(continued from last time), \( Y(K, L) = 0.5 K^{2/3} L^{1/3}, s = 0.2, \delta = 0.1, n = 0.03 \)

we had finished (a) to (g), now let’s finish the rest of this problem.

(a) show that the production function is constant return to scale

(b) argue that it has diminishing marginal return in capital and labor

(c) exhibit the growth accounting formula

(d) express the production function in the form of per worker: \( y = f(k) \)

(e) solve for the steady state level of capital per worker

(f) illustrate the graphs for the growth rate of capital and transition path of capital per worker, show the growth rate in the graphs drawn.

(g) what is the steady-state of real GDP per worker?

(h) what happens if saving rate increases to 0.3, compute the new \( k^{**} \).

soln: \( 0.3(0.5 k^{-1/3}) = 0.06, k^{**} = (2.5)^{-3} \)

(i) what is the new steady-state of output per worker?

soln: \( 0.5(2.5)^{-2} \)

(j) repeat item (f) with the information from item (h), graphically explain the effect of change in saving rate.

soln: pp52, figure 4.1

(k) what if the technology increase from \( A = 0.5 \) to \( A = 1.0 \), compute the new steady-state \( k^{**} and y^{**} \). (we use the old saving rate \( s = 0.2 \))

soln: old \( K^{**} \) is 8, the new steady state \( K = 64 \), and the new \( y^{**} \) is 16

(l) repeat (f)

soln: pp54, figure 4.2
(m) What if the population growth rate increase from $n=0.03$ to $n=0.08$, do the same thing for computing steady-state $k^{ss}$ and $y^{ss}$ and graph the effect. (using old $A=0.5$)

soln: $k^{ss}$ and $y^{ss}$ is 1 and 0.5.

(n) What if the depreciation rate change from $\delta = 0.1$ to $\delta = 0.15$? (using the old $n=0.03$)

soln: $k^{ss}$ and $y^{ss}$ is $(5/3)^3$, and $0.5(5/3)^2$

(o) Can you think of a simultaneous change of $n$ and $\delta$ such that the $k^{ss}$ does not change? Why?

soln: as long as $s\delta + n$ does not change, $n=0.01$, and $\delta = 0.2$

(p) Big Congratulations, Good job!!! Now let’s move to Chapter 6.

**Markets, Prices, Supply and Demand**

**Market Structure**

@The Goods Market

We simply assume that each household runs a family business and uses labor ($L$) and capital ($K$) to produce goods ($Y$), through the production function $Y=A F(K,L)$. Households sell all the goods they produce on a goods market. Then households buy back from this market the goods that they want. One reason that a household buys a goods is for consumption. Another reason is to increase the stock of goods in the form of capital-machines, and buildings-used for production. This use of goods is called investment.

$P Y$ is the dollar value per year of the goods bought and sold on the goods market.

($P$ dollars buy 1 unit of goods, so 1 dollar buys $\frac{1}{P}$ units of goods). The expression $\frac{1}{P}$ is the value of $1$ in terms of the goods (real value).

@The Labor Market
Each household has two hats: Households supply labor on a labor market, $L^s$, and at the same time, Households, as managers of family businesses, demand labor in the quantity $L^d$ from the labor market. We assume that the quantity supplied, $L^s$ is a constant.

Nominal wage rate is $w$ (in unit of dollars per hour), and the real wage rate is $\frac{w}{p}$.

@The Rental Market

Similarly as the labor market, each household has two hats in the rental market: each household rents out all of its capital, $K^s=K$, and then rents back the quantity $K^d$. Our assumption that the supply of capital services is constant is analogous to our assumption that the supply of labor is constant.

In the rental market, households rent out capital, $K$, for dollars the nominal rental price, $R$. The real rental price is $\frac{R}{p}$.

@The Bond Market

The holder of a bond—the lender—has a claim to the amount owned by the borrower.

Money bears no interest. In contrast, bonds will earn interest.

