Human Capital Formation with Endogenous Credit Constraints: Government Student Loan Programs and Private Lenders

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Abstract

This paper studies the nature and impact of different forms of credit constraints in the market for human capital. We compare a standard model, in which credit constraints are exogenous and fixed, with models in which credit limits endogenously respond to the investment and ability of borrowers. We derive endogenous constraints from the design of existing government student loan programs and from the repayment incentives under limited enforcement in private markets.

We show that the rising empirical importance of familial wealth and income in determining college attendance is consistent with increasingly binding credit constraints in the face of rising tuition costs and returns to schooling. For empirical estimates of the intertemporal substitution elasticity for consumption, the standard model with exogenous constraints generates a counterfactual negative ability - investment relationship for constrained youth, while endogenous constraint models generate a positive ability - investment relationship. Endogenous constraint models also explain the dramatic rise in private lending for college in recent years.

1 Introduction

The presence of borrowing constraints is a common explanation for the positive relationship observed between family income and schooling.[1] As pointed out by Becker (1975), individuals with low financial resources will underinvest in human capital if they are unable to obtain outside funding. Limits on the credit available for investing in human capital may naturally arise for two main reasons. First, human capital is poor collateral, because it cannot be repossessed in response to default. Second, investing in human capital is more efficient when individuals are young and typically have not established a reputation in credit markets or accumulated other forms of collateral.

A number of recent empirical studies (e.g. Cameron and Heckman 1998, 2001, Keane and Wolpin 2001, Carneiro and Heckman 2002, Cameron and Taber 2004) have argued that borrowing constraints explain little of the college attendance gap by family income for American youth making their attendance decisions in the early 1980s (based on the 1979 Cohort of the National Longitudinal Survey of Youth, NLSY79). However, Belley and Lochner (2007) find a much larger impact of family income on college attendance rates for youth making their attendance decisions in recent years (based on the 1997 Cohort of the NLSY, NLSY97). For this recent cohort, Belley and Lochner (2007) estimate differences in college attendance rates by family income (highest vs. lowest quartile) of roughly sixteen percentage points after controlling for ability and family background.

In this paper, we argue that the rising importance of family income as a determinant of college attendance (from the early 1980s to the early 2000s) can be explained by increasingly binding credit constraints in response to three broad economic and policy trends: (i) rising returns to schooling (Katz and Autor 1999, Heckman, Lochner, and Todd 2007); (ii) rising costs of tuition, fees, room, and board at U.S. colleges and universities (College Board 2006); and (iii) stable or declining (in real dollars) borrowing limits associated with government student loan programs (Kane 2007). As we discuss below, all of these trends are likely to increase the population of youth that that are constrained by government student loan (GSL) limits. In fact, recent U.S. Department of Education studies (Berkner 2000 and Titus 2002) report that the fraction of undergraduate student borrowers who borrowed the maximum allowable amount nearly tripled from 18% in 1989-90 to 52% in 1999-2000. Among dependent undergraduates, nearly 70% of all borrowers were borrowing the maximum amount in 1999-2000.

Despite the vigorous debate about the empirical importance of borrowing constraints, the literature has paid little attention to the nature of these constraints. The standard approach either assumes a fixed and exogenously set limit on the maximum amount that can be borrowed or that

[1] Another common explanation posits that schooling provides some direct utility value and is a normal good. Thus, higher income parents choose to purchase more schooling for their children. Belley and Lochner (2007) show that this force cannot explain the observed increase in the effect of family income on college attendance from the early 1980s to the early 2000s.

interest rates increase with the amount borrowed.[2] Both approaches typically assume that credit limits or interest rate schedules are independent of individual characteristics and decisions, and both lead to similar conclusions about human capital investment behavior. In this paper, we argue that it is necessary to incorporate endogenous credit limits that respond to human capital investments in order to explain empirical relationships between investment, ability, and family income.[3]

While the standard model of exogenous borrowing constraints can explain the observed empirical patterns for college attendance rates by family income, it does a poor job of replicating a second key empirical regularity: that cognitive ability and educational attainment are strongly positively correlated (e.g. Carneiro and Heckman 2002 and Belley and Lochner 2007). As we show in Section 2, a positive cognitive ability - college attendance relationship exists for all family income (or wealth) levels in both the NLSY79 and NLSY97, even after conditioning on family background. Perhaps surprisingly, the exogenous constraint model cannot deliver this prediction for standard parameterizations of preferences. In particular, with a consumption intertemporal elasticity of substitution (CIES) below one, the model predicts a *negative* relationship between ability and human capital investment among constrained borrowers. This poses a serious challenge, since most empirical estimates of the CIES are less than one (Browning, Hansen, and Heckman 1999).

