The Impact of Productivity Growth on Labor Market Dynamics

Hiroaki Miyamoto*
University of Wisconsin-Madison

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Abstract

This paper studies the impact of productivity growth on the rates of worker flows, and thus the unemployment rate. I find that in the U.S. labor market, faster productivity growth reduces the separation rate and the unemployment rate in the long-run. I incorporate disembodied technological progress, on-the-job search and sunk costs of creating job positions into a model of endogenous job separation by Mortensen and Pissarides (1994). The incorporation of on-the-job search and sunk costs for job creation gives rise to new channels through which faster growth may reduce unemployment by reducing the separation rate and inducing more job creation. I demonstrate that introducing these two factors substantially improves the performance of the Mortensen and Pissarides model in accounting for the impact of growth on unemployment. Quantitative analysis of the model shows that these new channels magnify the reduction in unemployment due to growth compared to the standard matching model with productivity growth. The paper also provides empirical evidence on the long-run relationship among productivity growth, unemployment rates and rates of job finding and separation in the U.S. labor market.

Keywords: Growth; Unemployment; Search-matching model; On-the-job search; Vacancy creation

JEL Classification: E24; J64; O40

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Mailing Address: University of Wisconsin-Madison, Department of Economics, 1180 Observatory Dr, Madison, WI 53706-1393, USA. E-mail: hmiyamoto@wisc.edu.

1 Introduction

How does productivity growth influence the unemployment rate? Recent empirical studies find that there is a strong negative relationship between them.[1] The search and matching model with disembodied technological progress (DTP henceforth) generates a negative impact of productivity growth on unemployment through more job creation: faster growth raises the returns to job creation, thus firms are encouraged to post more vacancies and the job finding rate of unemployed workers rises, resulting in lower unemployment.[2] However, the recent study by Pissarides and Vallanti (2007) demonstrates that the matching model with DTP fails to explain the magnitude of the impact of growth on unemployment under standard assumptions and plausible parameter values.

While focusing extensively on the job creation side of the labor market, the search and matching model with DTP has downplayed the role of job separation.[3] In the model, the separation rate is exogenous and assumed to be constant. However, by examining the long-run relationship between productivity growth and labor market variables in the United States, I find that while the job finding rate is positively correlated with the growth rate, there is a strong negative correlation between the separation rate and growth. These empirical findings suggest that productivity growth reduces the unemployment rate through not only increased job finding but also decreased separation. Therefore, in the matching model, the negative impact of productivity growth on unemployment might be strengthened if job separation is endogenized. However, in a model with endogenous separation, productivity growth in fact tends to increase the separation rate by creating better job opportunities for currently employed workers.[4] As a result, the impact of productivity growth on unemployment becomes qualitatively ambiguous. Furthermore, under plausible parameter values, the impact becomes positive.

The purpose of this paper is to provide a model that explains the above-mentioned empirical facts. For this purpose, I extend the endogenous job separation model of Mortensen and Pissarides (1994) with DTP by introducing sunk costs of creating new jobs, and on-the-job search. Empirical evidence points out the important difference between job flows and worker flows (Davis, Haltiwanger and Schuh, 1996, Davis and Haltiwanger, 1999). Part of the difference comes from job-to-job flows of workers and a strong tendency of employers to maintain jobs when workers quit. For example, Fallick and Fleischman (2004) and Nagypál (2005) document the pervasive job-to-job flows in the U.S. economy, and Blanchard and Diamond (1990) argue that roughly 85

[1] See Bruno and Sachs (1985), Ball and Moffitt (2001), Muscatelli and Tirelli (2001) , Staiger, Stock and Watson (2001) , Tripier (2006) and Pissarides and Vallanti (2007).

[2] This effect is identified by Aghion and Howitt (1994) as the *capitalization effect*.

[3] It is important to note that a search and matching model with embodied technological progress focuses on the job destruction side. However, the model with embodied technological progress generates a positive impact of growth on unemployment.

[4] See Prat (2007) for the detail.

percent of quits are replaced. This empirical evidence suggests that on-the-job search and costs of creating new jobs are important elements in understanding labor market dynamics. Thus, the incorporation of sunk costs for job creation and on-the-job search into the standard search model is a natural extension, especially in light of empirical findings.

By introducing both sunk costs for job creation and on-the-job search, the model in this paper can capture the process of replacement after quitting and make a distinction between job flows and worker flows. In the standard search and matching model, firms incur no cost to create jobs and thus there is no distinction between the creation of jobs and filling of vacancies. However, by introducing costs for job creation explicitly, I can analyze the decision of firms to create a job position and distinguish it from the filling of vacancies. In the model, jobs differ by idiosyncratic productivity level, and the heterogeneity in productivity motivates on-the-job search. In the standard on-the-job search model, it is assumed that the job is destroyed when a worker quits. However, in my model, because of the presence of sunk costs related to job creation, jobs are valuable once they are created. This implies that firms may find it optimal to refill their vacant jobs when their workers quit.

The incorporation of sunk costs for job creation and on-the-job search gives rise to new channels through which faster growth may reduce unemployment. First, the incorporation of sunk costs and on-the-job search makes some otherwise unproductive jobs productive enough to survive, leading to decreased separation when the rate of productivity growth rises. Second, on-the-job search generates more vacancies by accelerating the reallocation of workers when the rate of productivity growth rises. When growth accelerates, a rise in search intensity of employed workers increases the value of newly created jobs by facilitating recruitment. Third, the introduction of sunk costs for job creation increases job creation with faster productivity growth by strengthening the capitalization effect. These new channels produce two main results. First, the model demonstrates that faster productivity growth reduces the separation rate and increases the job finding rate, leading to a lower unemployment rate. Second, the model magnifies the reduction in the unemployment rate due to productivity growth, compared to the standard search and matching model with DTP.

The incorporation of sunk costs and on-the-job search improves the ability of the matching model to fit the data. The model generates a negative impact of growth on unemployment comparable to empirical estimates. In the model, a one percentage point decline in productivity growth leads to a 0.21% increase in the unemployment rate. Blanchard and Wolfers (2000) estimate that a 1% decline in the growth rate leads to 0.25%-0.7% increase in the unemployment rate. Pissarides and Vallanti (2007) find the effect to be 1.3% to 1.5%. Thus, the model performs well in matching the estimates. Furthermore, the model magnifies the reduction in the unemployment rate due to productivity growth, compared with the standard matching model with DTP. Under plausible parameters, the standard endogenous job separation model with DTP generates a positive impact of productivity growth on unemployment, and a one percentage point decline in productivity growth leads to a decrease in unemployment rate of 0.35%. Pissarides and Vallanti (2007) find that, in the standard matching model with DTP and exogenous job separation, a 1% decrease in growth rate increases the unemployment by about 0.01%, under the assumption of Nash bargaining wage determination. Thus, my model generates an empirically consistent sign of the effect, and a larger size of magnitude compared with standard models.

Related Literature: The search and matching theory predicts that the impact of productivity growth on unemployment depends on the extent to which new technology is embodied in new jobs (Mortensen and Pissarides, 1998, Pissarides and Vallanti, 2007). The standard search and matching model with disembodied technological progress predicts that a faster rate of productivity growth reduces unemployment through the so called capitalization effect (Pissarides, 2000, chapter 3). On the other hand, in the model with embodied technological progress, faster productivity growth can lead to higher unemployment through the creative destruction effect (Aghion and Howitt, 1994, 1998; Postel-Vinay, 2002). Motivated by the empirical evidence that productivity growth negatively affects the unemployment rate, Pissarides and Vallanti (2007) demonstrate that the consistency between the evidence and the model requires totally disembodied technology. In the present paper, I follow the implication of Pissarides and Vallanti (2007), and study the impact of DTP on labor market dynamics. Prat (2007) studies the impact of DTP on the unemployment rate in a model of endogenous job separation. He demonstrates that under plausible parameter values faster productivity growth increases the unemployment rate due to an outside option effect. My model demonstrates that by introducing sunk costs for job creation and on-the-job search, the search and matching model can generate a negative impact of productivity growth on unemployment under the setup of endogenous job separation.

In order to analyze the decision of job creation by firms, I incorporate sunk costs of creating new job positions into the matching model. Fujita and Ramey (2007) consider the matching model in which firms incur sunk costs of creating new positions, and distinguish between job flows and worker flows. Although Fujita and Ramey (2007) distinguish between job flows and worker flows, they don't consider search by employed workers explicitly. Furthermore, they focus on cyclical adjustment of labor market rather than long-run economic growth.

My paper also incorporates on-the-job search into the search and matching model. Pissarides (1994), Mortensen (1994), Pissarides (2000, ch.4), Barlevy (2002) and Nagypál (2007) also consider the role of on-the-job search in the framework of a matching model. While these papers assume that jobs are destroyed when employees quit, I consider the decision of the employer to refill the job position when the worker quits. The model in this paper is close to the model of Faberman and Nagypál (2008). While they assume that the search effort of workers is exogenous and constant, I consider a model of an endogenous search effort. Furthermore, none of these papers discusses the impact of productivity growth on labor market dynamics, which is the main focus of this study.

This paper provides empirical evidence on the long-run relationships among productivity growth, unemployment rates, and rates of worker flows in the U.S. labor market. Recently, several studies investigate contributions of inflows and outflows to the dynamics of unemployment. Shimer (2007) and Fujita and Ramey (2007) study the cyclical behavior of rates of job finding and separation, and also measure the contributions of fluctuations in job finding and separation to the variability of unemployment in the U.S. economy. Similarly, Petrongolo and Pissarides (2008) study the contribution of inflows and outflows to the dynamics of unemployment in European countries. While they focus on labor market dynamics over the business cycle, in the present paper I investigate its long-run properties.

The remainder of the paper is organized as follows. Section 2 presents salient features of the U.S. aggregate labor market data in the long-run trend, and discusses the relationship between productivity growth and labor market variables. Section 3 describes the theoretical model. I develop a generalized Mortensen and Pissarides model with sunk costs for job creation and on-the-job search. In Section 4, I calibrate the model parameters and present the result of quantitative comparative statics exercises. Section 5 studies the relationship between productivity growth and unemployment. Conclusion and suggestions for future research are presented in Section 6. In the Appendix, the steady state equilibrium of the model is characterized.

2 U.S. labor market facts

In this section, I present some of the salient features of the U.S. aggregate labor market data in the long-run trend, and discuss the relationship between productivity growth and labor market variables. I focus on productivity growth g and three labor market variables: the unemployment rate u, the job finding rate f, and the separation rate s.

The labor productivity growth is measured by the first difference of logged labor productivity. I use real output per person in the non-farm business sector as labor productivity. The Bureau of Labor Statistics (BLS) constructs this quarterly time series as part of its Major Sector Productivity and Costs program. Note that using output per hour or total factor productivity as a measure of labor productivity yields similar results, but using this series is natural way to consider productivity in the standard search and matching model, which I develop in subsequent section. Unemployment rate is the quarterly average of seasonally adjusted monthly data constructed by the BLS using the Current Population Survey (CPS) data.