Constructing the Budget Constraint

- Household nominal income = nominal profit + nominal wage income + nominal net rental income + nominal interest income
  
  $$= \Pi + wL + \left( \frac{R}{p} - \delta \right) \cdot PK + i \cdot B$$
  
  $$= \Pi + wL + i \cdot (B + PK)$$

  using: rate of return on bonds = rate of return on ownership of capital, $i = \frac{R}{p} - \delta$

- Household nominal consumption = $PC$

- Household Budget Constraint

  Nominal value of assets = $M + B + PK$

  Nominal saving = $\triangle (\text{nominal assets}) = \triangle M + \triangle B + P \cdot \triangle K = \triangle B + P \cdot \triangle K$
Nominal consumption + nominal saving = nominal Income  (budget constraint in nominal terms)
\[ PC + \Delta B + P \cdot \Delta K = \Pi + wL + i \cdot (B + PK) \]

Consumption+real saving = real income  (budget constraint in real terms)
\[ C + (\frac{1}{P}) \cdot \Delta B + \Delta K = \frac{\Pi}{P} + (\frac{W}{P}) \cdot L + i \cdot (\frac{R}{P} + K) \]

Exercise(a): If a household has $M of money and the price level is $P.
What is the value of money(nominal terms), and what is the value of this money in terms of the goods(real terms)? What is the real value of each dollar?

soln: M; M/P; 1/P

Exercise(b): draw the household budget constraint line, with consumption, C on the x-axis, and real saving on the y-axis, show that the slope is -1. And show the effect of an increase in Real Income on the budget constraint line.

soln: pp102, Figure 6.2; pp102, Figure 6.3

Exercise(c): Y(K,L)=A\cdot K^{0.4} L^{0.6}; and price level P=5, and nominal wage rate w=10, and nominal rental rate R=5.δ = 0.01

(1) Write down the nominal profit, and real profit in terms of w, P, R, and K, L, A.

soln: nominal profit: \[ \Pi = PA \cdot K^{0.4} L^{0.6} - wL - RK \]
real profit:  \[ \frac{\Pi}{P} = A \cdot K^{0.4} L^{0.6} - (\frac{w}{P}) \cdot L - (\frac{R}{P}) \cdot K \]

(2) Write down the FOCs of both nominal profit and real profit equations by take the first derivative with respect to L, and show that they yield the same condition:  \[ (\frac{\Pi}{P}) = MPL \]

soln: trivia, just take the first derivative and set them equals to 0.

(3) draw the demand of labor curve

soln: pp105, figure 6.5

(4) Using the numerical value of market-clearing w, and P, find the demand of labor, L^d in terms of K, and A. here we fix the labor input \( \overline{K} \), and technology \( \overline{A} \).
soln: W/P=10/5=2 = MPL=(0.6) \( A^{0.4} K^{-0.4} L^{0.4} \), and solve for \( K^d \) and this \( K^d = K^S \) because \( w \) and \( P \) are market-clearing wage and Price level.

(5) show that the maximized profit is 0

soln: plug in the demand of capital \( L^d \) (remember \( L^d \) is in terms of fixed \( K \) and technology \( A \)), and use the fixed \( K \) and technology \( A \) level in the profit equation. We end up 0.

(6) Take the first derivative with respect to \( K \) this time, write out the FOCs of both nominal profit and real profit equations, show that they yield the same condition: \((\frac{\dot{K}}{P}) = MPK\)

soln: trivia, as part(2)

(7) Show the capital demand curve

soln: pp107, figure 6.7

(8) Using the numerical value of \( R \), and \( P \), find the demand of labor, \( K^d \) in terms of \( L \), and \( A \). here we fix the labor input \( L \) and technology \( A \).

soln: \( R/P=5/5=1 = MPK=(0.4) \ A^{0.6} K^{-0.6} L^{0.6} \), and solve for \( K^d \), and this \( K^d = K^S \) b/c \( R \) and \( P \) are market-clearing rental rate of price level.

(9) show that the maximized profit is 0

soln: plug in the demand of capital \( K^d \) (remember \( K^d \) is in terms of fixed \( L \) and technology \( A \)), and use the fixed \( L \) and technology \( A \) level. Same as (4)

(10) Find what the equilibrum interest rate.

soln: \( i=R/P - \delta \) (rate of return on bonds=rate of return on ownership of capital), here \( (R/P) \) is market-clearing real rental rate.