior to bequest decisions.

We assume complementarity between early investments in the form of parental time, and later investments in the form of goods. We model the final stage of human capital formation as a college enrollment decision coupled with skill acquisition at college age, as much of inequality can be attributed to differences in educational attainment. The college choice also allows us to separate cross-sectional inequality into differential skill prices and different levels of skill. Post-schooling life-cycle wage growth follows a Ben-Porath human capital accumulation specification.

So while educational attainment no doubt has a large impact on human capital, it depends not only on parental genetics but also their income and their own educational attainment. [Lee and Seshadri \(2014\)](#) show that all these features help capture empirical relationships in the U.S. that previous models failed to explain.

The initial distribution over the next generation's human capital and assets are determined by investments in children and bequest decisions, while the terminal distributions of human capital and assets over the parent generation are determined by own human capital investment and life-cycle savings decisions. Since parents make these decisions, we require individual behavior to be consistent with observing these distributions in a stationary equilibrium. In particular, rationalizing the distribution of human capital jointly with assets is critical, since a framework that can account for earnings persistence but inconsistent with wealth inequality and its transmission would be unconvincing. The stationary equilibrium, which is the long-run outcome of clearing physical and human capital markets, also creates difficulties in operationalizing EOP concepts.

In addition, we also model most of the conventional forms of government intervention. The data we observe comes from an economy in which the government taxes income progressively to subsidize education, fund social security and assist low income households through welfare payments. For our counterfactual analysis, we focus on the shifting transfers between education and welfare payments.

## 2.1 Environment

A period is 6 years. There is a unit measure of households whose members live through a life-cycle. where an individual is born at age 0. So each individual goes through 13 periods of life ( $j = 0, \dots, 12$ ) or 4 stages:

1. Child/College:  $j = 0, 1, 2, 3$ . Attached to parent, choice to enter the labor market or attain college education (human capital accumulation) at  $j = 3$ .
2. Young parent:  $j = 4, 5, 6, 7$ . Independence and bears child at  $j = 4$ , and continues own human capital accumulation. Simultaneously makes early childhood and subsequent investments in child. Joint college decision for child at  $j = 7$ .
3. Grandparent:  $j = 8, 9, 10$ . Continues to accumulate own human capital, and saves for retirement and bequests to child.

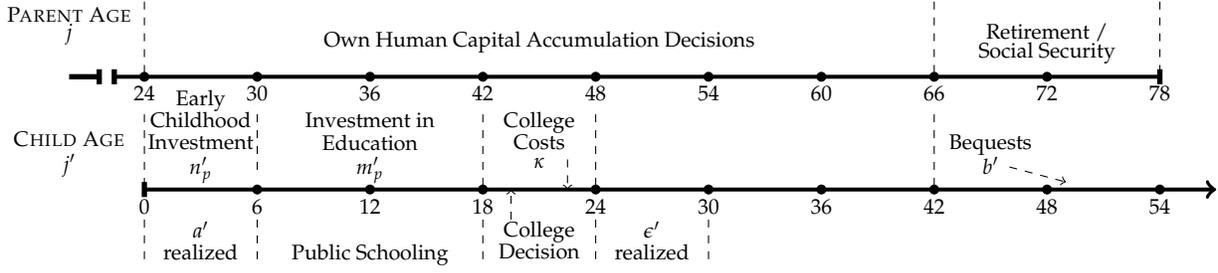


Figure 1: Sequence of Events

4. Retired:  $j = 11, 12$ . Consumes social security payouts, dies *after*  $j = 12$ .

Throughout the analysis, primes will denote next generation values.

Fertility is exogenous, and one parent is assumed to give birth to one child at  $j = 4$ , so  $j = j' + 4$  is fixed throughout the lifetimes of a parent-child pair. Until the child's full independence at  $j' = 4$ , the parent-child pair solve a Pareto problem. These decisions involve consumption, savings, investment in human capital, and college. The family decides whether the child should attend college when he becomes 18 years old ( $j' = 3$ ). The direct cost of college is fixed at  $\kappa$ , but there is also an implicit cost of the child's forgone earnings by delaying entry into the labor market. In retirement, the grandparent consumes his savings and social security benefits, and dies at age 78. He also sets aside a bequest  $b'$  for his child at  $j = 11$ , which is transferred to his child at  $j = 12$ . The sequence of events is depicted in Figure 1.

## 2.2 Human Capital Formation

The crux of our model-based approach is the elaborate human capital formation process. Previous studies analyze EOP of the current generation as an endogenous outcome across exogenously defined social background groups, which are proxied by parental circumstances. In contrast, in our model human capital investments occur across infinite generations, so that such circumstances themselves are endogenous outcomes. The only purely exogenous variables are the ability and market shocks,  $(a, \epsilon)$ , respectively. We will later show that this endogeneity of parental types can significantly alter EOP measures.

A child is born with innate ability  $a'$ , which determines his proficiency at accumulating human capital. The child's ability is stochastic, and depends on the parent's ability. This will capture the "genetic" part of intergenerational transmission. An adult's earnings, or returns to human capital, are subject to a stochastic "market luck shock"  $\epsilon$  that depends on his own ability, and remains

constant throughout his lifetime. This shock captures idiosyncratic risk associated with human capital accumulation, while the dependence on abilities captures both the potential persistence in luck across generations, and a human capital returns premium (or discount) for high ability individuals. Formally, denoting a young parent's ability as  $a$ , at  $j = 4$  (age 24) he draws  $(a', \epsilon)$  from a joint probability distribution conditional on  $a$ :

$$(a', \epsilon) \sim F(a', \epsilon | a).$$

In addition to exogenous transmissions, we also include adulthood human capital accumulation and most importantly three important aspects of child human capital formation that are well appreciated in the literature: i) parental time and good investments, ii) complementarity between inputs, and iii) the child's own investments.

From age 24 until retirement, we assume a Ben-Porath human capital accumulation function,

$$h_{j+1} = a(nh_j)^{\gamma_S} + h_j, \quad \text{for } j = 4, \dots, 10, \quad (1)$$

where  $h_{j+1}$  is human capital tomorrow and  $n_j$  is the time investment in own human capital accumulation. The parameter  $\gamma_S$  depends on whether the adult is college educated ( $S = 1$ ) or not ( $S = 0$ ).

During the early childhood of his offspring (ages 0-5,  $j' = 0$ ), a young parent invests  $n_4$  units of time in his own human capital accumulation (on-the-job accumulation of human capital), and spends  $n_p$  units of time with his child. When the child goes to secondary school ( $j' = 3$ ), he supports his education with  $m_p$  units of consumption goods.<sup>3</sup> The education production function (later childhood human capital formation) is CES in time and goods inputs:

$$h'_3 = \left[ \gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1 - \gamma_k)^{\frac{1}{\phi}} (n_p h_4)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

where  $d$  is a government subsidy, and  $\gamma_k$  captures the expenditure share of education. We have assumed that the time investment in the child,  $n_p$ , interacts with the human capital level of his parent,  $h_4$ . This is to capture the fact that more educated households (mothers) spend more time with their children (Behrman et al. (1999)). The CES parameter  $\phi$  is the elasticity between parental

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<sup>3</sup>It is not the timing of time and good investments that is germane to our model, but the complementarity between them.

time and good inputs, which can also be interpreted as complementarity between early and later childhood investments. [Cunha et al. \(2010\)](#), estimate this parameter in a model with multiple stages of child investments but only in terms of goods, while [Del Boca et al. \(2013\)](#) provide evidence that time is the dominant input when the child is young with its importance declining with age, and that the reverse is true for goods.

From ages 18-23 ( $j' = 3$ ), a child chooses whether or not to attend college, and invests own time  $n'_3$  into human capital production according to:

$$h'_4 = a' n'_3{}^{\gamma_{S'}} h_3{}^{\gamma_p} = a' n'_3{}^{\gamma_{S'}} \left[ \gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1 - \gamma_k)^{\frac{1}{\phi}} (n_p h_4)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi \gamma_p}{\phi-1}},$$

where  $\gamma_{S'}$  depends on whether the child is in college ( $S' = 1$ ) or not ( $S' = 0$ ), and  $\gamma_p$  captures the returns to parental inputs. This composite function is intended to capture the fact the child's own time investment becomes more important in later years ([Del Boca et al. \(2012\)](#)).

Notice that there are four mechanisms through which human capital is transmitted across generations. First, the ability to *learn* is transmitted across generations, that is, the child's  $a'$  is drawn from a distribution that depends on the parent's  $a$ . Second, there is partial inheritance of market luck through the correlation of  $(a, \epsilon)$ . This can be viewed as heritable disabilities that affect the ability to *earn*.<sup>4</sup> Third, the resources devoted to child human capital formation,  $m_p$ , will be a function of the parent's resources. If capital markets are complete, the investment will be independent of parental resources. However, with capital market imperfections (which we assume), parental resources will matter. Finally, we assume that a higher human capital parent (large  $h$ ) is better at transmitting human capital beyond his ability to pay for more resources per unit of  $n_p$ . While all these channels can be regarded as circumstantial which are not an individual's own responsibility, in the next section we will separate those variables that should be compensated for and those that should not, according to philosophical definitions.

The interaction between child and adult human capital accumulation is what differentiates us the most from other models. In [Lee and Seshadri \(2014\)](#) we show that the simultaneity of own and child investments, and in particular in the form of time, is key to generating empirically observed IGE's and retirement wealth patterns across groups. The assumption that the human capital level of the parent affects both the parent *and* the child is also important. In the data, wages may be

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<sup>4</sup>In general,  $\epsilon' \sim F(\epsilon' | a', \epsilon)$ , but we ignore the direct dependence of  $\epsilon'$  on  $\epsilon$  because i) it is not clear what a persistence of "luck" means conceptually, and ii) such persistence would not be identifiable without multiple generations of data. The current formulation leaves only one pure channel for genes.

thought of as reflecting both innate ability as well as acquired human capital. Viewed this way, it will be affected by unobservable parental (potential) human capital in addition to observables such as parental education or earnings. In particular, the non-linear human capital formation process is pivotal—our previous paper shows that when complementarity between parental time and goods investments are strong enough, innate abilities may play no role at all.

## 2.3 Government Policies

The government makes no decisions, and its only feedback on the economy comes through budget-balance conditions. Our counterfactual experiments will focus on altering the parameters of the government’s parametrized policies. The policies include a progressive income tax, the proceeds of which are partially used for a lumpsum transfer  $g$  and an education subsidy  $d$  too all individuals,<sup>5</sup> and a balanced-budget social security program. The first three are redistributive by nature (not the taxes themselves, but the fact that it is progressive), while social security is redistributive across generations.

