

## Supplementary Appendix 1 to

### THE LONG-TERM EFFECTS OF YOUTH UNEMPLOYMENT

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#### **A Theoretical Model and Conceptual Framework for Investigating the Impacts of Unemployment on Youths' Human Capital Acquisition**

##### **Notation and Setup:**

Define time to be discrete and finite,  $t=1, 2, 3, \dots, T$ .

Define  $P$  to be the probability of being unemployed in any particular time period  $t$ , where  $0 \leq P < 1$ .

Define  $s_t$  to be the fraction of each working period devoted to (planned) training, where  $0 \leq s_t \leq 1$ .

Define  $\lambda$  to be the fraction of the year an individual is not unemployed, where  $0 \leq \lambda \leq 1$ .

Define  $H_t$  to be the human capital stock (or productivity) of an individual at period  $t$ .

Define  $w$  to be the rental rate for human capital, which is constant over time.

Define  $\beta$  to be the rate at which individuals discount future incomes.

Define  $f(s_t)$  to be the production function of new human capital. We assume that  $f'(\cdot) > 0$ ,  $f''(\cdot) < 0$ , and that the production function satisfies the Inada conditions (Inada, 1964).<sup>1</sup> These conditions imply that the marginal product of training is large when an individual does at least a little training, and, therefore, given the possibility of training, it is always optimal to devote some fraction of time to training.

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<sup>1</sup> Given the finite time horizon, we in fact only rely on the condition that  $\lim_{k \rightarrow 0} f(k) = \infty$ .

Human capital evolves as  $H_{t+1} = H_t + f(s_t)$ .

We assume that all training takes place on the job and that the fraction of time devoted to training is determined before an individual learns whether he is to become unemployed. If he does not experience unemployment, his potential income at time  $t$  is  $wH_t$  and his disposable income is potential income minus the opportunity cost of training,  $w(1-s_t)H_t$ . On the other hand, if he becomes unemployed, then potential and disposable incomes are given by  $\lambda wH_t$  and  $\lambda w(1-s_t)H_t$ , respectively. We assume the fraction of time that is spent unemployed, if unemployment is experienced, is not stochastic.

### **Period T**

At the last time period, it is never optimal to train because there is no future return to training. In this case,

$$\text{Disposable Income} = \begin{cases} wH_T & \text{if the individual works full-time.} \\ \lambda wH_T & \text{if the individual is unemployed.} \end{cases}$$

We focus on the situation in which an individual maximizes the present discounted value of his disposable income. Therefore, the individual's expected utility as a function of the human capital stock at the start of the final period  $T$  is

$$EV_T(H_T) = (1-P)(wH_T) + P(\lambda wH_T) .$$

### **Period T-1:**

In period  $T-1$ , an individual makes a decision about how much time to devote to training. Time spent training is not paid, but training in period  $T-1$  increases the amount of human capital in period  $T$ . This in turn increases disposable income in the last period. According to our formulation of the model, training is possible only when employed.

Therefore, at the beginning of period T-1 an individual faces the following optimization problem:

$$\begin{aligned} \max_{S_{T-1}} EV_{T-1} (H_{T-1}) = & \\ & P \{ \beta^{T-1} \lambda (1-S_{T-1}) w H_{T-1} + \beta^T V_T (H_{T-1} + f(\lambda S_{T-1})) \} + \\ & (1-P) \{ \beta^{T-1} (1-S_{T-1}) w H_{T-1} + \beta^T V_T (H_{T-1} + f(S_{T-1})) \} \end{aligned}$$

This optimization can be rewritten as:

$$\begin{aligned} \max_{S_{T-1}} EV_{T-1} (H_{T-1}) = & \\ & P \{ \beta^{T-1} \lambda (1-S_{T-1}) w H_{T-1} + \beta^T P \lambda w (H_{T-1} + f(\lambda S_{T-1})) + \beta^T (1-P) w (H_{T-1} + f(\lambda S_{T-1})) \} \\ + & \\ & (1-P) \{ \beta^{T-1} (1-S_{T-1}) w H_{T-1} + \beta^T P \lambda w (H_{T-1} + f(S_{T-1})) + \beta^T (1-P) w (H_{T-1} + f(S_{T-1})) \} \end{aligned}$$

The first order condition (FOC) for the optimal choice of the fraction of time to devote to training is given by:

$$\begin{aligned} \frac{\partial V_{T-1}(H_{T-1})}{\partial S_{T-1}} = & - \beta^{T-1} P \lambda w H_{T-1} + \beta^T P^2 \lambda^2 w f'(\lambda S_{T-1}) + \beta^T P (1-P) \lambda w f'(\lambda S_{T-1}) \\ & - \beta^{T-1} (1-P) w H_{T-1} + \beta^T P (1-P) \lambda w f'(S_{T-1}) + \beta^T (1-P)^2 w f'(S_{T-1}) \\ = & 0 \end{aligned}$$

The second order condition (SOC) for a maximum is satisfied because of the concavity of the human capital production function:

$$\begin{aligned} \frac{\partial^2 V_{T-1}}{\partial S_{T-1}^2} = & \beta^T P^2 \lambda^3 w f''(\lambda S_{T-1}) + \beta^T P (1-P) \lambda^2 w f''(\lambda S_{T-1}) + \\ & \beta^T P (1-P) \lambda w f''(S_{T-1}) + \beta^T (1-P)^2 w f''(S_{T-1}) < 0 \end{aligned}$$

These conditions ensure that there exists a unique level of training that maximizes an individual's objective function at time period T-1. After re-arrangement, the decision-

rule for optimal training at T-1, given the amount of capital accumulated prior to this period, is:

$$\beta [P\lambda w + (1-P)w] [f'(\lambda S_{T-1})P\lambda + f'(S_{T-1})(1-P)] = [P\lambda w + (1-P)w] H_{T-1}$$

or

$$wH_{T-1} = \beta w[f'(\lambda S_{T-1})P\lambda + f'(S_{T-1})(1-P)]$$

The economic interpretation of this condition is that the expected income loss from additional training today should be equal to the expected present discounted value of the gain from the addition human capital available in the next time period.

Let  $S_{T-1}^*(H_{T-1})$  be the optimal choice of training as a function of the level of the human capital stock at the beginning of period T-1. By the Implicit Function Theorem (IFT):

$$\frac{dS_{T-1}^*}{dH_{T-1}} = - \frac{\partial^2 V_{T-1} / \partial S_{T-1}^* \partial H_{T-1}}{\partial^2 V_{T-1} / \partial S_{T-1}^{*2}} < 0$$

because

$$\frac{\partial^2 V_{T-1}}{\partial S_{T-1}^* \partial H_{T-1}} = -\beta^{T-1} P\lambda w - \beta^{T-1}(1-P)w < 0$$

and

$$\frac{\partial^2 V_{T-1}}{\partial S_{T-1}^{*2}} < 0 \quad \text{from the second order condition.}$$

Therefore, those entering period T-1 with lower human capital stocks will train more than those entering the period with higher human capital stocks. In anticipation of the main theoretical result, suppose there were two identical individuals at the start of period T-2, and one experienced unemployment while the other did not. Upon entering

period T-1, the individual who experienced unemployment in T-2 would choose to train more during period T-1 than the individual who did not experience unemployment.

