

Risk Matters:
The Real Effects of Volatility Shocks*

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Abstract

This paper shows how changes in the volatility of the real interest rate at which small open emerging economies borrow have a quantitatively important effect on real variables like output, consumption, investment, and hours worked. To motivate our investigation, we document the strong evidence of time varying volatility in the real interest rates faced by a sample of four small emerging open economies: Argentina, Ecuador, Venezuela, and Brazil. We postulate a stochastic volatility process for real interest rates using T-Bill rates and country spreads and estimate it with the help of the Particle filter and Bayesian methods. Then, we feed the estimated stochastic volatility process for real interest rates in an otherwise standard small open economy business cycle model. We calibrate eight versions of our model to match basic aggregate observations, two versions for each of the four countries in our sample. We find that an increase in real interest rate volatility triggers a fall in output, consumption, investment, and hours, and a notable change in the current account of the economy.

Keywords: Small Open Economy, DSGE Models, Stochastic Volatility.

JEL classification numbers: C32, C63, F32, F41.

1. Introduction

This paper shows how changes in the volatility of the real interest rate at which emerging economies borrow have a substantial effect on real variables like output, consumption, investment, and hours worked. These effects appear even when the level of the real interest rate itself remains constant. We argue that, consequently, the changing volatility of real interest rates is an important force behind the distinctive size and pattern of business cycle fluctuations of emerging economies.

To prove our case the paper makes two points. First, we document the strong evidence of time varying volatility in the real interest rates faced by a sample of four small emerging open economies: Argentina, Ecuador, Venezuela, and Brazil. We postulate a stochastic volatility process for real interest rates using T-Bill rates and country spreads and estimate it with the help of the Particle filter and Bayesian methods. We uncover large movements in the volatility of real interest rates and a systematic relation of those movements with output, consumption, and investment. Second, we feed the estimated stochastic volatility process for real interest rates in an otherwise standard small open economy business cycle model. We calibrate eight versions of our model to match basic aggregate observations, two versions for each of the four countries in our sample. Having eight different calibrations is most convenient because it teaches us about how different parameter values drive our findings. We solve the model by computing a third order approximation to the policy functions of the agents. This third order approximation allows the agents to keep track of time varying risk. We find that an increase in real interest rate volatility triggers a fall in output, consumption, investment, and hours, and a notable change in the current account.

We think of our exercise as capturing the following sequence of events. Prior to period t , households live in an environment characterized by the average standard deviation of interest rates. At time t , the standard deviation of the innovation associated to the country's spread increases by one standard deviation. In this experiment, only the variance of the spread goes up but not the level of the interest rate itself. Then, agents optimally adjust their consumption, labor, investment, and savings decisions to face the new level of risk of real interest rates. The effects are more salient for Argentina and Ecuador and milder for Venezuela and Brazil.

The intuition for our result is clear. Small open economies rely on foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. However, when the volatility of real interest rates raises, debt becomes a riskier asset as the economy gets exposed to potentially fast fluctuations in the real interest rate and their associated and unpleasant movements in marginal utility. To reduce this exposure, the economy must pay back part

of its outstanding debt by cutting on consumption. Moreover, as debt is suddenly a worse hedge for the productivity shocks that drive returns to physical capital, investment falls. A lower investment reduces output and, through a fall in the marginal productivity of labor, hours worked.

To strengthen our argument, we perform a battery of robustness checks. First, we highlight that movements in the volatility of real interest rates are highly correlated with variations in levels. We reestimate our stochastic volatility model while allowing for this correlation and recompute the model with the new processes. Our main conclusion that changes in risk affect real variables remains unchallenged, if anything reinforced by the correlation of shocks to levels and volatility. Second, we assess the importance of several parameter values for our quantitative conclusions. This check reinforces many of the lessons learned in the main part of the paper. Finally, we explore the consequences of the imposing different priors in our estimation exercise. Again, for a wide class of reasonable priors, our results are basically unaltered.

Our investigation begets a number of interesting additional points. First, we document a channel through which changing second moments impact the first moments of the variables. This is not only crucial to understand business cycles but also for the empirical implementation of the model. Volatility moves the ergodic distribution of the endogenous variables of the model away from their deterministic steady state. Hence, volatility forces us to calibrate the model according to that ergodic distribution and not, as commonly done, to match steady state values.

Second, due to the non-linear nature of stochastic volatility, we apply the Particle filter to evaluate the likelihood function of the process driving the real interest rates (see the description of the Particle filter in Doucet, de Freitas, and Gordon, 2001, and, applied to economics, in Fernández-Villaverde and Rubio-Ramírez, 2007 and 2008). By doing so, we introduce a new technique that can be of much application in international finance where nonlinearities abound (sudden stops, exchange rate regime switches, large devaluations, etc.).

Third, capturing time-varying volatility creates an interesting computational challenge. Since we are interested on the implications of a volatility increase while keeping the level of the real interest rate constant, we have to consider a third order Taylor expansion of the policy functions of the representative agent in the model. In a first order approximation, volatility would not even play a role since the policy rules of the representative agent follows a certainty equivalence principle. In the second order approximation, only the product of the innovations to the level and to the volatility of real interest rates appears in the policy function. It is only in the third order approximation when the innovations to the volatility play a role by themselves.

Our paper does not offer a theory of why real interest rate volatility evolves over time. Instead, we model it as an exogenously given process. By doing so, we join an old tradition in macroeconomics, going from Kydland and Prescott (1982), who took their productivity shocks as exogenous, to Mendoza (1995), who did the same with his terms of trade shocks, or Neumeyer and Perri (2005), who assume their interest rate process is given. Part of the reason we follow this approach is because an exogenous process for volatility concentrates our attention sharply in the mechanism through which real interest rate risk shapes the trade offs of economic agents in small open economies. Part of the reason is because the literature has not developed, even at the prototype level, an equilibrium model to endogenize volatility shocks. If we tried to build such model in this paper simultaneously with our empirical documentation of volatility and the measurement of its effects, we would lose focus and insights in exchange for a most uncertain reward. In comparison, a thorough understanding of the effects of volatility *per se* will be a solid foundation for more elaborated theories of time dependent variances.

Besides, the literature on financial contagion has appeared precisely to understand phenomena that distinctively look like exogenous shocks to small open economies (Kaminsky, Reinhart, and Végh, 2003). For instance, after Russia defaulted on its sovereign debt in the summer of 1998, Argentina, Brazil, or Hong Kong, countries that have little if anything in common with Russia or Russian fundamentals besides appearing in the same table at the back pages of *The Economist* as an emerging market, suffered a significant increase in the volatility of the real interest rates at which they borrowed. At a first pass, thinking about those volatility spikes as exogenous events and tracing their consequences within the framework a standard business cycle model seems empirically plausible and a worthwhile exercise.

Our paper links with two literatures. First, we engage with the discussion of why the business cycle of emerging economies present peculiar characteristics that are divergent from the pattern of business cycle fluctuations in developed small open economies (Aguiar and Gopinath, 2007, Neumeyer and Perri, 2005, and Uribe and Yue, 2006, among others). Our paper suggests that the higher real interest rate volatility faced by Argentina in comparison, let's say, with Canada, is an important source of differences. Volatility may go a long way towards explaining, for example, why consumption is more volatile than output in emerging economies.

However, we do not postulate real interest rate volatility as a substitute for any of the theories proposed by previous authors. Instead, we see it as a complement, as many of the channels explored by the literature may become stronger in the presence of time varying volatility. For instance, we document that this is precisely the case for the real interest rates shocks that are the attention of Neumeyer and Perri (2005).

Second, we relate with the literature on volatility in finance and macroeconomics. Stochastic volatility has been a widely studied concept in financial economics (Shephard, 2005). Disappointingly, there has been less work done in macroeconomics assessing the possible consequences of time-varying volatility. Justiniano and Primiceri (2007) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models where heteroscedastic shocks drive the dynamics of the economy to account for the “Great Moderation” that has characterized the last twenty years in the U.S. economy (Stock and Watson, 2002). The conclusion of both papers is that time-varying volatility helps to explain the reduction observed in the standard deviation of output growth and other macroeconomics variables. However, these papers also show that for the U.S. economy, stochastic volatility mainly affects the second moments of the variables with little effect on their first moments. In comparison, Bloom (2007) exploits firm level data to estimate a model where a spike in uncertainty affects real variables by freezing hiring and investment decisions. Bloom’s contribution is innovative for it builds an empirical testable mechanism through which volatility matters. Our paper nicely accompanies Bloom’s work by offering a second mechanism through which volatility has a first order impact.

The rest of the paper is organized as follows. Section 2 presents our data, the stochastic volatility process for real interest rates that we estimate, and the relation of this process with other aggregate variables. Section 3 lays down our benchmark small open economy model and explains how to calibrate and compute it. Section 4 discusses our results and section 6 offers some sensitivity analysis. Section 6 concludes.

2. Estimating the Law of Motion for Interest Rates

In this section, we estimate the laws of motion characterizing the evolution of the real interest rates for four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. We select our countries based on data availability and because they represent a relatively coherent set of South American economies. Our approach builds the real interest rate faced by each of these countries as the sum of the international risk free real rate and a country specific spread. After this step, we estimate the law of motion of the international risk free real rate, which, of course, is identical across countries, and the laws of motion of the country spread, one for each economy.

This section plays a dual role. First, it documents our assertion that changes in the volatility of the real interest rates are quantitatively important. Second, it provides us with the processes that we feed, later in the paper, into the calibrated versions of our model. To meet these two goals, the section is divided in four parts. First, we discuss our data

on interest rates. Second, we specify a process for interest rates. Third, we estimate these processes following a Bayesian approach. Fourth, we report some of the empirical regularities uncovered by our econometric exercise.

2.1. Data on Interest Rates

For any given country, we decompose the real interest rate, r_t , it faces on loans denominated in U.S. dollars as the international risk free real rate plus a country specific spread. We use the T-Bill rate as a measure of the international risk free nominal interest rate. This is a standard convention in the literature. We build the international risk free real rate by subtracting expected inflation from the T-Bill rate. Following Neumeyer and Perri (2005), we compute the expected inflation as the average U.S. CPI inflation in the current month and in the eleven preceding months. This assumption is motivated by the fact that the process for inflation in the U.S. is well approximated by a random walk (Atkeson and Ohanian, 2001). Both the T-Bill rate and the inflation series are obtained from the St. Louis Fed's FRED database. We use monthly rather than the more popular quarterly data because monthly data is more appropriate to capture the volatility in interest rates as required by our investigation. Otherwise, quarterly means would smooth out much of the variation of interest rates.

For data on country spreads, we use the Emerging Markets Bond Index (EMBI) Global Spread reported by J.P. Morgan at a monthly frequency. This index tracks secondary market prices of actively traded emerging market bonds denominated in U.S. dollars. Neumeyer and Perri (2005) explain in further detail the advantages of EMBI data in comparison with the existing alternatives. Unfortunately, except for Brazil monthly, EMBI is only available from 1998. Thus our sample misses the Tequila crisis and the beginning of the Asian crisis. Yet the sample is large enough to cover interesting developments such as the 1998 equity price correction in U.S. and the Argentinean crisis of 2002-2002. The EMBI data coverage is as follows: Argentina 1997.12 - 2008.02; Ecuador 1997.12 - 2008.02; Brazil 1994.04 - 2008.02; and Venezuela 1997.12 - 2008.02.

We plot our data set in figure 1 using annual rates in percentage points to facilitate comparison with the most commonly quoted rates. We see that the international risk free real rate is relatively low and stable over the sample (including a period of negative interest rates in 2002-2006). In comparison, all the country spreads are large and volatile. The spreads are nearly always larger than the real T-Bill rate itself and fluctuate at least an order of magnitude more. The most prominent case is Argentina, where the 2001-2002 crisis raised the country spreads to 70 percentage points. We also see in the figure the problems of Ecuador in 1998-1999 and the common turbulences for all four countries during virulent international turmoil of 1998.

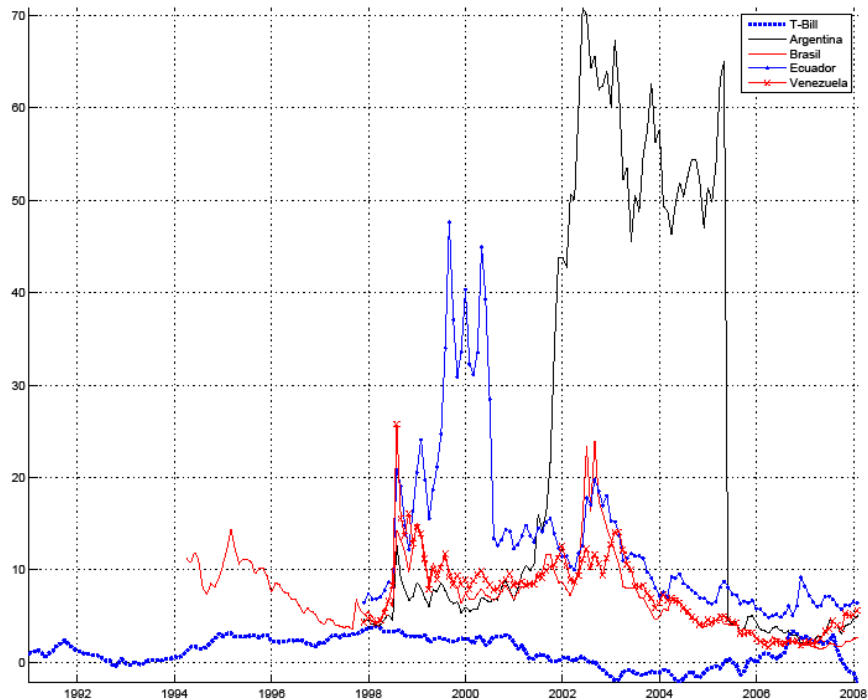


Figure 1: Country Spreads and T-Bill Real Rate

2.2. The Law of Motion for Interest Rates

According to our previous discussion, we write the law of motion driving the real interest rate faced by domestic residents in international markets as:

$$r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t} \quad (1)$$

In this equation, we define r as the the mean of the international risk free real rate plus the mean of the country spread. The term $\varepsilon_{tb,t}$ equals the international risk free real rate subtracted of its mean and $\varepsilon_{r,t}$ equals the country spread subtracted of its mean. These two terms decompose the deviations of the interest rate with respect to its mean into a common component and a country specific component. To ease notation, we omit a subindex for the country specific variables and parameters.