The dynamics of the unemployment rate are determined by the underlying flows into and out of unemployment, particularly by the rates at which workers match with and separate from jobs. In this paper, I define the job finding rate as the rate of transition from unemployment to employment, and the separation rate as the rate of transition from employment to unemployment. Shimer (2007) uses short-term unemployment data and total unemployment data to pin down these rates. Following Shimer's (2007) time aggregation correction, I measure job finding and separation rates from the CPS over the 1948-2005 period.

Since my focus is the long-run relationship among productivity growth, unemployment rate, and rates of worker flows, I consider the band-pass filtering as the method for smoothing the data. The band-pass filter is a linear filter which retains the cyclical components of each series within a specific band of frequency, and removes other components. By using the band-pass filter, I can isolate long-term components of the labor productivity growth and labor market variables, and study their relation. Let y_t be a quarterly time series, and let y_t^* denote its trend. Following Staiger et al. (2001), y_t^* is estimated by passing y_t through a two-sided low pass filter, with a cutoff frequency corresponding to 15 years.[5] Essentially, this estimates y_t^* as a long two-sided weighted moving average of y_t with weights that sum to one. Estimates of the trend at the beginning and end of the sample are obtained by extending the series with autoregressive forecasts and backcasts of y_t, constructed from an estimated AR(4) model for the first difference of y_t.

Figure 1 presents quarterly time series data and their estimated trends for (a) labor productivity growth, (b) the unemployment rate, (c) the job finding rate, and (d) the separation rate. Table 1 summarizes the relationship among smoothed series of the four variables that are my focus here. For the purpose of comparison, I also use the HP filtering and report the relationship among smoothed series in Table 1. However, it is worth noting that using the HP filter is not suitable for the analysis of the long run components of an economic series. The HP filter is best interpreted as a high-pass filter isolating frequencies of 8 years and higher in economic data and is not intended for frequencies falling into other bands. Moreover, although the HP filter produces cyclical components that are covariance stationary for raw series that are integrated up to order four, the trend component of it reflects the non-stationarirty of the raw data.

Table 1: Summary Statistics

Band-pass filtered Data					HP-filtered Data				
	u	g	f	s		u	g	f	s
u	1	-0.659	-0.841	0.854	u	1	-0.819	-0.744	0.860
g	-	1.	0.310	-0.842	g	-	1	0.575	-0.759
f	-	-	1	-0.447	f	-	-	1	-0.305
s	-	-	-	1	s	-	-	-	1

Figure 1 and table 1 show several important features of these data. First I look at the relationship between productivity growth and unemployment rate. In Figure 2, I reproduce the smoothed series for these two variables. The smoothed series move closer together until early 1960s, then apart during the 1970s, and then come slowly back to near their starting levels. The

[5]When I adopt the definition of the business cycle as the cyclical components between 1.5 years and 8 years following Baxter and King (1999) and Stock and Watson (1999) and use these limits as the definition of business cycles so to isolate the trend or low frequency of the data, I get similar results.

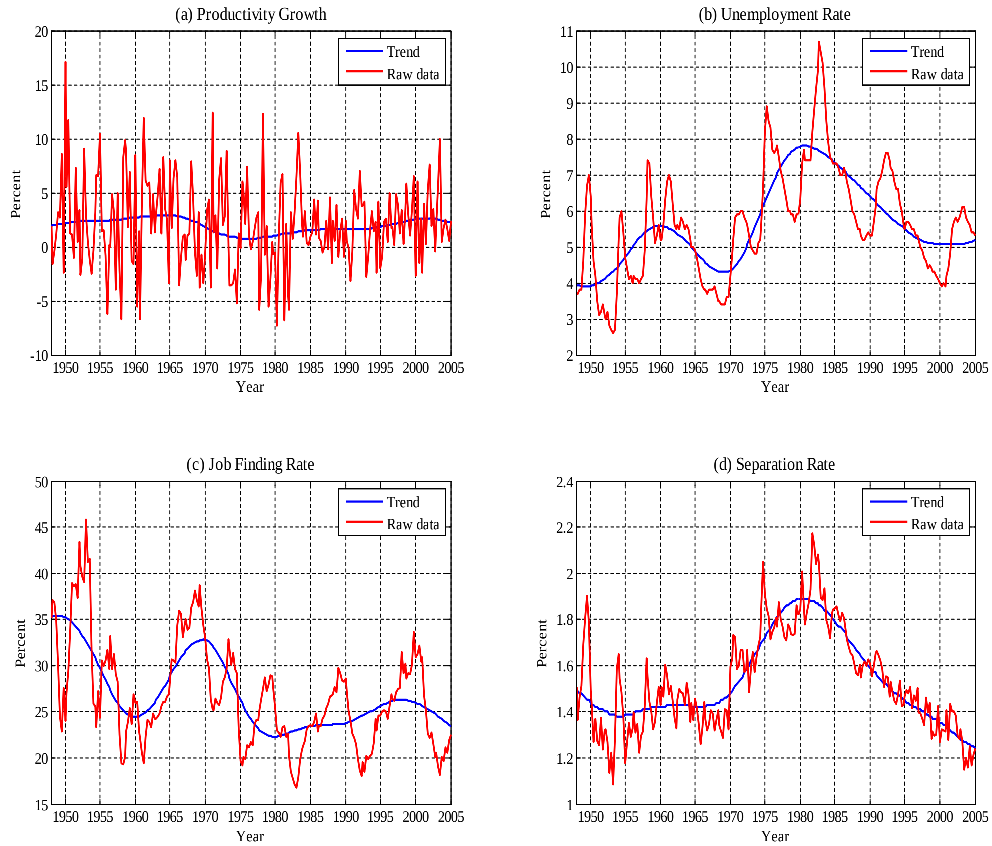

Figure 1: Macro series and their trend values

correlation between these two series during the sample period is -0.659 in band-pass filtered series and -0.819 in HP filtered series. Thus, there is a strong negative relationship between productivity growth and unemployment rate at low frequencies. This finding confirms the empirical evidences from the previous studies[6], and supports theories predicting a negative relationship between productivity growth and unemployment, such as Pissarides (2000) and Pissarides and Vallanti (2007).

Figure 1-(c) and 1-(d) show the smoothed series for job finding and separation rates. The striking fact is the difference in the time series properties of job finding and separation rates. The job finding rate exhibits large, averaging 27 percent during the sample period. In contrast,

[6]Staiger et al. (2001) find a strong negative relationship between productivity growth and unemployment rate in the U.S. economy. Muscatelli and Tirelli (2002) find a negative correlation between these two variables for Japan, Germany, Italy, France, and Canada. Pissarides and Vallanti (2007) also find the negative relationship between total factor productivity (TFP) and unemployment in the US, Japan and European countries.

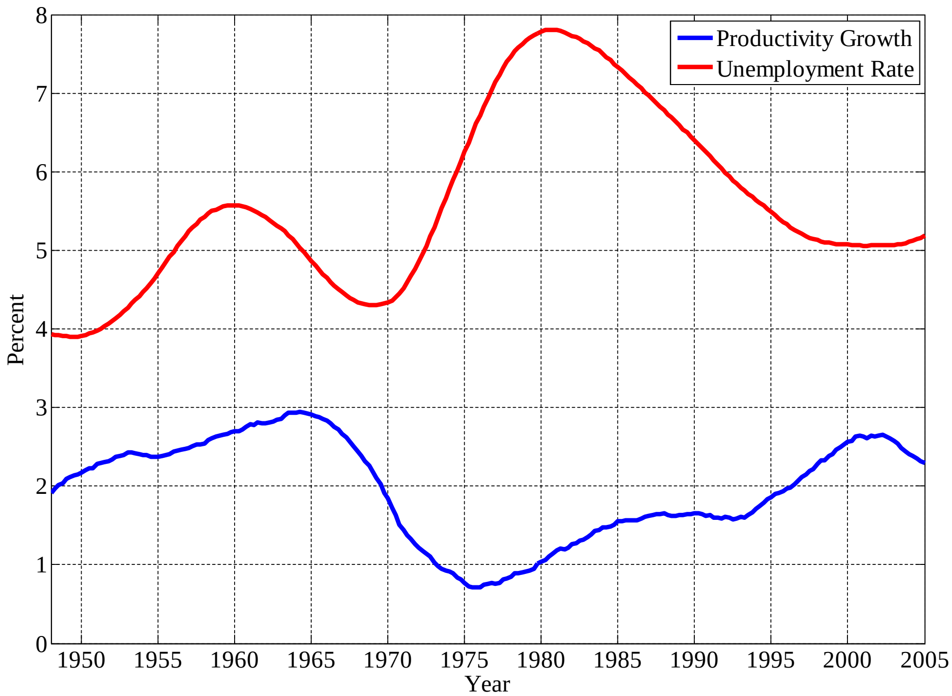

Figure 2: Productivity growth and unemployment rate

the separation rate is low and averaged 1.5 percent during the sample period. The separation rate has upward trend in the 1960s and 70s and downward trend subsequently. It is important to note that the separation rate co-move positively with unemployment rate and negatively with productivity growth. The correlation between the separation rate and unemployment rate is 0.854 in band-pass filtered series and 0.860 in HP filtered series. The correlation between the separation rate and the rate of productivity growth is -0.842 in band-pass filtered series and -0.759 in HP filtered series. On the other hand, the job finding rate is negatively related with unemployment rate and its correlation is -0.841 in band-pass filtered series and -0.744 in HP filtered series. I can also see the positive relationship between job finding rate and productivity growth. Their correlation is 0.310 in band-pass filtered series and 0.575 in HP filtered series.

In the above I have seen that job finding rates co-move negatively with unemployment rates and separation rates co-move positively with unemployment rates in the long-run. Now I quantify the contributions of separation and job finding rates to overall unemployment variability in the long-run following Shimer (2007) and Fujita and Ramey (2007). To analyze the contribution of the hazard rates to unemployment variability, Shimer (2007) and Fujita and Ramey (2007) approximate the unemployment rate by using the theoretical steady-state value associated with the contemporaneous job finding and separation rates. Thus,

$$u_t \simeq \frac{s_t}{s_t + f_t} \equiv \tilde{u}_t$$

where u_t is unemployment rate, s_t is the rate of transition from employment to unemployment (i.e., separation rate), and f_t is the rate of transition from unemployment and employment (i.e., job finding rate). Let $\bar{f}$ and $\bar{s}$ denote the average value of f_t and s_t during the sample period. Then I compute the hypothetical unemployment rates

$$u_t^F \equiv \frac{\bar{s}}{\bar{s} + f_t} \text{ and } u_t^S \equiv \frac{s_t}{s_t + \bar{f}}$$

as measures of the contributions of fluctuations in the job finding and employment exit rates to overall fluctuations in the unemployment rate.