Exogenous constraint models neglect any potential responsiveness of available credit to investment in human capital. However, a key feature of GSL programs is that credit is directly tied to the level of investment – students can borrow to help finance college-related expenses only if they are enrolled in school. We show that private lenders will also tend to link credit limits to the level of investment, as well as observable individual characteristics like cognitive ability. These features of endogenously determined (or variable) borrowing limits help to generate a positive relationship between ability and investment while still predicting a positive relationship between family resources and investment.[4] Thus, endogenous constraint models are consistent with both empirical regularities.

GSL programs have two distinct forms of credit limits: (i) a pre-specified maximum loan limit (denote this fixed limit by d_0), and (ii) an endogenous limit that restricts students from borrowing more than they spend on their education.[5] The second constraint is typically neglected, but it

[2]Studies assuming a fixed limit on borrowing include Aiyagari, Greenwood, and Seshadri (2002), Belley and Lochner (2007), Caucutt and Kumar (2003), Hanushek, Leung, and Yilmaz (2003), and Keane and Wolpin (2001). Studies assuming variable interest rates (or heterogeneous interest rates) include Becker (1975), Cameron and Taber (2004), Card (1995).

[3]It is worth noting that Keane and Wolpin (2001) allow borrowing limits to depend on the human capital level and age of youth in their estimated model. Consistent with our argument for incorporating endogenous constraints, they estimate that borrowing limits are increasing in human capital levels.

[4]We refer to these borrowing limits as 'endogenous', because they are a function of the borrower's investment behavior. We do not model the determination of these limits in the GSL system; however, borrowing limits set by private lenders are optimally determined from the incentives of borrowers to default.

[5]Under the Stafford Loan Program, students face a cumulative loan limit as well as annual borrowing limits which increase somewhat with year of post-secondary school.

alters investment behavior in important ways. As we show, youth that would like to borrow more than they spend on their schooling (i.e. those constrained by the second limit) invest the same amount in their human capital as if they were completely unconstrained. As such, their investment is increasing in ability in the same way it is for those who are unconstrained. When credit is tied directly to investment, there is no tradeoff between additional investment and consumption while in school, since every additional dollar of investment can be borrowed (provided investment remains below d_0). This implies that consumption decisions may be severely distorted even when schooling and investment decisions are not.[6] Even among more able youth from low income families who would like to invest more than the maximum loan limit, d_0, investment is likely to be non-decreasing in ability due to the second constraint that borrowing cannot exceed investment. Among these youth, however, family income (or initial wealth) should be positively related to investment, as has been observed for recent student cohorts. Thus, the direct link between borrowing and investment embodied in GSL programs can deliver the observed empirical patterns regarding college attendance, ability, and family income.

Beginning in the mid-1990s, student borrowing from private lending institutions (outside the GSL system) skyrocketed from negligible amounts to almost $14 billion (nearly 20% of all student loans distributed) in the 2004-05 academic year (College Board 2005). It is more important than ever to understand how private lenders determine the credit of student borrowers. We, therefore, analyze the incentives faced by private lenders and the resulting loan contracts they offer. Even if human capital cannot be directly repossessed by lenders, creditors can punish defaulting borrowers in a number of ways (e.g. lowering credit scores, seizing assets, garnishing a fraction of labor earnings), which tend to have a greater impact on debtors with high post-school earnings. In a life-cycle setting, we show how these mechanisms effectively link the borrowing limits of students to both their abilities and human capital investment levels. Smarter students who spend more on their education will be offered more credit by private lenders, since they can credibly commit to re-pay more given the punishments they will face if they default. As with the GSL system, the fact that private lenders link credit limits to investment decisions can generate a positive ability - investment relationship for constrained borrowers with standard parameterizations of preferences. Yet, family income is negatively related to investment among constrained borrowers. Thus, this model of credit constraints is also consistent with the two key empirical patterns for educational attainment described above (i.e. positive income - attendance and ability - attendance relationships).

Our preferred framework for analyzing current human capital investment behavior incorporates the lending opportunities provided by both GSL programs and private lenders. Given stable or declining borrowing limits attainable within the GSL system and a rise in student demand for

[6] Thus, evidence that family resources do not affect educational attainment or financial returns does not necessarily imply that credit constraints are non-binding.

credit, the recent emergence and expansion of private student lending is not surprising. This would be expected if (i) the GSL maximum borrowing limits were high enough to finance unrestricted levels of investment in the early 1980s, (ii) these maximum loan limits are too low to cover the higher levels of investment desired today, and (iii) current students can credibly commit to re-pay more than current GSL limits allow them to borrow. The evidence on family income - college attendance patterns in the NLSY79 and NLSY97 is consistent with the first two conditions. The higher earnings potential of recent graduates, coupled with higher costs of schooling, can explain why more college students are bunching up against GSL maximum borrowing limits (Berkner 2000 and Titus 2002). This creates new demand for private lenders to step in, offering more credit to those who can credibly commit to repay. With rising returns to schooling, commitments to repay become credible for more and more college students, suggesting that condition (iii) is also likely to be met.