It is important to note that we have a single value function at the start of this time period, which can be evaluated at different levels of the human capital stock. It is useful to rewrite the value function in a general form and prove its concavity in  $H_{T-1}$ .

$$V_{T-1}(S^*_{T-1}(H_{T-1}), H_{T-1}) = \\ P \{ \beta^{T-1} \lambda (1 - S_{T-1}) w H_{T-1} + \beta^T P \lambda w [H_{T-1} + f(\lambda S_{T-1})] + \beta^T (1 - P) w [H_{T-1} + f(\lambda S_{T-1})] \} + \\ (1 - P) \{ \beta^{T-1} (1 - S_{T-1}) w H_{T-1} + \beta^T P \lambda w [H_{T-1} + f(S_{T-1})] + \beta^T (1 - P) w [H_{T-1} + f(S_{T-1})] \}$$

By the Envelope Theorem the first total derivative of this value function is:

$$\frac{dV_{T-1}(S^*_{T-1}(H_{T-1}), H_{T-1})}{dH_{T-1}} = \frac{\partial V_{T-1}}{\partial S^*_{T-1}} \frac{dS^*_{T-1}}{dH_{T-1}} + \frac{\partial V_{T-1}}{\partial H_{T-1}} = \frac{\partial V_{T-1}}{\partial H_{T-1}}$$

since  $\frac{\partial V_{T-1}}{\partial S^*_{T-1}} = 0$  by the FOCs for a maximum.

Then

$$\frac{\partial V_{T-1}}{\partial H_{T-1}} = \beta^{T-1} P \lambda w (1 - S^*_{T-1}) + \beta^T P^2 \lambda w + \beta^T P (1 - P) w + \beta^{T-1} (1 - P) w (1 - S^*_{T-1}) + \beta^T P (1 - P) \lambda w + \\ \beta^T (1 - P)^2 w = \text{positive constant.}$$

By the Envelope Theorem the second total derivative is:

$$\frac{d^2 V_{T-1}(S^*_{T-1}(H_{T-1}), H_{T-1})}{dH_{T-1}^2} = \frac{\partial^2 V_{T-1}}{\partial S^*_{T-1}^2} \left( \frac{dS^*_{T-1}}{dH_{T-1}} \right)^2 + \frac{\partial V_{T-1}}{\partial S^*_{T-1}} \frac{d^2 S^*_{T-1}}{dH_{T-1}^2} + \frac{\partial^2 V_{T-1}}{\partial H_{T-1}^2} = \frac{\partial^2 V_{T-1}}{\partial S^*_{T-1}^2} \left( \frac{dS^*_{T-1}}{dH_{T-1}} \right)^2 < 0$$

because  $\frac{\partial V_{T-1}}{\partial S^*_{T-1}} = 0$  by FOC,

$$\frac{\partial^2 V_{T-1}}{\partial H_{T-1}^2} = 0 \quad \text{because} \quad \frac{\partial V_{T-1}}{\partial H_{T-1}} = \text{positive constant, and}$$

$$\frac{\partial^2 V_{T-1}}{\partial S_{T-1}^*{}^2} < 0 \quad \text{by SOCs of optimization problem at period T-1.}$$

The concavity of the value function in time period T-1 is sufficient to ensure that the FOCs at time T-2 define behaviors necessary to maximize the objective function at T-2.

**Period T-2:**

The individual's optimization problem in T-2:

$$\begin{aligned} \max_{S_{T-2}} EV_{T-2}(H_{T-2}) = & P \{ \beta^{T-2} \lambda (1-S_{T-2}) w H_{T-2} + \beta^{T-1} V_{T-1}(H_{T-2} + f(\lambda S_{T-2})) \} \\ & + (1-P) \{ \beta^{T-2} (1-S_{T-2}) w H_{T-2} + \beta^{T-1} V_{T-1}(H_{T-2} + f(S_{T-2})) \}, \end{aligned}$$

where

$$\begin{aligned} V_{T-1}(H_{T-2} + f(\lambda S_{T-2})) = & \\ = P & \left\{ \begin{aligned} & \beta^{T-1} \lambda (1-S_{T-1}) w [H_{T-2} + f(\lambda S_{T-2})] + \\ & + \beta^T P \lambda w [H_{T-2} + f(\lambda S_{T-2}) + f(\lambda S_{T-1})] + \beta^T (1-P) w [H_{T-2} + f(\lambda S_{T-2}) + f(\lambda S_{T-1})] \end{aligned} \right\} \\ + (1-P) & \left\{ \begin{aligned} & \beta^{T-1} (1-S_{T-1}) w [H_{T-2} + f(\lambda S_{T-2})] + \\ & + \beta^T P \lambda w [H_{T-2} + f(\lambda S_{T-2}) + f(S_{T-1})] + \beta^T (1-P) w [H_{T-2} + f(\lambda S_{T-2}) + f(S_{T-1})] \end{aligned} \right\} \end{aligned}$$

and

$$\begin{aligned} V_{T-1}(H_{T-2} + f(S_{T-2})) = & \\ = P & \left\{ \begin{aligned} & \beta^{T-1} \lambda (1-S_{T-1}) w [H_{T-2} + f(S_{T-2})] + \\ & + \beta^T P \lambda w [H_{T-2} + f(S_{T-2}) + f(\lambda S_{T-1})] + \beta^T (1-P) w [H_{T-2} + f(S_{T-2}) + f(\lambda S_{T-1})] \end{aligned} \right\} \\ + (1-P) & \left\{ \begin{aligned} & \beta^{T-1} (1-S_{T-1}) w [H_{T-2} + f(S_{T-2})] + \\ & + \beta^T P \lambda w [H_{T-2} + f(S_{T-2}) + f(S_{T-1})] + \beta^T (1-P) w [H_{T-2} + f(S_{T-2}) + f(S_{T-1})] \end{aligned} \right\} \end{aligned}$$

The value function in T-2 is concave when both  $V_{T-1}(H_{T-2} + f(\lambda S_{T-2}))$  and

$V_{T-1}(H_{T-2} + f(S_{T-2}))$  are concave in human capital stock, a result which was proved above for arbitrary levels of the human capital stock at the start of T-1.

### **Conclusions for T, T-1, and T-2:**

1.  $V'_T(.) =$  positive constant and  $V''_T(.) = 0$  because the value function in the last period T is linear in human capital/income.
2.  $V'_{T-1}(. ) > 0$  and  $V''_{T-1}(. ) < 0$  because the value function of period T-1 is concave in the human capital stock (or income).
3.  $V'_{T-2}(. ) > 0$  and  $V''_{T-2}(. ) < 0$  because the value function of period T-2 is concave in human capital as it is a linear combination of a linear utility function and a concave value function in period T-1.