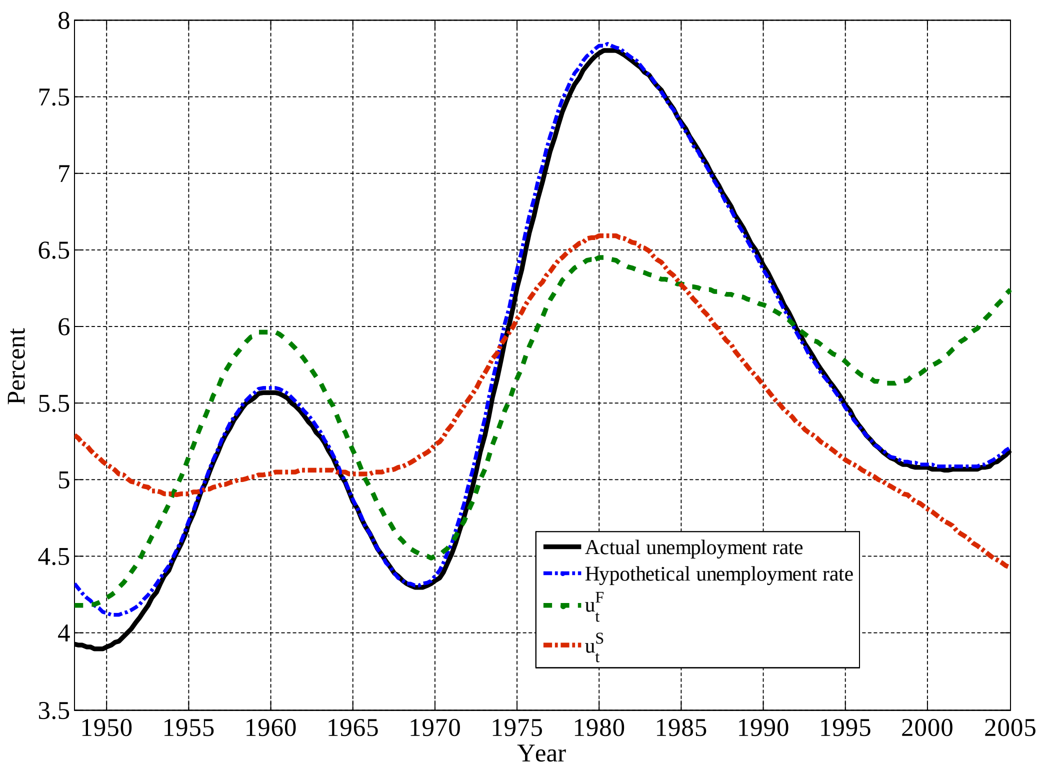

Figure 3: Contribution of Unemployment Rate Variability

Figure 3 plots hypothetical unemployment rates $\tilde{u}_t$, u_t^F and u_t^S together with the unemployment rate. Figure 3 shows that both the job finding rate and the separation rate tend to move with the unemployment rate during the sample period.

To quantify the contribution of separation and job finding rates to the overall unemployment variability, I look at the co-movement of long-run component of the data. Over the sample period, the covariance of the cyclical component of u_t and u_t^F accounts for about the half of the variance of the cyclical component of u_t. Similarly, cyclical fluctuation in the separation rate also explains 47 percent of the fluctuation in unemployment rate. Thus, my finding suggests that job finding rate and separation rate account for roughly similar proportions of overall unemployment variability in the long-run.

Since both job finding and separation rates contribute to the overall unemployment variability in the long-run, the fact that the job finding rate is positively correlated with productivity growth seems to support the capitalization effect story of Pissarides (2000). Furthermore, it is important to note that the fact of the negative correlation between the separation rate and productivity growth suggests a new channel of the effect of productivity growth on unemployment, which has not been pointed out in the literature.

3 The Model

I consider a search and matching model with endogenous job separation, in the spirit of Mortensen and Pissarides (1994). In order to study the impact of long-run productivity growth on labor market dynamics, I introduce disembodied technological progress, as in Pissarides (2000) and Pissarides and Vallanti (2007). My model differs from the standard endogenous job destruction model with respect to job search and job creation. First, I allow for search by employed workers. Jobs differ by idiosyncratic productivity level, and the heterogeneity in productivity motivates on-the-job search. The search intensity of workers is endogenously determined. Second, I introduce sunk costs for job creation, which captures physical capital required to create a new job. In the standard search model, firms incur no cost to create jobs and thus there is no distinction the creation of jobs and filling vacancies. However, by introducing sunk costs for job creation explicitly, I can analyze the decision of firms to create a job and distinguish between it from the filling of vacancies. By introducing both on-the-job search and sunk costs for job creation, my model can capture the process of replacement after quitting and makes a distinction between job flows and worker flows.

3.1 The environment

I consider an economy consists of firms and workers who are both risk-neutral, infinitely lived and maximize the present discounted value of an income stream with discount rate r. Time is continuous.

Production technology Every moment of time, there is a measure n of potential firms born into the economy. Each potential firm has one project, and each project is endowed with a parameter x which represents the idiosyncratic productivity of the project. The idiosyncratic productivity x is distributed according to a distribution $F : [\underline{x}, \bar{x}] \rightarrow [0, 1]$ with corresponding survival function $\bar{F} = 1 - F$. Subsequently, let $\{x\}$ be a jump process characterized by arrival rate λ and a distribution of new realizations F. When an entrant receives a project, it observes the productivity of the project, and then chooses whether or not to open a job position and to enter the vacancy pool. If a firm starts a new project, then it incurs fixed sunk costs K_t, called start-up costs. The firm does not have to pay the sunk cost for this job position in the future. Given this structure, a firm chooses a reservation productivity x^* and posts a job position whose productivity is above it. If the firm does post a vacant job, it then pays a flow cost of posting a vacancy γ_t and searches for a worker.

When an idiosyncratic shock changes the productivity of a vacant job, a firm chooses to keep the vacancy open or close a current opening and exit from the market. The vacancy posting decision of the firm is determined by a reservation rule, and a firm posts a vacancy when its productivity is above some threshold $\hat{x}$.

Production takes place in one firm-one worker pairs. The match produces a flow of output px where p is general productivity parameter common to all producing jobs and x is an idiosyncratic productivity specific to each job. Suppose that the leading technology in the economy is driven by an exogenous invention process that grows at the rate $g < r$. Let the general productivity parameter be a function of time, and suppose that it grows at the constant rate g, i.e.,

$$p_t = e^{gt} p_0$$

where $p_0 > 0$ is some initial productivity level and is assumed to be normalized to one.

Facing the post shock productivity x, the worker and the firm have to choose either to continue producing at the new productivity or to terminate the employment relationship. Job destruction is driven by a reservation productivity rule on the idiosyncratic productivity parameter. Each worker-pair of a firm and a worker chooses the reservation value R and destroys the job if the idiosyncratic productivity falls below it. The reservation productivity is chosen so as to maximize the value of the job to both the firm and the worker.[7] When the job is destroyed, the firm leaves the labor market and the worker moves from employment to unemployment and searches for another job.

Matching technology There is a single matching market with a matching function that determines the number of meetings between workers and firms as a function of the total amount of search effort of workers, $\bar{e}$, and the number of vacancies posted, v:

$$m_t = m(v_t, \bar{e}_t)$$

[7] Therefore, there is no inefficient job separation.

The matching function $m(v, \bar{e})$ is continuous, twice differentiable, increasing in its arguments and has constant returns to scale. The meeting rate per unit of vacant jobs is

$$q(\theta_t) = \frac{m(v_t, \bar{e}_t)}{v_t} = m\left(1, \frac{1}{\theta_t}\right)$$

where

$$\theta_t \equiv \frac{v_t}{\bar{e}_t}$$

is labor market tightness in the model at time t. The meeting rate per unit of search effort for workers is

$$\theta_t q(\theta_t) = \frac{m(v_t, \bar{e}_t)}{\bar{e}_t} = m\left(\frac{v_t}{\bar{e}_t}, 1\right).$$

Search decision A worker can be either employed or unemployed. If a worker is employed, he produces output and earns an endogenous wage w_t and he is allowed to search for other jobs. If he is unemployed, he receive a flow utility z_t and searches for a job. Both unemployed and employed workers can choose to engage in search for a new job at a flow cost $c_t(e)$, where $c_t(e)$ is a strictly increasing, strictly convex, twice continuously differentiable function with $c_t(0) = c_t'(0) = 0$. Workers exerting search intensity e encounter new job opportunities at the Poisson rate $\theta q(\theta)e$. When an employed worker quits a job, the firm either destroys the job and leaves the market or re-enters the vacancy pool and searches for a worker.

Because of on-the-job search and sunk costs of creating vacancies, the distribution of idiosyncratic productivity over filled jobs differs from the distribution over vacant jobs. The measure of filled jobs with an idiosyncratic productivity less than or equal to x is denoted by $\Psi(x)$. Thus, $\Psi(x)$ is the distribution of filled jobs. Since one job is filled with one worker, the distribution of employed workers is also represented by $\Psi(x)$. Then, $\Psi(\bar{x}) = 1 - u$. Similarly, the measure of a vacant job with an idiosyncratic productivity less than or equal to x is denoted by $\Gamma(x)$, so that $\Gamma(\bar{x}) = v$. Thus, the $\Gamma(x)$ is the un-normalized distribution of firm productivity across vacancies. Then, I denote the normalize distribution of firm productivity across vacancies by $\tilde{\Gamma}(x) = \Gamma(x)/v$.

The total amount of search effort $\bar{e}$ exerted by all workers is

$$\bar{e} = ue_u + (1-u)\int_R^{\bar{x}} e(x')d\Psi(x').$$

Rendering the growth model stationary I focus on the steady state of the normalized economy. This corresponds to a balanced growth path where the actual economy grows at the rate of disembodied technological progress g. To make the model stationary, I restrict my attention to the case where all exogenous variables grow at the rate of disembodied technological progress g.[8] Thus, I define three positive exogenous parameters z, γ and K such that $z_t = p_t z$, $\gamma_t = p_t\gamma$

[8]In order to ensure the existence of a balanced growth path, usually all the exogenous parameters are assumed to follow the pace of productivity growth in the literature. See, for example, Mortensen and Pissarides (1999) and Pissarides and Vallanti (2007).

and $K_t = p_t K$. Furthermore, the search cost function also can be rewritten as $c_t(e) = p_t c(e)$.

3.2 Bellman equations

The values of workers and firms in the market are described by a series of Bellman equations. I start by examining the worker's side. Let the value for an employed worker in a job with idiosyncratic productivity x be $W(x)$ and the value for an unemployed worker be U.

The value of employment to be a worker in a job with productivity x is characterized by the following Bellman equation:

$$\begin{aligned}(r-g)\,W(x) &= \max_e \Big\{ w(x) - pc(e) + \lambda \left[\int \max[W(x'), U] dF(x') - W(x) \right] \\ &\quad + e\theta q(\theta) \int \mathbb{I}\left\{W(x') > W(x)\right\} \left[W(x') - W(x)\right] d\tilde{\Gamma}(x') \Big\}\end{aligned} \tag{1}$$

where $\mathbb{I}\{\cdot\}$ is an indicator function and it equals one if its expression is true, zero otherwise. The value of an employed worker in a job with productivity x is determined by several factors. The worker receives wage $w(x)$. The match draws a new value of idiosyncratic productivity at rate λ, in which case the worker loses the current asset value $W(x)$ and gains the asset value associated with working at the new productivity level x' or being unemployed, whichever is greater. Moreover, the worker optimally chooses his search intensity at cost $pc(e)$ and obtain the benefit of meeting new job opportunities at rate $e\theta q(\theta)$. If the worker encounters a new firm, he accepts any job that has a higher asset value $W(x')$ than the current asset value $W(x)$. Finally, the asset value of a match is expected to change over time due to exogenous technological progress.