Now, we specify that both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow $AR(1)$ processes described by:

$$\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}}u_{tb,t} \quad (2)$$

and:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \quad (3)$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed shocks with mean zero and variance equal to one.

The main feature of our process is that the standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ are not constant, as commonly assumed, but follow an $AR(1)$ processes:

$$\sigma_{tb,t} = (1 - \rho_{\sigma_{tb}}) \sigma_{tb} + \rho_{\sigma_{tb}} \sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t} \quad (4)$$

and

$$\sigma_{r,t} = (1 - \rho_{\sigma_r}) \sigma_r + \rho_{\sigma_r} \sigma_{r,t-1} + \eta_r u_{\sigma_r,t} \quad (5)$$

where both $u_{\sigma_r,t}$ and $u_{\sigma_{tb},t}$ are normally distributed shocks with mean zero and variance equal to one. Thus, our process for interest rates displays stochastic volatility. Our specification is parsimonious yet powerful enough to capture some salient peculiarities of the data (Shepard, 2005). Alternative specifications, like estimating realized volatility, are of difficult implementation because we do not have intraday data and because we need a parametric law of motion for volatility to feed into the equilibrium model of section 3.

Given values of persistence parameters, ρ_{tb} and $\rho_{\sigma_{tb}}$, σ_{tb} and η_{tb} control the degree of volatility and stochastic volatility in the international risk free real rate. Hence, given ρ_{tb} and $\rho_{\sigma_{tb}}$, a high σ_{tb} implies high volatility of the international risk free real rate and a high η_{tb} delivers a high degree of stochastic volatility. The same can be said about σ_r and η_r and the volatility and stochastic volatility in the country spread given values for ρ_r and ρ_{σ_r} .

Two shocks affect each of the components of the real interest rate: one influencing its level and another its volatility. For instance, the deviation due to the international risk free real rate, $\varepsilon_{tb,t}$, is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. Conditional on $u_{\sigma_{tb},t}$, the first innovation, $u_{tb,t}$, changes the level of the deviation, while the second innovation, $u_{\sigma_{tb},t}$, only affects the standard deviation of $u_{tb,t}$. The shocks $u_{r,t}$ and $u_{\sigma_r,t}$ have a similar reading. We call the first type of innovations, i.e., $u_{tb,t}$ and $u_{r,t}$, shocks to the level of the international risk free real rate and the country spread respectively.¹ We call the second type of innovations, i.e., $u_{\sigma_{tb},t}$ and $u_{\sigma_r,t}$, shocks to the volatility (or standard deviation) of international risk free real rate and the country spread respectively. Sometimes, for simplicity, we call this second type of innovations stochastic volatility shocks.

¹Strictly speaking, they are shocks to the deviation of the real interest rate with respect to its mean due to the country spreads and the international risk free rate respectively. Hereafter, to facilitate exposition, we omit the word “deviation” where we do not risk ambiguity.

Following the literature, we can interpret a shock to the volatility of real interest rates in at least two different views. First, as reflecting more risk surrounding the economy. Times generally understood as uncertain, like the Asian and the Long Term Capital Management (LTCM) crises, as associated with strong increases in volatility. A second interpretation builds around the idea that volatility is related to information (Ross, 1989, and Andersen, 1996). A rising real interest rate volatility may reflect the arrival of more information about the economy’s health. During turbulent times, news arrive frequently (or perhaps more attention is devoted to them) inducing high volumes of trade in foreign debt and, consequently, raising volatility in interest rates.

As our benchmark exercise, we assume that $u_{tb,t}$, $u_{r,t}$, $u_{\sigma_{tb},t}$, and $u_{\sigma_r,t}$, are independent of each other. How strong is this assumption? On one hand, we determine that $u_{tb,t}$ and $u_{r,t}$ are uncorrelated in the data. This result confirms the findings of Neumeier and Perri (2005). On the other hand, we find that 1) the pair $u_{tb,t}$ and $u_{\sigma_{tb},t}$ is strongly correlated and 2) the pair $u_{r,t}$ and $u_{\sigma_r,t}$ is strongly correlated as well. Motivated by this evidence, we later reestimate our stochastic volatility process allowing for correlation. However, we keep the case without correlation as our benchmark case because it separates more neatly the effects of the changes to levels from the effects of changes to volatility.

2.3. Estimation

We estimate first the parameters of the process driving the international risk free real rate deviation defined by equations (2) and (4). Second, for each of the four countries in our sample, we estimate the process driving the country spread deviation defined by equations (3) and (5).

The likelihood of these processes is challenging to evaluate because of the presence of two innovations, the innovation to levels and to volatility, that interact in a nonlinear way. We address this problem using the Particle filter. This filter is a Sequential Monte Carlo algorithm that allows for the evaluation of the likelihood through resampling simulation methods. The appendix offers some further details and references about the Particle filter.

We follow a Bayesian approach to inference by combining the likelihood function with a prior. In our context, Bayesian inference is convenient because we have short samples that can be complemented with pre-sample information and because the alternative of maximizing the likelihood function often delivers unreliable and unstable estimates. This problem is particularly relevant in our context because the Particle filter delivers an evaluation of the likelihood function that is not differentiable with respect to parameter values because of the inherent discreteness of resampling. We start, then, by specifying priors.

2.3.1. Priors

We first concentrate on the priors for the parameters driving the law of motion of the country spread deviation. Then, we analyze the priors for the parameters of the process for international risk free real rate deviations.

Table 1: Priors

	ρ_r	σ_r	ρ_{σ_r}	η_r
Argentina	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.30, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}(0.6, 0.3)$
Brazil	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.60, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}(0.6, 0.3)$
Ecuador	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-5.80, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}(0.6, 0.3)$
Venezuela	$\mathcal{B}(0.9, 0.02)$	$\mathcal{N}(-6.50, 0.4)$	$\mathcal{B}(0.9, 0.1)$	$\mathcal{N}(0.6, 0.3)$
Note: 1) \mathcal{B} and \mathcal{N} stand for beta and normal distributions.				
2) Mean and standard deviation in parentheses.				

Table 1 reports our priors for the parameters of the processes corresponding to each of the four countries spreads. Except for σ_r , we adopt the same prior for all four countries. This similarity facilitates the comparison of the different posteriors.

For ρ_r and ρ_{σ_r} we choose a Beta prior with mean 0.9 and a moderate standard deviation, 0.02, for ρ_r , and a fairly large one, 0.1, for ρ_{σ_r} . This prior reflects our view that there is a mild persistence in interest rates (since we have a monthly model, a value of 0.9 monthly is equivalent to 0.73 quarterly). The moderate standard deviation for ρ_r pushes the posterior towards lower values of this parameter. Otherwise, the median of the posterior would get virtually identical to 1, exacerbating the effects of stochastic volatility. Hence, our choice is conservative in the sense that it biases the results *against* our hypothesis that stochastic volatility is quantitatively important. The value of 0.1 for the standard deviation for ρ_{σ_r} embodies our relative ignorance regarding the persistence of the shock to volatility.

For η_r , we pick a truncated normal, where the truncation ensures that the parameter is positive. The mean of the prior for η_r implies that, on average, the standard deviation of the innovation to the level of the country spread increases by a factor of roughly two after a positive stochastic volatility shock of one standard deviation ($\exp(0.6) = 1.8$). This rise is modest compared to the large swings in interest rate volatility hinted by figure 2.1. For the case of Argentina, the standard deviation change on the country spread is 7 times larger in the period 2002 – 2005 compared to that of the years 1998 – 2002. The standard deviation of 0.3 allows the posterior to substantially move away from the mean of the prior. Finally, σ_r is chosen to be country specific. At the prior mean, the unconditional variance of $\varepsilon_{r,t}$ matches

that of the data if we assumed no stochastic volatility shocks. Therefore, our priors capture the observation that Argentina and Ecuador have larger country spread variances than Brazil and Venezuela.

Overall, we view our priors are sufficiently loose to accommodate all countries in our sample. Indeed, we found that increasing the standard deviation of the priors for σ_r , ρ_{σ_r} , and η_r had no significant impact on our findings, while increasing the the standard deviation of the prior for ρ_r favors our results. We further elaborate on the effects of the priors in the robustness section.

The priors for the parameters of the law of motion of the international risk free real rate are chosen following an identical approach than for the country specific spreads. Thus, the justifications we provided before for these priors also hold here. We choose Beta priors for ρ_{tb} and $\rho_{\sigma_{tb}}$ with mean 0.9 and standard deviations of 0.02 and 0.1 respectively. For η_{tb} , we picked a truncated normal with mean 0.6 and standard deviation 0.3. Finally, σ_{tb} is such that, at the prior mean, the unconditional variance of $\varepsilon_{tb,t}$ matches the one observed in the data without stochastic volatility shocks.

2.3.2. Posterior Estimates

We draw 5000 times from the posterior of the each of the five processes that we estimate (one for the international risk free real rate and one for each country spread) using a random walk Metropolis-Hastings. This draw was implemented after an exhaustive search for appropriate initial conditions and an additional 5000 burn-in draws. We select the scaling matrix of the proposal density to induce the appropriate acceptance ratio of proposals (Roberts, Gelman and Gilks, 1997). Each evaluation of the likelihood is performed with 2000 particles. We implemented standard tests of convergence of the simulations, both of the Metropolis-Hastings and of the Particle filter. Given the low dimensionality of the problem, even a relatively short draw like ours converges without further problems.

The sample mean for the real return of the T-Bill, our measure of the international risk free real interest rate, is 0.001, a number that coincides, for example, with the computations in Campbell (2003). Table 2 presents the mean of the real interest rate for each country, r , in a monthly base. Clearly, each country in the sample pays an important risk premium, from the 0.006 of Brazil and Venezuela to the 0.019 of Argentina. In an annual base, the mean differential varies from 740 basis points to 2530 basis points.

Table 2: Mean of Real Interest Rate

	Argentina	Ecuador	Venezuela	Brazil
r	0.02	0.011	0.007	0.007

Table 3 reports the posterior medians of the parameters for the law of motion of the country spread. First, for the case of Argentina and Ecuador (and for Brazil and Venezuela in a lesser degree), the average standard deviation of a shock to the level of country spread, σ_r , is large in comparison with r . This ratio implies a large degree of volatility in the country spread data. Moreover, the posterior is concentrated, which indicates that our belief about the size of the volatility is tight. Second, for all four countries in our sample, there is a substantial presence of stochastic volatility in the country spread series, i.e., a large η_r . Finally, the shocks to the level and standard deviation of country spread are highly persistent, i.e., a large ρ_r and ρ_{σ_r} . The standard deviation of the posteriors of ρ_r is small. The standard deviation of the posteriors of ρ_{σ_r} is larger, but even at the fifth percentile, we have a persistent process in the range of 0.8 to 0.89.

Table 3: Posterior Medians
(95 percent set in parenthesis)

	Argentina	Ecuador	Venezuela	Brazil		T-Bill
ρ_r	0.97 [0.96,0.98]	0.95 [0.93,0.96]	0.94 [0.92,0.96]	0.95 [0.94,0.97]	ρ_{tb}	0.95 [0.93,0.97]
σ_r	-5.70 [-6.31,-4.95]	-6.05 [-6.63,-5.51]	-6.88 [-7.25,-6.38]	-6.96 [-7.44,-6.30]	σ_{tb}	-8.06 [-8.38,-7.71]
ρ_{σ_r}	0.93 [0.85,0.99]	0.96 [0.89,0.99]	0.91 [0.80,0.98]	0.94 [0.87,0.99]	$\rho_{\sigma_{tb}}$	0.94 [0.82,0.99]
η_r	0.47 [0.35,0.60]	0.35 [0.22,0.46]	0.34 [0.20,0.46]	0.30 [0.20,0.38]	η_{tb}	0.13 [0.05,0.26]

We now spend a bit more time examining each case in particular. We start with Argentina, the most volatile country in our sample. The estimated value of σ_r implies that the innovation to the level of the spread has an average annualized standard deviation of 400 basis points ($= 120000 \exp(\sigma_r)$), where the loading factor of 120000 transforms the estimate into annualized basis points. A positive stochastic volatility shock of one standard deviation magnifies the standard deviation of the innovation to the level of the spread by a factor of 1.6 ($= \exp(\eta)$). Consequently, a combined positive shock to both the level and volatility would raise Argentina's spread by 640 basis points ($= 120000 \exp(\sigma_r + \eta)$).

Let us now turn to Brazil, the less volatility country. Its innovation to the level of the spread has a average standard deviation of 110 basis points in annualized terms. Furthermore, a positive volatility shock amplifies the effects of a level shock by a factor of 1.35, indicating that a combined positive shock to both the level and volatility would raise Brazilian's spread by 149 basis points.

Ecuador and Venezuela lay in the middle of our sample. Ecuador has an average standard deviation of 283 basis points and a combination of positive shocks increases the spread by 401

basis points. As we will see later in our simulations, these results put Ecuador in line with Argentina. Venezuela's numbers are closer to Brazil's. It has an average standard deviation of 123 basis points and a combined positive shock increases the interest rate spread by 173 basis points.

In comparison with the country spread, the international risk free real rate has both lower average standard deviation of the innovation to its level, i.e., σ_{tb} is smaller than σ_r for any of the four countries, and there is less stochastic volatility, i.e., η_{tb} is also smaller than η_r for any of the four countries. In particular, we find that posterior median for σ_{tb} equals -8.06 and for η_{tb} equals 0.13 . Thus, the innovation to the level of the international risk free real rate has an average annualized standard deviation of only 40 basis points and when combined with a positive shock to volatility, the international risk free real rate increases to 44 basis points. The persistence, 0.95 , is in line with other estimates in the literature (for example, Neumeyer and Perri, 2005, find a persistence of 0.81 at a quarterly rate).

If we compare the volatility of the international risk free real rate and the volatility of the country spreads, the latter is between 3 to 10 times more volatile than the former and has between 2 to 4 times bigger time-varying component. These relative sizes justify why we concentrate in our theoretical model on the study of shocks to the level and volatility of country spreads and forget about shocks to the international risk free real rate.

2.4. Empirical Regularities

We now exploit the output from our econometric exercise to document several empirical regularities about business cycles and country spread volatility in our four economies. The objective is to analyze the correlations between country spreads, output, investment, and consumption with country spread volatility.