We first describe the tax system. Denote by  $e_j$  the earnings in any working period  $j$ . Then,  $e_j$  is taxed at a progressive rate  $\tau_e(e_j)$ , and also subject to a flat rate payroll tax  $\tau_s$  that is used to fund the social security benefits. The final after tax, after subsidy net earnings of a working adult is

$$f(e_j) = [1 - \tau_s - \tau_e(e_j)] e_j. \quad (2)$$

Capital income is taxed at a flat rate  $\tau_k$ , so we can define  $\tilde{r} = [1 + (1 - \tau_k)r]^6 - 1$ , the 6 year compounded effective interest rate faced by a household, where  $r$  is the pretax annual interest rate.<sup>6</sup>

Social security benefits  $p$  are modeled as a function of  $\bar{e}$ , the average lifetime earnings from ages 24-65 ( $j = 5$  to 10).<sup>7</sup> We model it as an affine function:

$$p = p(\bar{e}) = p_0 + p_1 \bar{e}, \quad \bar{e} = \frac{\sum_{j=5}^{10} e_j}{6}$$

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<sup>5</sup>The rest would capture all other government expenses that do not directly affect the utility of domestic agents; metaphorically, it is “thrown in the ocean.”

<sup>6</sup>Of course a more realistic system would also capture capital income tax progressivity, but to the extent that capital income is barely accrued by households not in the very top percentiles of the income distribution, and that many loopholes exist for capital income for the very wealthy (Gruber and Saez, 2002), the tractability we gain by this assumption should justify what we miss by this assumption.

<sup>7</sup> Social security benefits in the U.S. are based on the 35 years of an individual’s highest earnings.

where  $(p_0, p_1)$  are parameters governing the social security regime. Social security benefits are not subject to any tax. The government runs a balanced budget on these benefits by financing them with the payroll tax revenue.

We assume that the initial distribution  $F_0$  for  $(a', a, \epsilon)$  is the stationary distribution of  $F(a', \epsilon|a)$ . The steady state equilibrium and budget balance conditions are explained in detail below.

## 2.4 Household's Problem

We assume that an adult faces natural borrowing constraints for all savings and education decisions, and a non-negative bequest constraint. In [Lee and Seshadri \(2014\)](#) we present the full-blown structure of the individual problem and then explain in detail how, given the timing of shocks, we can split the life-cycle into two sub-periods: young and old. This simplifies greatly the structure of the household decision problem. Here we only present the simplified problem.

Let  $y = 4$  and  $o = 8$ , the 2 nodes in the lifecycle that an individual needs to make decisions. Since there are no other time costs on and after stage  $y + 1$ , an adult at this stage would simply maximize the present discounted value (PDV) of lifetime income, which admits deterministic decisions which are unaffected by what happens in his old age.<sup>8</sup> Hence the PDV lifetime income is a function of her own education level, ability, market luck, and human capital:

$$z_{h,y+1}(S, a, \epsilon, h_{y+1}) = \max_{\{n_{y+l}\}_{l=1}^6} \left\{ \sum_{l=1}^6 \frac{1}{(1+\tilde{r})^{l-1}} \cdot \left[ f(e_{y+l}) + \frac{(2+\tilde{r})p_1}{6(1+\tilde{r})^{8-l}} \cdot e_{y+l} \right] \right\},$$

where

$$e_{y+l} = w_S h_{y+l} \epsilon (1 - n_{y+l}),$$

is her labor market earnings at stage  $y + l$ , and her human capital production and earnings net of taxes  $f(\cdot)$  are specified in (1) and (2), respectively. This problem can be easily solved recursively, so the young parent can take  $z_{h,y+1}$  as given.

To solve the rest of the household's problem, there are three value functions we need to keep track of: two value functions at stage  $y$ ,  $W_y(\cdot)$  and  $V_y(\cdot)$ , before and after the young parent decides whether or not to send her child to college, and  $V_o(\cdot)$ , the value function at stage  $o$ , when

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<sup>8</sup>In laymen terms, it means a grandparent does not change his labor supply decisions depending on whether his child was lucky or not in the labor market, or his grandchild turned to be smart or dumb. This does not seem to be such an unrealistic framework.

her child has his own child. The arguments of these functions are the state variables, which form the circumstances of an individual. The choices made based on the states can be viewed as efforts, which, conditional on the state, should be rewarded based on Roemer's criterion. In particular of interest for us are the states of  $V_y$ , which are an outcome of parental investments and intergenerational transmissions, a major part of which is  $\mathbf{x} = (S, a, \epsilon, h_y)$ , the states that determine a young parent's lifetime earnings. These states are also important when we discuss how to apply EOP concepts to our model in the next section.

We assume that a young parent knows already at childbirth whether or not she will send her child to college, i.e., the value of  $S'$  is known.<sup>9</sup> We also assume that she (perfectly) anticipates the present discounted value of bequests,  $z_y = b/(1 + \tilde{r})^3$ , that will be decided by her now old parent (the grandparent). In addition to  $(S'; \mathbf{x}; z_y)$ , choices are also affected by the child's ability. The young parent makes a joint consumption choice  $C_y$  for her and her child, human capital investments  $(n_p, m_p)$  in her child and  $n_y$  in herself, and carries  $z_o$  of wealth into her old age. The child makes his first human capital accumulation decision  $n_k$ .

$$\begin{aligned}
W_y(S', a'; \mathbf{x}; z_y) &= \\
&\max_{C_y, z, n_y, n_p, m_p, n_k} \left\{ q_y u(C_y) + \beta^4 \int_{a'', \epsilon'} V_o(a''; \mathbf{x}'; z_o) dF(a'', \epsilon' | a') \right\} \\
C_y + \frac{z_o}{(1 + \tilde{r})^4} + \frac{m_p}{(1 + \tilde{r})^2} &= f(e_y) + \frac{z_{h,y+1}(S, a, \epsilon, h_{y+1})}{1 + \tilde{r}} + \frac{f(e_k) - S' \cdot \kappa}{(1 + \tilde{r})^3} + z_y + G, \quad (3) \\
e_y &= w_S h_y \epsilon (1 - n_y - n_p) \quad \text{and} \quad n_y, n_p \geq 0, \quad n_y + n_p \leq 1 \\
e_k &= w_S h_k (1 - n_k) \quad \text{and} \quad n_k \in \left[ \frac{2S'}{3}, 1 \right], \\
h'_y &= a' n_k^{\gamma_S'} h_k^{\gamma_p}, \\
h_k &= \left[ \gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1 - \gamma_k)^{\frac{1}{\phi}} (n_p h_y)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},
\end{aligned}$$

where  $q_y$  adjusts utility to account for adult-equivalent family size and subjective discounting, and  $(e_k, h_k)$  denote, respectively, the child's earnings and human capital level at college age. The parent's own human capital in the next period is determined by (1), which, in turn, determines her PDV lifetime income  $z_{h,y+1}$ . The parameter  $\kappa$  is a fixed cost for college, which is in addition

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<sup>9</sup>This means that the young parent *plans ahead of time* to send the child to college, given her own level of human capital, income and wealth, and the revealed ability of her child. This may be a rather strong assumption given that not everything may go as planned, but we need to simplify the life-cycle dimension somewhat to operationalize the intergenerational dimension.

to the cost of forgone earnings incurred by the child remaining in school: if  $S' = 1$ , the child is committed to spend at least two-thirds of a period, or 4 years, out of the labor market and accumulating human capital. The last term in the budget constraint

$$G = g \cdot \left[ \sum_{l=y}^{y+2} \frac{q_A}{(1+\tilde{r})^{l-y}} + \frac{2}{(1+\tilde{r})^3} + \sum_{l=0}^{o+4} \frac{1}{(1+\tilde{r})^{l-y}} \right] + \frac{p_0(2+\tilde{r})}{(1+\tilde{r})^8},$$

is the lifetime PDV of the lumpsum portion of subsidies, where  $q_A$  is an adult-equivalent scales that adjusts for family size. The value function when old,  $V_o$ , is specified below.

We assume that the grandparent makes bequest decisions before the child decides whether or not to attend college. So given  $z_y$ , the young parent-child pair simply choose

$$V_y(a'; \mathbf{x}; z_y) = \max_{S' \in \{0,1\}} \{W_y(S', a'; \mathbf{x}; z_y)\}.$$

Hence, the grandparent indirectly controls whether or not his grandchild goes to college by leaving more or less bequests (within his budget constraint). This is in fact the only decision he makes, as his lifetime earnings decision is made when he is young. His states are the ability of his now newly-born grandchild,  $a''$ , the lifetime earnings of his child (the young parent) which is determined by  $(S', a', \epsilon', h'_y)$ , and the wealth he carried over from when he was young:

$$V_o(a''; \mathbf{x}'; z_o) = \max_{C_o, z'_y} \left\{ q_o u(C_o) + \theta V_y(a''; \mathbf{x}'; z'_y) \right\}$$

$$C_o + z'_y = z_o$$

$$z'_y \geq 0,$$

where the last inequality is the intergenerational non-negative bequest constraint. Whatever is not left as bequests is consumed ( $C_o$ ), which yields utility to the grandparent discounted by  $q_o$ .<sup>10</sup> The parameter  $\theta$  is the altruism factor, which (for positive values) makes the problem dynastic.

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<sup>10</sup>Bequests are *decided upon* before the grandchild's college decision, but occurs after the realization of the grandchild's market luck shock and great-grandchild's ability shock. Given the time horizon of a dynasty, it does not seem unreasonable to assume that bequest decisions are not affected by these.

## 2.5 Firm and Stationary Equilibrium

We assume a standard neoclassical firm that takes physical and human capital as inputs to produce the single consumption good. It solves

$$\max_{K, H_h, H_l} F(K, H_0, H_1) - RK - w_0 H_1 - w_1 H_1,$$

where  $R = (1 + r + \delta)^6 - 1$  is the competitive rental rate and  $(K, H_0, H_1)$  are the aggregate quantities of capital and effective units of labor by skill in the economy, respectively. The inclusion of this stand-in firm is what creates general equilibrium effects that may amplify or dampen the effect of policy changes on the resulting long-run equilibrium.