The value for an unemployed worker is

$$(r-g)\,U = \max_e \left\{ pz - pc(e) + e\theta q(\theta) \int \left\langle \max[W(x'), U] - U \right\rangle d\tilde{\Gamma}(x') \right\} \tag{2}$$

An unemployed worker also chooses his search effort e at cost $p_t c(e)$.

I now turn to the firm side. Let $J(x)$ denote the asset value for the firm with a filled job with idiosyncratic productivity x. Given the optimal search intensity of an employed worker in a job with productivity x by $e(x)$, the value of a filled job with an idiosyncratic productivity x satisfies

$$\begin{aligned}(r-g)\,J(x) &= px - w(x) + \lambda \left[\int \max[J(x'), V(x')] dF(x') - J(x) \right] \\ &\quad + e(x)\theta q(\theta) \int \mathbb{I}\left\{W(x') > W(x)\right\} [V(x) - J(x)]\, d\tilde{\Gamma}(x')\end{aligned} \tag{3}$$

where V is the value of posting a vacancy. A firm with a filled job receives flow revenues of $px - w(x)$, which is the productive output of the match minus the wage paid to the worker. The match draws a new value of idiosyncratic productivity at rate λ. Facing the changed productivity, the firm decides to continue producing if $J(x')$ is larger than the value of a vacant job. The match may be destroyed by worker's quitting at rate $e(x)\theta q(\theta)\int_{\underline{x}}^{\bar{x}}\mathbb{I}\{W(x') > W(x)\}d\tilde{\Gamma}(x')$, in which case the firm loses the asset value of the filled job and obtains the value of a vacant job. Finally, the asset value of a match is expected to change over time due to exogenous technological progress. Note that when the match chooses to terminate the employment relationship, the job will be destroyed and the firm exits the market. On the other hand, when the employee quits the firm due to on-the-job search, the firm can keep the job position and search for a new worker to fill the position.

The value of a vacancy is

$$(r-g)\,V(x) = \max\left\{0, -p\gamma + q(\theta)A(x)\,[J(x) - V(x)] + \lambda\left[\int \max[V(x'), 0]dF(x') - V(x)\right]\right\}. \tag{4}$$

where $A(x)$ is the probability that a searching worker accepts a job with productivity x. This is the ratio of search effort by workers who are willing to accept a match with initial productivity x to the total amount of search effort $\bar{e}$ exerted by all workers. Thus,

$$A(x) = \begin{cases} \dfrac{1}{\bar{e}}\left[ue_u + (1-u)\int_R^x e(x')d\Psi(x')\right] & \text{if} \quad x \geq R \\ 0 & \text{if} \quad x < R \end{cases}. \tag{5}$$

The firm can choose to shut down a current vacancy or to open the vacant job. When the firm chooses to close the job position, its payoff is zero. When the firm keeps the vacant job, the value of a vacant job is determined by three factors. The firm pays a flow cost of posting the vacancy $p\gamma$. The firm meets a worker and starts production with probability $q(\theta)A(x)$. In the case, the firm obtains the net value $J(x) - V(x)$. Moreover, the firm draws a new value of idiosyncratic productivity at rate λ, in which case the firm loses the current asset value and gains the asset value associated with new productivity level x'. In order to ensure that it is never optimal for a firm to keep a vacancy if the job cannot be profitable when it is filled at current productivity, I assume that the cost of posting a vacancy is large enough so that $p\gamma \geq \lambda \int V(x')dF(x')$.

3.3 Surplus sharing and wage determination

Wages are determined by the sharing of the surplus from the match where the worker and the firm receive share β and $1-\beta$, respectively. It is assumed that wages can be revised continuously at no cost, so long-run contracts are ruled out.[9] Furthermore, I assume that matches cannot be recalled. Note that the outside option of the worker is unemployment under these assumptions. It is important to note that I assume at each instant: the worker first decides the level of search intensity in anticipation of wage outcomes and then the surplus sharing takes place.

[9]Thus, even if an employed worker could start negotiations with a new employer before resigning from the current job, this would not affect the equilibrium outcome. The new employer would immediately renegotiate the wage once the worker breaks the relationship with the previous employer.

The key object for the characterization of the model is the match surplus function. Let $S(x)$ be the joint gross return from a match with job-specific productivity x. Then the surplus function is

$$S(x) = J(x) + W(x) - U - V(x). \tag{6}$$

Surplus sharing implies

$$\begin{aligned} J(x) - V(x) &= (1-\beta)S(x) \\ W(x) - U &= \beta S(x) \end{aligned}$$

Because of the timing structure of the model and the nature of bargaining, there is no role for a wage in reducing the likelihood of a worker quitting. Therefore, the non-convexity of the Pareto frontier discussed in Shimer (2006) does not arise in my model. In the model, at each instant a worker decides how much to search in anticipation of the wage outcome and then the sharing of surplus takes place. Thus, the surplus sharing rule does not allow for the wage to be used to influence the search behavior of the worker. This allows us to determine the wage as an outcome to Nash bargaining, since the feasible payoff set is convex.[10]

By making use of equations (1),(3) and (6), I obtain the equation characterizing $S(x)$ as follows:

$$\begin{aligned} (r-g)\left[S(x)+V(x)+U\right] &= px - pc(e(x)) + \lambda\left[\int \left\langle \max[S(x'),0] + V(x')\right\rangle dF(x') - S(x) - V(x)\right] \\ &\quad + e(x)\theta q(\theta)\int \mathbb{I}\left\{S(x') > S(x)\right\}\left[\beta S(x') - S(x)\right] d\tilde{\Gamma}(x') \end{aligned} \tag{7}$$

Since the surplus function $S(\cdot)$ is strictly increasing, the firm and the worker will choose to adopt a reservation policy, i.e., they will choose to continue their match if $S(x) \geq 0$ but stop if $S(x) < 0$. Thus, a separation takes place at $x = R$, where R is defined by

$$S(R) = 0 \Leftrightarrow W(R) = U \Leftrightarrow J(R) = V(R).$$

3.4 Optimal search choice

Since $W(x)$ is strictly increasing in x, an employed worker accepts all newly encountered jobs that have a higher initial productivity than the productivity of his current job, and reject all other newly encountered jobs. Thus,

$$\int_{\underline{x}}^{\bar{x}} \mathbb{I}\{W(x') > W(x)\}\left[W(x') - W(x)\right] d\tilde{\Gamma}(x') = 0 \text{ for } x \geq \bar{x},$$

[10] It may be helpful to consider a discrete time version of the model to understand this argument. In the discrete time model, the search behavior of a worker is governed by returns to search that accrue in the next period. Thus, to reduce the probability of the worker's quitting, the firm should commit to higher wages in the future. However, with continuous renegotiation, such a commitment cannot be made. Therefore, under these assumptions, the payoff set is convex, and surplus sharing is equivalent to the solution of Nash bargaining.

and

$$\int_{\underline{x}}^{\bar{x}} \mathbb{I}\{W(x') > W(x)\} \left[W(x') - W(x)\right] d\tilde{\Gamma}(x') = \int_{x}^{\bar{x}} \left[W(x') - W(x)\right] d\tilde{\Gamma}(x') \text{ for } x < \bar{x}.$$

Then, equation (1) can be rewritten as

$$\begin{aligned}(r-g)\,W(x) \quad &= \quad \max_e \left\{ w(x) - pc(e) + \lambda \left[\int \max[W(x'), U] dF(x') - W(x) \right] \right. \\ &\left. + e\theta q(\theta) \int_{x}^{\bar{x}} \left[W(x') - W(x)\right] d\tilde{\Gamma}(x') \right\}.\end{aligned}$$

The optimal search intensity of an employed worker is found by the use of the above equation. The first-order condition of the above equation is

$$pc'(e) = \theta q(\theta) \int_{x}^{\bar{x}} \left[W(x') - W(x)\right] d\tilde{\Gamma}(x') = \theta q(\theta) \beta \int_{x}^{\bar{x}} \left[S(x') - S(x)\right] d\tilde{\Gamma}(x'). \tag{8}$$

By the strict convexity of the search cost function and by the fact that $S(x)$ is strictly increasing, the optimal search effort by employed workers is strictly decreasing in x for $x < \bar{x}$. Furthermore, by the convexity of $c(\cdot)$ and $c'(0) = 0$, $e(x) = 0$ for $x \geq \bar{x}$.

Since $W(\cdot)$ is strictly increasing, an unemployed worker accepts any job with productivity $x \geq R$. Thus, the value function of an unemployed worker can be rewritten as

$$(r-g)\,U = \max_e \left\{ pz - pc(e) + e\theta q(\theta) \int_{R}^{\bar{x}} \left[W(x') - U\right] d\tilde{\Gamma}(x') \right\}. \tag{9}$$

Denote the optimal search intensity of an unemployed worker by e_u. Then, the first-order condition of (9) yields

$$-pc'(e_u) + \theta q(\theta) \int_{R}^{\bar{x}} \left[W(x') - U\right] d\tilde{\Gamma}(x') = 0 \Leftrightarrow pc'(e_u) = \beta \theta q(\theta) \int_{R}^{\bar{x}} S(x') d\tilde{\Gamma}(x'). \tag{10}$$

Under the assumption that the cost of search effort is the same whether employed or not, a comparison of equations (8) and (10) implies that the optimal search effort when unemployed equals search effort when employed at $x = R$, i.e.,

$$e_u = e(R).$$

3.5 Job creation and vacancy shut-down conditions

A potential firm opens a new job position if its productivity is high enough. Given that $V(x)$ is increasing, the firm's decision on opening a new job is determined by a reservation rule, and a potential firm enters the labor market and posts a vacancy when its productivity is above some threshold x^*. This threshold satisfies

$$V(x^*) = pK. \tag{11}$$

Then, in every moment the entry of new jobs into the economy is determined by

$$n \int_{x^*}^{\bar{x}} dF(x) = n\left[1 - F(x^*)\right].$$

When an idiosyncratic shock changes the productivity of a vacant job, a firm chooses to keep the vacancy open or close a current opening and exit from the market. The vacancy re-posting decision of the firm is also determined by a reservation rule. A firm posts a vacancy when its productivity is above some threshold $\hat{x}$ such that

$$V(\hat{x}) = 0. \tag{12}$$

It is important to note that $\hat{x} > R$ since $J(x) > V(x)$ and $J(\hat{x}) > 0$. Furthermore, the monotonicity of the function V implies that $x^* \geq \hat{x}$. In this structure, a firm with productivity $x \in [R, \hat{x}]$ finds it profitable to continue existing employment relationships but does not find it profitable to re-post a vacancy if the employee quits. When the worker leaves, the firm chooses to close the job position and exit the labor market. Thus, we might call jobs in the range $[R, \hat{x}]$ "obsolete" jobs as they are waiting to die, unless technology saves them. In contrast, the optimal behavior of a firm with productivity $x \geq \hat{x}$ is to re-post a vacancy and refill the position when the worker leaves. Because of the presence of sunk costs for job creation, job positions are valuable once they are created. Thus, some firms try to replace job positions by reposting vacancies when their workers quit through on-the-job search.