We obtain aggregate data from the International Financial Statistics (IFS) service of the International Monetary Fund, except for Venezuela, where data comes from the Central Bank of Venezuela (Venezuela is not a member of the International Monetary Fund). The data coverage is: Argentina: 1993.Q1 - 2004.Q3; Brazil: 1995.Q1 - 2004.Q1; Ecuador: 1992.Q1 - 2001.Q2; and Venezuela: 1991.Q1- 2004.Q4. Consumption corresponds to household expenditure on goods and services; investment is the sum of gross fixed capital formation and changes in inventories; net exports equals exports of goods and services minus imports of goods and services; finally, output equals the addition of consumption, investment, and net exports. Real variables were obtained by dividing nominal ones by the GDP deflator. All variables were seasonally adjusted using the U.S. Census Bureau's X-12 program. Unless otherwise mentioned, output, consumption, and investment are H-P filtered.

The challenge for the researcher is that the country spread volatility, $\sigma_{r,t}$, is not an observable variable but an inherently latent object. However, we can take advantage of our model for country spreads, i.e., equations (3) and (5), and the Particle filter to smooth the distribution of country spread volatilities conditional on our whole sample. We report the value of the average smoothed volatility conditional on the median of posterior of the parameters. Since we use monthly data for interest rates and quarterly data for output, we linearly interpolate output, investment, and consumption in order to produce the figures in this section.

A first exercise is to plot, in figure 2, the time series of output and the smoothed country spread volatility in annualized basis points. The figure indicates that in our set of emerging economies there is a clear negative correlation between output and country spread volatility. For all four countries, times of high volatility are times of low output. A similar picture would emerge if we printed volatility against consumption or investment in each of the four countries of our sample.

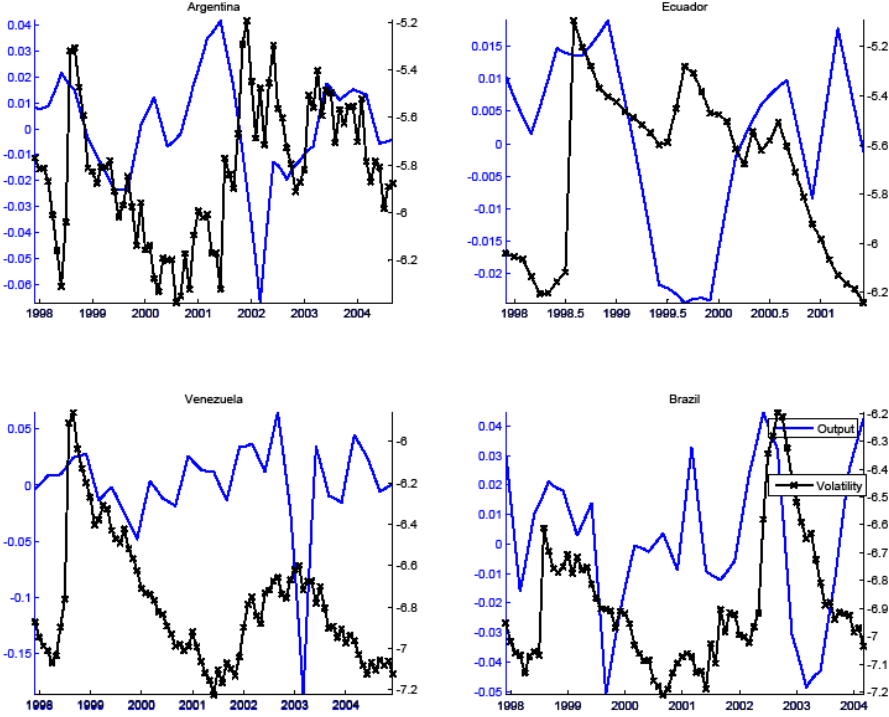


Figure 2: Output and Volatility

An alternative view of this negative correlation is to plot, in figure 3, the cross-correlation between output and country spread volatility at different lags for the countries in our sample. The main result is that country spread volatility is countercyclical and leads the cycle

by about five months. The contemporaneous correlation coefficients between output and volatility range from around zero in Brazil to -0.3 in Argentina or Ecuador. The average contemporaneous correlation is -0.15.

Figure 3 also plots the cross-correlation between investment and country spread volatility and consumption and country spread volatility. As it was the case with output, country spread volatility is countercyclical and leads the cycle with respect to investment and consumption. For the case of consumption, the contemporaneous correlation varies from slightly below zero for Brazil to -0.17 in Ecuador. The average is -0.11. For the case of investment, the contemporaneous correlation moves from -0.05 for Brazil to -0.2 in Argentina. The average is -0.12.

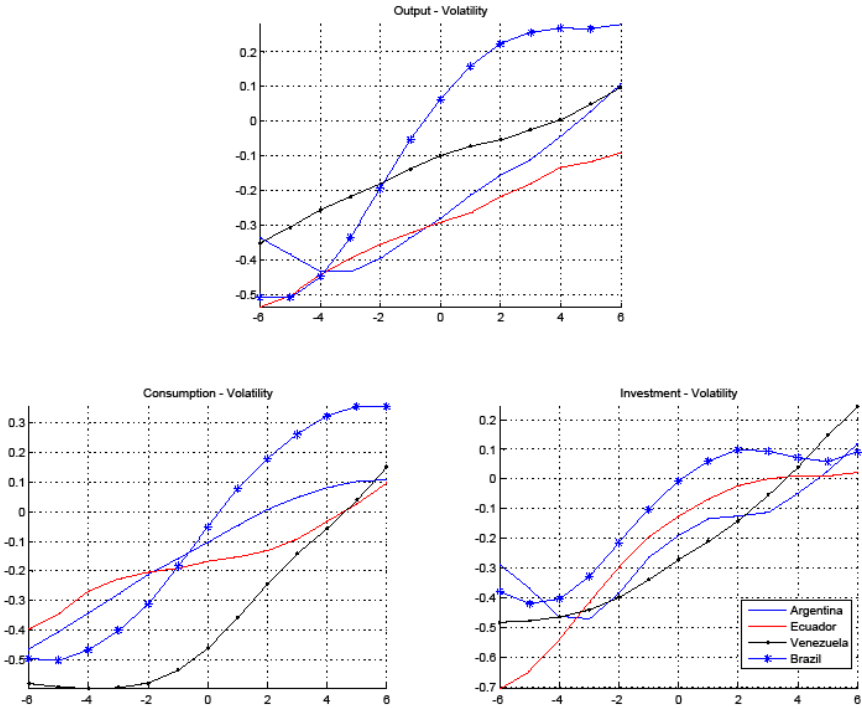


Figure 3: Cross-correlations: Output-Volatility, Consumption-Volatility, Investment-Volatility

Finally, figure 4 plots the time series of country spread and the computed average country spread volatility. The figure immediately reveal that in emerging economies there is a positive comovement between country spread and country spread volatility. Hence, periods of high country spreads are associated with periods of high country spread volatility. Note than when we estimated our model of stochastic volatility for country spread, we assumed the innovation to the level and volatility of the country spread are uncorrelated. Figure 4 indicates that our

assumption is too strong. In the next subsection, we relax it. Interestingly, this generalization only makes our case stronger.

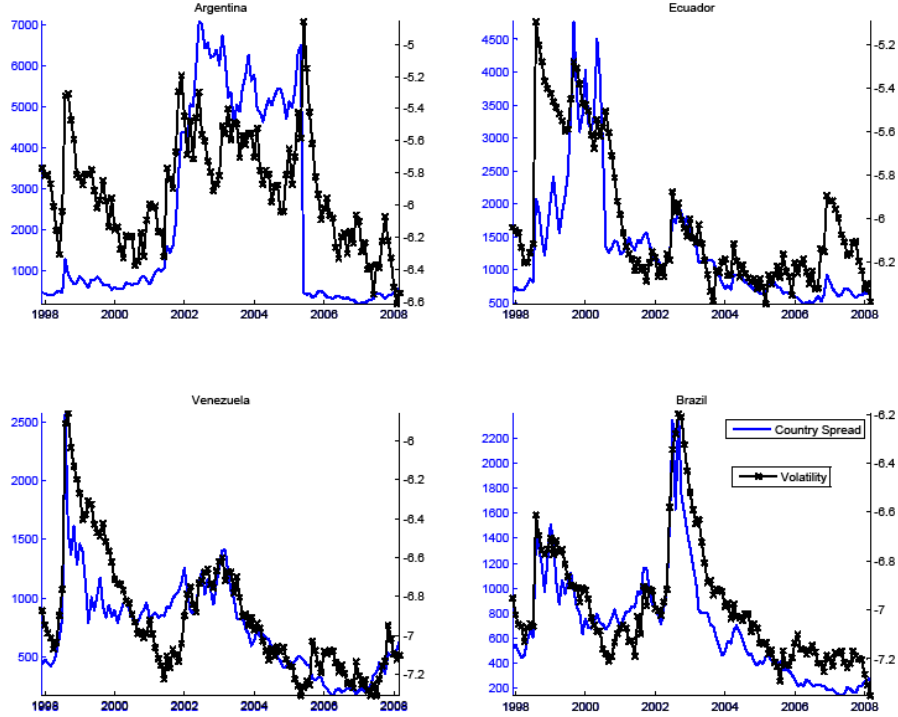


Figure 4: Country Spread and Volatility

2.5. Re-estimating the Processes with Correlation of Shocks

The evidence in figure 4 indicates that our benchmark assumption that the shocks to the level and the volatility are uncorrelated is too strong. We repeat our exercise assuming now that the shocks come from a multivariate normal:

$$\begin{pmatrix} u_{r,t} \\ u_{\sigma_r,t} \end{pmatrix} \sim \mathcal{N} \begin{pmatrix} 0 & 1 & \kappa \\ 0 & \kappa & 1 \end{pmatrix}$$

We do not correlate the shocks to levels and volatility of the international risk free real rate since their empirical size is small and they would not play a quantitatively significant role in the simulation of the model. We impose a uniform prior for κ in $(-1, 1)$ to reflect an roughly neutral stand on the size of the correlation.

Table 4: Posterior Medians with Correlation
(95 percent set in parenthesis)

	Argentina	Ecuador	Venezuela	Brazil		T-Bill
ρ_r	0.97 [0.96,0.98]	0.95 [0.92,0.96]	0.95 [0.93,0.96]	0.96 [0.94,0.97]	ρ_{tb}	0.95 [0.93,0.97]
σ_r	-5.78 [-6.18,-5.35]	-5.71 [-6.10,-5.26]	-6.10 [-6.62,-5.57]	-6.6 [-6.80,-6.28]	σ_{tb}	-8.06 [-8.38,-7.71]
ρ_{σ_r}	0.90 [0.81,0.96]	0.93 [0.85,0.95]	0.94 [0.90,0.97]	0.91 [0.86,0.94]	$\rho_{\sigma_{tb}}$	0.94 [0.82,0.99]
η_r	0.46 [0.30,0.61]	0.35 [0.25,0.46]	0.28 [0.21,0.41]	0.29 [0.23,0.37]	η_{tb}	0.13 [0.05,0.26]
κ	0.69 [0.44,0.87]	0.90 [0.82,96]	0.81 [0.70,0.91]	0.88 [0.79,0.95]		

Table 4 reports our posterior. Direct inspection reveals that the median values of the posterior of the parameters ρ_r , σ_r , ρ_{σ_r} and η_r for each of the four countries are quite similar to our benchmark estimates. Thus, the quantitative patterns of figures 2.4 to 2.4 remain virtually identical and we skip including them to save space. The new parameter, κ , is estimated to be highly positive, between 0.69 and 0.88 depending on the country. As we will see in next section, the clustering of level and volatility shocks only makes our case stronger. We will argue below that, in a standard small open economy model, volatility shocks have real effects. If those shocks are positive correlated with interest rate shocks (which have the same sign), both shocks reinforce each other. By keeping as a benchmark scenario the situation without correlation, we isolate more clearly the direct effects of volatility. At the same time, for completeness, we will also report the consequences of feeding to the model the processes with correlation of the shocks.

2.6. Summary of Empirical Results

In this section we have estimated the law of motion for country spread and international risk free rates for the four countries in our sample. We have reached four important conclusions. First, the average standard deviation of a shock to the level of country spread is large. Second, there is substantial stochastic volatility in the country spread data. Third, international risk free rates have both less mean volatility and less stochastic volatility than the country spread for any of the four countries. Fourth, country spread volatility is countercyclical and leads the cycle with respect to output, investment, and consumption. Given these findings, in the rest of the paper we use a canonical small open economy model to measure the business cycle implications of the large degree of volatility and stochastic volatility that we find in country spreads.

3. The Model

We formulate a prototypical small open economy with incomplete asset markets in the spirit of Mendoza (1991), Correia *et al.* (1995), Neumeyer and Perri (2005), and Uribe and Yue (2006). The small open economy is populated by a representative household whose preferences are captured by the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t - \omega^{-1} H_t^\omega]^{1-v} - 1}{1-v}. \quad (6)$$

Here, \mathbb{E}_0 is the conditional expectations operator, C_t denotes consumption, H_t stands for hours worked, and $\beta \in (0, 1)$ corresponds to the discount factor.

Our choice of the Greenwood-Hercowitz-Huffman (GHH) preferences follows the finding by Correia *et al.* (1995) that such utility function is better suited to match the second moments of small open economies. The main appealing feature of the GHH preferences is the absence of wealth effects on the labor supply decision. In this way, labor supply depends only on the real wage and the model, as suggested by the data, is capable of generating a contraction in consumption, labor, and output after a positive shock to the interest rate level.

The real interest rate r_t faced by domestic residents in financial markets follows the equations (2) to (5) that we specified in section 2. This assumption, motivated by our empirical evidence, is the main difference of our model with respect to the standard small open economy business cycle model.

The household can invest in two types of assets: the stock of physical capital, K_t , and an internationally traded bond, D_t . We join the convention that positive values of D_t denote debt. Then, the household's budget constraint is given by:

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t H_t - R_t K_t + C_t + I_t + \frac{\Phi_D}{2} (D_{t+1} - D)^2 \quad (7)$$

where W_t represents the real wage, R_t denotes the real rental rate of capital, I_t is our notation for gross domestic investment, $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position, and D is a parameter that determines the steady state debt. The cost is paid to some foreign international institution. We assume that the household faces this costs of holding a net foreign asset position with the sole purpose of eliminating the unit root otherwise built in the dynamics of the small open economy model. This unit root is inconvenient because it makes difficult to analyze transient dynamics. Schmitt-Grohé and Uribe (2003) compare a number of standard alternative ways to induce stationarity in the small open economy framework and conclude that all of them produce virtually identical

implications for business fluctuations.²

The stock of capital evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta)K_t + \left(1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$$

where δ is the depreciation rate and the process of capital accumulation is subject to adjustment costs. The parameter $\phi > 0$ controls the size of these adjustment costs. The introduction of capital adjustment costs is commonplace in business cycle models of the small open economy. They are a convenient and plausible way to avoid excessive investment volatility in response to changes in the real interest rate. Finally, the representative households is also subject to the typical no-Ponzi-game condition.