### **Periods $t < T$ :**

The individual's optimization problem in each earlier period t is given by:

$$\max_{S_t} EV_t(H_t) = P\{\beta^t \lambda(1 - S_t)wH_t + \beta^{t+1}V_{t+1}(H_t + f(\lambda S_t))\} + (1-P)\{\beta^t(1-S_t)wH_t + \beta^{t+1}V_{t+1}(H_t + f(S_t))\}$$

FOC:

$$\frac{\partial V_t(H_t)}{\partial S_t} = -P\beta^t \lambda wH_t + P\beta^{t+1}V'_{t+1}(H_t + f(\lambda S_t))f'(\lambda S_t)\lambda - (1-P)\beta^t wH_t + (1-P)\beta^{t+1}V'_{t+1}(H_t + f(S_t))f'(S_t) = 0;$$

After rearrangement, the FOC for optimal training in an arbitrary period t is:

$$\beta[V'_{t+1}(H_t + f(\lambda S_t))f'(\lambda S_t)P\lambda + V'_{t+1}(H_t + f(S_t))f'(S_t)(1-P)] = [P\lambda w + (1-P)w]H_t$$

Now we prove that the value function of an arbitrary period  $t < T$  is concave in human capital stock (or income). To do this, we show that concavity of the value function in the next period guarantees concavity in any arbitrary period  $t < T$ .

By the Envelope Theorem, the first total derivative of the value function in period  $t$  is:

$$\frac{dV_t(S_t^*(H_t), H_t)}{dH_t} = \frac{\partial V_t(S_t^*(H_t), H_t)}{\partial S_t^*} \frac{dS_t^*}{dH_t} + \frac{\partial V_t(S_t^*(H_t), H_t)}{\partial H_t} = \frac{\partial V_t(S_t^*(H_t), H_t)}{\partial H_t}.$$

Now,

$$\begin{aligned} \frac{\partial V_t(S_t^*(H_t), H_t)}{\partial H_t} &= P[\beta^t \lambda(1 - S_t^*)w + \beta^{t+1} V'_{t+1}(H_t + f(\lambda S_t^*))] \\ &\quad + (1 - P)[\beta^t (1 - S_t^*)w + \beta^{t+1} V'_{t+1}(H_t + f(S_t^*))] \end{aligned}$$

Since  $V'_{t+1}(\cdot) > 0$  is true for time period T-3, it holds by induction for all earlier time periods and the value function is everywhere an increasing function of the human capital stock.

By the Envelope Theorem, the second total derivative is:

$$\frac{d^2 V_t(S_t^*(H_t), H_t)}{dH_t^2} = \frac{\partial^2 V_t}{\partial S_t^{*2}} \left( \frac{dS_t^*}{dH_t} \right)^2 + \frac{\partial V_t}{\partial S_t^*} \frac{d^2 S_t^*}{dH_t^2} + \frac{\partial^2 V_t}{\partial H_t^2} = \frac{\partial^2 V_t}{\partial S_t^{*2}} \left( \frac{dS_t^*}{dH_t} \right)^2 + \frac{\partial^2 V_t}{\partial H_t^2} < 0$$

where

$$\frac{\partial V_t}{\partial S_t^*} = 0 \quad (\text{by FOC});$$

$$\frac{\partial^2 V_t}{\partial H_t^2} = P\beta^{t+1}V''_{t+1}(H_t + f(\lambda S_t^*)) + (1 - P)\beta^{t+1}V''_{t+1}(H_t + f(S_t^*)) < 0$$

provided that the value function of the next period is concave in the stock of human capital. Again, this condition holds for T-3 and, by induction, for all earlier time periods.

The period  $t$  second order condition for the maximal choice of training intensity is given by:

$$\begin{aligned} \frac{\partial^2 V_t(H_t)}{\partial S_t^2} &= P\beta^{t+1}V''_{t+1}(H_t + f(\lambda S_t))(f'(\lambda S_t)\lambda)^2 + P\beta^{t+1}V'_{t+1}(H_t + f(\lambda S_t))f''(\lambda S_t)\lambda^2 + \\ &\quad + (1-P)\beta^{t+1}V''_{t+1}(H_t + f(S_t))(f'(S_t))^2 + (1-P)\beta^{t+1}V'_{t+1}(H_t + f(S_t))f''(S_t) \end{aligned}$$

The concavity of the value function of human capital in the next period  $t+1$ , the positive first derivative of this value function, and the concavity of the human capital production function guarantee the concavity of the value function at an arbitrary period. This concavity, in turn, ensures that the first order conditions describe the behaviors necessary to maximize the expected present discounted value of disposable earnings at each point in time.

Next, we establish that a larger stock of human capital results in lower investment in additional human capital at each point in time. Since

$$\begin{aligned} \frac{\partial^2 V_t}{\partial S_t \partial H_t} &= -P\beta^t \lambda W + P\beta^{t+1}V''_{t+1}(H_t + f(\lambda S_t))f'(\lambda S_t)\lambda \\ &\quad - (1-P)\beta^t W + (1-P)\beta^{t+1}V''_{t+1}(H_t + f(S_t))f'(S_t) < 0 \end{aligned}$$

because of the concavity of the value function in  $t+1$  and human capital production function. Therefore, by the Implicit Function Theorem:

$$\frac{dS_t^*}{dH_t} = -\frac{\partial^2 V_t / \partial S_t^* \partial H_t}{\partial^2 V_t / \partial S_t^{*2}} = -\frac{(-)}{(-)} < 0 \text{ for any arbitrary period } t < T.$$

The result derived for period  $T-1$  with an explicit formulation of the utility and value functions holds for an arbitrary period as well. Therefore, an individual's optimal behavior is to invest less in training at higher levels of human capital holding age,  $t$ , constant. Since an “exogenous” unemployment shock reduces the human capital stock at the start of the next time period, those who experienced unemployment will choose to

undertake more training in the next time period. This establishes Proposition 1 in the main text.

To establish Proposition 2 in the text, suppose that, at the start of period  $t+1$ , two otherwise identical individuals have human capital stocks that differ by an arbitrary amount  $\Delta$ . This difference in human capital stocks could have arisen because these two individuals had different unemployment experiences during period  $t$ . At the start of  $t+1$ , their potential earnings would differ by  $w\Delta$ . By Proposition 1, the individual with the lower human capital stock in  $t$  will choose to invest more in additional human capital at  $t+1$ . Let  $s_{t+1}^N$  and  $s_{t+1}^U$  be these two optimal decisions, where  $s_{t+1}^N < s_{t+1}^U$  and the superscripts N and U stand, respectively, for not having been unemployed at  $t$  and having been unemployed at time period  $t$ . At the start of period  $t+2$ , if neither individual experienced unemployment during  $t+1$ , then the potential earnings would differ by  $w(\Delta + [f(s_{t+1}^N) - f(s_{t+1}^U)]) < w\Delta$  because higher levels of training increase the stock of human capital. If both individuals had experienced unemployment during  $t+1$ , then potential earnings at the start of  $t+2$  would differ by  $w(\Delta + [f(\lambda s_{t+1}^N) - f(\lambda s_{t+1}^U)]) < w\Delta$ , which also is a convergence of the potential human capital stocks. So, provided that unemployment experiences do not increase the propensity to experience future unemployment too severely (the model assumes a zero effect), there will be a convergence in expected potential earnings from  $t+1$  to  $t+2$ . By induction, there will be continued convergence in expected earnings for  $t+3$  and for later periods.