3.6 Labor market dynamics

The steady-state unemployment rate is determined by equating the flow into and out of unemployment. Thus,

$$\lambda F(R)(1-u) = \theta q(\theta) e_u u.$$

To complete the characterization of the equilibrium, I need to derive the stationary distribution of productivity across filled jobs and vacancies. These distributions can be obtained from the steady-state balance equations that arise from equating flows into and out of the relevant pool.

I start by considering the distribution of productivity across filled jobs. The measure of filled jobs with idiosyncratic productivity less than or equal to x is denoted by $\Psi(x)$. The flow into this subset consists of newly filled vacancies and filled jobs that had a change in their productivity. The flow out of this subset consists of filled jobs that had a change in their productivity and filled jobs that lost their employees by quitting. Thus, I have

$$\lambda\left[F(x) - F(R)\right]\Psi(\bar{x}) + \int_R^x q(\theta) A(x') d\Gamma(x') = \lambda \Psi(x) + \int_R^x \alpha(x') d\Psi(x'). \tag{13}$$

where

$$\alpha(x') = e(x)\theta q(\theta)[1 - \tilde{\Gamma}(x)]$$

which captures the probability that workers in jobs with productivity x quit.

The distribution of productivity across vacancies can be obtained in the similar manner. The inflow into $\Gamma(x)$ consists of the following three flows. First is the flow of newly created vacancies. Second is the flow of filled jobs that lost their workers and re-post vacancies. Third is the flow of existing vacancies that had a change in their productivity. The flow out of this subset is made up of filled vacancies and vacancies that had a change in their productivity. Equating these two flows gives

$$\mathbb{I}(x \geq x^*)\, n \int_{x^*}^{x} dF(x') + \lambda \left[F(x) - F(\hat{x})\right] \Gamma(\bar{x}) + \int_{\hat{x}}^{x} \alpha(x') d\Psi(x') = \int_{\hat{x}}^{x} q(\theta) A(x') d\Gamma(x') + \lambda \Gamma(x). \tag{14}$$

The total number of vacant jobs that are filled has to be equal to the total number of jobs taken by workers. Thus,

$$q(\theta) \int_{\hat{x}}^{\bar{x}} A(x') d\Gamma(x') = e_u \theta q(\theta) u + \int_{R}^{\bar{x}} \alpha(x') d\Psi(x').$$

It is useful to fomally define a stationary equilibrium of this economy.

Definition 1 *A stationary (balanced growth) equilibrium is a collection of unemployment u, vacancy rate v, total search intensity $\bar{e}$, asset values $\{J(x), W(x), V(x), U\}$, wage equation $w(x)$, search intensity of unemployed workers e_u, search intensity of employed workers $e(x)$, three reservation thresholds R, $\hat{x}$, and x^*, and a distribution of productivity across filled jobs $\Psi(x)$ and vacancies $\Gamma(x)$, such that*

- *The asset values $\{J(x), W(x), V(x), U\}$ satisfy equations (1)-(4) in above.*
- *Wages are determined by Nash bargaining, and are set by sharing of the surplus of the employment relationship in fraction β and $1 - \beta$, given $e(x)$.*
- *Vacant jobs are created until all profits are exhausted. Thus, the equilibrium reservation productivity x^* satisfies $V(x^*) = pK$.*
- *The optimal reservation productivity $\hat{x}$ satisfies the condition $V(\hat{x}) = 0$.*
- *The optimal reservation productivity R satisfies conditions $S(R) = 0$.*
- *The distributions $\Psi(x)$ and $\Gamma(\cdot)$, the unemployment rate u, the vacancy rate v, and the total search intensity $\bar{e}$ are consistent with the decisions of the agents in the economy.*

Characterization of equilibrium can be found in the Appendix.

4 Quantitative Analysis

In this section, I calculate the equilibrium of the above model by using numerical methods, since it is not possible to solve analytically. First, I calibrate the model to match the data in several dimensions. Then, I perform quantitative comparative statics exercises by calculating the steady-state response to an increase in the rate of disembodied technological growth.

4.1 Basic Calibration

In order to calculate the impact of disembodied technological progress on unemployment, I calibrate the model to match U.S. labor market facts. The following 13 parameters have to be determined: the discount rate r, the level of productivity p, the value of leisure z, the worker's bargaining power β, two matching function parameters m_0 and ϕ, two search cost function parameters c_0 and μ, technological growth rate g, the flow cost of posting a vacancy γ, the arrival rate of idiosyncratic productivity shocks λ, the measure of newly born potential firms n, and sunk costs for job creation K.

I choose the model period to be one-year and set the discount rate $r = 0.05$ to match the fact that the annual real interest rate has been around 5%. Since the level of productivity does not influence the steady state, I normalize $p = 1$ without loss of generality. I set g to 2%, the average productivity growth in the US over 1948-2007.

The matching function is assumed to be Cobb-Douglas,

$$m(v, \bar{e}) = m_0 v^{1-\phi} \bar{e}^{\phi}$$

where m_0 is the matching constant and ϕ is the matching elasticity with respect to the total search effort of workers. Then, the job finding rate is $\theta q(\theta) = m_0 \theta^{1-\phi}$ and the worker finding rate is $q(\theta) = m_0 \theta^{-\phi}$. Shimer (2005) calculated the elasticity with respect to a vacancy to equal to 0.28. However, with on-the-job search, market tightness is no longer equal to the vacancy-unemployment ratio, so this value is not the appropriate value. In the model, employed workers contact vacancies at the same rate as unemployed workers, so market tightness is proportional to the number of vacancies. Following Mortensen and Nagypál (2007), I choose the elasticity of the matching function to equal 0.5. I also set the workers' bargaining power β to 0.5.

In order to pin down the scale parameter m_0, I combine the monthly job finding rate, $f = 0.45$, estimated in Shimer (2005) with the monthly vacancy filling rate, $q = 0.71$, proposed by Hagedorn and Manovskii (2006). The matching function dictates that the equilibrium labor market tightness θ must be equal to the value of their ratio, namely 0.634. The matching scale parameter $m = 6.78$ is chosen to match the vacancy filling rate.

The search cost function is specified by

$$c(e) = c_0 e^{1+\mu}$$

where c_0 is a scale parameter and $\mu > 0$. Since the value of c_0 can be eliminated from the equilibrium condition, I set it to equal to 1. Moreover, I assume $\mu = 1$. Thus, the search cost function is quadratic.

I now determine the unemployment flow utility parameter z. In a calibration of search and matching models, this parameter has been the subject of some discussion.[11] In this paper, following Shimer (2005) and Hall (2005), I set the unemployment flow utility parameter equal to match the replacement rate. Thus, z is set to 0.4. The distribution of idiosyncratic productivity F is assumed to be uniform on support $[0, 1]$ as it is commonly used in the literature. Following Prat (2007), I directly target the average recruitment cost and set the flow cost of posting a vacancy equal to the monthly vacancies filling rate, so that $\gamma = 0.71$.

The average unemployment rate in the U.S. is 6% per annum and the job separation rate is 12% per annum. I choose sunk cost K, the rate of arrival of the idiosyncratic shocks λ, and the measure of potential firms n to match the rate of job finding, unemployment rate and job separation rate. The parameter values are summarized in column (1) of Table 2.

Table 2: Parameter Values

Parameter	Interpretation	(1)	(2)
r	Discount rate	0.05	0.05
m_0	Scale parameter of Matching function	6.78	6.135
ϕ	Elasticity of Matching function	0.5	0.72
β	Workers's bargaining power	0.5	0.72
z	Flow value of unemployment	0.4	0.4
γ	Cost of posting a vacancy	0.71	0.284
c_0	Scale parameter in search cost function	1	–
μ	Parameter in search cost function	1	–
K	Sunk costs for job creation	0.292	–
λ	Arrival rate of idiosyncratic shock	0.851	0.058
n	The measure of potential firms	0.968	–
p	Aggregate productivity	1	–
g	The rate of productivity growth	0.02	0.02

11. Shimer (2005) sets z equal to 0.4 in order to capture unemployment benefits. Hagedorn and Manovskii (2007) argue that Shimer's choice of the value of opportunity cost of employment is too low because it does not allow for the value of leisure, home production, disutility of work forgone when employed as well as unemployment benefit and they calibrate the opportunity cost of employment and the worker's bargaining power to match the observed cyclical response of wages and average profit rate. Their results are $z = 0.943$ and $\beta = 0.061$. Mortensen and Nagypál (2007) criticize Hagedorn and Manovskii (2007) for using these parameters because these parameters yields workers only gain 2.8% of flow utility by going from unemployment to employment and this implication is economically implausible.

4.2 Results

The equilibrium values of the model under the chosen parameterization are reported in Table 3.

Table 3: Model solutions

Variable	Description	Solution	Variable	Description	Solution
θ	Labor market tightness	0.693	$\theta q(\theta)$	Job finding rate	5.64
u	Unemployment rate	0.058	$q(\theta)$	Worker finding rate	8.14
v	Vacancy rate	0.041	e_u	Search effort by unemployed	0.342
$\bar{e}$	Total search effort	0.059	$\lambda F(R)$	Separation rate	0.119
R	Reservation productivity	0.140	$-$	Average quit rate	0.238
$\hat{x}$	Reservation productivity	0.451	$-$	EU-transition	0.112
x^*	Reservation productivity	0.716	$-$	Job-to-job transition	0.210

The job finding rate and the job separation rate are almost same to their calibrated value, while the unemployment rate is very close. The reservation productivity R is 0.140, which is much lower than the value of unemployment flow utility. Thus, unemployed workers are willing to match with low productivity jobs. This result comes from the option value of working.

The calibrated job-to-job transitions and flows of workers from employment to unemployment are 0.210 and 0.112, respectively. This implies that the ratio of job-to-job to employment-to-unemployment transition is 1.88. Fallick and Fleischman (2004) and Nagypál (2005) report that job-to-job transitions are twice as large as employment-to-unemployment transitions in data from the Current Population Survey. Thus, the prediction of the model is slightly lower than what is observed in data.

The equilibrium search intensity as a function of idiosyncratic productivity is plotted in Figure 4. The search intensity of workers is decreasing with productivity. Furthermore, employed workers in jobs with productivity less than $\hat{x}$ search for jobs much more intensively compared with employed workers in jobs with productivity greater than $\hat{x}$. In the literature, it is reported that only a small fraction of employed workers searches for jobs intensively, and these actively searching employed workers account for a small portion of job-to-job transitions. In the model, the optimal search intensity of an unemployed worker is 0.342. Employed workers in jobs with productivity lower than 0.23 search for jobs with an intensity level that equals 80% of the intensity level of unemployed workers. In equilibrium, 97.8% of employed workers are above this productivity. Thus, the model can match the fact that the fraction of employed workers actively searching for a job is small.