Firms rent capital and labor from households to produce output in a competitive environment according to the technology:

$$Y_t = K_t^\alpha (e^{X_t} H_t)^{1-\alpha} \quad (8)$$

where X_t corresponds to a labor-augmenting technology shock that follows an $AR(1)$ process:

$$X_t = \rho_x X_{t-1} + e^{\sigma_x} u_{x,t}. \quad (9)$$

where $u_{x,t}$ is a normally distributed shock with mean zero and variance equal to one.

Firms maximizes profits by equation wages and the rental rate of capital to inputs marginal productivities. This implies that we can rewrite equation (7) as:

$$NX_t = Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1 + r_t} + \frac{\Phi_D}{2} (D_{t+1} - D)^2$$

where NX_t are net exports. Also, we can define the current account as $CA_t = D_t - D_{t+1}$ where the order of the terms is switched from conventional notation because of positive values of D_t denote debt. Combining the definitions of net exports and current account:

$$CA_t = (1 + r_t) NX_t - r_t D_t - (1 + r_t) \frac{\Phi_D}{2} (D_{t+1} - D)^2$$

²In an earlier version of the paper, we found that closing the model with Uzawa preferences delivered qualitatively similar results to those reported here. We could also have closed the economy with a debt-elastic interest rate like $r_t = r + \Phi_d (e^{D_{t+1}/D^s} - 1) + \varepsilon_{r,t} + \varepsilon_{tb,t}$. Under such representation, the responses after a shock to volatility would contain an indirect effect. After a volatility shock, the level of debt would change. This change, as a consequence of the presence of debt in the interest rate rule, would affect the interest rate level. It is more transparent to avoid this indirect effect.

3.1. Equilibrium

A competitive equilibrium can be defined in a standard way as a sequence of allocations and prices such that both the representative household and the firm maximize and markets clear. The set of equilibrium conditions that characterize the time paths for C_t , H_t , D_{t+1} , K_{t+1} , and I_t are given by the first order conditions for the household and the firm:

$$\begin{aligned} \left[C_t - \frac{H_t^\omega}{\omega} \right]^{-v} &= \lambda_t, \\ \frac{\lambda_t}{1+r_t} &= \lambda_t + \Phi_D (D_{t+1} - D) + \beta \mathbb{E}_t \lambda_{t+1}, \\ -\varphi_t + \beta \mathbb{E}_t \left[(1-\delta) \varphi_{t+1} + \alpha \frac{Y_{t+1}}{K_{t+1}} \lambda_{t+1} \right] &= 0, \\ H_t^\omega &= (1-\alpha) Y_t, \\ -\lambda_t + \varphi_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta \mathbb{E}_t \left[\varphi_{t+1} \phi \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right] &= 0, \end{aligned}$$

together with the resource constraint, the law of motion for capital, the production function, and the stochastic processes for the interest rate. The Lagrangian λ_t is associated with the debt level and the Lagrangian φ_t with physical capital.

The deterministic steady state is given by the solution to the following set of equations:

$$\begin{aligned} \left[C - \frac{H^\omega}{\omega} \right]^{-v} &= \lambda, \\ \beta \left[(1-\delta) \varphi + \alpha \frac{Y}{K} \lambda \right] &= \varphi, \\ H^{\omega-1} \left[C - \frac{H^\omega}{\omega} \right]^{-v} &= (1-\alpha) \lambda \frac{Y}{H}, \\ \lambda &= \varphi, \\ \frac{D}{1+r} &= D - Y + C + I, \\ Y &= K^\alpha H^{1-\alpha}, \\ I &= \delta K. \end{aligned}$$

Note that the steady level of debt D will be hand picked below to match the mean value of debt observed in the data. In addition, r is set at the mean of the country's interest rate (T-Bill plus EMBI). Hence, we have a system of 7 equations for 7 unknowns: C , H , λ , φ , K , I , and Y .

3.2. Solving the Model

We solve the model by relying on perturbation methods to approximate the policy functions of the agents and the laws of motion of exogenous variables around the deterministic steady state defined above. Perturbation was introduced in economics by Judd (1992) and it is well described by Schmitt-Grohé and Uribe (2002). Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) report that perturbation methods are highly accurate and deliver fast solution in a closed economy version of the model considered here.³

One of the exercises we are keenly interested in this paper is to measure the effects of a volatility increase, i.e., a positive shock to either $u_{\sigma_r,t}$ or $u_{\sigma_{ib},t}$, while keeping the level of the real interest rate unchanged, i.e., fixing $u_{r,t} = 0$ and $u_{tb,t} = 0$. Consequently, we need to obtain a *third* approximation of the policy functions.

A first order approximation to the model would miss all the dynamics induced by volatility because this approximation satisfies a certainty equivalence principle. Thus, the policy functions would exclusively depend on the shocks $u_{tb,t}$, $u_{r,t}$, and $u_{X,t}$ (all three of which are normal variables with zero mean and unit variance). At the same time, the shocks to volatility, $u_{\sigma_r,t}$ and $u_{\sigma_{ib},t}$, do not appear in this approximation (or more precisely, the coefficient in front of this variables is exactly equal to zero). A second order approximation would only capture the volatility effect indirectly via cross product terms of the form $u_{r,t}u_{\sigma_r,t}$ and $u_{tb,t}u_{\sigma_{ib},t}$, i.e., through the joint interaction of both shocks. Thus, in the second order approximation, volatility does not have an effects as long as the real interest rate does not change. It is only in a third-order approximation that the stochastic volatility shocks, $u_{\sigma_r,t}$ and $u_{\sigma_{ib},t}$, enter as independent arguments in the policy functions with a coefficient different from zero. Hence, if we want to explore the direct role of volatility, we need to consider cubic terms.

The intuition is simple. The first order approximation to the policy function would correspond to the exact solution of the problem when the household has a quadratic utility function. But agents with quadratic preferences only care about means and not about variances. A second order approximation to the policy function corresponds to a cubic utility function. In this case, the household cares about variances (the third derivative of the utility function is different from zero) but not about how this variance changes by itself. It is in the third order approximation to the policies and the associated quartic utility function that the evolution of the variance matters as an independent term.

³We could have solved the model using Value function iteration or projection methods. However, there is a considerable time cost to implement those with the required level of accuracy when the variance of the shocks is changing over time. Moreover, as we will argue momentarily, we need to repeatedly solve the model while we calibrate it since we are dealing with a situation where the moments of ergodic distribution of endogenous variables are not well approximated by the deterministic steady state. Employing a time intensive solution method would make this task too onerous.

Furthermore, the cubic terms in the policy functions are quantitatively important. A particularly relevant point is that, given the level of volatility that we found in section 2, the third order approximation will imply that the mean of the ergodic distributions of the endogenous variables of the model and the deterministic steady state values are quite different. Thus, it will be important that our calibration targets the moments of interest generated by the ergodic distributions and not the moments of the deterministic steady state, which are not representative of the stochastic dynamics of interest. In the appendix we show how the simulation paths of the model are affected by these higher order terms.⁴

The states of the model are given by $\varphi_t = \left(\widehat{K}_t, \widehat{I}_{t-1}, \widehat{D}_t, X_{t-1}, \varepsilon_{r,t-1}, \varepsilon_{tb,t-1}, \sigma_{r,t-1}, \sigma_{tb,t-1}, \Lambda \right)'$ and the exogenous shocks are $\xi_t = (u_{X,t}, u_{r,t}, u_{tb,t}, u_{\sigma_r,t}, u_{\sigma_{tb},t})'$ where \widehat{K}_t , \widehat{I}_{t-1} , and \widehat{D}_t are deviation of the log of K_t , I_{t-1} , and D_t with respect to the log of their steady state values and Λ is the perturbation parameter that premultiplies all the variances in the model.

We take a perturbation solution around $\Lambda = 0$, i.e., around the steady state implied by the equilibrium conditions of the model when all the variances of the shocks are equal to zero. Since the optimal decision rules depend on the states and the exogenous shocks, we define $s_t = (\varphi_t', \xi_t')'$ as the vector of arguments of the policy function. Also, we call s_t^i to the i -th entry of s_t and ns to the number of states. Thus, we can write the third order approximation to the laws of motion of the endogenous states. First, we have a law of motion for capital:

$$\widehat{K}_{t+1} = \psi_i^K s_t^i + \frac{1}{2} \psi_{i,j}^K s_t^i s_t^j + \frac{1}{6} \psi_{i,j,l}^K s_t^i s_t^j s_t^l,$$

where each term $\psi_{i,\dots}^K$ is a scalar and where we have followed the tensor notation:

$$\begin{aligned} \psi_i^K s_t^i &= \sum_{i=1}^{ns} \psi_i^K s_t^i \\ \psi_{i,j}^K s_t^i s_t^j &= \sum_{i=1}^{ns} \sum_{j=1}^{ns} \psi_{i,j}^K s_t^i s_t^j \\ \psi_{i,j,l}^K s_t^i s_t^j s_t^l &= \sum_{i=1}^{ns} \sum_{j=1}^{ns} \sum_{l=1}^{ns} \psi_{i,j,l}^K s_t^i s_t^j s_t^l \end{aligned}$$

that eliminates the symbol $\sum_{i=1}^{ns}$ when no confusion arises. Similarly, we have a law of motion

⁴This observation raises two possible objections to our perturbation solution. First, whether approximating the policy function around the steady state is the best choice. Second, whether a third order solution is accurate enough. The first objection can be dealt with by noting that the approximation around the steady state is the asymptotically valid one (something that cannot be said for sure about other approximation points) and that the second and third order terms include constants that correct for precautionary behavior. The second objection is answered by relying on the evidence from Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), which shows the large range of accuracy of perturbation methods.

of investment:

$$\widehat{I}_t = \psi_i^I s_t^i + \frac{1}{2} \psi_{i,j}^I s_t^i s_t^j + \frac{1}{6} \psi_{i,j,l}^I s_t^i s_t^j s_t^l,$$

and a law of motion of foreign debt level:

$$\widehat{D}_t = \psi_i^D s_t^i + \frac{1}{2} \psi_{i,j}^D s_t^i s_t^j + \frac{1}{6} \psi_{i,j,l}^D s_t^i s_t^j s_t^l$$

Finally, we have the law of motions for the technology shock, (9), the deviation of the real interest rate due to the country spread, (3), the deviation of the real interest rate due to the international risk free real rate, (2), and the volatilities, (4) and (5). For the case of the law of motion for the deviation of the real interest rate due to the country spread, (3), and the deviation of the real interest rate due to the international risk free real rate, (2), we also consider third order approximations instead of their exact form to keep the order of the approximation consistent across equations. Our solution, including calculating all the analytic derivatives, is implemented in `Mathematica`.

3.3. Calibration

We calibrate eight versions of model, two for each country; one using our benchmark estimates of the laws of motion for interest rates (without correlation of the shocks to level and volatility) and one for the alternative estimates (with correlation). Thereafter, we call the first version of the model, the benchmark process without correlation of shocks, M1, and the second version, where we feed in the processes with correlation, M2.⁵ Since the processes for the interest rate that we estimate are monthly, we also set one period in our model to be one month and calibrate the parameters accordingly. Below, when we compare the moments of the model with the moments of the data, we aggregate three periods of the model to create a quarter.

We fix the value of the following five parameters in all eight calibrations: the parameter that determines the elasticity of labor to wages, $\omega = 1.6$, the depreciation factor, $\delta = 0.014$, capital share, $\alpha = 0.32$, the inverse of the elasticity of intertemporal substitution, $v = 2$, and $\rho_g = 0.95$, the autoregressive of the productivity process. The values for ω , α , and v are those used in Mendoza (1991), Schmitt-Grohé and Uribe (2003), and Aguiar and Gopinath (2007). The depreciation rate is taken from Neumeyer and Perri (2005), who find this relatively high value appropriate for Argentina. The absence of equivalent measures for the other countries forces us to use Argentina's depreciation rate across the four different versions of our model. The autoregressive process is more difficult to pin down because of the

⁵Ideally, we would like to estimate most of the structural parameters in our model. However, the lack of reliable data at the monthly basis and the non-linear nature of our solution method make such an enterprise unfeasible.

absence of good data on the Solow residual. Following the suggestion of Mendoza (1991), we select a value slightly lower than the one commonly chosen for rich economies. We checked that our results are robust to variations in the value of ρ_g .⁶

The rest of the parameters differ across each version of the model. First of all, we set the parameters for the law of motion of the real interest rate equal to the median of the posterior distributions reported in section 2. Second, we set the discount factor equal to the inverse of the gross mean real interest rate of each country

$$\beta = \frac{1}{1+r}$$

Conditional on all the previous parameters, we begin to match moments of the model with moments of the data. Remember that, as explained before, the moments of the model are the ones implied by a third order approximation and not the steady state. We start with D , the parameter that controls the steady state value of debt, to match the mean observed value of debt and the adjustment cost of debt, Φ_D , to control the ratio of net exports over output and the volatility of consumption. To discipline the exercise, we pick only two levels of Φ_D , one for the two more volatile countries, Argentina and Ecuador, and another for Venezuela and Brazil which is 50 percent of the first value. It is reassuring that our choices for Φ_D is consistent with the empirical estimates reported in Uribe and Yue (2006). In any case, this small value helps to close the model without significantly affecting its dynamic properties.⁷ Finally, the two last parameters, σ_x , the standard deviation of productivity shocks and ϕ , the adjustment cost of capital, are chosen to match as closely as possible output volatility and the volatility of investment with respect to output.

The empirical moments to be matched are reported in Table 5 and they are based on H-P filtered quarterly data from we sources that we described in section 2. The row nx/y displays the average of net exports as a percentage points of output. A positive value means that the country is running a trade surplus.

Table 5: Empirical Second Moments

	Argentina	Ecuador	Venezuela	Brazil
σ_y	4.80	2.50	6.72	4.79
σ_i/σ_y	3.80	9.32	2.88	1.65
nx/y	1.80	3.90	4.2	0.1

⁶We recalibrated and recomputed the model for values of ρ_g as low as 0 without finding important differences in the effects of volatility shocks.

⁷Nevertheless, this parameter has a small impact because the third order approximation. In a linear approximation, this parameter is irrelevant.