Let  $X$  denote the aggregate state spanning all generations, and denote its stationary distribution by  $\Phi(X)$ . Let  $\Gamma(\cdot)$  denote the law of motion for  $\Phi$ , which is derived from the agents' policy functions. In a stationary equilibrium, prices  $(r, w_0, w_1)$  solve

1. Market clearing and stationarity:

$$\begin{aligned}\Phi &= \Gamma(\Phi) \\ K &= \int_X (s_j^* + b^*) \Phi(dX) \\ w_S H_S &= \int_X e_j^* \Phi(dX, S, 3 \leq j \leq 10).\end{aligned}$$

where  $s_j^*$  are the optimal savings decisions of an individual at stage  $j$ ,  $b^*$  the bequest decisions of individuals at stage 11, and  $e_j^*$  the earnings resulting from the optimal human capital and time investment decisions of adults in stage  $j$ .

2. Government budget balance:

$$g = \pi_g \bar{\mathbf{e}}^*, \quad d = \pi_d \bar{\mathbf{e}}^*,$$

i.e. the subsidies  $(g, d)$  are fixed fractions  $(\pi_g, \pi_d)$  of equilibrium average earnings  $\bar{\mathbf{e}}^*$ :

$$\bar{\mathbf{e}}^* = \int_X e_j^* \Phi(dX, 3 \leq j \leq 10).$$

The social security regime is also balanced:

$$\tau_s \bar{e}^* = 2 \left[ p_0 + p_1 \int_X \bar{e}^* \Phi(dX) \right],$$

where  $\bar{e}^*$  is the past 6-stage average earnings of individuals in stages 11 and 12.

### 3. Calibration

The model is calibrated to 1990 United States. We choose 1990 mainly due to data availability issues, and also to avoid extreme temporal events (such as the oil crisis of the 70s or Great Recession of recent). For a more detailed description of the calibration procedure and robustness checks, refer to [Lee and Seshadri \(2014\)](#).

#### 3.1 Parametrization

**Exogenous processes** We assume an AR(1) process for ability shocks:

$$\log a' = -\frac{(1 - \rho_a)\sigma_a^2}{2} + \rho_a \log a + \eta, \quad \eta \sim \mathcal{N}(0, (1 - \rho_a^2)\sigma_a^2) \quad (4)$$

where  $\sigma_a^2$  is the unconditional variance of abilities. For luck we shocks assume:

$$\log \epsilon' = -\sigma_v^2/2 + \rho_\epsilon (\log a' + \sigma_a^2/2) + v, \quad v \sim \mathcal{N}(0, \sigma_v^2),$$

so both abilities and luck are assumed to have a mean of 1. Abilities and luck are discretized into 4 and 3 grid points, respectively, using the Rouwenhorst method.

**Preferences and Technology** First, the parameters  $(\beta, v)$  are found in equilibrium so that we hit exactly an annual interest of  $r = 4\%$  and a college earnings premium of 46.8%, which is computed from the 1990 IPUMS CPS. In experiments, we fix  $(\beta, v)$  to their calibrated values and find the interest rate  $r$  that clears the asset market and wage ratio  $w$  that clears the labor market by skill. Period utility is modeled as a standard CRRA utility function,

$$u(c) = (1 - \theta\beta^4) \cdot \frac{c^{1-\chi}}{1-\chi},$$

for all individuals, including children and retirees. The normalization by  $(1 - \theta\beta^4)$  facilitates interpreting the value function in terms of consumption equivalent values. The aggregate production function and capital stock evolution are parameterized as:

$$F(K, H_0, H_1) = K^\alpha H^{1-\alpha}, \quad H = \left[ v^{\frac{1}{\sigma}} H_1^{\frac{\sigma-1}{\sigma}} + (1-v)^{\frac{1}{\sigma}} H_0^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

$$K' = (1 - \delta)K + I.$$

This parameterization is consistent with [Heckman et al. \(1998\)](#), and we use their point estimate for the elasticity between skilled and unskilled labor,  $\sigma = 1.441$ .

The parameters for human capital production are calibrated within the model, except for  $\phi$ . We set the elasticity of substitution between age 6 and 18 human capital investments  $\phi = 2.569$  as estimated in [Cunha et al. \(2010\)](#).

**Policy Parameters** Tax rates should capture marginal tax rates, not average tax rates. Hence,  $\tau_k$  is set as 0.31, the 1990 value from [Gravelle \(2007\)](#)'s study on effective marginal tax rates on capital income. For labor income taxes, we follow the log specification of average taxes in [Guner et al. \(2013\)](#), which they show retains attractive properties for the marginal tax rate:

$$\tau_e(e) = \tau_0 + \tau_1 \log(e/\bar{e}),$$

where again,  $\bar{e}$  is the average earnings in the economy. We set  $(\tau_0, \tau_1)$  to the values estimated in their study, and calibrate  $\bar{e}$  to the average earnings in our model economy.<sup>11</sup>

We then use  $\bar{e}$  to compute transfers and college costs. Parameters for the social security system,  $(p_0, p_1, \tau_s)$ , are set as follows. First, we fix  $p_1 = 0.32$ , the median replacement rate for social security payments. Given  $p_1$ , we set  $(p_0, \tau_s)$  to balance the social security budget and match a replacement rate of 40%, as reported in [Diamond and Gruber \(1999\)](#). To simplify the numerical procedure, we then assume that earnings in model periods 3 – 4 (ages 18-29) are negligible in the aggregate so that  $\bar{e} \approx \int_X \bar{e} \Phi(dX)$ , where  $\bar{e}$  was individual average earnings from ages 30-65. Given this assumption,  $\tau_s = 0.1$  and  $p = \pi_p \bar{e}$  with  $\pi_p = 0.133$ .

Transfers as a fraction of average earnings,  $(\pi_g, \pi_d)$ , are set to  $(2\%, 5/(1 - \alpha)\%)$ , respectively. We view lumpsum transfers mainly as welfare for the poor. The size of welfare transfers in the

<sup>11</sup>Although their estimates are based on 2000 IRS tax returns, the 1990s displayed significantly modest changes to tax policies compared to other decades.

Parameter	Value	Description
$\chi$	2	CRRA coefficient, $u(c) = \frac{c^{1-\chi}}{1-\chi}$
$\phi$	2.569	CES age 6 vs 18 human capital investments, <a href="#">Cunha et al. (2010)</a>
$\sigma$	1.441	CES high vs low skill labor, <a href="#">Heckman et al. (1998)</a>
$(\alpha, \delta)$	(0.322, 0.067)	capital income share / depreciation rate, <a href="#">Huggett et al. (2011)</a>
$(\tau_0, \tau_1)$	(0.099, 0.035)	earnings tax constants $\tau(e) = \tau_0 + \tau_1 \log \frac{e}{\bar{e}}$ , <a href="#">Guner et al. (2013)</a>
$(q_A, \pi_g)$	(1.7, 0.02)	adult equivalence scale, lumpsum subsidies as fraction of average earnings $\bar{e}$
$(\pi_d, \pi_\kappa)$	(0.05, 0.3)	education subsidies and cost of college, as fraction of average earnings $\bar{e}$
$(p_1, \pi_p, \tau_s)$	(0.32, 0.08, 0.12)	social security parameters implied by balanced social security budget and median replacement rate of 40% ( <a href="#">Diamond and Gruber (1999)</a> ), refer to text.
$(\bar{r}, \bar{EP})$	(4%, 46.8%)	rate of return on capital / earnings premium in 1990 IPUMS CPS

Table 1: Fixed Parameters

U.S. is approximately 1-2% of total GDP throughout the late 1980s to mid 1990s, of which we take the upper-bound for two reasons: the model transfers captures more than welfare transfers, and is modeled as a fraction of average earnings which is smaller than GDP/capita. Education transfers in the model are assumed to be public spending on secondary education and below in the data, which is obtained from the 1990 Digest of Education Statistics. As a fraction of GDP, this value is 5%, and the division follows from the labor income share of total output, since  $\pi_d$  is a fraction of average earnings.

Annual college costs are set to equal  $\pi_\kappa = 30\%$  of average earnings, which is roughly equal to the ratio of college costs from the National Center of Education Statistics and average earnings from the CPS throughout the 1980s-1990s, so that

$$\kappa = \frac{1 - \beta^4}{1 - \beta^6} \cdot \pi_\kappa \bar{e}.$$

Since all policy parameters are expressed as a fraction of  $\bar{e}$ , we solve the fixed point problem (2) such that in equilibrium, individual choices given the implied values of tax, subsidies and college costs leads to the average earnings used to compute those values.

Lastly,  $q_A$ , the adult equivalence scale for households with children, is set to 1.7, which is the adjustment factor for two-adult households with two children versus those without children, used by the OECD.

$M_{s,i}$	Description	value
$i = 1.$	Average wage ratio at ages 48-53 vs. 30-35, $S = 0$	141.0%
2.	Average wage ratio at ages 48-53 vs. 30-35, $S = 1$	153.7%
3.	College enrollment rate	41.3%
4.	(Secondary and below) education expenses-GDP ratio	6.9%
5.	Education persistence $P(S' = 1 S = 1)$ (refer to text)	63.4 %
6.	IGE of lifetime earnings	0.6
7.	Gini coefficient of lifetime earnings	0.488
8.	Gini coefficient of retirement wealth	0.62
9.	Fraction of population receiving zero bequests	45%

Parameter	$\gamma_0$	$\gamma_1$	$\gamma_p$	$\gamma_k$	$\rho_a$	$\rho_\epsilon$	$\sigma_a$	$\sigma_v$	$\theta$
Value	0.63	0.74	0.97	0.64	0.35	-0.55	0.17	0.52	0.54
Chosen in	$\beta$	$v$							
Equilibrium	0.99	0.60							

Table 2: SMM Moments and Calibrated Parameters. The probability measure  $P(\cdot)$  is taken over the equilibrium stationary distribution.

### 3.2 Simulated Method of Moments

In sum, 16 parameters are taken from the literature or exogenously fixed by the data, see Table 1. As explained above, three parameters,  $(\beta, v, \bar{e})$ , are found in equilibrium. The remaining 9 parameters are chosen by using the model to simulate 9 equilibrium moments to match an exactly identified number of empirical moments. Specifically, the parameter vector is

$$\Theta = [\gamma_0 \ \gamma_1 \ \gamma_p \ \gamma_k \ \rho_a \ \sigma_a^2 \ \rho_\epsilon \ \sigma_v^2 \ \theta]'$$

and the vector of empirical moments,  $M_s$ , is summarized in Table 2. Moments 1-3 and 5 are computed from the NLSY79. College,  $S = 1$ , is defined as at least one year of education beyond high school graduation, and high school graduates or below are defined as  $S = 0$ .<sup>12</sup> To obtain the persistence of education we refer to the Children of NLSY79, from which we can compute  $\mathbb{P}(S' = 1|S = 1)$  in the sample.