The first part of Proposition 3 in the text follows directly because the fraction of time spent earning income (i.e., not spent training) is higher for those who did not experience unemployment at time  $t$ . Therefore, the observed disposable earnings

differential at t+1 is larger than that that implied if there were no training differential in response to experiencing unemployment at t. In particular,

$$\begin{aligned} (1-s_{t+1}^N)w(H_t + \Delta) - (1-s_{t+1}^U)w(H_t) &= (1-s_{t+1}^U)w\Delta + (s_{t+1}^N - s_{t+1}^U)w(H_t) \\ &> (1-s_{t+1}^U)w\Delta \end{aligned}$$

The second part of Proposition 3 in the text follows from two observations. First, for a given human capital differential between two individuals, there will be a larger training differential at t+1 than at t+2 because the future benefit of additional human capital declines as t approaches T, holding constant one of the individual's human capital stock. The second observation follows from the concavity of the human capital production function. Consider holding constant the human capital at t+2 for the individual with the higher level of human capital at the start of period t+1 by examining a particular type of (un)employment experience during t+1. Following from the optimal catch-up response in Proposition 1, at higher levels of the other individual's human capital stock for the same type of t+1 (un)employment experience, the differentials in the training responses at t+2 will be smaller than otherwise in the absence of the immediate catch-up response. Because of the model's assumption that future unemployment propensities do not depend on previous unemployment events, this implies that we can use equal "weights" for the two individuals when integrating over possible subsequent unemployment experiences.

These two observations, in conjunction, imply that the subsequent average training intensities for the two types of individuals, defined by the type experiencing unemployment at period t, will become more similar over time. Therefore, the convergence of their disposable earnings over time will reflect not only the convergence

in their human capital stocks but also the convergence in the optimal share of time that is spent training. The observed convergence in their disposable earnings after time  $t+1$ , therefore, would happen at a faster rate than would be implied by solely the convergence in their human capital stocks.

### *Model Limitations*

This model is a simple but useful tool. It directly links the present and the future through the process of human capital investment and accumulation. By establishing equivalence between an involuntary unemployment spell and an exogenously constrained human capital stock, it can examine the spell's effects on future behavior and outcomes. The duration of unemployment spells, however, will vary by the intensity and duration of job search. While the duration of search is potentially observable, intensity is not. Search intensity is a component of the unobserved heterogeneity that makes unemployment a potentially endogenous variable in statistical analyses. It is unclear, however, what search theory would contribute to this simple framework in particular. The inverse relationship between search intensity and duration is unlikely to yield unambiguous theoretical predictions. In this case, the answers to the questions posed here are entirely empirical. On the other, this simple model uses a standard human capital framework to analyze these issues. Most labor economists probably accept that one mechanism through which current unemployment can affect future behavior is the human capital stock. Notwithstanding this acceptance, a model like this has not been found in the youth labor market literature. Further, even if all youth unemployment is simply time spent watching television, it may still be relevant to ask whether there are long-term consequences, especially for future earnings.

Finally, the model is expressed in terms of involuntary unemployment. We note that much of the literature on job search views the distinction between quits and layoffs to have little economic content. See, for example, McLaughlin (1991). As is trenchantly noted by Gottschalk and Maloney (1985), however, much of this debate is tautological. To summarize their argument, even coerced decisions can be viewed as voluntary since they result from re-optimization under an alternate set of constraints. In this case, all unemployment may be considered voluntary. It is not possible to distinguish the nature of unemployment using NLSY79 data. Total unemployment, however, is identically the sum of its involuntary and voluntary components. Isolating one of these components is sufficient to distinguish them empirically, since the other is identically the residual. Local variation in labor market conditions over time and exogenous changes in mandated minimum wages over time are potentially suitable instruments to make this empirical distinction.

Gottschalk, Peter and Tim Maloney (1985): "Involuntary Terminations, Unemployment, and Job Matching: A Test of Job Search Theory," *Journal of Labor Economics*, 3(2), 109-123.

McLaughlin, Kenneth (1991): "A Theory of Quits and Layoffs with Efficient Turnover," *Journal of Political Economy*, 1991, 99(1), 1-29.

## Supplementary Appendix 2 to

### THE LONG-TERM EFFECTS OF YOUTH UNEMPLOYMENT

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Identification, Specifications, Point Estimates and Standard Errors

#### *Identification*

This study treats training, school attendance, work experience, prior job changes, and unemployment as potentially endogenous variables that evolve as the young men in the sample age. These variables are outcomes as well as determinants of later outcomes. Therefore, it is important to demonstrate that there is sufficient information to obtain identification of the effect of lagged outcomes, prior unemployment in particular, on current labor market events.

Because we treat the youths' places of residence as exogenous, this analysis contains numerous non-deterministically varying, time-dependent exogenous variables. These include local unemployment rates and the real level of minimum wages, an urban residence dummy, region dummy variables, state-level college undergraduate tuition levels, and separately real per-pupil state expenditures on secondary and post-secondary education (see Table 2). It is important to ask whether these variables are sufficient to achieve identification of the approximation to the structural model.

As discussed in Bhargava (1991) and Mroz and Surette (1998), panel-data relationships like those examined here implicitly provide many additional identification conditions than one might infer by simply counting the number of contemporaneous

exogenous variables (e.g., instruments) excluded from a structural equation of interest. There are two primary reasons for this.

First, consider the case of linear dynamic models examined by Bhargava (1991), in which one is willing to impose stability on the structural parameters over time. In the empirical model, we also impose this restriction. Bhargava derives the reduced form equations for a system of dynamic equations and demonstrates that *every lag* of each instrumental variable could have a separate impact on the “contemporaneous” value of an endogenous explanatory variable. The time dimension for the exogenous time-varying instruments, therefore, creates a multiplicity of “instruments” associated with each “exclusion restriction,” resulting in significantly more degrees of freedom to control for endogenous determinants. His analysis demonstrates that over-identification can be obtained under quite weak conditions in linear dynamic models.

A second source of identification arises in the context of dynamic nonlinear models. Mroz and Surette (1998) discuss this in greater detail. Their discussion exploits the fact that variations in the time ordering of the exogenous variables provide even higher degrees of over-identification than would be obtained by a simple reference to Bhargava’s (1991) observation discussed above. It is especially appropriate for economic relationships like school attendance and work decisions, in which there can be considerable fixed costs of changing status over time.