To compute the moments of the ergodic distribution generated by our model we proceed as follows. First, we simulate the model, starting from the steady state, for 2096 periods. We disregard the first 2000 periods as a burn-in and use the last 96 periods, which corresponds to 8 years in the data, to compute the mean of the posterior. Since our macro data come in a quarterly frequency, we transform the model simulated variables from a monthly to a quarterly basis and, then, we H-P filter them. We repeat this exercise 10 times to obtain the mean of the moments over the 10 simulations. We checked that our estimates of the moments were stable. The country specific results of our calibration are summarized in table 6.

Table 6: Summary Calibration

	Argentina		Ecuador		Venezuela		Brazil	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
β	0.979	0.979	0.989	0.989	0.993	0.993	0.993	0.993
Φ_D	$1.4e - 4$	$1.4e - 4$	$1.4e - 4$	$1.4e - 4$	$7e - 5$	$7e - 5$	$7e - 5$	$7e - 5$
D	27	24	60	50	95	75	10	10
ϕ	280	240	60	55	20	80	65	150
σ_x	0.0075	0.0072	0.0014	0.001	0.007	0.0075	0.0075	0.0075

4. Results

In this section, we analyze the quantitative implications of our model. First, we report the moments generated by our model and compare them with the data. Second, we look at the impulse response functions of shocks to the level and volatility of country spreads. Third, we asses the robustness of our findings.

4.1. Moments

Our first exercise is to compute the moments of the model based on our simulations. Table 7 reports the results for both versions of the model (without and with correlation and shocks) and reproduces the data moments for comparison purposes. For both calibrations, the model does a fair job at matching the moments of the data. Even if we have used some of the moments for calibration, the relative success of the model is no small accomplishment, as small open economy models often have a tough time matching moments in the data for any parameter value.

We highlight two results. First, the model roughly accounts for the relative volatility of net exports over output. This finding is important because this was a moment that we

did not use in the calibration. Second, it is interesting that the moments with and without correlation of shocks are quite similar.

Table 7: Second Moments

	Argentina			Ecuador			Venezuela			Brazil		
	Data	M 1	M 2	Data	M 1	M 2	Data	M 1	M 2	Data	M 1	M 2
σ_y	4.80	4.82	4.64	2.50	2.90	3.14	4.72	4.43	4.85	4.79	4.59	4.65
σ_c/σ_y	1.30	1.90	1.70	2.50	1.60	2.10	0.87	0.85	1.22	1.10	0.78	0.87
σ_i/σ_y	3.80	3.34	3.50	9.32	9.40	9.40	3.42	3.79	3.66	1.65	1.70	1.55
σ_{nx}/σ_y	0.39	0.23	0.20	0.65	0.21	0.16	0.19	0.13	0.09	0.22	0.34	0.12
nx/y	1.80	1.50	1.94	3.90	4.00	3.12	4.00	3.20	2.62	0.1	0.39	0.31

4.2. Impulse Responses

Our second exercise looks at the impulse responses functions (IRFs) of the model to shocks in the level and volatility of country spreads. Computing these IRFs in a non-linear environment is somehow involved since the IRFs are not invariant to re-scaling and to the previous history of shocks. We refer the reader to the appendix for details on how we construct them.

4.2.1. Argentina

We find convenient to start our discussion by analyzing the effects of shocks in Argentina. All graphs and results for the other three countries will follow the same format in the order of presentation. In figure 5 we plot the IRFs to three shocks (rows) of consumption (first column of panels), investment (second column), output (third column), labor (fourth column), the interest rate (fifth column), and debt (the sixth column). Interest rates are expressed in basis points while all other variables are expressed as percentage deviations from the mean of their ergodic distributions.

The first row of panels plot the IRFs to a one standard deviation shock to the level of the Argentinean country spread, $u_{r,t}$ in the M1 version of the model. Following a 34 basis point rise in the level of Argentina's monthly spread, the country experiences a persistent contraction with consumption dropping by 1.6 percent upon impact and investment falling for two years. To match the second moments found in the Argentinean data, our model requires a significant degree of adjustment costs in investment. Consequently, we find that the decline in output is highly persistent. Only after 66 months output reaches its lowest level (-0.66 percent). Labor tends to mimic the dynamics of output, which results from our reliance on

GHH preferences. Finally, debt falls for five years and a half, with a total reduction of nearly 20 percent of the value of the liability.

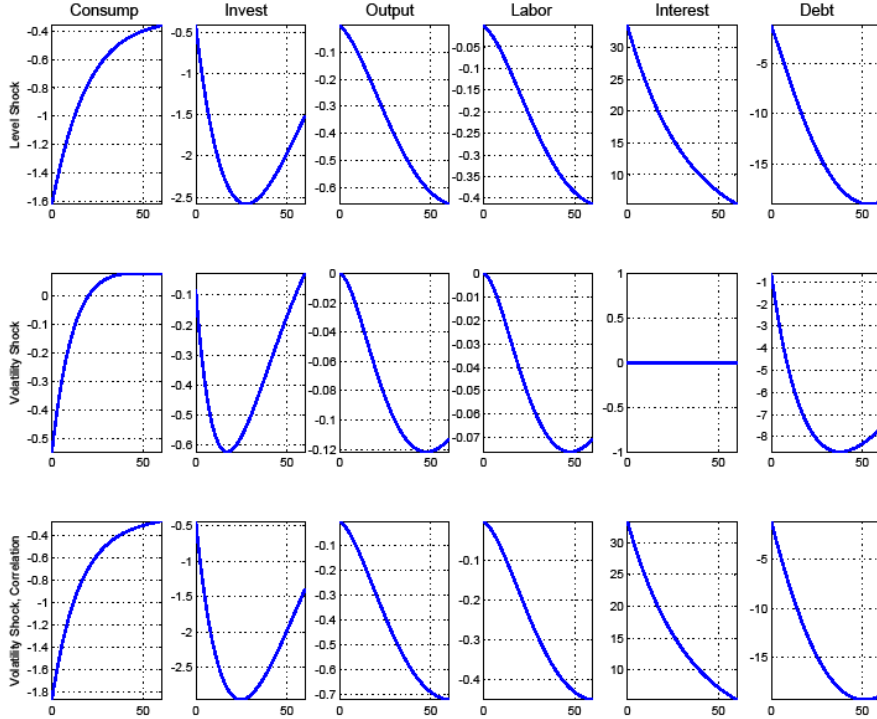


Figure 5: IRFs Argentina

The intuition for the drop in output, consumption, and investment is well understood (see, for example, Neumeyer and Perri, 2005). A higher r_t has a negative wealth effect that reduces consumption, forces a decrease in the level of debt (since now it is more costly to finance it), and lowers investment through a non-arbitrage condition between the returns to physical capital and to foreign assets. We include this exercise to show that the model delivers the same answers than the standard model when hit by equivalent level shocks and to place the size of the IRFs to volatility shocks in context.

The contraction in economic activity may seem large relative to those found in the literature. Uribe and Yue (2006), for instance, estimate that a 1 percentage point rise in the country spread dwarfs output and investment by 0.15 percent and 0.5 percent, respectively. However, we must keep in mind that our time frame is a month, which implies that the interest rate in fact rises by 4.1 percentage points in an annual basis. When we normalize the spread shock so that the interest rate increases by 9 basis points upon impact (or a 1 percentage point in a yearly basis), we find that consumption falls by 0.4 percent while output

and investment contracts by 0.15 and 0.7 percent, respectively. These findings are more in line with the empirical estimates reported by Uribe and Yue (2006). Furthermore, Uribe and Yue find that it takes about 2 years for output to reach its lowest level. Their result raises the question of whether our model may overpredict the persistence of output because of a large investment adjustment cost. We will discuss the effects of smaller adjustment costs in section 5.

The second row of panels plots the IRFs to a one standard deviation shock to the volatility of the Argentinean country spread, $u_{\sigma,t}$. To put a shock of this size in perspective, our econometric estimates of section 2 indicate the collapse of LTCM in 1998 meant a positive volatility shock of a 1.5 standard deviation and that the 2001 financial troubles amounted to two repeated shocks of roughly 1 standard deviation size.

This second row is perhaps the most important finding of our paper. First, note that there is no movement on the level of the domestic interest rate faced by Argentina. What changes is the volatility of future changes, the risk with respect to the future, not the level or the expected level of the country spread. Second and most strikingly, there is a) a pronounced contraction in monthly consumption (0.55 percent at impact), b) a slow fall of investment (after 1.5 years it falls -0.62 percent), c) a slow fall in output (after 5 years, it falls 0.12 percent and labor, and d) debt shrinks upon impact and keeps declining until it reaches its lowest level (-8.7 percent), roughly 3 years after the shock. These IRFs show how increments in risk have real effects in the economy even when the level of the real interest rate remains constant.

To understand the economic logic behind the reaction of real variables to changes in risk, we can go back to the equilibrium conditions of the model. To facilitate the discussion, we assume for a moment that $\phi = 0$. Nothing serious on our argument depends on this assumption since adjustments costs to investment only slow moderate the fluctuations of capital. Then, we can simplify the relevant first order conditions to:

$$\begin{aligned} \left[C_t - \frac{H_t^\omega}{\omega} \right]^{-v} &= \lambda_t \\ \frac{\lambda_t}{1+r_t} &= \Phi_D (D_{t+1} - D) \lambda_t + \beta \mathbb{E}_t \lambda_{t+1}, \\ \beta \mathbb{E}_t \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \frac{\lambda_{t+1}}{\lambda_t} &= 1 \end{aligned}$$

The starting point of the reasoning is the second equation, which we can write as:

$$\frac{1}{1+r_t} - \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \Phi_D (D_{t+1} - D) \tag{10}$$

A volatility shock leaves r_t unchanged but it raises $\mathbb{E}_t \lambda_{t+1} / \lambda_t$, as illustrated in figure 6. Why? The Lagrangian λ_t is the marginal utility of consumption and labor. Higher real interest rate risk implies more volatile consumption in the future. Our estimate for η implies that a typical stochastic volatility shock in Argentina raises the magnitude of a shock to the level of interest rates by a factor of 1.5 ($= \exp(\eta)$). Thus, households may face a 51 (1.5×34) basis point surge in the monthly interest rates on their debt obligations if a one standard deviation level shock to interest rates materializes tomorrow. Since marginal utility is convex, Jensen's inequality tells us that $\mathbb{E}_t \lambda_{t+1}$ raises. The total increment of the ratio $\mathbb{E}_t \lambda_{t+1} / \lambda_t$ is smaller because, as we saw in the IRFs, consumption drops at impact and recovers in the following periods, decreasing marginal utility. Nevertheless, this second effect is dominated by the dispersion of marginal utilities. Hence, the left hand side of (10) falls and we can only make the equation hold with equality if D_{t+1} falls as well. The intuition is simple: holding foreign debt is now riskier than before. Hence, the representative household wants to reduce its exposure to this risk.

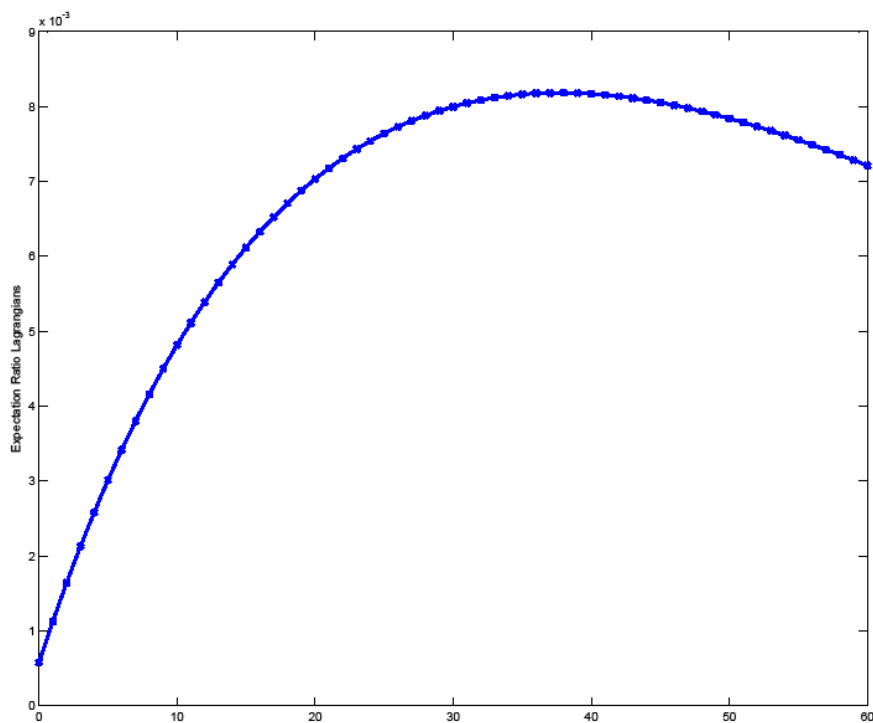


Figure 6: Evolution of $\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}$

The next step is to rewrite:

$$\beta \mathbb{E}_t \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \frac{\lambda_{t+1}}{\lambda_t} = 1$$

as

$$\beta \mathbb{E}_t \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} + cov_t \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}}, \frac{\lambda_{t+1}}{\lambda_t} \right) = 1 \quad (11)$$

In this expression, the conditional covariance of the return to capital and the ratio of Lagrangians decreases when volatility raises. The household uses debt to smooth productivity shocks. Let us imagine that we are in a situation with low volatility. Then, after a negative shock to X_t and the subsequent fall in the return to capital, consumption drops by a small amount (and hence the ratio of Lagrangians raises by a small amount) because debt increases to smooth consumption. However, when volatility is high, the household accepts a bigger reduction in consumption after a productivity shock since increasing debt is a less attractive option as it opens the door to a large interest rate risk.

At the same time, we saw before that $\mathbb{E}_t \lambda_{t+1}/\lambda_t$ only increases by a small amount because of the interaction of mean reverting consumption with the increased dispersion of marginal utilities. Therefore, the only term that can change in equation (11) to accommodate the lower covariance is to raise

$$\mathbb{E}_t \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right)$$

which can only be accomplished with a lower investment today.