In Figure 5, I plot the initial density of the distribution of initial idiosyncratic productivity draws F' and the endogenous equilibrium distributions of productivity across filled jobs Ψ' and vacancies $\tilde{\Gamma}'$. Note that the distribution of productivity across filled jobs is equal to that of employed workers across productivity. Since potential firms with productivity above x^* enter the

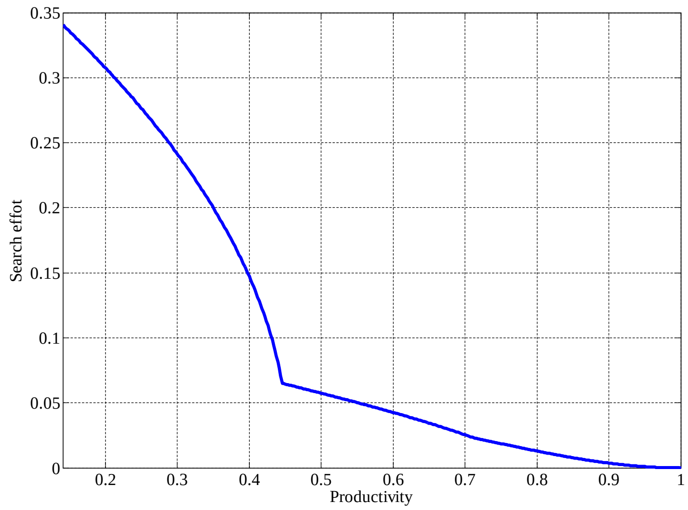

Figure 4: Optimal Search Intensity

market, the distribution of vacancies has a kink point at x^*. Expect at the productivity level, the distribution of vacancies is decreasing with productivity due to on-the-job search.

5 The impact of productivity growth on the labor market

I now calculate steady-state response to an increase in the rate of disembodied technological progress growth. Figure 6 reports the results.

The main result from the numerical analysis is that a faster rate of productivity growth leads to a substantial fall in the unemployment rate, due to the incorporation of on-the-job search and sunk costs for job creation.

First, the incorporation of on-the-job search and sunk costs make otherwise unproductive jobs productive enough to survive, leading to decreased separation when productivity growth increases. After considering costs and expected benefit, each firm chooses whether to enter labor market by paying costs of creating a new job position or not. Because of the job creation costs, the firm becomes more resistant to negative productivity shocks and tends to hoard labor. Thus, the labor hoarding effect is strengthened to economize the job creation costs that would be incurred if the firm was to re-enter the market. This reduces job separation. Furthermore, because of on-the-job search, productivity growth reduces the outside option effect. In the model, workers who work in firms with low productivity jobs search for better job intensively. Part of the benefit from on-the-job search is shared with the firm through wage sharing rule.[12] Thus, the worker

[12] Since wages is determined by surplus sharing, the outside option effect arises in the model. If wages are

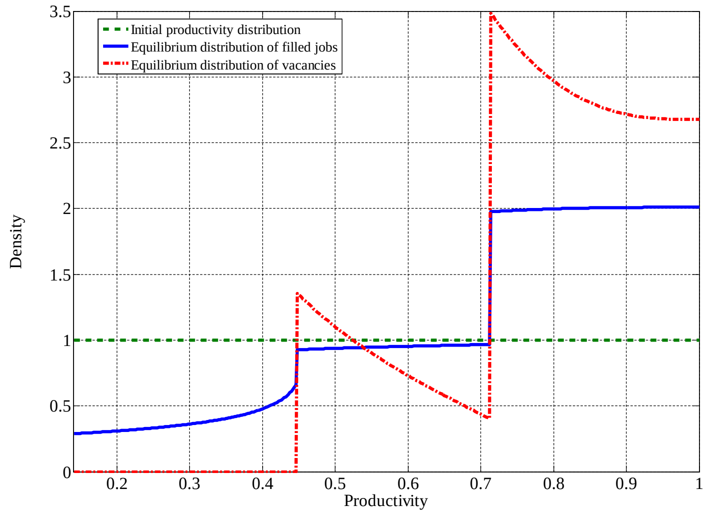

Figure 5: Distribution of jobs across productivity

gets a lower wage than he would be paid if he could not search on the job. This increases the value of the match and reduces the reservation threshold R. Since the labor hoarding effect is strengthened and the outside option effect is weakened, the reservation productivity R decreases when the rate of productivity growth rises. As a consequence, the separation rate decreases.

Second, on-the-job search generates more vacancies by accelerating the reallocation of workers when productivity growth increases. When growth accelerates, a rise in search intensity of employed workers increases the value of newly created jobs by facilitating recruitment. In a model without on-the-job search, all job creation has to be fed from the pool of unemployed, which is quickly exhausted in a high growth economy. Instead, in my model, when productivity growth is high, increasing search activity by employed workers expands the pool of potential hires for firms. This creates more job creation.

Third, the introduction of sunk costs for job creation generates a greater job creation with faster productivity growth by strengthening the capitalization effect. This is because a firm has to pay the cost for job creation at entering the labor market in addition to the vacancy cost.

When productivity growth increases, the job finding rate raises and the separation rate falls, lowering unemployment. These results are consistent with the empirical observation in Section 2. Thus, incorporation of on-the-job search and sunk costs improves the ability of the matching

determined by the output sharing rule, in which the non-convexity of the Pareto frontier discussed in Shimer (2006) does not arise, there is no outside option effect. Thus, the impact of growth on separation may be strengthened. However, under output sharing rule, inefficient separation arises. To consider an alternative wage setting mechanism in an endogenous job destruction model with on-the-job search is a fruitful avenue for research.

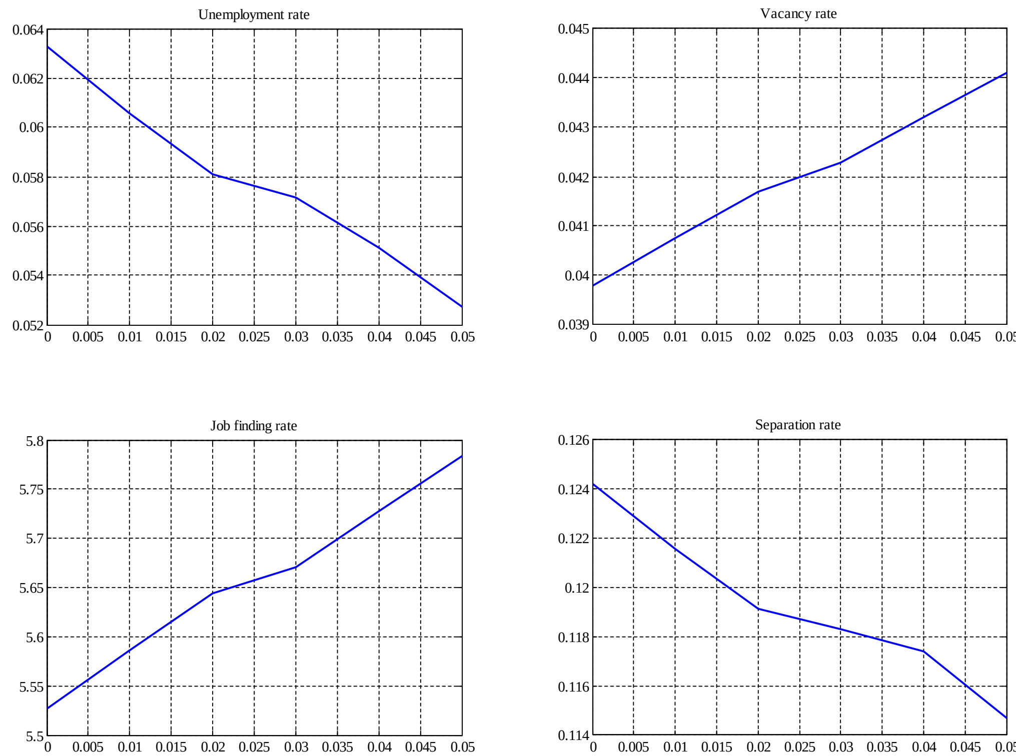

Figure 6: Comparative statics for productivity growth

model to fit the data. Furthermore, the model generates a negative impact of growth on unemployment comparable to empirical estimates. In my model, a one percentage point decline in productivity growth leads to a 0.21% increase in the unemployment rate. Blanchard and Wolfers (2000) estimate that a 1% decline in the growth rate leads to 0.25%-0.7% increase in the unemployment rate. Pissarides and Vallanti (2007) find the effect to be 1.3% to 1.5%. Moreover, the model magnifies the reduction in the unemployment rate due to productivity growth, compared with the standard matching model with DTP. While the standard endogenous job separation model by Mortensen and Pissarides generates a positive impact of growth on unemployment under plausible parameter values, my model generates a negative relationship between growth and unemployment in the setting of endogenous job separation. Furthermore, Pissarides and Vallanti (2007) find that even when job separation is exogenous, under plausible parameters and the standard assumption of Nash bargaining on wages, in the standard search model a 1% decline in the growth rate leads to an increase in unemployment rate of less than 0.01%.

It is also important to understand how the optimal search intensity of workers changes in response to productivity growth. In order to illustrate this, in Figure 7 I plot the equilibrium search intensity as a function of the idiosyncratic productivity with different productivity growth rates, $g = 0.02$ and $g = 0.04$. Figure 7 shows that the optimal search intensity is higher when productivity growth is higher. This is because a faster rate of productivity growth increases the benefits of on-the-job search and reduces its cost.

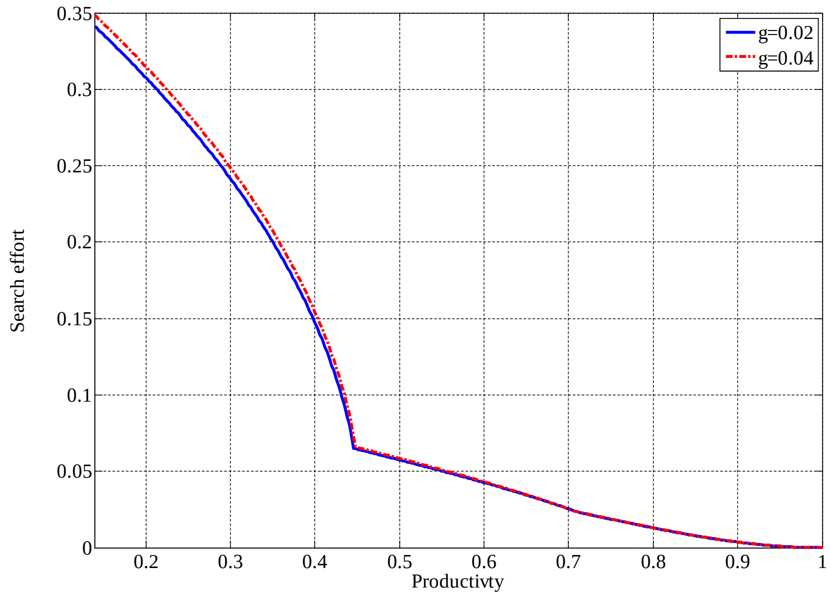

Figure 7: The effect of productivity growth on optimal search intensity

6 Productivity growth and unemployment

As shown in the above numerical analysis, in my model with both on-the-job search and sunk cost for creating new jobs, faster productivity growth reduces the separation rate, leading to a fall in the unemployment rate. Now I assess the contribution of these two factors to this key result by examining a model with neither on-the-job search nor sunk costs. This experiment allows us to determine whether or not the results of my complete model crucially depend on the incorporation of on-the-job search and sunk costs.