The basic idea underlying their argument of additional identification is that, in dynamic nonlinear models of the type used here, the impact of any lagged exogenous variable on a current endogenous explanatory variable depends crucially on the precise forms of the prior time series of all exogenous variables. Implicitly, the impact of any

single lagged exogenous variable is modified by prior lagged values of all other exogenous variables. For example, the impact of low college tuition in 1984 on school attendance for a 21 year old in 1984 would depend explicitly on school attendance at age 20; the magnitude of the 1984 tuition effect in the “reduced form equation,” then, would depend on the level of tuition in 1983. Further, the magnitude of the impact of the tuition variable at age-20 on age-20 attendance depends on the lagged (age 19) attendance decision; and so this reduced form effect depends interactively on tuition levels at that as well as prior ages. As long as subsequent values of the lagged exogenous variables cannot be perfectly forecasted in time-separable non-linear models, there should be an even greater degree of identification than that discussed by Bhargava (1991).

As another example, consider the local unemployment rate. At any point in time, such a variable is exogenous to young people. In 1985, variation in this rate has a direct impact on 1985 labor market outcomes. Similarly, variation in 1983 has a direct impact on 1983 outcomes. Because of the timing of decision-making, however, the 1983 rate has no direct impact on 1985 outcomes except through the accumulated stock of human capital as of 1985. As a consequence, the 1983 rate is, theoretically, an instrument for human capital stocks observed in 1985.

By using an explicit sequential dynamic modeling framework one can incorporate all such interactions that depend on the precise timing and sequencing of the values of the time-varying exogenous variables. The maximum likelihood approach we use here automatically incorporates these interactions among the time series properties of the sets of exogenous variables. They do so efficiently, without one having to resort to including

numerous time-varying interactions of the exogenous variables in an arbitrary fashion, as would be the case with a more static instrumental variables approach.

Identification in this model is also secured through contemporaneous, theoretical exclusion restrictions and functional form. Some of the time-varying exogenous variables already mentioned, for example, can be assumed to affect indirectly the schooling and training decisions and labor supply but have no direct impact on wages other than through the human capital stock. They are, therefore, excluded from the wage equation. And, of course, it is certainly the case that our assumed functional forms for index functions do provide some additional “over-identification” above that which could be achieved in a fully nonparametric model that only incorporated the dynamic exclusion restrictions discussed above and interactions among the sequences of lagged exogenous variables.

Bhargava, Alok (1991): “Identification and Panel Data Models with Endogenous Regressors,” *The Review of Economics Studies*, 58(1), 129-140.

Mroz, Thomas and Brian Surette (1998): “Post-Secondary Schooling and Training Effects on Wages and Employment,” mimeo, Department of Economics, University of North Carolina at Chapel Hill

Equation by Equation Specifications with Point Estimates  
and Standard Errors for the Discrete Factor Likelihood Model

**Note:** A “probit”-type model refers to a binary outcome model that, conditional on the unobserved heterogeneities, uses the standard normality assumptions of a probit model with independent errors. An “OLS”-type model refers to classical regression model with independent normal errors after conditioning on the unobserved heterogeneities.

Log-likelihood function value: -220859.39

Number of parameters: 444

	COEFF.	STD.ERR	T-RATIO
<b>Any Schooling (as) (“probit”-type):</b>			
1 cons_sch	-5.40282	2.30871	-2.34019
2 year	0.05649	0.01169	4.83332
3 afqt	0.00741	0.00056	13.15392
4 readmat	-0.03012	0.03315	-0.90864
5 libcard	0.06536	0.02585	2.52829
6 livpar	0.06613	0.02518	2.62634
7 prot	-0.02088	0.02485	-0.84026
8 black	0.16552	0.03096	5.34641
9 hisp	0.04341	0.03396	1.27802
10 nc	0.07289	0.04575	1.59320
11 so	-0.00419	0.06025	-0.06959
12 we	0.04072	0.05643	0.72156
13 urb	0.03687	0.03246	1.13591
14 ur	0.00670	0.00433	1.54613
15 mw	0.79964	0.48936	1.63403
16 mwwage	-0.15555	0.02050	-7.58685
17 mwhgc	0.24087	0.02147	11.22086
18 ugtuit	-0.12351	0.25891	-0.47706
19 expsec	-0.26184	0.20309	-1.28928
20 expps	-0.21402	0.11324	-1.88995
21 age	0.04831	0.11669	0.41398
22 age2	0.00673	0.00163	4.13370
23 exp	-0.17608	0.01167	-15.08939
24 exp2	0.00670	0.00057	11.69062
25 hgc	-0.42516	0.06735	-6.31241
26 dum12y	-0.57403	0.04710	-12.18772
27 coldeg	-1.07555	0.05241	-20.52079
28 lag1sc	1.58061	0.03599	43.91313
29 lag1wu	-0.00688	0.00143	-4.80965
30 lag2wu	-0.00688	0.00151	-4.54923
31 lag3wu	-0.00456	0.00171	-2.66375

32	lag4wu	-0.00432	0.00180	-2.39687
33	lag5wu	0.00064	0.00178	0.36010
34	cumtr	0.02549	0.01142	2.23258
35	year79	-0.31345	0.10275	-3.05065
36	year8082	-0.08663	0.06421	-1.34918
37	year9294	-0.11862	0.06743	-1.75917
38	age1415	0.94031	0.24527	3.83375
39	age16	1.15128	0.19533	5.89391
40	age17	0.53692	0.15164	3.54075
41	age1819	-0.11913	0.09632	-1.23677
42	age2021	-0.36704	0.06483	-5.66170
43	rhod11	-0.12899	0.08088	-1.59496
44	rhod12	-0.16098	0.06922	-2.32545

**Any Training (tr) ("probit"-type):**

45	cons_tra	-7.62289	1.86784	-4.08113
46	year	0.01437	0.01215	1.18265
47	afqt	0.00109	0.00051	2.13251
48	readmat	0.01565	0.03109	0.50341
49	libcard	0.04166	0.02390	1.74324
50	livpar	-0.01075	0.02421	-0.44401
51	prot	0.01949	0.02231	0.87363
52	black	-0.00854	0.03073	-0.27794
53	hisp	0.02800	0.03277	0.85448
54	nc	0.13633	0.03974	3.43093
55	so	0.04304	0.05299	0.81225
56	we	0.08520	0.05038	1.69113
57	urb	-0.01042	0.02599	-0.40090
58	ur	-0.00846	0.00372	-2.27217
59	mw	0.76114	0.37831	2.01196
60	mwage	-0.00273	0.01451	-0.18787
61	mwhgc	-0.05418	0.01885	-2.87400
62	ugtuit	-0.32935	0.22278	-1.47837
63	expsec	0.35550	0.16719	2.12634
64	expps	-0.26458	0.09504	-2.78380
65	age	0.18231	0.07622	2.39182
66	age2	-0.00391	0.00085	-4.59464
67	exp	0.02128	0.01014	2.09937
68	exp2	-0.00004	0.00044	-0.09536
69	hgc	0.20305	0.05946	3.41472
70	dum12y	0.11290	0.03762	3.00108
71	coldeg	0.15048	0.04559	3.30105
72	lag1wu	0.00351	0.00134	2.61187
73	lag2wu	0.00111	0.00154	0.72009
74	lag3wu	-0.00188	0.00159	-1.18583
75	lag4wu	0.00061	0.00158	0.38600
76	lag5wu	-0.00054	0.00165	-0.32558