Again the intuition is straightforward. After a volatility shock, and fearing an scenario of substantially higher real interest rates, the representative household wants to pay back its debt. Since the country is not more productive than before the shock, the only way to do so is to increase net exports by either working more or by consuming and investing less. The first alternative is precluded by our GHH preferences that do not have a wealth effect. Hence, the household must consume and invest less. The reduction in consumption is bigger because lowering investment has the negative consequence of reducing future output and hence making more costly in utility terms to service the debt. Figure 5 captures these intertemporal trade offs.⁸

To quantify the debt reduction mechanism, we find informative to show in figure 7 the evolution of debt, current account, and net exports (which are linked with debt through the

⁸A third possibility could be to reduce consumption and increase investment to expand output in the future. However, this option reduces notably consumption, which has to suffer the burden of both increased net exports and higher investment. The desire for consumption smoothing eliminates this third possibility for the parameter values that we explored.

budget constraint) all three of them as a percentage of monthly output. After a volatility shock, debt falls 6.5 points of monthly output, or slightly over half a year of output, the current account improves 0.61 percent at impact and net exports raise to 0.63 percent. This figure suggest that volatility is a potentially important factor behind movements in current account and net exports in countries like Argentina.

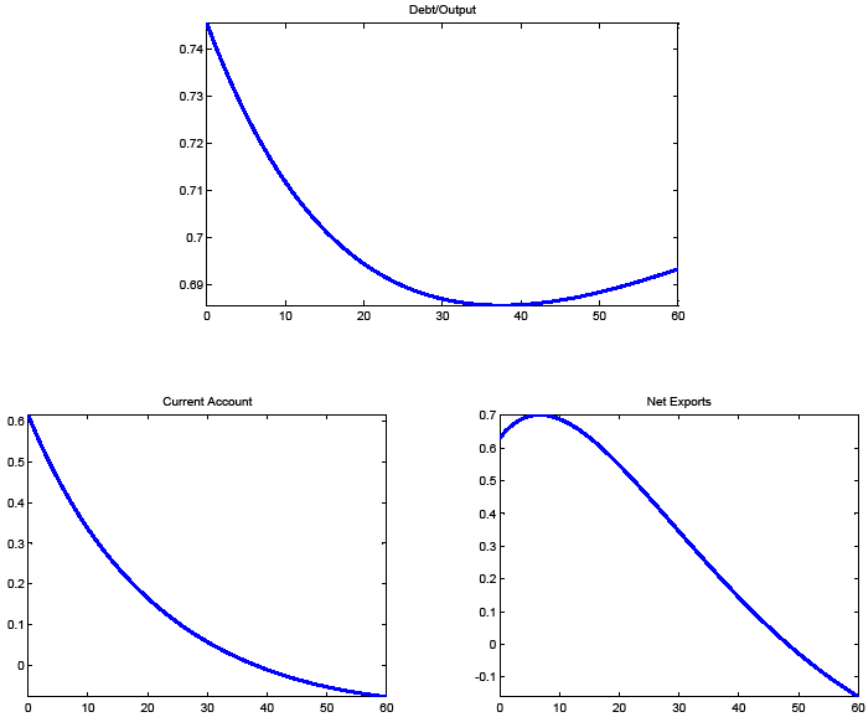


Figure 7: IRFs Debt/Output, Current Account, Net Exports

A slightly different way to understand the fall in investment after a volatility shock is to note that foreign debt allows the household to hedge against the risk of holding physical capital since the shocks to productivity and to the interest rate are uncorrelated. This hedging property raises the desired level of physical capital. The total effect is, however, small because at the same time, debt allows the representative household to rely less on physical capital as a self-insurance device. For example, for the benchmark calibration for Argentinian, the presence of debt increases the average holdings of capital by 1.25 percent in comparison with a closed economy version of our model. A raise in the volatility of the real interest rate makes the hedge provided by foreign debt less attractive, it induces the household to reduce its level of debt, and, hence, it also lowers its holdings of physical capital, goal that is accomplished by a fall in investment.

The last row in figure 5 plots the IRFs in the M2 version of the model where there correlation in the shocks to the level and volatility of r_t . In this row, we plot the IRFs after a one standard deviation level shock that is accompanied by a κ -standard deviation shock to volatility. The pattern of the IRFs is qualitatively the same than in the first row. The quantitative size is now bigger as we combine two shocks. The lesson from this third row is that the results in our paper are fundamentally robust to the presence of correlation between shocks to level and volatility of r_t .

4.2.2. Ecuador

Now, we turn to Ecuador, whose IRFs are plotted in figure 8. Again, we find a very similar set of patterns than in the Argentinian case. There is a decline in economic activity with responses qualitatively similar although somehow smaller than those for Argentina.

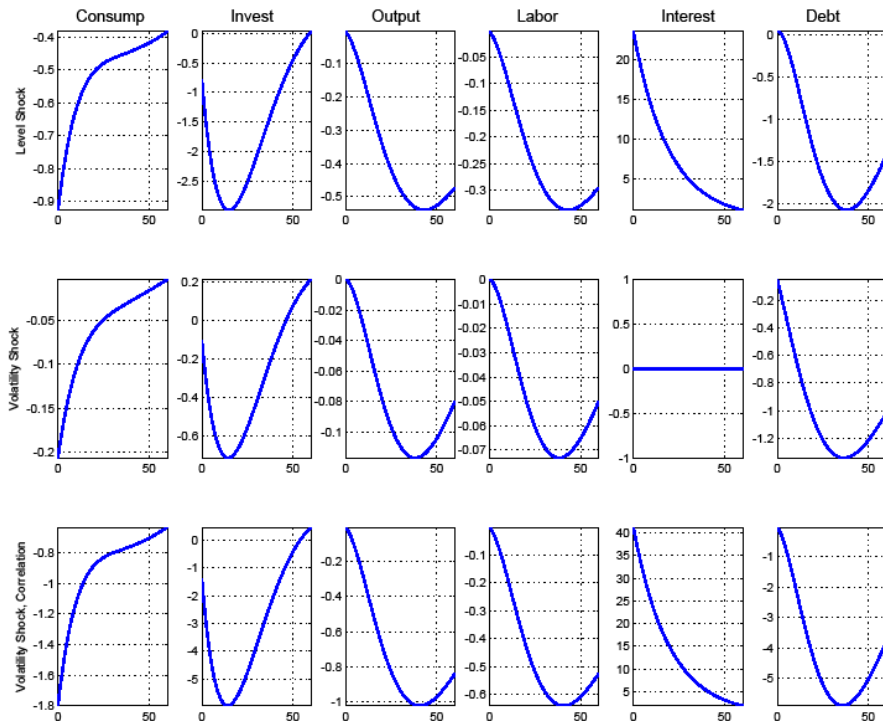


Figure 8: IRFs Ecuador

After a shock to volatility, consumption, for example, drops by 0.20 percent upon impact, investment a 0.10 percent, and debt a 0.05 percent. Then, as in the Argentinian case, investment keeps falling for 16 months and output and labor for approximately three years and a half, when debt also reaches its lowest level, 1.34 percent below its original level.

The relatively modest responses reported in figure 8 are perhaps a little surprising given Ecuador's large debt-to-output ratio (net exports are 3.9 percent of output). The key for these small responses is that Ecuador enjoys a small standard deviation in the innovation to volatility shocks, η_r , and specially, a very low volatility of productivity shocks ($\sigma_r = 0.0014$).

It is interesting, however, to look at the third row of IRFs, when the shocks to the level and to volatility are correlated. While a shock to the level raises the interest rate only by 14 basis points, a correlated shock does it by 0.41. This is due to the high estimated correlation of 0.9. Then, when we have simultaneously a one standard deviation shock to levels and a 0.9 standard deviation shock to volatility, output takes a real dive, by falling over 1 percent after 4 years. When we evaluate this last row in conjunction with the results of our econometric exercise, we can adventure the hypothesis that Ecuador's debacle in the late 1990s started with a sharp volatility shock of 1998 of a 2.5 standard deviations size.

4.2.3. Venezuela

Our next IRFs are those of Venezuela in figure 9. Although the qualitative shape of the IRFs is similar to the two previous cases, the response in Venezuela to a volatility shock are very mild. This is surprising because the similar net export-to-output ratios in Ecuador and Venezuela would made us suspect that these countries should experience similar contractions following a volatility shock. Yet a quick look at figures 8 and 9 reveals that consumption drops ten times as much in Ecuador as in Venezuela. This remark is important because it indicates that large indebtedness alone cannot generate large recessions. Furthermore, the size of the volatility shock, η , is essentially the same for the two countries. Therefore, it must be the case that the departures across the countries' impulse responses come from differences in the steady state interest rates and in the size of the shock level shock, σ_r .

To better understand the implications of the steady state interest rate on the model predictions, we propose the following experiment. At time t , the economy is hit by a one standard deviation volatility shock, which is followed by a shock to the interest rate level, u_r , at time $t + 1$. An Ecuadorian household facing this scenario understands that annualized interest rates will increase tomorrow by as much as 4 percentage points. The same sequence of events imply that Venezuelans will see an increase in annualized interest rates of 1.7 percentage points.⁹ Clearly, Ecuador faces a rather stringent situation, which explains the larger recession in this country. A similar argument can be used to digest the strong decline in Argentina's real variables.

⁹We use $1200 \exp(\sigma_r + \rho_\sigma \eta)$ to arrive to those figures. As before, the loading term is needed to transform the interest rates into annualized percentage points.

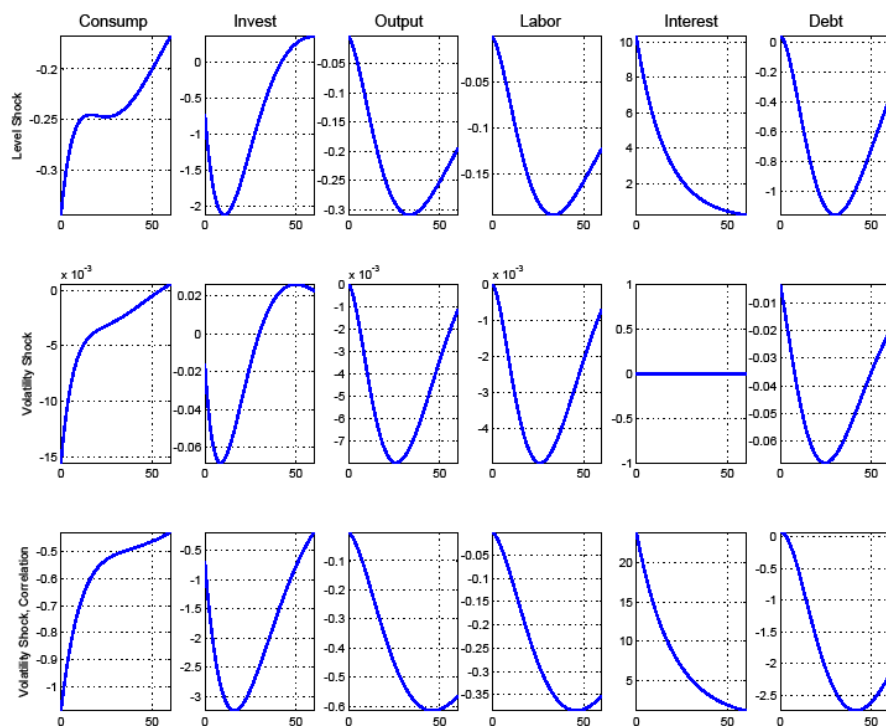


Figure 9: IRFs Venezuela

4.2.4. Brazil

Figure 10 presents Brazil's responses to level and volatility shocks. The main result for Brazil's case is, once more, the similarity of the IRFs to previous findings although now the response of output is quite muted, even more so than in the case of Venezuela. The stronger response in Venezuela to volatility shocks than in Brazil can be accounted for by Venezuela's larger shocks and debt-to-output ratio. This remark further illustrates how the mechanism through which volatility affects real variables is the increased exposure to consumption risk implied by D_t when the real interest rate volatility raises.

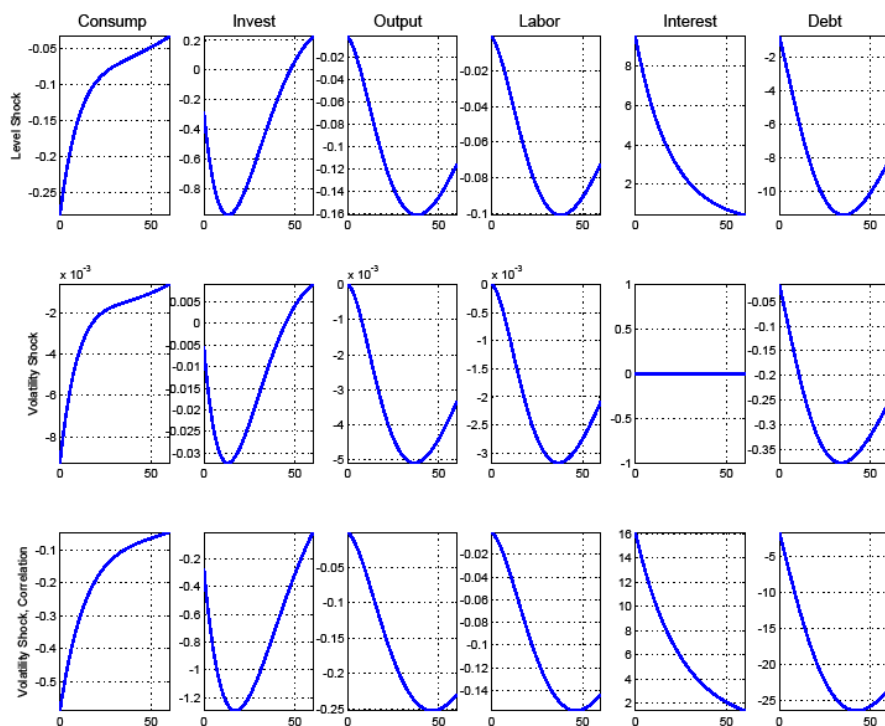


Figure 10: IRFs Brazil

4.3. Variance Decomposition

An additional exercise we can undertake is to measure the contribution of each of the three shocks in our model to aggregate fluctuations. The task is complicated because, with a third order approximation to the policy function and their associated nonlinear terms, we cannot neatly divide total variance among the three shocks as we would do in the linear case.

A possibility is to set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. We explore five possible combinations: 1) the benchmark case with all three shocks, 2) when we only have a shock to productivity, 2) when we have a shock to productivity and to the level of the interest rate (with volatility fixed at its unconditional value), 3) when we only have a shock to the level of the interest rate, 4) when we have shocks to levels and to volatility, 5) when we have shocks only to volatility.