6.1 Generalized Mortensen and Pissarides model

I now consider a model of an endogenous job separation with productivity growth. Since neither on-the-job search nor sunk costs is incorporated, the model makes no distinction between the creation of jobs and fillings of vacancies. Therefore, I assume that there is no heterogeneity among vacant jobs. Then, the basic structure of the model is the same to that of Mortensen and Pissarides (1994). To analyze the impact of productivity growth on unemployment, I incorporate disembodied technological progress into the model of Mortensen and Pissarides (1994). Furthermore, to facilitate comparison between my original model and this model, I assume that initial value of idiosyncratic productivity x is drawn from a distribution F, and subsequently let $\{x\}$ be a jump process characterized by arrival rate λ and a distribution of new realization G. Note that in Mortensen and Pissarides (1994), all new jobs are created at the highest productivity. The detail of the model can be found in Appendix.

Without sunk costs and on-the-job search, the following job creation and job destruction conditions determine endogenous variables θ and R.

$$\frac{\gamma}{q(\theta)} = \frac{(1-\beta)}{r+\lambda-g} \int_R^1 \left(x' - R\right) dF(x') \tag{15}$$

and

$$0 = R - z - \frac{\beta\theta\gamma}{1-\beta} + \frac{\lambda}{r+\lambda-g} \int_R^1 \left(x' - R\right) dG(x'). \tag{16}$$

Given the equilibrium θ and R, the unemployment rate is determined by

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta) \bar{F}(R)}.$$

Note that in the model without on-the-job search, the labor market tightness θ is equal to the vacancy-unemployment ratio.

Now I examine the impact of disembodied technological progress on unemployment. Total differentiating (15) and (16) shows that the labor market tightness is an increasing function of g. The Intuition is that a faster growth raises the return from job creation, thus firms are encouraged to post more vacancies, resulting in higher labor market tightness. In a model with exogenous job separation, this is the only channel, and faster productivity growth leads to a fall in unemployment rate. However, once endogenous job separation is incorporated, the impact of DTP on unemployment rate turns out to be ambiguous.

The ambiguity is due to a higher workers' outside option from a higher rate of job finding. Since faster productivity growth increases the job finding rate, workers have better outside opportunities and so ask for higher wages. This reduces the value of the match and the firm raises the reservation value of productivity R. This effect is identified by Prat (2007) as the outside option effect. On the other hand, faster productivity growth increases the option value of the match, and leading to a fall in the reservation productivity through the labor hoarding effect. When the outside option effect dominates the labor hoarding effect, faster productivity growth increases the reservation value of productivity, and thus the separation rate. Then, since faster productivity growth increases both job finding rate and separation rate, the impact of growth on unemployment becomes ambiguous.

The impact of productivity growth on the separation rate can be understood by examining the job destruction condition. Totally differentiation of (16) yields

$$\frac{dR}{dg} = \left(\frac{r+\lambda-g}{r+\lambda F(R)-g}\right)\frac{\beta\gamma}{1-\beta}\underbrace{\frac{d\theta}{dg}}_{(+)} - \frac{\lambda\int_R^1 (x'-R)\,dG(x')}{[r+\lambda F(R)-g]\,(r+\lambda-g)}.$$

The first term and the second term of RHS capture the outside option effect and the labor hoarding effect, respectively. From the above equation, we can see that the sign of the impact of productivity growth on the separation rate depends on which effect dominates. When the labor hoarding effect dominates, faster productivity growth reduces the separation rate, and thus unemployment rate. On the other hand, when the outside option effect dominates, faster productivity growth increases the separation rate, and the impact of growth on unemployment rate becomes ambiguous.

Now I calibrate the model in order to examine the impact of productivity growth on unemployment. Basic parameter values are the same to those discussed before. Since the labor market tightness is equal to the vacancy-unemployment ratio, I set the elasticity with respect to unemployment to 0.72, which is estimated by Shimer (2005). This implies that the matching scale parameter $m = 6.135$. To satisfy "Hosios (1990) condition" the worker's bargaining power β is chosen to be 0.72. The cost of posting a vacancy γ and the arrival rate of idiosyncratic shocks λ are chosen to match the job finding rate and the average unemployment rate. Furthermore, I assume that both F and G are uniform on support $[0, 1]$ as it is commonly used in the literature, and $F = G$. The parameter values are reported in column (2) of Table 2.

Again I perform quantitative comparative statics exercises by calculating the steady-state responses to an increase in the rate of disembodied technological progress growth. Figure 8 reports the results.

As we expected, a faster productivity growth increases the labor market tightness. This result is the same to that of the model with both sunk costs and on-the-job search. However, the effect of DTP on the reservation productivity is opposite to that of my complete model, and a faster growth increases the reservation productivity R. This implies that the outside option effect prevails over the labor hoarding effect under my calibrated parameter values.

The overall effect of DTP on unemployment rate is determined by the interactions between the labor market tightness and the reservation productivity. While an increase in labor market tightness due to the capitalization effect tends to reduce unemployment, more job destruction due to higher reservation productivity increases unemployment. From Figure 8, one can see that faster productivity growth increases the unemployment rate under my parameter values. The model shows that a one percentage point rise in productivity growth leads to a 0.32% increase in unemployment rate. Thus, the model fails to generate the empirical consistent sign of the impact of growth on unemployment rate.

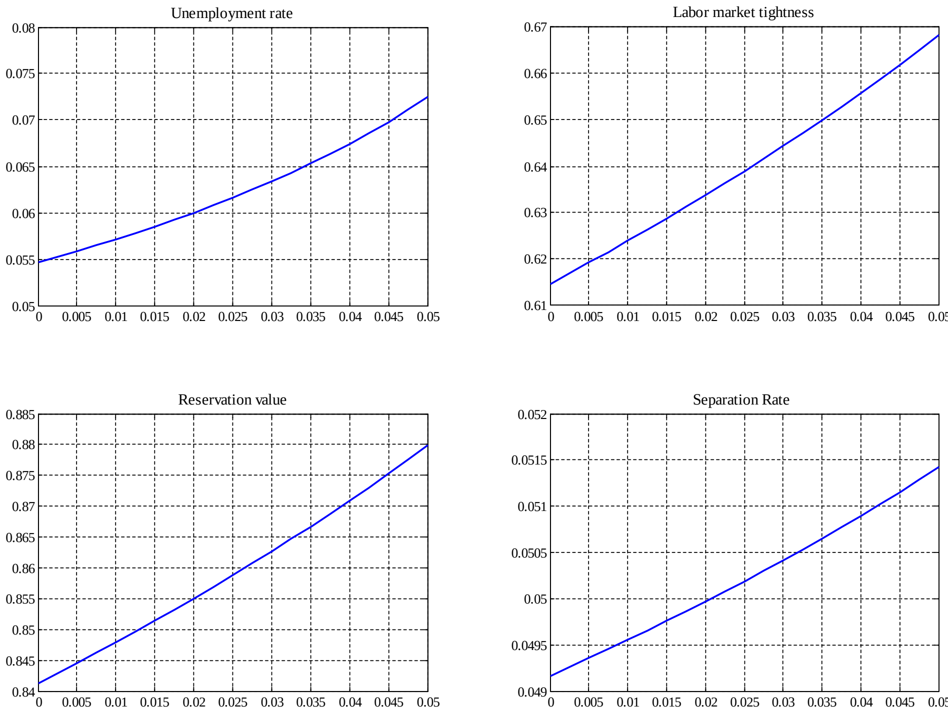

Figure 8: Comparative Statics for productivity growth in the model with neither sunk cost nor on-the-job search

Qualitatively, the impact of disembodied technological progress on unemployment rate is ambiguous in the search and matching model with endogenous job separation. Under my parameter values, the model generates a positive impact of growth on unemployment rate. Prat (2007) demonstrates that the sing of the impact of DTP on unemployment rate depends on the degree of uncertainty. In the model, the degree of uncertainly is captured by the parameter λ. In order to examine the robustness of the result, I plot the derivative of the unemployment rate with respect to the rate of productivity growth as a function of g and λ in Figure 7.

Figure 7 shows that productivity growth and unemployment are positively correlated for the plausible parameter values of λ. Thus, under plausible parameter values, the endogenous job separation model with DTP fails to generate the empirically consistent sign of the impact of growth on unemployment.

To summarize, the incorporation of sunk costs for job creation and on-the-job search improves the ability of the matching model with endogenous job separation to fit the data. In the standard model of endogenous job separation, the impact of growth on unemployment is ambiguous due

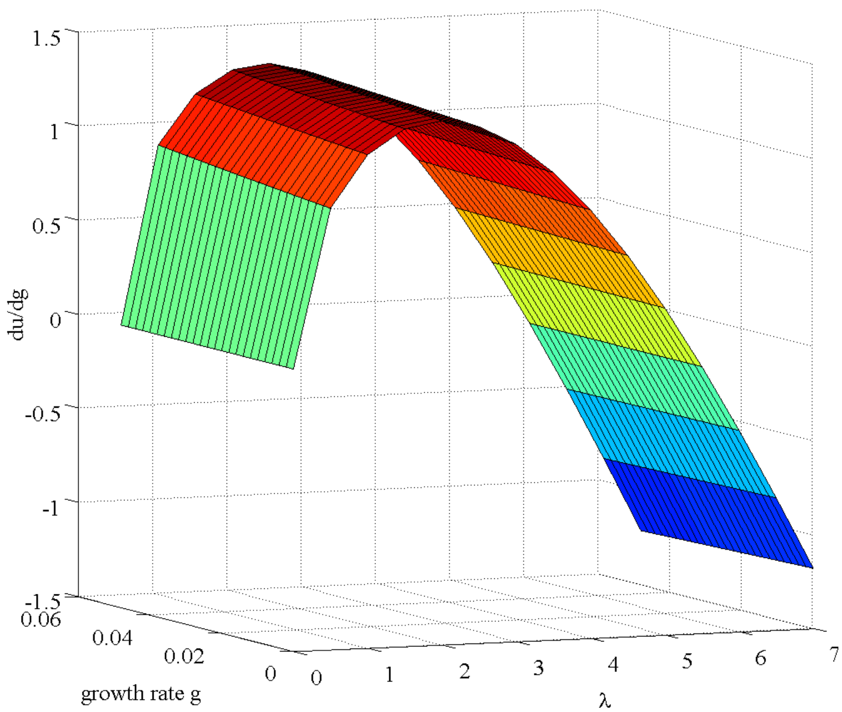

Figure 9: Derivative of the unemployment rate with respect to the rate of productivity growth.

to the outside option effect. Furthermore, under plausible parameters, the model generates a positive impact of productivity growth on unemployment. On the other hand, the incorporation of sunk costs and on-the-job search gives rise to new channels through faster growth reduce unemployment. The model with these two elements generates not only an empirically consistent sign of the impact of growth on unemployment but also a larger size of magnitude compared with the standard model.