77	cumtr	0.25118	0.00930	27.02045
78	year79	-0.27332	0.09459	-2.88957
79	year8082	0.16708	0.05659	2.95256
80	year9294	-0.10617	0.04705	-2.25641
81	oldcoh	-0.01503	0.03663	-0.41017
82	lag1ch	0.03625	0.05080	0.71350
83	lag2ch	0.01289	0.05598	0.23032
84	lag3ch	-0.11927	0.06042	-1.97416
85	lag4ch	-0.00804	0.06231	-0.12911
86	lag5ch	-0.12287	0.06438	-1.90856
87	rhod21	0.16462	0.07470	2.20368
88	rhod22	0.08093	0.05749	1.40782

**Any Work (work) ("probit"-type):**

89	cons_wo	22.32405	5.19979	4.29326
90	year	-0.12317	0.03333	-3.69558
91	afqt	0.00389	0.00155	2.51672
92	readmat	-0.10216	0.06703	-1.52404
93	libcard	-0.09337	0.05690	-1.64089
94	livpar	0.06955	0.05639	1.23347
95	prot	-0.02522	0.05907	-0.42704
96	black	-0.42809	0.07549	-5.67083
97	hisp	-0.20362	0.08292	-2.45555
98	nc	0.34945	0.10242	3.41181
99	so	0.44976	0.13748	3.27137
100	we	0.27055	0.12232	2.21181
101	urb	-0.03057	0.07020	-0.43552
102	ur	-0.04419	0.00861	-5.13382
103	mw	-3.77947	1.07507	-3.51554
104	mwage	0.11122	0.04210	2.64217
105	mwhgc	0.10043	0.04463	2.25040
106	ugtuit	0.51153	0.57046	0.89670
107	expsec	1.11080	0.41570	2.67213
108	expps	-0.02553	0.28613	-0.08921
109	age	-0.06406	0.20003	-0.32028
110	age2	-0.00832	0.00206	-4.03865
111	exp	0.54387	0.02508	21.68812
112	exp2	-0.01910	0.00172	-11.09092
113	hgc	-0.20883	0.14067	-1.48456
114	dum12y	0.03572	0.08737	0.40881
115	coldeg	0.58514	0.22084	2.64967
116	lag1wu	-0.00144	0.00233	-0.62084
117	lag2wu	0.00259	0.00292	0.88722
118	lag3wu	0.00877	0.00317	2.76588
119	lag4wu	0.00004	0.00310	0.01310
120	lag5wu	0.00379	0.00269	1.41008
121	cumtr	0.15554	0.03191	4.87432

122	year79	-0.24914	0.27897	-0.89308
123	year8082	0.14971	0.14845	1.00847
124	year9294	0.20260	0.12490	1.62207
125	oldcoh	0.04065	0.10252	0.39653
126	lag1ch	-0.37553	0.07427	-5.05657
127	lag2ch	-0.07009	0.08997	-0.77907
128	lag3ch	-0.16504	0.09275	-1.77949
129	lag4ch	-0.01650	0.09935	-0.16612
130	lag5ch	-0.00535	0.10855	-0.04926
131	rhod31	-3.78445	0.23214	-16.30238
132	rhod32	-0.24393	0.12996	-1.87694

**Any Unemployment(un) ("probit"-type):**

133	cons_un	-0.74671	4.69515	-0.15904
134	year	0.06530	0.02525	2.58576
135	afqt	-0.00604	0.00101	-5.98804
136	readmat	-0.04357	0.05226	-0.83374
137	libcard	-0.00557	0.04256	-0.13093
138	livpar	-0.04730	0.04267	-1.10830
139	prot	0.05083	0.04439	1.14523
140	black	0.18790	0.05589	3.36207
141	hisp	-0.00868	0.06349	-0.13677
142	nc	0.29733	0.08151	3.64801
143	so	0.27573	0.10801	2.55272
144	we	0.32793	0.10133	3.23614
145	urb	0.08488	0.05182	1.63807
146	ur	0.06484	0.00735	8.81730
147	mw	0.35621	1.06182	0.33547
148	mwage	-0.02910	0.04007	-0.72628
149	mwhgc	0.04306	0.04101	1.04981
150	ugtuit	0.08569	0.42106	0.20352
151	expsec	1.28847	0.34161	3.77172
152	expss	0.38318	0.18632	2.05657
153	age	-0.38773	0.19468	-1.99156
154	age2	0.00726	0.00212	3.42409
155	exp	0.00083	0.02038	0.04049
156	exp2	-0.00013	0.00115	-0.11505
157	hgc	-0.20773	0.12884	-1.61228
158	dum12y	-0.09233	0.07082	-1.30378
159	coldeg	0.01134	0.11492	0.09868
160	lag1wu	0.09265	0.00386	23.98149
161	lag2wu	0.00411	0.00281	1.46375
162	lag3wu	0.00688	0.00274	2.51465
163	lag4wu	0.00851	0.00279	3.05033
164	lag5wu	0.00302	0.00252	1.19717
165	cumtr	-0.05241	0.01995	-2.62756
166	year79	0.12579	0.31275	0.40221

167	year8082	0.46921	0.14090	3.33015
168	year9294	-0.17808	0.09167	-1.94250
169	oldcoh	0.10904	0.07092	1.53750
170	lag1ch	0.30694	0.08554	3.58822
171	lag2ch	0.22186	0.08693	2.55207
172	lag3ch	0.12513	0.08415	1.48708
173	lag4ch	0.22416	0.09049	2.47731
174	lag5ch	0.00448	0.08995	0.04984
175	rhod41	1.87575	0.14864	12.61938
176	rhod42	-1.36867	0.10659	-12.84061

**Any Job Change (ch) ("probit"-type):**

177	cons_ch	4.39813	5.19413	0.84675
178	year	-0.00547	0.03051	-0.17931
179	afqt	-0.00460	0.00128	-3.58572
180	readmat	-0.03884	0.06086	-0.63824
181	libcard	0.02831	0.05195	0.54486
182	livpar	-0.06131	0.05110	-1.19976
183	prot	0.05564	0.05487	1.01414
184	black	0.19311	0.06956	2.77607
185	hisp	0.04618	0.07570	0.61012
186	nc	0.08828	0.10065	0.87706
187	so	0.03647	0.13019	0.28009
188	we	0.13910	0.11870	1.17189
189	urb	0.06088	0.06629	0.91851
190	ur	0.04999	0.00893	5.59698
191	mw	-0.15158	1.06520	-0.14231
192	mwage	-0.00485	0.03991	-0.12158
193	mwhgc	0.03470	0.04340	0.79968
194	ugtuit	-0.36683	0.54402	-0.67430
195	expsec	0.44405	0.41192	1.07800
196	expps	-0.28266	0.22398	-1.26197
197	age	-0.33359	0.20315	-1.64209
198	age2	0.00643	0.00223	2.89025
199	exp	0.00292	0.02412	0.12115
200	exp2	-0.00092	0.00124	-0.74296
201	hgc	-0.16177	0.13824	-1.17021
202	dum12y	-0.09284	0.08463	-1.09706
203	coldeg	0.05012	0.14111	0.35518
204	lag1wu	0.01654	0.00236	7.01810
205	lag2wu	0.00645	0.00245	2.63323
206	lag3wu	-0.00361	0.00247	-1.46429
207	lag4wu	0.00527	0.00267	1.97501
208	lag5wu	0.00010	0.00248	0.04087
209	cumtr	-0.01271	0.02598	-0.48897
210	year79	-1.87776	0.28855	-6.50768
211	year8082	0.04009	0.13133	0.30531