Table 8: Variance Decomposition: Argentina

	All Three Shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.82	4.51	4.64	0.98	1.81	0.16
σ_c	8.89	3.93	7.20	6.10	8.09	2.86
σ_i	15.3	1.44	9.17	9.08	15.3	2.24
σ_{nx}	1.90	0.13	2.00	1.13	0.84	0.19

The results of this exercise are included in table 8 for the Argentinian case. When we only allow productivity to change over time, the economy has fluctuations that are around 94 percent of the observed ones. Remember that, in the absence of good data on the Solow residual, we are calibrating productivity shocks to match output volatility, and hence this 94 percent is not *sensu stricto* a measurement of the impact of productivity innovations. A more informative result is that, counterfactually, the standard deviation of consumption falls below the standard deviation of output. In a model with such a strong consumption smoothing desire as the real business cycle model, it is difficult to get around this result when we only have productivity shocks. This result is important because one of the most salient characteristics of the business cycle of emerging economies is that consumption is more volatile than output. Investment and net exports fluctuate very little because of our relatively high adjustment cost that we need to match the volatility of capital formation when we have all three shocks.

When we add an real interest rate level shock, volatility does not increase much, with output standard deviation going up a mere 3 percent to 4.64. The reason is that, in the simulation, since both shocks are independent, their effects often cancel each other (i.e., a positive technological shock happens at the same time than a raise in the real interest rate). However, the presence of both shocks simultaneously substantially raises the volatility of consumption, which now becomes bigger than output. The intuition is that, while the household wants to smooth out productivity shocks, it prefers to pay back the debt and adjust consumption as a response to a positive level shock to the real interest rate. For a similar reason, investment becomes more volatile, as the household reduces its holdings of physical capital. These two mechanisms are seen more clearly in the next column, where we report the case with only level shocks. While output variability drops to only 0.98, the standard deviation of consumption is still 6.1 and the standard deviation of investment 9.08.

The fourth case is when we have level and volatility shocks. The standard deviation of output raises to 1.81, a 38 percent of the observed volatility, consumption goes to 8.09, and investment to 15.3. The final case is when we only have volatility shocks. In this situation, the standard deviation of output is low, only 0.18 (after all, volatility per se only appears in

the third order term of the policy function) . We highlight how important is the interaction of the level and volatility shocks: jointly they generate a standard deviation of 1.81 while we separate they induce standard deviations of 0.98 and 0.16. The difference is accounted for by the cross terms of level and volatility shocks that appear because both shocks affect the economy in the same direction. Volatility alone, however, makes a substantial contribution to the fluctuations of consumption (the standard deviation is 2.86 with volatility shocks alone) and investment (standard deviation of 2.24).

For completeness, we include the results of the variance decomposition in the other three countries of our sample. Table 9 reports the results for Ecuador. The main lesson to learn from Ecuador is that productivity shocks are less important than the level and volatility shocks at explaining output, consumption, and investment volatility.

Table 9: Variance Decomposition: Ecuador

	All three shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	2.24	0.85	1.49	1.23	2.09	0.22
σ_c	5.10	0.69	4.50	4.48	5.08	0.98
σ_i	20.7	0.70	16.2	16.2	20.6	2.84
σ_{nx}	0.47	0.12	0.54	0.54	0.47	0.43

Table 10 includes the results for Brazil:

Table 10: Variance Decomposition: Brazil

	All Three Shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.59	4.40	4.49	0.42	0.62	0.01
σ_c	3.57	3.15	3.46	1.33	1.38	0.04
σ_i	7.24	2.82	6.65	6.06	6.75	0.15
σ_{nx}	0.85	1.40	1.69	0.58	0.50	0.70

and table 11 for Venezuela.

Table 11: Variance Decomposition: Venezuela

	All Three Shocks	Prod.	Prod. and Level	Level	Level and Volatility	Volatility
σ_y	4.43	4.21	4.37	0.94	1.51	0.04
σ_c	3.81	3.07	3.69	1.95	2.15	0.01
σ_i	15.8	5.00	14.7	13.8	15.2	0.45
σ_{nx}	0.50	3.49	0.64	0.54	0.45	0.32

5. Robustness Checks

In the interest of space, we only consider robustness analysis for Argentina. However, the lessons that we learn from the Argentinian case are general for all four countries in our sample.

The first, and perhaps the most natural, experiment is to gauge the effects of risk aversion in our results for it is this parameter the one that makes the variance of the shocks matter for the policy functions of the agents. In the first row of panels of figure 11 we plot the IRFs of Argentina after a one standard deviation volatility shock when we lower risk aversion, v , from 2 to 1, while keeping the rest of the parameters at their original levels. As the representative household becomes less risk adverse, the ratio $\mathbb{E}_t \lambda_{t+1} / \lambda_t$ raises less than in the benchmark case, while debt, consumption, investment, and output drop less. We can undertake the opposite exercise by raising risk aversion to 5. We report the new IRFs in the second row of figure 11. Inspection of this second row shows that again the qualitative patterns of the IRFs are unchanged. The reader may see the evolution of debt as having the opposite sign than in the benchmark case. However, this is a product of having defined debt as a positive number. When we set risk aversion to 5, the mean of debt in the ergodic distribution becomes negative, i.e., the country holds positive foreign assets. Then, as the household wants to reduce its exposure to the increased real interest rate risk induced by a higher volatility, it will unload part of these assets.

Our third robustness experiment is motivated by the observation that, relative to the empirical evidence (Uribe and Yue, 2006), our model predicts a more persistent response of investment following a shock to the interest rate spread. This persistence arises from the large adjustment cost in investment required to match the second moment properties found in the Argentinean data. To understand the consequences of such a cost, we repeat our simulations with an adjustment cost that makes investment's response to a spread shock consistent with the evidence in Uribe and Yue. The results are reported in the third panel in figure 11. We observe that 1) all variables become more responsive to a stochastic volatility shock and 2) investment peaks one year after the shock. The faster response of investment is a direct implication of the smaller adjustment costs.

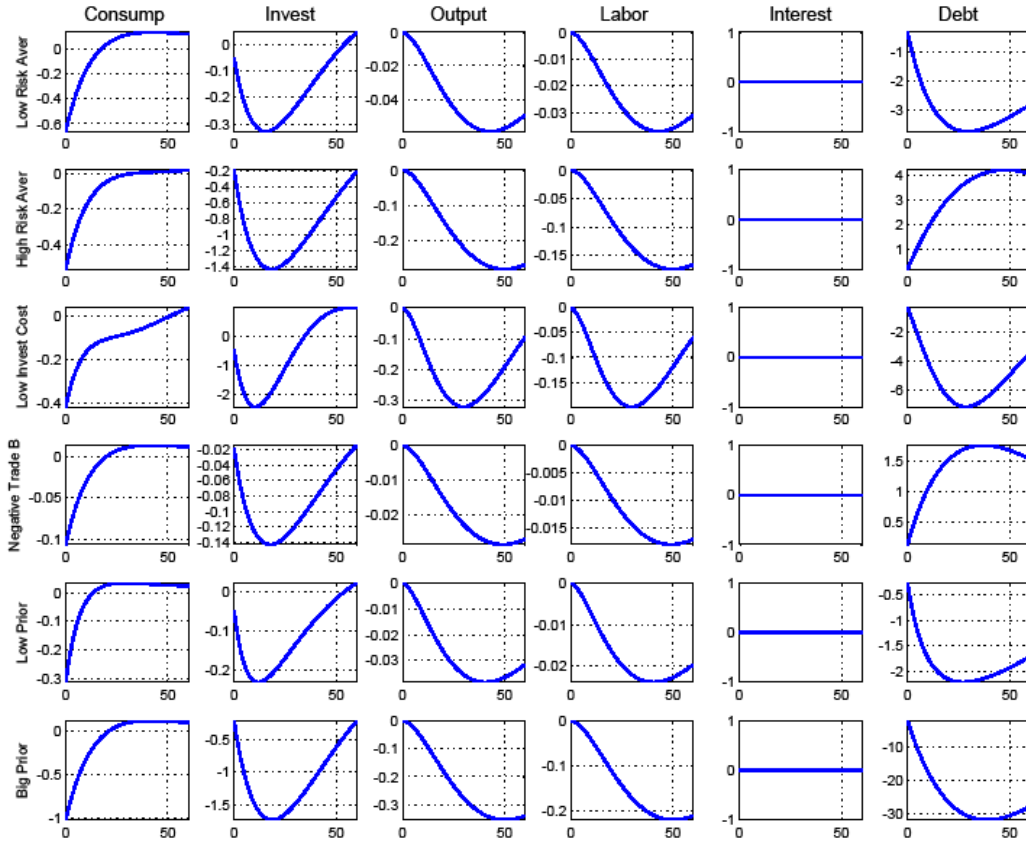


Figure 11: Robustness Checks, Argentina

The large contraction in economic activity can be understood as follows. A smaller adjustment cost implies that investment can easily drop following the volatility shock. Such an event causes two effects on households. On one hand, the large decline in investment ameliorates the need to reduce consumption in the aftermath of the shock for households can use the additional proceeds from lower investment to buy back debt. On the other hand, capital will significantly shrink tomorrow thanks to smaller investment. Low capital in turn implies low labor productivity, which reduces the demand for labor and hence households' wealth. This decline in income ultimately exacerbates the contraction in consumption. The evidence in figure 11 indicates that this second effect dominates giving raise to a large recession in the economy.

We previously argued that a volatility shock to the interest rate is contractionary because households consume less and save more in anticipation of possibly larger interest rate shocks in the future. An extension of this argument therefore suggests that a country with a positive net assets, $D < 0$, should experience a boom after a volatility shock. This is so because if the

rise in volatility were accompanied by a positive shock to the level of the interest rate, the country would end up receiving higher returns from their asset holdings. Facing this scenario, households should increase their consumption while depleting the country’s foreign position.

To test for this hypothesis, we repeat the experiment for the Argentinean calibration, model M1, but we now assume that the economy starts with a net export-to-output ratio of -1.63 (the negative value of what we previously used). We report the results in the first row of panels in figure 11. Consistent with our intuition, the economy experiences a temporal boom in consumption, investment, and output accompanied by a decline in foreign assets. However, the boom is significantly smaller, in absolute terms, than the recession reported in the first column. This asymmetric behavior is likely a consequence of risk aversion; we explore this hypothesis next.

As a final robustness check, we discuss the implications of the priors on our model’s predictions. To that end, we re-estimate the processes (3) and (5) for Argentina with two alternative priors. For the first option (Case I), we select relatively uninformative priors for ρ_r and ρ_σ centered in 0.5 while the other parameters’ priors remain the same than in the original exercise. For the same reason than in the original prior (i.e., to minimize the impact of stochastic volatility), we endow ρ_r with a tighter prior. The resulting posterior medians are reported in Table 13. We still find under this prior that the posterior ρ_r and ρ_σ concentrate around 1. For the second alternative (Case II), we center ρ_r around its OLS estimates and the other priors are left as in the baseline setup. Overall, the estimates are again similar to those in Table 3.

Table 12: Alternative Priors

	Case I		Case II	
	Prior	Median Posterior	Prior	Median Posterior
ρ_r	$B(0.5, 0.1)$	0.98 [0.97, 0.99]	$B(0.96, 0.02)$	0.97 [0.96, 0.98]
σ_r	$\mathcal{N}(-5.3, 0.4)$	-5.89 [-6.49, -5.45]	$\mathcal{N}(-5.3, 0.4)$	-5.72 [-6.35, -4.98]
ρ_σ	$B(0.5, 0.2)$	0.87 [0.76, 0.95]	$B(0.9, 0.1)$	0.94 [0.86, 0.99]
η	$\mathcal{N}(0.6, 0.3)$	0.54 [0.41, 0.69]	$\mathcal{N}(0.6, 0.3)$	0.50 [0.37, 0.63]

We present the results from using the new priors in the fourth (case I) and fifth (case II) panels in Figure (5). For the first alternative, note that the impulse responses are quantitatively similar those that we found under our benchmark formulation. For example, consumption experiences a decline of 0.73% while investment contracts by half a percentage point. The higher persistence in ρ_r have a strong effect in the response of debt.

More interestingly are the results from the second set of priors. Note the strong response of all variables following the volatility shock. The decline in consumption is 0.4 percentage points larger than in our baseline scenario. Similarly, investment's largest response is about 4 times bigger than the one we observe in Figure (4.2.1). To understand these results, notice that the substantially high posterior medians for ρ_r and ρ_σ . If a level shock, $u_{r,t}$, follows the volatility shock, interest rates will remain above its pre-shock level for quite a few periods. As a consequence, households will endure substantially larger payments on its debt obligations. Furthermore, even if the shock level does not materialize tomorrow, households know that the large persistence of the volatility process imply that future level shocks will be almost equally painful. In anticipation of either of these scenarios, households choose to make large debt repayments today and therefore contract substantially consumption and investment.

6. Summary and Directions for Future Research

Our empirical evidence shows that time-varying volatility is an important feature of the real interest rate faced by emerging economies. This changing volatility has a quantitatively important effect on the dynamics of the economy as measure by an otherwise standard business cycle model even when the level of the real interest rate remains constant.

Our investigation opens the door to many interesting questions. First, and most obviously, why does volatility change over time? Is it related with some states of the economy? How does it interact with other phenomena like debt default, debt renegotiation, or financial market integration? Second, we would like to evaluate the possibilities of having time varying volatilities in other aspects of the economy. For example, in a recent and influential paper, Aguiar and Gopinath (2007) have argued that an important factor behind business cycle fluctuations in emerging economies are recurrent changes in the productivity growth trend, possibly caused by policy. We could explore the consequences of introducing stochastic volatility in these changes.

7. Appendix

For completeness, this appendix includes a brief introduction to the Particle filter that we use to evaluate the likelihood of the stochastic volatility process of interest rates, a more detailed discussion of the consequences of using a third order approximation for the dynamics of the model, and the explanation of how we compute the IRFs of the model.

7.1. Particle Filter

We present a brief introduction to the particle filter. We will concentrate on the main idea of the algorithm and skip most of the technical details. As mentioned in the paper, Doucet, de Freitas, and Gordon (2001) is an excellent reference of the interested reader. Fernández-Villaverde and Rubio-Ramírez (2007 and 2008) are examples of application of the Particle filter in economics.

We want to evaluate the likelihood of the country spread deviations $\varepsilon_{tb,t}$ and of the international risk free real interest rate $\varepsilon_{r,t}$. Since the explanation of the filter for one likelihood or the other is equivalent, we just take the first case.