The education expense ratio is from the 1990 Digest of Education Statistics. Note that this is the aggregate of both public and private expenses. The IGE of lifetime earnings is from Mazumder (2005) using data from the SER. The number 0.6 is the coefficient obtained by running a regression

<sup>12</sup> Hence, we make no distinction between high school dropouts and high school graduates, nor college dropouts, some college, college graduates and beyond. Increasing the education categories would perhaps make the study more interesting, however the numerical analysis becomes exponentially costly.

on the logarithm of fathers' earnings averaged over the years 1970-1985, where the dependent variable is the logarithm of children's earnings (both sons and daughters) averaged over the years 1995-1998. The Gini coefficient of lifetime earnings is from [Leonesio and Bene \(2011\)](#). Their data set is based on Social Security Administration data from 1980-2004, for more than 3 million people. The number we take is the Gini coefficient of a 12-year present discounted value of earnings using data on men aged 31-50 in 1993. The retirement wealth Gini is computed in [Hendricks \(2007\)](#) using data from the PSID. [McGarry \(1999\)](#) documents detailed information on intergenerational transfers using data from the HRS and AHEAD. The AHEAD survey elicits the subjective probability of leaving a bequest, of which the sample average is 0.55.<sup>13</sup>

We find the point estimate  $\hat{\Theta}$  by

$$\hat{\Theta} = \arg \min_{\Theta} [M(\Theta) - M_s]' W_s [M(\Theta) - M_s], \quad (5)$$

where  $M(\Theta)$  are the simulated model moments and  $W_s$  is a weighting matrix. Most of the empirical moments come from different data sources, so we set  $W_s = \mathcal{I}$ , the identity matrix, as it is not clear how to obtain an optimal weighting matrix  $\hat{W}_s$ .<sup>14</sup> The resulting benchmark parameters are summarized in the top panel of [Table 2](#).

## 4. Operationalizing EOP

Previous EOP concepts focus on the distribution of an outcome variable  $Y$ . This is viewed as the outcome of (relative) efforts  $E$  that an individual is solely responsible for, and therefore be rewarded, and heritage or background types  $B$  that an individual played no role in shaping, i.e. circumstances that an individual cannot be held responsible for ([Roemer, 1993](#)). More recently [Vallentyne \(1997, 2002\)](#) theorized how luck should be rewarded, arguing that some types of luck should be (relatively) rewarded ( $L_e$ ) while some should not ( $L_b$ ). Formally, outcome  $Y$  is then defined as a function:

$$g : (E, B; L; \Omega) \mapsto Y, \quad L = [L_e \ L_b] \quad (6)$$

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<sup>13</sup> In comparison, 43% of respondents in the HRS respond that affirmatively to the question on whether they "expect to leave a sizable inheritance."

<sup>14</sup> Since the parameters are recovered using an identity weighting matrix, it makes no sense to present standard errors.

where  $\Omega$  is the environment which can also be shaped by a planner. It is implicitly assumed that this function is known, in particular how it may respond to the environment  $\Omega$ : this is also true in our model. Since  $(E, L)$  are difficult to observe in the data, EOP has been discussed via means of the conditional distribution  $F(Y|B)$ . In a model without luck, [Roemer \(1993\)](#) argues that although justice requires that these conditional distributions should be equal, it is infeasible to achieve this so that some measure (median, mean, or quantile) be equalized. [Lefranc et al. \(2009\)](#) presents how to operationalize such concepts empirically, cautioning that luck cannot be separated from effort in the data.

Our goal is not to define a new concept but to operationalize these concepts in our model. For this, we need to decide on what corresponds to  $(Y, E, B, L_e, L_b)$ . For what follows, a variable with the subscript  $-1$  denotes the variable of the previous generation.

#### 4.1 Luck

One advantage of having a model is that we know exactly what  $L_b$  is as opposed to  $L_e$ . Our model has two sources of luck, genetic ability  $a$  and market luck  $\epsilon$ , which are exogenous to any individual choices by construction. Most authors agree that own ability should be rewarded ([Vallentyne, 1997](#); [Arneson, 2010](#)). Otherwise, it could violate other ethical values, such as the libertarian value of self-ownership of natural endowments. However, having a high ability *parent* does not constitute self-ownership. In other words, only the part of ability that is uncorrelated with the parent should be rewarded, and relatively.

The market luck shock can be interpreted as [Dworkin \(1981\)](#)'s notion of "brute luck," in particular "later brute luck" ([Vallentyne, 2002](#)) since it occurs in adulthood. Notice that conditional on parental ability, all individuals face the same amount of risk: parental ability is the sole source of *ex ante* inequality in terms of market luck. So according to ([Vallentyne, 2002](#); [Arneson, 2012](#)), what should be compensated for is *not* any of these shocks per se, but the ability level of the parent.

This is not the end of the story. In our intergenerational setup, we need to consider not only the luck of the previous and current generations but also the next. Should an individual be rewarded or compensated for having a high ability child, or child with great luck in the market? If we are consistent with how we separated the luck between generations  $t - 1$  and  $t$ , it becomes clear that there is no need to compensate generation  $t$  for whatever happens in generation  $t + 1$ : the correlation between generations  $t$  and  $t + 1$  come solely through  $a$ , which is a natural endowment, and any shocks to generation  $t + 1$  is then "later brute luck."

In summary,

1.  $F(a, \epsilon | a_{-1}) \sim L_\epsilon$ : conditional on parental ability, all luck should be rewarded relatively;
2.  $(a_{-1}, \epsilon_{-1}) \sim L_b$ : parental luck should be compensated for;

and note that 1. includes luck occurring to one's offspring as well.

## 4.2 Outcomes and Efforts

While it was rather straightforward how to separate luck by construction of the model, mapping  $(Y, E, B)$  into our model is less obvious because all such variables in our model are endogenous. This also raises some insight into intergenerational concerns that have not received much attention in the literature.

In philosophical applications investigating what constitutes EOP and economic models that focus on childhood,<sup>15</sup> the wage processes or outcomes of the parent generation are taken as exogenous and children's outcomes considered final. What gives us a novel perspective on EOP is that these processes themselves are the outcomes of efforts of previous generations, and that the choices of the child generation forms the bases for the grandchild generation. Formally, in (6),  $(E, B)$  themselves are in fact (assuming that  $\Omega$  remains constant)

$$B \equiv B(E_{-1}, B_{-1}, L_{-1})$$

while although  $E$  is in principle also functions of previous generation variables, since it is relative given  $B$  we can assume this has been already accounted for. However, even then, such efforts would not only depend on one's self but also their offspring, so

$$E \equiv E(E', L').$$

This raises the concern that if  $B$  is compensated away, previous generations' efforts are not being rewarded. Conversely, if we are to ignore that some of  $B$  reflects previous generations' efforts, the part of  $g(E, \cdot)$  that is affected by future generations should not be considered. In other words, previous generations should be compensated for what current generations are compensated for, or if not, current generations should not be punished for what previous generations are not rewarded

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<sup>15</sup>E.g., [Caucutt and Lochner \(2012\)](#); [Del Boca et al. \(2013\)](#).

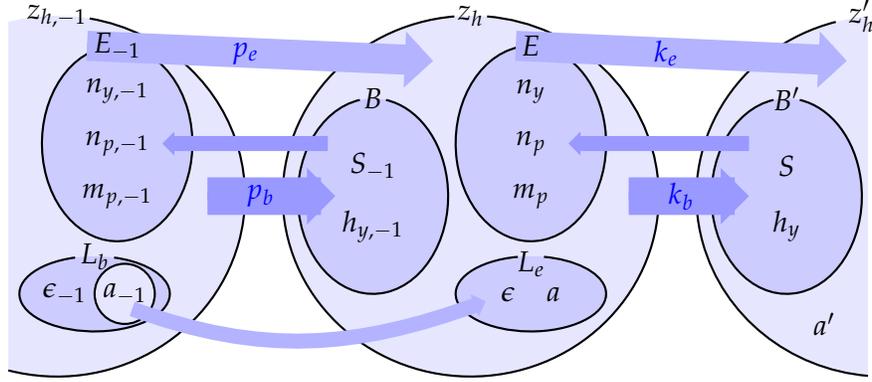


Figure 2: Overlapping generations of luck, background and effort.

Part of outcome  $Y$  is formed by parental efforts  $E_{-1}$  through  $p$ , which itself depends on  $B$ . Furthermore,  $B$  depends on  $Y_{-1}$ , which includes previous generation's efforts. Hence if  $p$  is not rewarded then,  $k$  should be compensated for. The only luck that should be compensated is parental ability,  $a_{-1}$ . All other outcomes are either "later brute luck" in the sense of Vallentyne (2002), or natural endowments that constitute the individual.

for. This is best depicted graphically Figure 2: if the  $p$ -links are ignored, so should the  $k$ -links, and vice versa. Note that we are not advocating whether or not such links should be ignored, but only for logical consistency.<sup>16</sup> In our quantitative experiments, we consider both stances and show that they can have vastly different implications.

Let us formalize the graphical depiction in Figure 2. An advantage of our model-based approach is that we can easily compare the states and choices of the young and old age adults: all variables have clear interpretations, from which the authority making ethical judgments can choose. Choices that occur from age 24 onward constitute effort, which, conditional on  $B$ , form the effort variables  $E$ . For such choices, then, the individual should be rewarded according to Roemer's EOP definition. It remains to determine  $B$ .

Since abstract discussions as well as previous empirical studies have focused on income or earnings, let us first inspect the lifetime earnings of a young adult in (3) as a potential candidate for  $Y$ . This is conducive since earnings are observable in the data. Ignoring  $f(e_k)$ , which is earned by the child and not the parent, and also time allocation choices which is part of  $E$ , the primary determinants of earnings are  $\mathbf{x} = (S, a, \epsilon, h_y)$ : one's education level, ability, market luck, and human capital level upon independence. We already argued above that  $a_{-1}$  and not  $(a, \epsilon)$  should be compensated for. Own  $(S, h_y)$ , in turn, depend on both parental investments, which are part of  $E_{-1}$ , and the parent's  $(S_{-1}, h_{y,-1})$ . This is represented by  $p_b$  and  $B$  in Figure 2, respectively.

<sup>16</sup>In reality, the interdependency of states and choices are much more complex than the few linkages we depict in Figure 2. We are not depicting all the linkages here for the sake of not cluttering the entire diagram with arrows.