Our model of private lending is related to the existing literature on endogenous credit constraints. Much of this literature has focused on implications for risk sharing and asset prices in endowment economies (e.g. Alvarez and Jermann 2000, Fernandez-Villaverde and Krueger 2005, Krueger and Perri 2002, Kehoe and Levine 1993 and Kocherlakota 1996), or production economies but applied to firm financing and investment (e.g. Albuquerque and Hopenhayn 2004, Monge-Naranjo 2007a, 2007b). Our model differs by considering a life-cycle economy in which earnings are endogenously determined through human capital investment and the penalties of default have a finite horizon. Moreover, we include the amounts available from government-backed programs in the feasible set of credit. Andolfatto and Gervais (2006) also study human capital accumulation with limited commitment but their focus is on the optimal set of intergenerational transfers and not on the implications of credit constraints across individuals of different abilities and familial wealth.

The rest of this paper proceeds as follows. In the next section, we briefly review the empirical literature on credit constraints and provide new evidence on educational attainment by cognitive ability and family income from the NLSY79 and NLSY97. In Section 3, we describe the borrowing opportunities embodied in U.S. GSL programs, as well as recent trends in private lending. In Section 4, we compare the qualitative predictions of alternative models of credit constraints using a simple two period model, focusing on the models' implications for ability - investment and initial wealth - investment (empirically, family income/wealth - investment) patterns. We construct and calibrate a continuous time life-cycle model in Section 5 and show that its predictions are consistent with observed cross-section patterns for human capital investments. We also show that the model is useful for understanding the impact of changes in tuition costs and returns to schooling on the observed patterns of human capital investment, as well as the increasing role of private lending. Section 6 concludes with a summary and discussion of avenues for future research.

2 Evidence on the Role of Ability and Family Income

This section begins with a brief review of the empirical literature on credit constraints and human capital. This literature suggests that borrowing constraints and family income had little effect on college attendance decisions in the early 1980s, but that this is unlikely to be true today. The family income - attendance relationship has become substantially stronger for recent college cohorts. We next provide new evidence from the NLSY79 and NLSY97 on the strong positive correlation between cognitive ability/achievement and college attendance for youth from different family income backgrounds.

The increased importance of family income as a determinant of college attendance and the strong positive correlation between ability and schooling (even among youth from low income families) will serve as two 'stylized facts' with which we evaluate different models of credit constraints in Sections 4 and 5.

The Empirical Debate on Credit Constraints and Schooling

The empirical literature aimed at measuring the impact of credit constraints on human capital investment has focused on two basic implications of constraints, both originating from the seminal work of Becker (1975). One strand of the literature tests whether, conditional on ability, proximity to college, local tuition rates, and family background, individuals from different family income levels have differential college enrollment and completion rates (e.g. Cameron and Heckman (1998, 1999), Ellwood and Kane (2000), Carneiro and Heckman (2002), and Belley and Lochner (2007)). The second strand compares the *returns* to schooling for individuals who are expected to face different interest rates or constraints on their borrowing (e.g. Lang 1994, Card 1995, and Cameron and Taber 2004)).

Disagreement about the importance of credit constraints in determining college-going abounds. On one hand, Ellwood and Kane (2000) argue that differences in family income are responsible for important differences in college enrollment rates, suggesting that borrowing constraints inhibit college-going for youth from low income families. Card (1995) argues that individuals most likely to face constraints receive higher returns to schooling, which suggests that constraints prevent poor youth from pursuing highly productive investments. On the other hand, Cameron and Heckman (1998, 1999) find that after controlling for scores on the Armed Forces Qualifying Test (AFQT) and unobserved heterogeneity, family income has little effect on college enrollment rates.[7] Carneiro and Heckman (2002) also estimate differences in college enrollment rates and other college-going outcomes by family income after accounting for differences in family background and AFQT in

[7] The AFQT is a composite score from four different tests used by the U.S. military: arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. These tests are taken by nearly all individuals in both the NLSY79 and NLSY97 during their teenage years as part of the survey process. See the NLSY79 or NLSY97 User's Guides for details.

the NLSY79. Cameron and Taber (2004) find little evidence of differential returns to school that would be consistent with borrowing constraints. These authors argue that short-term borrowing constraints have negligible impacts on human capital formation, and that differences in college enrollment rates are mostly driven by familial environment or the constraints faced early in a child's youth.