The likelihood is costly to evaluate because of the nonlinear interaction of volatility and levels. Let us start by stacking all the observations of $\varepsilon_{tb,t}$ in ε_{tb}^T and the parameters of the process in Ψ . Given the Markov structure of our state space representation, we can factorize the likelihood function as:

$$p(\varepsilon_{tb}^T; \Psi) = \prod_{t=1}^T p(\varepsilon_{tb,t} | \varepsilon_{tb}^{t-1}; \Psi)$$

Now, we can derive the factorization:

$$p(\varepsilon_{tb}^T; \Psi) = \int p(\varepsilon_{tb,1} | \varepsilon_{tb,0}, \sigma_{tb,0}; \Psi) d\sigma_{tb,0} \prod_{t=2}^T \int p(\varepsilon_{tb,t} | \varepsilon_{tb,t-1}, \sigma_{tb,t}; \Psi) p(\sigma_{tb,t} | \varepsilon_{tb}^{t-1}; \Psi) d\sigma_{tb,t}$$

and using equation (3):

$$p(\varepsilon_{tb}^T; \Psi) = \int \frac{1}{(2\pi)^{0.5}} \exp \left[-\frac{1}{2} \left(\frac{\varepsilon_{tb,1} - \rho_{tb} \varepsilon_{tb,0}}{e^{\sigma_{tb,0}}} \right)^2 \right] d\sigma_{tb,0} * \\ * \prod_{t=2}^T \int \frac{1}{(2\pi)^{0.5}} \exp \left[-\frac{1}{2} \left(\frac{\varepsilon_{tb,t} - \rho_{tb} \varepsilon_{tb,t-1}}{e^{\sigma_{tb,t}}} \right)^2 \right] p(\sigma_{tb,t} | \varepsilon_{tb}^{t-1}; \Psi) d\sigma_{tb,t} \quad (12)$$

Consequently, if we had access to $\{p(\sigma_{tb,t} | \varepsilon_{tb}^{t-1}; \Psi)\}_{t=1}^T$, we could compute (12). Unfortunately, in general, this sequence of conditional densities cannot be characterized analytically.

The Particle filter is a sequential Monte carlo procedure that substitutes the density

$p(\sigma_{tb,t}|\varepsilon_{tb}^{t-1}; \Psi)$ by an empirical draw from it. In other words, the filter relies on the observation that if we have available a draw of N simulations $\{\sigma_{tb,t|t-1}^i\}_{i=1}^N$ from $p(\sigma_{tb,t}|\varepsilon_{tb}^{t-1}; \Psi)$, then a Law of Large numbers ensures that:

$$\int p(\varepsilon_{tb,t}|\varepsilon_{tb,t-1}, \sigma_{tb,t}; \Psi) p(\sigma_{tb,t}|\varepsilon_{tb}^{t-1}; \Psi) d\sigma_{tb,t} \simeq \frac{1}{N} \sum_{i=1}^N p(\varepsilon_{tb,t}|\varepsilon_{tb,t-1}, \sigma_{tb,t|t-1}^i; \Psi)$$

where our notation for each draw i indicates in the subindex the conditioning set (i.e., $t|t-1$ is a draw at moment t conditional on information until $t-1$) and where

To draw from $p(\sigma_{tb,t}|\varepsilon_{tb}^{t-1}; \Psi)$, the Particle filter uses the idea of sequential important sampling by Rubin (1988):

Proposition 1. Let $\{\sigma_{tb,t|t-1}^i\}_{i=1}^N$ be a draw from $p(\sigma_{tb,t}|\varepsilon_{tb}^{t-1}; \Psi)$. Let the sequence $\{\tilde{\sigma}_{tb,t}^i\}_{i=1}^N$ be a draw with replacement from $\{\sigma_{tb,t|t-1}^i\}_{i=1}^N$ where the resampling probability is given by

$$\omega_t^i = \frac{p(\varepsilon_{tb,t}|\varepsilon_{tb,t-1}, \sigma_{tb,t|t-1}^i; \Psi)}{\sum_{i=1}^N p(\varepsilon_{tb,t}|\varepsilon_{tb,t-1}, \sigma_{tb,t|t-1}^i; \Psi)},$$

Then $\{\sigma_{tb,t|t}^i\}_{i=1}^N = \{\tilde{\sigma}_{tb,t}^i\}_{i=1}^N$ is a draw from $p(\sigma_{tb,t}|\varepsilon_{tb}^t; \Psi)$.

The proposition 1, which is just a simple application of Bayes' theorem, builds the draws $\{\sigma_{tb,t|t}^i\}_{i=1}^N$ recursively from $\{\sigma_{tb,t|t-1}^i\}_{i=1}^N$ by incorporating the information on $\varepsilon_{tb,t}$. The resampling step is crucial. If we just draw a whole sequence of $\left\{ \left\{ \sigma_{tb,t|t-1}^i \right\}_{i=1}^N \right\}_{t=1}^T$ without resampling period by period, all the sequences would become arbitrarily far away from the true sequence of volatilities, since it is a zero measure set. Then, the sequence that happened to be closer to the true states would dominate all the remaining ones in weight and the evaluation of the likelihood would be most inaccurate. Evidence from simulation shows that this degeneracy problem already appears after a very small number of observations.

Now that we have $\{\sigma_{tb,t|t}^i\}_{i=1}^N$, we can draw N exogenous shocks $u_{\sigma_{tb,t+1}}^i$ from a standard normal distribution and find:

$$\sigma_{tb,t+1|t}^i = (1 - \rho_{\sigma_{tb}}) \sigma_{tb} + \rho_{\sigma_{tb}} \sigma_{tb,t|t}^i + \eta_{tb} u_{\sigma_{tb,t+1}}^i \quad (13)$$

to generate $\{\sigma_{tb,t+1|t}^i\}_{i=1}^N$. This forecast step places us back at the beginning of proposition 1, but with one period ahead in our conditioning.

The following pseudocode summarizes the description of the algorithm:

-
- Step 0, Initialization:** Set $t \rightsquigarrow 1$. Sample N values $\left\{ \sigma_{tb,0|0}^i \right\}_{i=1}^N$ from $p(\sigma_{tb,0}; \Psi)$.
- Step 1, Prediction:** Sample N values $\left\{ \sigma_{tb,t|t-1}^i \right\}_{i=1}^N$ using $\left\{ \sigma_{tb,t-1|t-1}^i \right\}_{i=1}^N$, the law of motion for states and the distribution of shocks $u_{\sigma_{tb,t}}$.
- Step 2, Filtering:** Assign to each draw $\left(\sigma_{tb,t|t-1}^i \right)$ the weight ω_t^i in proposition 1.
- Step 3, Sampling:** Sample N times with replacement from $\left\{ \sigma_{tb,t|t-1}^i \right\}_{i=1}^N$ using the probabilities $\left\{ \omega_t^i \right\}_{i=1}^N$. Call each draw $\left(\sigma_{tb,t|t}^i \right)$. If $t < T$ set $t \rightsquigarrow t + 1$ and go to step 1. Otherwise stop.
-

With the output of the algorithm, we just substitute into our formula

$$\begin{aligned}
 p(\varepsilon_{tb}^T; \Psi) &\simeq \frac{1}{N} \sum_{i=1}^N \frac{1}{(2\pi)^{0.5}} \exp \left[-\frac{1}{2} \left(\frac{\varepsilon_{tb,1} - \rho_{tb} \varepsilon_{tb,0}}{e^{\sigma_{tb,0|0}^i}} \right)^2 \right] * \\
 &* \prod_{t=2}^T \frac{1}{N} \sum_{i=1}^N \frac{1}{(2\pi)^{0.5}} \exp \left[-\frac{1}{2} \left(\frac{\varepsilon_{tb,t} - \rho_{tb} \varepsilon_{tb,t-1}}{e^{\sigma_{tb,t|t-1}^i}} \right)^2 \right]
 \end{aligned} \tag{14}$$

and we obtain the estimate of the likelihood. Del Moral and Jacod (2002) and Künsch (2005) provide weak conditions under which the right-hand side of the previous equation is a consistent estimator of $p(\varepsilon_{tb}^T; \Psi)$ and a central limit theorem applies.

7.2. Computation

In the main part of the paper, we argued that a third order approximation was important if we wanted to evaluate the effects of volatility shocks independently of real interest rate shocks. In this appendix, we provide some evidence that the effects on allocations of the higher order terms are non-trivial.

We simulate the Argentinian economy for 500 periods (after a period of burn-in to eliminate the effect of initial conditions) at the benchmark calibration parameter values and we followed the results for the deviations of consumption, investment, output, labor, and debt with respect to the steady state when we have a first order approximation, a second order approximation, and a third order approximation. The interest rate evolution was kept the same in all three simulations. We plot the results in figure A1. We see how, even if the general pattern of behavior is similar, there are non-trivial differences, in particular in investment, debt, an output. The differences are particularly salient between, on one hand, the

first approximation, and the other hand, the second and third approximation. The presence of constants in the higher order approximation that reflect precautionary behavior are largely responsible for the permanent differences in level that we see, for example, in output.

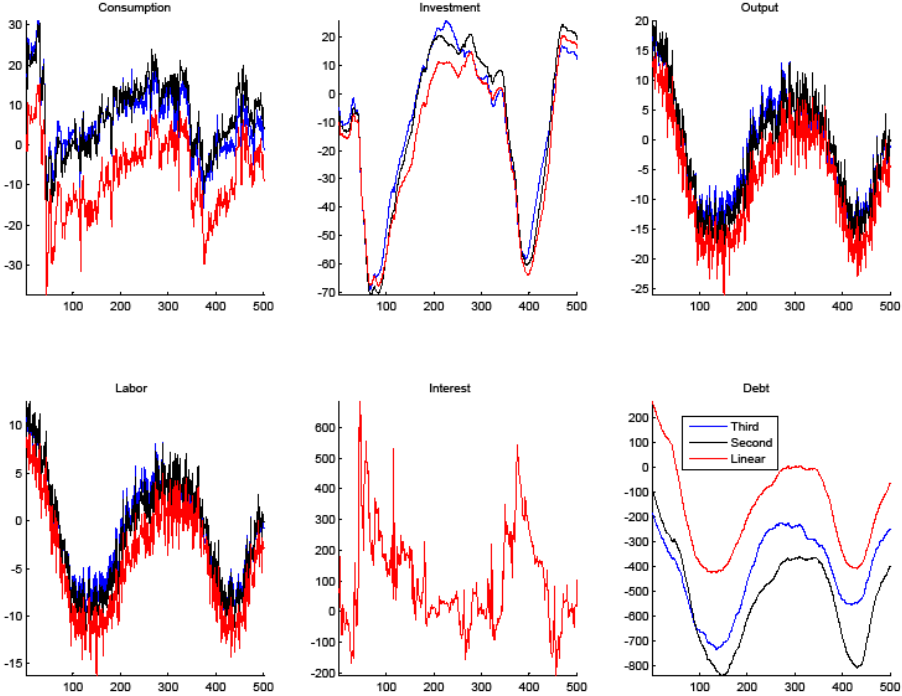


Figure A1: Simulation, different Approximations

Because the scale of figure A1 may make difficult to appreciate our point, we zoom in figure A2 a section of the simulation for investment in the center of the sample. We can see how around periods 30 to 40, in the first order approximation, investment is stable around 10 percent above the steady state, in the second order approximation, it is falling from around 20 percent above steady state to around 15 percent, and in the third order approximation, investment is raising up to 25 percent. We could hardly have a clearer picture: as a response to the same real interest rate shocks, each level of approximation tells us a different history of the evolution of investment.

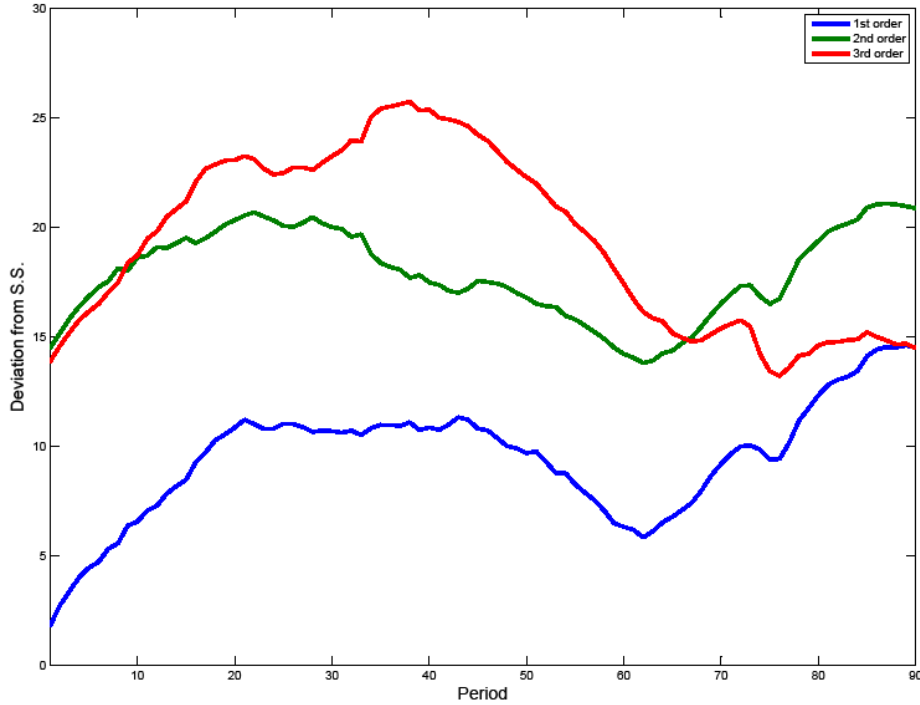


Figure A2: Evolution of Investment

7.3. Computing Impulse Responses

As argued in the main section and in Fernandez-Villaverde and Rubio-Ramirez (2008), our higher order approximation makes the simulated paths of states and controls in the model move away from their steady state values. Consequently, computing impulse responses as percentage deviations of the model's steady state is not a terribly interesting exercise. To compute the impulse responses reported in the paper, we proceed as follows:

1. We simulate the model, starting from its steady state, for 2096 periods. We disregard the first 2000 periods as a burn-in.
2. Based on the last 96 periods, we compute the mean of the ergodic distribution for each variable in our model. Adding additional periods have essentially no impact on the mean.
3. Starting from the ergodic mean and in the absence of shocks, we hit the model with a one standard deviation shock to the volatility process $u_{\sigma,t}$.
4. We report the resulting impulse responses as percentage deviations from the variables' ergodic means.

In the context of a threshold model, Koop *et al.* (1996) have argued that the use of the standard impulse response definition may be misleading. These authors urge the use of the so-called generalized impulse response to overcome the drawbacks reported in their manuscript. We computed the generalized impulse response but we essentially found no differences between the two procedures to compute the impulse responses. We choose to report the traditional impulse responses as their computation and interpretation are neater than theirs.

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