At first glance, it seems that  $B$  should be equalized away according to Roemer's notion of EOP. However,  $E_{-1}$  is a function of  $B$ , so then the parent would not be rewarded for efforts that he made not for himself, but for his child. This is the direct linkage,  $p_e$ . Furthermore,  $B$  is also partially a function of the previous generation's efforts through  $Y_{-1}$  and  $p_b$ . Then, full compensation would violate the principle that efforts—not of the current generation, but of the previous generation, and the one before that, and *ad infinitum*—should be rewarded. This is the indirect linkage.

If all generations' efforts are to be rewarded, then, there is, in fact, no role for  $B$ : it can be expressed as an infinite regress of genetic and brute luck. The only element of  $Y$  that should be compensated for is whatever inequality that stems from  $L_b \sim (\epsilon_{-1}, a_{-1})$  alone. The problem in practice is that such infinite histories are not observed, but our model offers a solution. Since it is solved so that intergenerational investments are consistent with a long-run equilibrium, we can consider whether a far away ancestor is rewarded for his dynastic efforts and examine EOP accordingly.

Besides practical limitations, one could still argue that when considering justice across generations, such far-away generational efforts should not be rewarded so that  $B$  should still remain a primary concern (Arneson, 1998). Nonetheless, if a planner decides to ignore the  $p$ -linkages for whichever reason, for redistributive justice across generations to hold the outcome variable  $Y$  should also be purged of  $k$ -linkages as well. In laymen terms, if at least part of the motivation of an individual to achieve a higher social status was to better the prospects of his offspring, this should be rewarded. Otherwise, when comparing incomes of the child generation, one should first deduct the education expenses the child incurs for the *grandchild* generation.

To take all this into consideration, all EOP measures are analyzed by examining the conditional distributions of three objects. The first two are

1.  $z_h$ : children's lifetime earnings, which is observable in the data and has been the conventional object of analysis in previous studies.
2.  $z_w \equiv z_o - z'_y$ , i.e., implied wealth at age 42 less bequests to be left for the grandchild. This is free of investments the child has made in the grandchild, both in terms of human capital and assets.

Our structural model allows us to do more. Since individuals in our model are rational, the reason they invest more or less in their children is solely because they derive utility from doing so. The continuation utility of the child factors in all costs and benefits that come from previous and

Parent Type (quartiles)	Child's Outcome Distribution		
	$z_h$	$z_w$	$c_v$
$z_{h,-1}$	Case 1	Case 3	Case 5
$a_{-1}$	Case 2	-	Case 4

Table 3: Types and outcome distributions considered.

subsequent generations. Since  $z_{h,-1}$  include efforts from previous generations, by comparing the conditional utilities to the financial measures in items 1. and 2, and the utilities when dividing types by  $a_{-1}$  types, we can analyze how such efforts are being rewarded *dynamically*. Indeed, it turns out that in our model, most of such efforts manifest themselves as utility gained from future generations (but at a decreasing rate), not one's own consumption. So specifically, our third object is

3.  $c_v \equiv [(1 - \chi)V_y]^{\frac{1}{1-\chi}}$  : to make the units comparable with  $z_h$  and  $z_w$ , we transform the variable into consumption equivalent units.

The types that are chosen for conditioning are based on either: i) parental  $z_{h,-1}$  (lifetime earnings) quartiles, which is a one-dimensional stratification of  $x_{-1}$  and also applicable to real data, or ii) the four discrete points of  $a_{-1}$ , parental ability, which is not observed in the data.<sup>17</sup> These cases are labeled 1-5 as shown in Table 3. If cases 2. and 3. closely mirror cases 4. and 5., respectively, we can conclude that factoring in the intergenerational investments brings us closer to utility measures of outcomes, rather than dollar outcomes.

### 4.3 Ordinal Measures of EOP

Given the numerically simulated stationary distribution  $\Phi$ , we compute both ordinal and cardinal measures of EOP. The ordinal measures are developed in Lefranc et al. (2008, 2009) and applied to the variables discussed above. Their goal was not to propose a new concept for EOP, but to operationalize how concepts such as Roemer's would be applied to real data. We simply apply the same methods to our simulated data, but to a wider range of variables as discussed above. For each of the 5 cases, we examine the following for each pair of conditional distributions:

1. conditional C.D.F.'s and Lorenz curves (for visual inspection)

<sup>17</sup>Of course in theory we can increase the number of types to as many as we wish, but we keep the number small so that the comparisons are easily comprehensible. In particular, note that we are ignoring conditioning on  $\epsilon_{-1}$ .

2. Equality and stochastic dominance (FSD,SSD) tests
3. Lorenz dominance tests

Here we give a brief explanation of items 2.-3.; we refer the interested reader to [Lefranc et al. \(2008\)](#) for details. Item 2 is conducted as follows:

1. Test whether we can statistically reject that the distributions are equal using a two-sample Kolmogorov-Smirnov test. If we cannot, EOP is supported in the strong sense.
2. If not, test SSD for the two distributions. Check, statistically, whether neither dominates the other or both dominate each other. If they do, EOP is supported in the weak sense. For checking stochastic dominance, we simulate  $p$ - values via recentered bootstrap according to [Barrett and Donald \(2003\)](#).

Note the hierarchy of these tests. The first test amounts to saying that not only are different types compensated for, but also relative efforts and luck are equally rewarded within each type. This corresponds to an ideal notion of EOP that even in a simple stylized setup as in [Roemer \(1993\)](#) cannot be expected to be feasible. For the second test, there is a sense that the case when both dominate each other is "more equal" than when neither dominate each other; however given that the first test rejects equality, there is no formal method to rank the two outcomes. The FSD test only applies when EOP is violated, meaning one distribution dominates the other at the second-order; if in addition it dominates at the first-order it implies a strong violation of EOP since one type would be preferred to the other type for *any* unobserved effort-luck combination. The Lorenz dominance test also only applies when EOP is violated. A Lorenz curve normalizes all outcome variables so that a mean effect is removed, revealing whether comparable effort-luck combinations in the sense of [Roemer \(1993\)](#) are being equally rewarded, conditional on circumstances.

#### 4.4 Cardinal Measures of EOP

Since the above measures are all ordinal (qualitative) and binary (equal or unequal), we also compute cardinal measures so that we can quantify the degree of inequality of opportunity. Since policy changes shift the entire stationary equilibrium, this is especially conducive for policy experiments since we can compare the outcomes quantitatively. We somewhat deviate from previous studies and use a normalized Theil index for a cardinal comparison. While a Gini coefficient or similar variants may be conceptually closer to stochastic dominance tests, we opt for the Theil

index because it is readily decomposable into a Theil index across the mean of each type, plus the weighted average of Theil indices within types. Hence it is ideal for giving a sense of whether inequality stems from inequality across types or unequal compensations across effort-luck combinations within types.

Besides this, the Theil index, which is defined as maximum minus observed entropy, is ideal for positive economic analysis. [Grünwald and Dawid \(2004\)](#) have shown (subject to some regularity conditions) that the problem of maximizing entropy is equivalent to minimizing worst-case expected loss. Hence, an unconstrained Rawlsian planner would be equivalent to an entropy maximizer. Then, similarly to [Sen \(1974, 1976\)](#)'s social welfare function that considers both average incomes and the Gini coefficient, one could consider a mixed utilitarian-Rawlsian planner that maximizes a utilitarian objective function subject to an entropy constraint. The attractiveness of such an approach is that the planner's concern for equality can be represented by a constant parameter, which the economist can take as given, and that a dynamic decision problem can be represented in a simple recursive form ([Hansen and Sargent, 2007](#)).

Suppose there are  $J$  types indexed by  $j = 1, \dots, J$ , each of mass  $\mu_j$ , so total population is  $\mu = \sum_{j=1}^J \mu_j$ — $\mu$  does not have to equal 1. Denote the conditional distribution of outcomes,  $y$ , within each type by the c.d.f.  $F_j$ . The Theil index in this setup is

$$\begin{aligned} T &= \sum_{j=1}^J \frac{\mu_j}{\mu} \cdot \int \frac{y}{\bar{y}} \cdot \left( \log \frac{y}{\bar{y}} \right) dF_j(y) = \sum_{j=1}^J s_j \left[ \frac{\bar{y}_j}{\bar{y}} \cdot \left( \log \frac{\bar{y}_j}{\bar{y}} \right) + \int \frac{y}{\bar{y}_j} \cdot \left( \log \frac{y}{\bar{y}_j} \right) dF_j(y) \right] \\ &= T_b + \sum_{j=1}^J s_j T_j \end{aligned}$$

where

$s_j$  = type  $j$ 's share of total  $y$  in the population,

$\bar{y}$  = mean of  $y$  in the population,

$\bar{y}_j$  = conditional mean of  $y$  in type  $j$ ,

$T_b$  = Theil index across the means of each type, and

$T_j$  = Theil index within each type.

Then the problem of maximizing the expected outcomes across groups as in [Roemer \(1993\)](#) is equivalent to minimizing  $T_b$ . More objectively, given a total level of inequality  $T$ , EOP can be

quantified by the contribution of between-type inequality  $T_b/T$ . Furthermore, by cross-examining the  $T_j$ 's of each type  $j$ , we can also compare the extent to which similar relative efforts are being rewarded similarly.

We also compute the change in a few simple aggregate statistics as is typically done in the economics literature: i) the IGE, or intergenerational elasticity of lifetime earnings, and ii)  $\Delta \log \bar{y} = \log \bar{y} - \log \bar{y}_B$  where  $\bar{y}$  is the mean of  $y \in \{\bar{z}_h, \bar{z}_w, \bar{c}_v\}$ , and  $y_B$  the mean of  $y$  in the benchmark: this is a utilitarian measure of gain (loss) in efficiency. Since our equilibrium is stationary, the square of the IGE measures the intergenerational component of the cross-sectional variance of lifetime earnings, while  $\Delta \log \bar{c}_v$  measures the consumption equivalent welfare gain from a policy change in a utilitarian sense.