212	year9294	-0.12556	0.10816	-1.16090
213	oldcoh	-0.06027	0.09013	-0.66868
214	lag1ch	0.17112	0.07020	2.43763
215	lag2ch	0.10180	0.08085	1.25909
216	lag3ch	0.25839	0.07755	3.33208
217	lag4ch	0.24215	0.08722	2.77629
218	lag5ch	0.17742	0.09300	1.90765
219	rhod51	1.77302	0.18589	9.53822
220	rhod52	-0.98839	0.12069	-8.18933

**Annual Hours Work (h, Conditional on Working)  
("OLS"-type):**

221	cons_hw	3732.28816	959.17900	3.89113
222	year	-17.08248	7.64238	-2.23523
223	afqt	0.91746	0.31697	2.89449
224	readmat	-17.59887	17.79130	-0.98918
225	libcard	-9.01697	14.17603	-0.63607
226	livpar	49.54895	15.13751	3.27326
227	prot	-1.21263	13.91489	-0.08715
228	black	-94.30124	18.05057	-5.22428
229	hisp	-31.05630	19.69059	-1.57722
230	nc	-32.60257	22.34989	-1.45873
231	so	18.17003	27.16728	0.66882
232	we	-10.92480	25.86992	-0.42230
233	urb	-51.91571	13.25442	-3.91686
234	ur	-17.19556	1.63393	-10.52406
235	mw	-163.06607	192.18122	-0.84850
236	mwage	-1.58275	7.32497	-0.21608
237	mwhgc	13.85266	7.48268	1.85130
238	ugtuit	322.02056	102.98940	3.12673
239	expsec	-59.09172	78.00897	-0.75750
240	expps	-39.11304	45.31896	-0.86306
241	age	54.30585	35.77315	1.51806
242	age2	-1.81219	0.37444	-4.83976
243	exp	112.38616	4.71022	23.86007
244	exp2	-2.60099	0.17962	-14.48028
245	hgc	11.62469	23.61971	0.49216
246	dum12y	-12.82351	20.41242	-0.62822
247	coldeg	56.21566	32.23167	1.74411
248	lag1wu	-6.13522	0.55146	-11.12538
249	lag2wu	1.48764	0.56054	2.65397
250	lag3wu	1.30891	0.55155	2.37315
251	lag4wu	1.39288	0.55675	2.50183
252	lag5wu	1.24083	0.53497	2.31943
253	cumtr	25.07666	4.79220	5.23280
254	year79	-80.94356	51.77019	-1.56352
255	year8082	-110.73201	24.69076	-4.48475

256	year9294	54.12616	19.64162	2.75569
257	oldcoh	44.50441	23.64567	1.88214
258	lag1ch	-115.63616	16.58982	-6.97031
259	lag2ch	-34.60553	16.23511	-2.13152
260	lag3ch	-26.36610	18.11034	-1.45586
261	lag4ch	12.13116	19.14107	0.63378
262	lag5ch	35.69885	21.25885	1.67925
263	sdhw	464.13570	2.79081	166.30857
264	rhoc11	-1558.49060	32.12707	-48.51019
265	rhoc12	150.36015	32.08095	4.68690

**Annual Weeks of Unemployment (wun, Conditional on Unemployment) ("OLS"-type):**

266	cons_wu	-1.21723	37.19787	-0.03272
267	year	-0.01714	0.21620	-0.07927
268	afqt	-0.02301	0.00927	-2.48110
269	readmat	-0.18629	0.39441	-0.47232
270	libcard	-0.02405	0.36048	-0.06672
271	livpar	-0.71227	0.33642	-2.11722
272	prot	0.51331	0.37418	1.37183
273	black	1.22613	0.46920	2.61321
274	hisp	0.82405	0.52764	1.56176
275	nc	0.79769	0.69494	1.14785
276	so	-1.02733	0.92372	-1.11216
277	we	-1.23083	0.87079	-1.41346
278	urb	-0.05863	0.44295	-0.13236
279	ur	0.39704	0.05794	6.85263
280	mw	1.46456	7.39617	0.19802
281	mwage	-0.07298	0.30176	-0.24184
282	mwhgc	0.08580	0.30575	0.28061
283	ugtuit	-6.08689	4.00414	-1.52015
284	expsec	-1.87526	2.93697	-0.63850
285	exppts	1.00824	1.69214	0.59584
286	age	0.12530	1.59188	0.07871
287	age2	0.00457	0.01806	0.25292
288	exp	0.23105	0.17888	1.29165
289	exp2	-0.00871	0.01266	-0.68788
290	hgc	-0.49446	1.00435	-0.49232
291	dum12y	0.47143	0.56712	0.83127
292	coldeg	0.70045	1.14054	0.61414
293	lag1wu	0.14474	0.01425	10.15383
294	lag2wu	0.02528	0.01519	1.66455
295	lag3wu	0.03026	0.01570	1.92667
296	lag4wu	0.00138	0.01576	0.08732
297	lag5wu	0.02328	0.01595	1.45945
298	cumtr	-0.34769	0.18970	-1.83286
299	year79	-0.49721	1.78280	-0.27890

300	year8082	1.93042	0.80413	2.40063
301	year9294	0.14282	0.81495	0.17525
302	oldcoh	-1.01979	0.60431	-1.68753
303	lag1ch	-0.19298	0.44556	-0.43312
304	lag2ch	-0.61154	0.50298	-1.21583
305	lag3ch	-0.63270	0.58094	-1.08910
306	lag4ch	0.25363	0.62387	0.40655
307	lag5ch	0.50435	0.64186	0.78576
308	sdwun	9.03644	0.13109	68.93370
309	rhoc21	12.51703	1.35019	9.27057
310	rhoc22	-3.14764	0.90937	-3.46135