7 Conclusion

This paper studies the impact of productivity growth on rates of job finding and separation, and unemployment rates in the search and matching model. By examining the long-run relationship between the productivity growth and labor markets variables in the US, I find that, while the job finding rate is positively correlated with the productivity growth rate, there is a strong negative correlation between the separation rate and growth. Furthermore, I find that both job finding and separation rates contribute to the overall unemployment variability in the long-run. These empirical findings suggest that productivity growth reduces the unemployment rate through not only increased job finding but also decreased separation.

In order to explain the empirical facts, I incorporate sunk cost for job creation and on-the-job search into the endogenous job separation model of Mortensen and Pissarides (1994) with disembodied technological progress. The incorporation of sunk costs for job creation and on-the-job search gives rise to new channels through which faster growth may reduce unemployment by reducing separation rate and inducing more job creation. My model demonstrates that faster productivity growth reduces the separation rate and increases the job finding rate, leading to a lower unemployment rate. This result is consistent with my empirical findings. Furthermore, the model magnifies the reduction in the unemployment rate due to productivity growth, compared to the standard search and matching model with DTP.

A number of important issues remain for future research. One issue to be considered is the magnitude of the impact of productivity growth on the unemployment rate. Although the model in this paper generates a larger magnitude of the effect compared to the standard models, it is still smaller than the estimated one in the literature. Also, the short-run relationship between productivity growth and labor market variables is an important issue. Several empirical studies point out that the impact of productivity growth on unemployment is different in the short-run than in the long-run. To understand this difference is important as well.

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8 Appendix

8.1 Characterization

By using the fact that $S(x)$ and $W(x)$ are strictly increasing in x and the value of an unemployed worker, equation (7) can be rewritten as

$$\begin{aligned}(r+\lambda-g)\left[S(x)+V(x)\right] &= p\left(x-z\right)-pc(e(x))+pc(e_u)+\lambda\left[\int \max[S(x'),0]dF(x')+V(x')dF(x')\right] \\ &\quad +e(x)\theta q(\theta)\int_x^{\bar{x}}\left[\beta S(x')-S(x)\right]d\tilde{\Gamma}(x')-e_u\theta q(\theta)\beta\int_R^{\bar{x}}S(x')d\tilde{\Gamma}(x') \qquad (17)\end{aligned}$$

Evaluating (17) at $x=R$ and using the optimal separation rule $S(R)=0$, I have

$$0=p\left(R-z\right)+\lambda\int\left\langle \max[S(x'),0]+V(x')\right\rangle dF(x') \qquad (18)$$

This equation implies that the difference between R and z comes from the option value of changing idiosyncratic productivity.

By using the surplus function, the value of a vacancy can be rewritten as

$$\left(r+\lambda-g\right)V(x)=\begin{cases}0 & \text{for } x<\hat{x} \\ -p\gamma+q(\theta)A(x)\left(1-\beta\right)S(x)+\lambda\int_{\hat{x}}^{\bar{x}}V(x')dF(x') & \text{for } x\geq\hat{x}\end{cases}. \qquad (19)$$

From (11) and (19), I have the following job creation condition.

$$\left(r+\lambda-g\right)pK=-p\gamma+q(\theta)A(x^*)\left(1-\beta\right)S(x^*)+\lambda\int_{\hat{x}}^{\bar{x}}V(x')dF(x') \qquad (20)$$

By making use of (12) and (19), I obtain

$$p\gamma=q(\theta)A(\hat{x})\left(1-\beta\right)S(\hat{x})+\lambda\int_{\hat{x}}^{\bar{x}}V(x')dF(x') \qquad (21)$$

This is the equilibrium condition for the productivity threshod $\hat{x}$. This condition says that the optimal reservation value $\hat{x}$ should be set to so as to equalize the expected cost of posting a vacancy and the expected benefit of it.

Differentiating (19) with respect to x yields

$$\left(r+\lambda-g\right)V'(x)=q(\theta)\left(1-\beta\right)\left[A'(x)S(x)+A(x)S'(x)\right] \text{ for } x\in[\hat{x},\bar{x}] \qquad (22)$$

Differentiating with respect to x on both sides of the surplus equation and using the envelope theorem and (22) gives

$$\begin{aligned}&\left\langle r+\lambda-g+e(x)\theta q(\theta)\left[1-\tilde{\Gamma}(x)\right]+\mathbb{I}\left(x\geq\hat{x}\right)q(\theta)\left(1-\beta\right)A(x)\right\rangle S'(x) \qquad (23) \\ =\;& p+\left\{\left[e(x)\tilde{\Gamma}'(x)-e'(x)\left(1-\tilde{\Gamma}(x)\right)\right]\theta q(\theta)\left(1-\beta\right)-\mathbb{I}\left(x\geq\hat{x}\right)q(\theta)\left(1-\beta\right)A'(x)\right\}S(x).\end{aligned}$$

Since $e(x) = 0$ for $x \geq \bar{x}$, (23) gives

$$S'(x) = \frac{p}{r + \lambda - g + q(\theta)(1-\beta)} \text{ for } x \geq \bar{x}.$$

Totally differentiating the optimal search intensity condition (8) yields

$$e'(x) = \frac{-\theta q(\theta)\beta S'(x)[1 - \tilde{\Gamma}(x)]}{pc''(e(x))} \tag{24}$$

This is an ordinary differential equation for $e(x)$ with the boundary condition $e(\bar{x}) = 0$.

Differentiating equation (13) and (14) with respect to x and rearranging yields

$$\Psi'(x) = \frac{\lambda F'(x)(1-u) + q(\theta)A(x)\tilde{\Gamma}'(x)}{\lambda + \alpha(x)} \text{ for } x \in [R, \bar{x}]$$

and

$$\Gamma'(x) = \frac{\mathbb{I}(x \geq x^*)\, nF'(x) + \lambda F'(x)H(\bar{x}) + \alpha(x)\Psi'(x)}{q(\theta)A(x) + \lambda}.$$

Rearrangment of these two equations, I have

$$\Psi'(x) = \begin{cases} \dfrac{\lambda F'(x)(1-u)}{\lambda + \alpha(x)} & \text{for } x \in [R, \hat{x}] \\ \dfrac{\lambda F'(x)(1-u) + q(\theta)A(x)\Gamma'(x)}{\lambda + \alpha(x)} & \text{for } x \in [\hat{x}, \bar{x}] \end{cases} \tag{25}$$

and

$$\Gamma'(x) = \begin{cases} \dfrac{[\lambda + \alpha(x)]\, v + \alpha(x)(1-u)}{q(\theta)A(x) + \lambda + \alpha(x)} F'(x) & \text{For } x \in [\hat{x}, x^*] \\ \dfrac{n\left[\dfrac{\lambda + \alpha(x)}{\lambda}\right] + [\lambda + \alpha(x)]\, v + \alpha(x)(1-u)}{q(\theta)A(x) + \lambda + \alpha(x)} F'(x) & \text{For } x \in [x^*, \bar{x}] \end{cases}. \tag{26}$$

The equilibrium of the model is characterized by functions $S(x)$, $V(x)$, $e(x)$, $\Psi(x)$, $\Gamma(x)$, $A(x)$ and $\alpha(x)$ and endogenous variables R, $\hat{x}$, x^*, u, v, θ, and e_u. I can solve the system of equations from, (22) together with boundary condition $V(\hat{x}) = 0$, (23) together with boundary condition $S(R) = 0$, (24) together with boundary condition $e(\bar{x}) = 0$, (25) together with boundary condition $\Psi(R) = 0$, (26) together with boundary condition $\Gamma(\hat{x}) = 0$ and equations (5), (μ), (10), (18), (20) and (21). The number of unemployment u, the number of vacancies v and labor market tightness θ are determined by $u = 1 - \Psi(\bar{x})$, $v = \Gamma(\bar{x})$, and $\theta = v/\bar{e}$.

8.2 Generalized Mortensen and Pissarides (1994) Model

In this appendix, we develop a generalized endogenous job separation model of Mortensen and Pissarides (1994) with disembodied technological progress. The basic structure of the model is same to that of Mortensen and Pissarides (1994). While all new jobs are created at the highest productivity in Mortensen and Pissarides (1994), we assume that initial value of idiosyncratic productivity x is drawn from a distribution F, and subsequently let $\{x\}$ be a jump process characterized by arrival rate λ and a distribution of new realization G. Furthermore, in order to study the impact of long-run productivity growth on unemployment, we incorporate the disembodied technological progress into the model. We re-use same notations of our original model to describe the generalized Mortensen and Pissarides (1994) Model.

The value functions of firms and workers are

$$\begin{aligned}(r-g)\,J(x) &= px - w(x) + \lambda\left[\int \max[\langle J(x'),V\rangle\,dG(x') - J(x)\right] \\ rV &= -p\gamma + q(\theta)\left[\int \max[J(x'),V\,dF(x') - V(x)\right] \\ (r-g)\,W(x) &= w(x) + \lambda\left[\int \max[\langle W(x'),U\rangle\,dG(x') - W(x)\right] \\ (r-g)\,U &= pz + \theta q(\theta)\left[\int \max[\langle W(x'),U\rangle\,dF(x') - U\right]\end{aligned}$$

Given the free entry condition $V=0$, the surplus function $S(x)$ is characterized by

$$(r+\lambda-g)\,S(x) = px - pz + \lambda\int \max[\langle S(x'),0\rangle\,dG(x') - \theta q(\theta)\beta\int \max\langle S(x'),0\rangle\,dF(x'). \quad \text{(A1)}$$

This implies

$$S'(x) = \frac{p}{r+\lambda-g} > 0.$$

Since the surplus function is increasing in x, the firm and the worker will choose to form and continue any match that has am idiosyncratic productivity $x \geq R$ and the reservation productivity is determined by $S(R)=0$. By using integration by parts and the free entry condition, we have the following job creation condition

$$\frac{\gamma}{q(\theta)} = \frac{(1-\beta)}{r+\lambda-g}\int_R^1 \big(x'-R\big)\,dF(x').$$

Evaluating (A1) at $x=R$ and using the above job creation condition, we have the following job destruction condition

$$0 = R - z - \frac{\beta\theta\gamma}{1-\beta} + \frac{\lambda}{r+\lambda-g}\int_R^1 \big(x'-R\big)\,dG(x').$$

These two equations determine equilibrium value of θ and R. Given these values, the unemployment rate is determined by

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)\bar{F}(R)}.$$