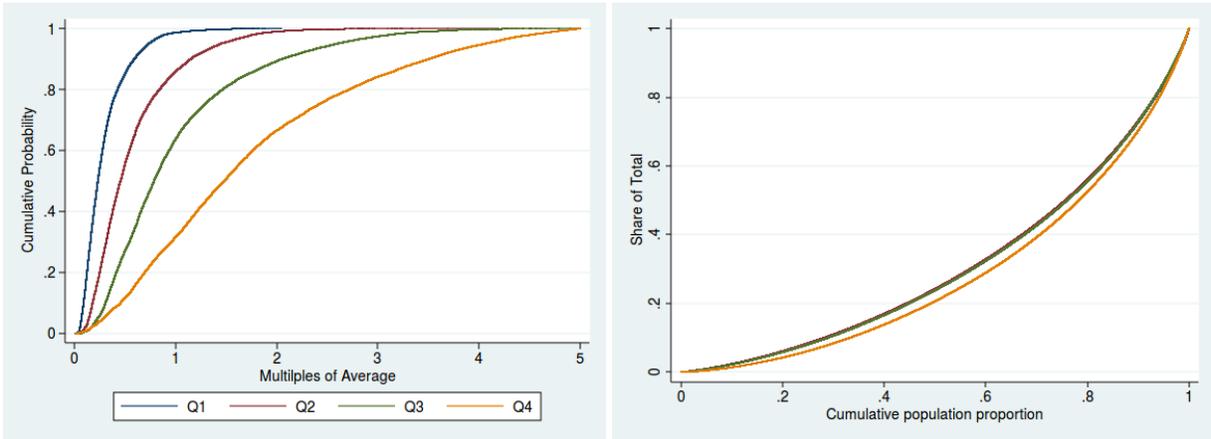
## 5. Results

First, we analyze the benchmark equilibrium to demonstrate that accounting for intergenerational investments matter for EOP, both between and within types. We then turn to our counterfactual policy experiments.

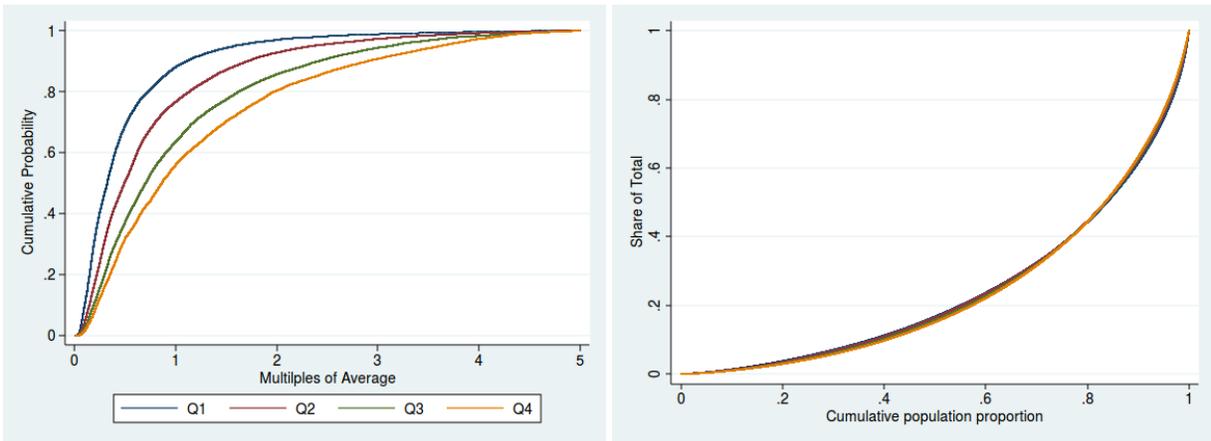
### 5.1 Visual Inspection and Lorenz Dominance

Figures 3 and 4 show the c.d.f.'s to the left and Lorenz curves to the right, for each of the cases 1.-5. Since the model is stylized, the equilibrium distribution is smooth and nicely shaped, unlike what is observed in the data. However, visual inspection reveals that our division of parental states, in all cases, qualitatively deliver an order of distributions that are comparable to what [Lefranc et al. \(2009\)](#) find for the empirical distributions in France, where they condition on parental occupations. What is visually clear, however, is that when we divide types by parental ability rather than lifetime earnings, the distance between the distributions shrink dramatically. It is also clear that when we condition on parental lifetime earnings but instead look at children's net wealth ( $z_w$ ) as the outcome variable, the distance between the distributions widen.

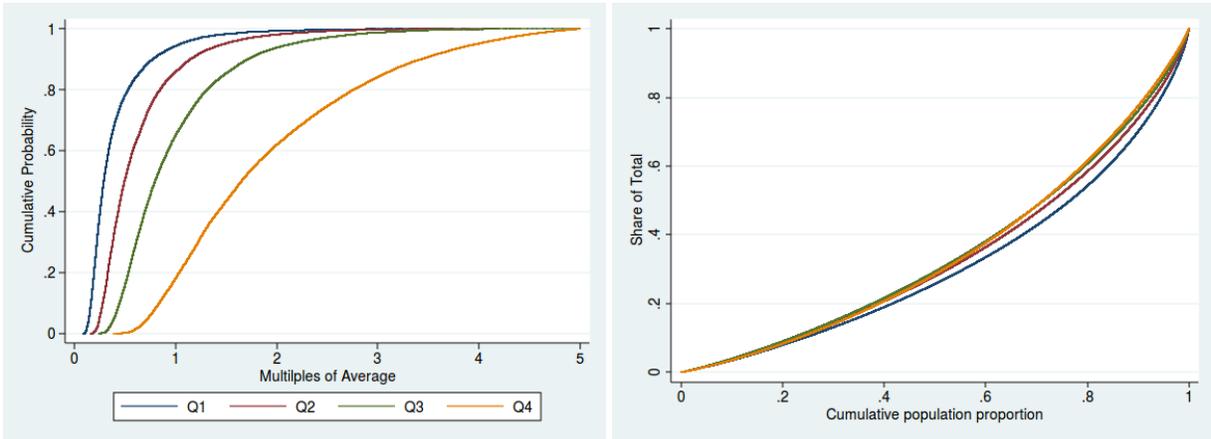
Unfortunately, the smoothness of our simulated distributions give clear dominance orderings regardless of the visual distances—it is always the case that the conditional distributions of the higher parental quantiles first-order dominate the lower ones, regardless of how we divide types. In other words, the dominance orderings do not give us a sense of how large the levels of dominance is. This would not be the case in the real data that is observed with noise and includes more



(a) Case 1: conditional distributions of  $z_h$  by  $z_{h,-1}$  quartiles

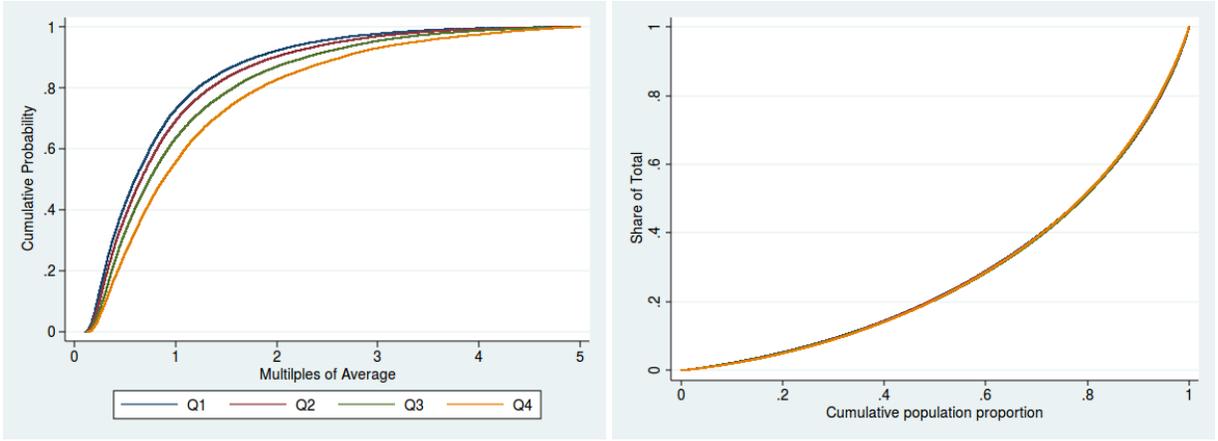


(b) Case 2: conditional distributions of  $z_h$  by  $a_{-1}$  quartiles

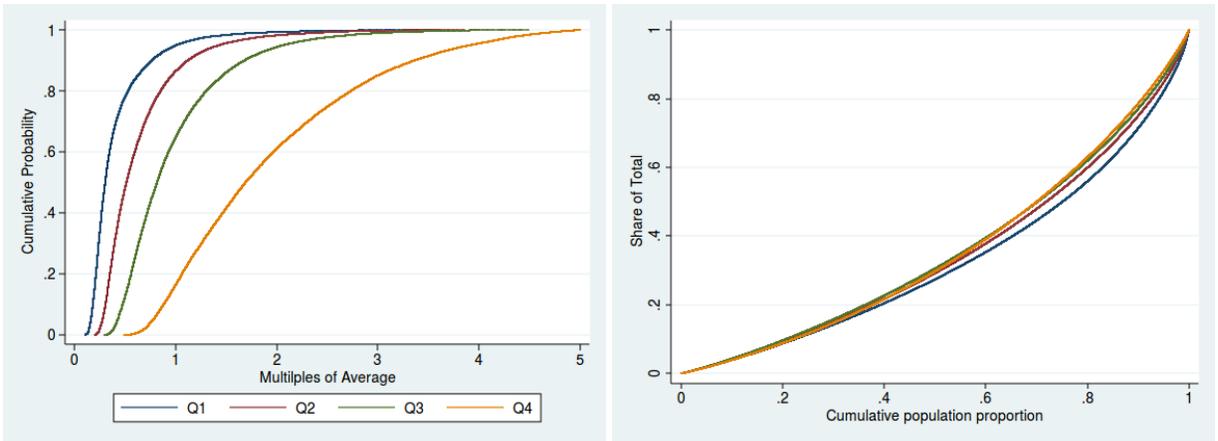


(c) Case 3: conditional distributions of  $z_w$  by  $z_{h,-1}$  quartiles

Figure 3: Outcome distributions and Lorenz curves by type, cases 1-3. For each case, the variables are demeaned by the unconditional mean.



(a) Case 4: conditional distributions of  $c_v$  by  $a_{-1}$  quartiles



(b) Case 5: conditional distributions of  $c_v$  by  $z_{h,-1}$  quartiles

Figure 4: Outcome distributions and Lorenz curves by type, cases 4-5. For each case, the variables are demeaned by the unconditional mean.

randomness than the channels we include in the model, or were we to assume some noise associated with observing the simulated variables. Instead of pursuing this route, below we instead turn to the Theil index for a cardinal comparison.