**Log of Hourly Average Earnings (w) ("OLS"-type):**

311	cons_lnw	-0.79345	0.70452	-1.12624
312	year	-0.01642	0.00547	-3.00415
313	afqt	0.00359	0.00022	16.03136
314	readmat	0.03431	0.01343	2.55570
315	libcard	0.02460	0.01048	2.34703
316	livpar	-0.02949	0.01067	-2.76306
317	prot	-0.04015	0.01080	-3.71902
318	black	-0.07863	0.01386	-5.67332
319	hisp	-0.00597	0.01487	-0.40132
320	nc	-0.14034	0.01366	-10.27481
321	so	-0.12452	0.01249	-9.96872
322	we	-0.06747	0.01377	-4.89873
323	urb	0.08424	0.01000	8.42671
324	ur	-0.00811	0.00130	-6.24792
325	mw	0.07405	0.14320	0.51712
326	mwage	0.00351	0.00553	0.63577
327	mwhgc	-0.00968	0.00640	-1.51177
328	age	0.13150	0.02625	5.00916
329	age2	-0.00267	0.00028	-9.43860
330	exp	0.06732	0.00409	16.46816
331	exp2	-0.00160	0.00018	-9.06741
332	hgc	0.07907	0.02086	3.78987
333	dum12y	-0.04068	0.01550	-2.62389
334	coldeg	0.09227	0.02476	3.72709
335	lag1wu	-0.00177	0.00040	-4.45569
336	lag2wu	-0.00126	0.00038	-3.32614
337	lag3wu	-0.00115	0.00040	-2.88001
338	lag4wu	-0.00081	0.00042	-1.90704
339	lag5wu	0.00024	0.00042	0.57447
340	cumtr	0.03255	0.00358	9.08066
341	year79	0.13989	0.04227	3.30958
342	year8082	0.04067	0.02026	2.00760
343	year9294	-0.00968	0.01594	-0.60745
344	oldcoh	-0.01013	0.01783	-0.56847

345	lag1ch	-0.03192	0.01211	-2.63596
346	lag2ch	-0.00084	0.01253	-0.06680
347	lag3ch	-0.00270	0.01371	-0.19669
348	lag4ch	-0.00832	0.01371	-0.60640
349	lag5ch	0.01062	0.01428	0.74410
350	sdlnw	0.33642	0.00175	192.79140
351	rhoc31	0.73388	0.02455	29.89510
352	rhoc32	1.50124	0.02059	72.90562

**Initial Schooling Level in 1979 (is) ("OLS"-type):**

353	cons_ini	-5.70145	3.87557	-1.47113
354	mohgc	0.01863	0.00690	2.69980
355	fahgc	0.01844	0.00574	3.21460
356	sibnum	-0.02429	0.00653	-3.72103
357	rbne	0.08320	0.08732	0.95281
358	rbnc	0.29234	0.09655	3.02799
359	rbso	0.02224	0.07750	0.28698
360	rbwe	0.22578	0.09657	2.33795
361	r14ne	1.02708	0.09308	11.03479
362	r14nc	0.78330	0.09515	8.23262
363	r14so	1.03586	0.08045	12.87626
364	r14we	1.10845	0.09235	12.00222
365	afqt	0.01262	0.00071	17.72058
366	readmat	0.19703	0.04181	4.71209
367	libcard	0.14383	0.04112	3.49746
368	livpar	0.05846	0.03765	1.55274
369	prot	0.06035	0.04388	1.37546
370	black	0.26490	0.04969	5.33104
371	hisp	0.11269	0.06146	1.83346
372	ugtuit	1.49251	0.63685	2.34358
373	expsec	-0.47857	0.32260	-1.48349
374	expps	-0.01924	0.18976	-0.10141
375	age	1.01264	0.43514	2.32714
376	age2	-0.01145	0.01230	-0.93044
377	age1415	-0.66638	0.18732	-3.55745
378	age16	-0.37201	0.11488	-3.23832
379	age17	-0.10472	0.07952	-1.31691
380	sdhgcini	0.92389	0.00732	126.24723
381	rhoc41	0.11976	0.13810	0.86726
382	rhoc42	0.32200	0.11723	2.74673

**Heterogeneity Information:**

POINT	PROB. WEIGHT	MASS POINT
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## FIRST PERMANENT HETEROGENEITY

1	0.02925	0.00000
2	0.08027	0.37403
3	0.32656	0.60835
4	0.44150	0.77702
5	0.12242	1.00000

## SECOND PERMANENT HETEROGENEITY

1	0.12959	0.00000
2	0.35603	0.58521
3	0.48150	0.31993
4	0.03288	1.00000

## NONLINEAR TRANSITORY HETEROGENEITY

## Equation 1: Any Schooling

1	0.17477	0.00000
2	0.03244	0.02102
3	0.01183	-0.12577
4	0.42026	0.22717
5	0.02763	-0.07135
6	0.33308	0.69062

## Equation 2: Any Training

1	0.17477	0.00000
2	0.03244	-0.00446
3	0.01183	0.07182
4	0.42026	0.04399
5	0.02763	0.05970
6	0.33308	-0.05100

## Equation 3: Any Work

1	0.17477	0.00000
2	0.03244	2.53530
3	0.01183	-3.18005
4	0.42026	-0.12075
5	0.02763	4.94166
6	0.33308	0.16036

## Equation 4: Any Unemployment

1	0.17477	0.00000
2	0.03244	-0.04904
3	0.01183	-0.75519
4	0.42026	-4.95586
5	0.02763	-1.23314
6	0.33308	-1.22591

## Equation 5: Any Job Change

1	0.17477	0.00000
2	0.03244	0.29726

3	0.01183	0.19501
4	0.42026	-2.23391
5	0.02763	-0.84130
6	0.33308	-99.00000

## Equation 6: Hours worked

1	0.17477	0.00000
2	0.03244	-952.28885
3	0.01183	255.23279
4	0.42026	398.01243
5	0.02763	-76.58800
6	0.33308	96.74391

## Equation 7: Weeks Unemployed

1	0.17477	0.00000
2	0.03244	15.27832
3	0.01183	27.98220
4	0.42026	21.25957
5	0.02763	0.00354
6	0.33308	-4.32722

## Equation 8: Log Wage

1	0.17477	0.00000
2	0.03244	1.25946
3	0.01183	-3.60136
4	0.42026	0.04553
5	0.02763	-1.58095
6	0.33308	0.10578

## Equation 9: Initial Schooling Level (no transitory heterogeneity)

1	0.17477	0.00000
2	0.03244	0.00000
3	0.01183	0.00000
4	0.42026	0.00000
5	0.02763	0.00000
6	0.33308	0.00000

## ST. DEV. OF FIRST PERMANENT HETEROGENEITY:

0.19800532191946

## ST. DEV. OF SECOND PERMANENT HETEROGENEITY:

0.21874620581648

## ST. DEV. OF NONLINEAR TRANSITORY HETEROGENEITY:

Equation:	1	St. Dev. :	0.42531946262434
Equation:	2	St. Dev. :	4.2893100665523D-02
Equation:	3	St. Dev. :	1.0086913177151
Equation:	4	St. Dev. :	3.2971649614189
Equation:	5	St. Dev. :	57.154377787081
Equation:	6	St. Dev. :	316.29440007216

Equation:	7	St. Dev. :	14.595081582500
Equation:	8	St. Dev. :	0.52774440325561
Equation:	9	St. Dev. :	0.0