The Lorenz curves are also similar to [Lefranc et al. \(2009\)](#)'s findings, in the sense that it appears as if at least visually, the outcome variables can be expressed as a fixed component that varies by type, multiplied by a random term that does not depend on type. This implies that relative luck and effort as a whole is rewarded similarly across types, once a first-order effect is accounted for, which is unexpected: although we assume type-independent shocks, remember in fact that all the shocks are intercorrelated so that it is not clear why conditional outcomes should have similar spreads, not to mention the immense non-linearity associated with human capital acquisition.

Case 1. $(z_{h,-1}, z_h)$				Case 2. $(a_{-1}, z_h)$				Case 3. $(z_{h,-1}, z_w)$			
	Q2	Q3	Q4		Q2	Q3	Q4		Q2	Q3	Q4
Q1	=	=	?	Q1	>	>	<	Q1	<	<	<
Q2		>	?	Q2		?	?	Q2		<	?
Q3			?	Q3			<	Q3			?

Case 4. $(a_{-1}, c_v)$				Case 5. $(z_{h,-1}, c_v)$			
	Q2	Q3	Q4		Q2	Q3	Q4
Q1	=	=	<	Q1	<	<	<
Q2		=	<	Q2		<	?
Q3			<	Q3			?

Table 4: Lorenz Dominance tests. Case  $I$ ,  $(x, y)$  refers to the each case  $I$  where  $x$  is the type variable and  $y$  is the outcome variable. Equality of the Lorenz curves is tested by Kolmogorov-Smirnov. Lorenz dominance is tested by testing 2nd order stochastic dominance of variables demeaned by type. We simulate  $p$ -values by recentered bootstrap following [Barrett and Donald \(2003\)](#). The null is accepted if not rejected at the 95% confidence level, so all our notions are weak orderings.

=: Lorenz curves are equal.

>: row Lorenz dominates column.

<: column Lorenz dominates row.

?: cannot be ranked according to either test.

This is also confirmed in Table 4, where unlike the stochastic dominance tests, it is impossible to discern a clear pattern across the conditional distribution for most cases. What is even more striking is that again, when we condition on parental ability as depicted for cases 2. and 4., the overlap is even more extreme for both  $z_h$  and  $c_v$ , while the lowest type displays visually less equality when taking  $z_w$  or  $z_h$  as the outcome variable conditional on  $z_{h,-1}$ . For cases 3. and 5., the dominance relationships also exactly overlap.

In summary, we have preliminary confirmation that indeed, accounting for intergenerational investments comes closer to measuring the EOP of welfare in our model, than simply looking at lifetime earnings across both generations.

## 5.2 Theil Index Decomposition

The first column in Table 5 shows the Theil indices for our benchmark equilibrium, all normalized to lie between 0 and 100—hence, the quantitative values can be directly compared across all cases and groups. The rows titled "Between" correspond to the between-type indices,  $T_b$ . Our visual inspections from the previous subsection are numerically confirmed: notice that the between-type inequality comprises 46% of total inequality when conditioning on parental lifetime earnings, but

drops to 8% when conditioning on parental abilities. Hence, if one takes the stance that intergenerational efforts should not be compensated away, the U.S. displays substantially higher EOP than might be expected. On the other hand, if we take the stance that intergenerational efforts *should* be compensated for, i.e., we define the child's outcome variable as  $z_w$  instead of  $z_h$ , EOP looks worse.

We argued in section 4 that, depending on which stance is taken on how types should be chosen, either of the two comparisons that should be made in light of intergenerational concerns. The Theil index offers a quantitative gauge of whether this is an appropriate measure by comparing the outcome distributions with the distributions of utility derived by the child generation. As seen in the first column of Table 5, the between-type Theil indices in cases 2. and 3. mirror cases 4. and 5., a confirmation that intergenerational concerns must be taken into account when measuring EOP—this is true regardless of whether one believes intergenerational efforts should be compensated for or not. The between index contribution for case 1. is 46%, while it is 52% for case 3.: looking at net wealth is more likely for us to conclude that EOP is less satisfied than lifetime earnings. Furthermore, this figure is similar to the 54% in case 5. Likewise, the between index contribution is a mere 8% in case 2., while in case 4. it is at its minimum, 2%. Hence EOP is almost satisfied if we believe that intergenerational efforts should be rewarded, and we get a similar picture if we looking at lifetime earnings directly as well (case 2).

But why is this the case? In Lee and Seshadri (2014), we show that most of the IGE, in fact, can be explained by human capital investments, especially by the parent. Genetic abilities are not so much reflected across intergenerational earnings once early childhood investments are accounted for. Once this is understood, it is not so surprising that conditioning on parental ability alone results in small differences in the lifetime earnings distributions of the child generation: the overlapping of the conditional distributions is an indicator that intergenerational human capital investments are being rewarded across multiple generations, and overwhelms any advantages that may come from having a high ability parent. This is confirmed by the fact that the between Theil contribution to the inequality between continuation values are minuscule when conditioning on parental ability. Perhaps even more surprisingly, the within-type Theil indices also tend to converge to each other, implying similar rewards to relatively similar levels of effort and luck.

In contrast, in cases 3. and 5. when we condition on parental lifetime earnings but instead look at  $z_w$ , the child's net wealth at middle age, as the outcome variable, EOP is even more strongly rejected than when looking at the child's lifetime earnings. This is again due to the importance of early childhood investments, but now interacting with intergenerational borrowing constraints.

		<i>B</i>	<i>E</i>	<i>G</i>			<i>B</i>	<i>E</i>	<i>G</i>
<b>TOTAL</b>		<b>4.32</b>	<b>3.52</b>	<b>4.56</b>	<b>TOTAL</b>		<b>2.66</b>	<b>2.14</b>	<b>2.79</b>
Between		46%	43%	47%	Between		2%	3%	2%
Within		54%	57%	53%	Within		98%	97%	98%
Case 1. ( $z_{h,-1}, z_h$ )	Q1	2.30	1.92	3.21	Case 4. ( $a_{-1}, c_v$ )	Q1	2.93	2.38	3.08
	Q2	2.22	2.00	2.22		Q2	2.95	2.36	3.10
	Q3	2.30	2.10	2.25		Q3	2.97	2.38	3.13
	Q4	2.94	2.51	2.92		Q4	2.91	2.30	3.08
Between		8%	8%	8%	Between		54%	52%	56%
Within		92%	92%	92%	Within		46%	48%	44%
Case 2. ( $a_{-1}, z_h$ )	Q1	4.59	3.90	4.90	Case 5. ( $z_{h,-1}, c_v$ )	Q1	2.11	1.90	2.31
	Q2	4.44	3.61	4.66		Q2	1.57	1.40	1.58
	Q3	4.44	3.59	4.76		Q3	1.28	1.19	1.28
	Q4	4.48	3.55	4.80		Q4	1.24	1.22	1.26
<b>TOTAL</b>		<b>2.87</b>	<b>2.37</b>	<b>3.02</b>					
Between		52%	49%	53%					
Within		48%	51%	47%					
Case 3. ( $z_{h,-1}, z_w$ )	Q1	2.38	1.99	2.59					
	Q2	1.76	1.55	1.79					
	Q3	1.45	1.32	1.45					
	Q4	1.42	1.17	1.43					

Table 5: Theil Index Decompositions. All indices are normalized to lie between 0 and 100. The rows "Between" and "Within" are the percentage contributions of the between- and within-type Theil indices to the total Theil index. Case  $I.(x, y)$  refers to the each case  $I$  where  $x$  is the type variable and  $y$  is the outcome variable.  $B$ ,  $E$  and  $G$  refer to different stationary equilibria: the benchmark, only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

Children with low  $z_{h,-1}$  parents have low  $z_h$  but even lower  $z_w$  because of the binding constraints. The incentive to invest in children is large and if possible the current child generation would want to borrow from the next (grandchild) generation, but they are not permitted to do so and hit the zero bequest constraint. Indeed, in [Lee and Seshadri \(2014\)](#) we show that parents hitting the constraint invest in their children as much as their counterparts who do not, implying a larger gap in wealth than in earnings. That this is the proper outcome variable for which EOP should be considered is again confirmed by the fact that the between Theil contribution to the inequality between continuation values mirrors closely that of the inequality between  $z_w$  rather than  $z_h$ .

The coincidence of the Lorenz curves in cases 2. and 4., and widening gaps between the curves in cases 3. and 5. is also confirmed by comparison of the within Theil indices. These are remark-

	<i>B</i>	<i>E</i>	<i>G</i>
<i>IGE</i>	0.60	0.53	0.60
Gini $z_h$	0.48	0.44	0.49
RW Gini	0.62	0.59	0.63
$\Delta \log \bar{z}_h$	-	34%	-1%
$\Delta \log \bar{z}_w$	-	29%	-1%
$\Delta \log \bar{c}_v$	-	28%	-1%

Table 6: Counterfactual moments (*IGE* and Gini's) and Average Outcomes. *IGE* is the correlation of lifetime earnings across generations.  $z_h$  is lifetime earnings, and  $(z_w, c_v)$  are defined in the text. RW stands for "Retirement Wealth." For each  $y$ ,  $\bar{y}$  refers to its cross-sectional average. *B*, *F*, *E* and *G* refer to different stationary equilibria: the benchmark, flat instead of progressive earnings taxation, only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

ably close to each other in the former case, reinforcing the strong dependence on early human capital investments of the model. In contrast, in cases 3. and 5., it is the lower types, and especially the lowest type, that has the least similar index. Given that lower types will have the more intergenerationally borrowing constrained parents, not only between all types but even within the lower types, inequality of opportunities are magnified.

### 5.3 Policy Analysis

Given our benchmark parametrization, we compare two different steady states: shifting all lump-sum subsidies to education subsidies, and shifting all education subsidies to lump-sum subsidies. In the tables that follow, we label these cases as *E* (only education subsidies) and *G* (only lump-sum subsidies), respectively, and *B* refers to the benchmark.

Both policies are inherently redistributive: education subsidies support the human capital formation of low income families, and lump-sum subsidies provide public insurance in the face of idiosyncratic risk, which is mostly important for low-income families. By design of holding the total amount of subsidies fixed, they may also crowd out each other: education subsidies may lead to too much education at the expense of other consumption-saving opportunities, while lump-sum subsidies can disincentivize human capital investment and reduce long-run per capita income or welfare.<sup>18</sup> To attain a sense of the effect that such changes will have on equity and efficiency,

<sup>18</sup>Of course there can be transition costs associated with increasing human capital when education subsidies are too high as well, but our goal is to demonstrate a divergence between equality of outcomes and EOP, not compute exact welfare gains or losses as in the conventional economics literature (e.g. [Krueger and Ludwig \(2013\)](#); [Badel and Huggett \(2014\)](#)).

we first present the IGE, average outcomes and Gini indices of lifetime earnings and retirement wealth for each case. These are the objects typically focused on in previous economic studies, and the outcomes are in line with previous results (including our own in [Lee and Seshadri \(2014\)](#)). However, a comparison of the Theil indices reveal that EOP is barely affected.

Table 6 shows that education subsidies reduce both the IGE and Gini's, consistent with the "Gatsby" curve found in empirical work ([Corak, 2013](#)). This is because they directly affect human capital investments by ensuring a certain level of human capital. The impact of the education subsidies must be analyzed in two directions: column *E* represents an increase, and *G* a reduction, while keeping the overall level of subsidies neutral relative to average earnings. Observe that even when education subsidies are compensated for with additional lump-sum subsidies (column *G*), it leads to an increase in inequality of outcomes, albeit small. The two columns together imply that enough insurance is being provided by the lump-sum subsidies, so that it is better to incentivize a higher level of education which lead to more equal outcomes.

Not only do education subsidies equalize outcomes, it is also substantively efficient—all variables ( $z_h, z_w, c_v$ ) increase by significant amounts, in particular, consumption equivalent welfare increases by as much as 28%. While the magnitude is somewhat large, we are not computing transition costs nor modeling exactly how the subsidies are financed. In any case, the fact that education subsidies can enhance both equity and efficiency itself is not so surprising, especially in light of the dynamic complementarities we model for human capital acquisition ([Benabou, 1996](#); [Cunha and Heckman, 2007](#)).

What is surprising is that, looking back at Table 5, EOP as measured by the contribution of the between-type Theil index toward the total Theil index barely changes compared to the benchmark. This is not because the Theil index measures something orthogonal to the Gini index: the total Theil decreases significantly with more education subsidies (column *E*). Despite this fact, for cases 2. and 4., there seems to be no change in EOP compared to the benchmark at all. This is for two reasons: for cases 2. and 4., EOP is strongly present to begin with, and the relatively small size of the education subsidies (approximately 5% of average income) does little to push it down further. Furthermore, as should be clear by now our outcome variables are highly dependent on how much human capital each individual accumulates and even more importantly, how much they invest in their children. Given that cases 2. and 4. measure EOP under the view that intergenerational efforts should be rewarded, education subsidies, which favor the offspring of those dynasties for whose ancestors exerted *less* effort, should not be expected to improve.

But then what about cases 3. and 5.? First note that while the changes are small, there still is a positive improvement in EOP (as measured by the "Between" row). Closer inspection of the within-type Theil indices reveal that this is associated with the lowest type "catching up" with the other types, in terms of rewarding relative luck and efforts equally. Recall that the lower types are the most intergenerationally constrained, so there within exists the largest room for improvements in EOP. The fact that education subsidies reduce this gap indicate that the subsidies are alleviating the intergenerational borrowing constraints, but that the magnitudes are not enough. So for both between and within types, the size of the subsidies must be much larger, and/or designed to be much more progressive, to achieve higher levels of EOP.

## 6. Conclusion

In this paper, we apply EOP concepts to a quantitative economic model featuring a rich mechanism of human capital investment, intergenerational borrowing constraints, and government policies. We emphasize that, rather than looking at the lifetime earnings across two generations, intergenerational investments should be accounted for before analyzing the extent to which EOP is present. In comparison to EOP implications when only considering lifetime earnings, we find little evidence for lack EOP if intergenerational investments should be rewarded, while it is even more lacking if such investments should not be rewarded. We propose which variables should be considered to arrive at these conclusions and validate our proposals by also considering the distributions of continuation values. We also showed that, focusing all subsidies on education can increase efficiency while reducing inequality, but may still be not enough to rectify EOP at empirically observed levels of subsidies.

## References

- Arneson, R. (1989). Equality and equal opportunity for welfare. *Philosophical Studies* 56(1), 77–93.
- Arneson, R. (1999). Against rawlsian equality of opportunity. *Philosophical Studies* 93(1), 77–112.
- Arneson, R. J. (1998). Real freedom and distributive justice. In N. G. Jean-Francois Laslier, Marc Fleurbaey and A. Trannoy (Eds.), *Freedom in Economics: New Perspectives in Normative Analysis*. London and New York: Routledge.
- Arneson, R. J. (2010, Winter). Self-ownership and world ownership: Against left-libertarianism. *Social Philosophy and Policy* (27), 168–194.
- Arneson, R. J. (2012, Spring). Rethinking luck egalitarianism and unacceptable inequalities. *Philosophical Topics* 40(1), 153–169.
- Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory* 2(3), 244 – 263.
- Badel, A. and M. Huggett (2014). Taxing top earners: A human capital perspective. Technical report, Federal Reserve Bank of St. Louis.
- Barrett, G. F. and S. G. Donald (2003). Consistent tests for stochastic dominance. *Econometrica* 71(1), 71–104.
- Becker, G. S. and N. Tomes (1986, July). Human capital and the rise and fall of families. *Journal of Labor Economics* 4(3), S1–39.
- Behrman, J. R., A. D. Foster, M. R. Rosenzweig, and P. Vashishtha (1999). Women’s schooling, home teaching, and economic growth. *Journal of Political Economy* 107(4), pp. 682–714.
- Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy* 75, 352.
- Benabou, R. (1996, April). Equity and efficiency in human capital investment: The local connection. *Review of Economic Studies* 63(2), 237–64.
- Caucutt, E. M. and L. Lochner (2012, October). Early and late human capital investments, borrowing constraints, and the family. Working Paper 18493, National Bureau of Economic Research.

- Corak, M. (2013). Income inequality, equality of opportunity, and intergenerational mobility. *Journal of Economic Perspectives* 27(3), 79–102.
- Cunha, F. and J. J. Heckman (2007, May). The technology of skill formation. *American Economic Review* 97(2), 31–47.
- Cunha, F., J. J. Heckman, and S. M. Schennach (2010, 05). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica* 78(3), 883–931.
- Del Boca, D., C. Flinn, and M. Wiswall (2013, April). Transfers to households with children and child development. Working Papers 2013-001, Human Capital and Economic Opportunity Working Group.
- Del Boca, D., C. Monfardini, and C. Nicoletti (2012, March). Children’s and parents’ time-use choices and cognitive development during adolescence. Working Papers 2012-006, Human Capital and Economic Opportunity Working Group.
- Diamond, P. and J. Gruber (1999). Social security and retirement in the united states. In *Social Security and Retirement around the World*, NBER Chapters, pp. 437–473. National Bureau of Economic Research, Inc.
- Dworkin, R. (1981). What is equality? part 2: Equality of resources. *Philosophy & Public Affairs* 10(4), pp. 283–345.
- Gravelle, J. G. (2007). Historical effective marginal tax rates on capital income. Technical report, Congressional Research Service Report for Congress.
- Gruber, J. and E. Saez (2002, April). The elasticity of taxable income: evidence and implications. *Journal of Public Economics* 84(1), 1–32.
- Grünwald, P. D. and A. P. Dawid (2004). Game theory, maximum entropy, minimum discrepancy and robust bayesian decision theory. *The Annals of Statistics* 32(4), pp. 1367–1433.
- Guner, N., R. Kaygusuz, and G. Ventura (2013, July). Income taxation of u.s. households: Facts and parametric estimates. Working Papers 705, Barcelona Graduate School of Economics.
- Hansen, L. P. and T. J. Sargent (2007). *Robustness*. Princeton University Press, Princeton NJ.

- Heckman, J., L. Lochner, and C. Taber (1998, January). Explaining rising wage inequality: Explanations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. *Review of Economic Dynamics* 1(1), 1–58.
- Hendricks, L. (2007, 05). Retirement wealth and lifetime earnings. *International Economic Review* 48(2), 421–456.
- Huggett, M., G. Ventura, and A. Yaron (2011, December). Sources of lifetime inequality. *American Economic Review* 101(7), 2923–54.
- Krueger, D. and A. Ludwig (2013). Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous. *American Economic Review* 103(3), 496–501.
- Lee, S. Y. T. and A. Seshadri (2014). On the intergenerational transmission of economic status. Technical report, University of Mannheim.
- Lefranc, A., N. Pistoiesi, and A. Trannoy (2008, December). Inequality Of Opportunities Vs. Inequality Of Outcomes: Are Western Societies All Alike? *Review of Income and Wealth* 54(4), 513–546.
- Lefranc, A., N. Pistoiesi, and A. Trannoy (2009). Equality of opportunity and luck: Definitions and testable conditions, with an application to income in france. *Journal of Public Economics* 93(11-12), 1189–1207.
- Lefranc, A., N. Pistoiesi, and A. Trannoy (2012). The link between intergenerational earnings mobility and equality of opportunity: Application to france and the u.s.
- Leonesio, M. V. and L. D. Bene (2011). The distribution of annual and long-run us earnings, 1981-2004. *Social Security Bulletin* 71(1).
- Lucas, R. E. (2003, March). Macroeconomic priorities. *American Economic Review* 93(1), 1–14.
- Mazumder, B. (2005, July). Fortunate sons: New estimates of intergenerational mobility in the united states using social security earnings data. *The Review of Economics and Statistics* 87(2), 235–255.
- McGarry, K. (1999, September). Inter vivos transfers and intended bequests. *Journal of Public Economics* 73(3), 321–351.

- Okun, A. M. (1975). *Equality and efficiency : the big tradeoff*. Washington, D.C. :: The Brookings Institution.
- Roemer, J. E. (1993). A pragmatic theory of responsibility for the egalitarian planner. *Philosophy and Public Affairs* 22(2), pp. 146–166.
- Roemer, J. E. (2002). Egalitarianism against the veil of ignorance. *The Journal of Philosophy* 99(4), pp. 167–184.
- Sen, A. (1970). *Collective Choice and Social Welfare*. Holden-Day.
- Sen, A. (1974, November). Informational bases of alternative welfare approaches : Aggregation and income distribution. *Journal of Public Economics* 3(4), 387–403.
- Sen, A. (1976). Real national income. *The Review of Economic Studies* 43(1), pp. 19–39.
- Vallentyne, P. (1997). Self-ownership and equality: Brute luck, gifts, universal dominance, and leximin. *Ethics* 107(2), pp. 321–343.
- Vallentyne, P. (2002). Brute luck, option luck, and equality of initial opportunities. *Ethics* 112(3), pp. 529–557.