Keane and Wolpin (2001) estimate a structural model of schooling and work that incorporates constraints on borrowing and parental transfers that may depend on child schooling decisions. While they estimate very tight borrowing limits (much more stringent than federal student loan limits), they find little effect of borrowing constraints on educational attainment. Instead, they argue that enrollment-contingent parental transfers and work while in school help offset the costs of college attendance for most youth. Poor youth also forego consumption while in school – a margin of adjustment we argue is important when analyzing GSL programs. Keane and Wolpin's (2001) findings, along with our model of GSL programs, suggest that constraints may be binding even when schooling decisions are largely unaffected. Thus, the empirical 'tests' described above are not necessarily tests of binding borrowing constraints; instead, they measure (under ideal conditions) the extent to which borrowing constraints impact educational outcomes.

All of the aforementioned studies that find little effect of borrowing constraints on educational attainment use the NLSY79 data (e.g. Cameron and Heckman 1998, 1999, Keane and Wolpin 2001, Carneiro and Heckman 2002, and Cameron and Taber, 2004), thereby analyzing the outcomes of youth who made their initial college-going decisions in the early 1980s (or earlier).[8] Yet, much has changed since then. Financial returns to schooling have risen dramatically (Katz and Autor 1999, Heckman, Lochner, and Todd 2007) as have costs of tuition, fees, room, and board at U.S. colleges and universities (College Board 2005). At the same time, real borrowing limits associated with government student loan programs have remained stable or declined (Kane 2007). As we discuss further below, all of these trends are likely to increase the role of borrowing constraints in determining educational attainment.

Consistent with this possibility, Belley and Lochner (2007) show that family income has become a much more important determinant of college attendance for the recent NLSY97 cohort, which made its college-going decisions in the early 2000s. They find that the estimated effects of income on attendance – conditional on the same AFQT and family background measures used by Carneiro

[8]Stinebrickner and Stinebrickner (2007) find little effect of borrowing constraints on overall college dropout rates for a recent cohort of students at Berea College. Despite low family income levels among most Berea students, only 20% are determined to be borrowing constrained in the sense that they report a desire to borrow additional money (at reasonable interest rates). However, among these 'constrained' students, subsequent drop out rates are roughly twice as high as among similar students who report that they would not like to borrow. Because Berea charges no tuition and provides all of its students with sizeable room and board subsidies, we might expect more than 20% of students enrolled in other institutions from low-income families (who must pay tuition, room, and board) to be constrained. Stinebrickner and Stinebrickner (2007) are unable to explore the effects of borrowing constraints on attendance, since their sample only includes youth already enrolled at Berea.

and Heckman (2002) – nearly doubled from the NLSY79 to the NLSY97. Youth from high income families in the NLSY97 are sixteen percentage points more likely to attend college than are youth from low income families, conditional on AFQT scores, family composition, parental age and education, race/ethnicity, and urban/rural residence.[9] The combined effects of family income and wealth are even more dramatic. Comparing youth from the highest family income and wealth quartiles to those from the lowest quartiles yields an estimated difference in college attendance rates of nearly 30 percentage points after controlling for ability and family background.

New Evidence on the Ability - College Attendance Relationship

We use the NLSY79 and NLSY97 data to examine the effects of ability (as measured by AFQT scores) on college attendance for youth from different family income (or wealth) backgrounds. The NLSY79 reflects a random survey of American youth ages 14-21 at the beginning of 1979, while the NLSY97 samples youth ages 12-16 at the beginning of 1997.[10] Since the oldest respondents in the NLSY97 recently turned 24 in the 2004 wave of data, we analyze college attendance as of age 21 (limiting our sample to the older age cohorts of this data).

Individuals are considered to have attended college if they *attended* at least 13 years of school by the age of 21.[11] For the 1979 cohort, we use average family income when youth are ages 16-17, excluding those not living with their parents at these ages. In the NLSY97 data, we use household income and net wealth reported in 1997 (corresponding to ages 13-17), dropping individuals not living with their parents that year.[12] We categorize individuals according to their family income, family net wealth (in NLSY97), and AFQT score quartiles.[13]

Figure 1 shows college attendance rates by AFQT quartiles and either family income or family wealth quartiles in the NLSY79 and NLSY97. For all family income or wealth categories in both NLSY samples, we observe dramatic increases in college attendance with AFQT. The difference in attendance rates between the highest and lowest ability quartiles range from .47 to .68 depending on the family income or wealth quartile. Most importantly for our theoretical analysis below, there is no indication that the effects of ability are systematically smaller (or negative) for lower income youth who are most likely to be constrained, especially in the NLSY97.

[9]Differences in attendance rates by family wealth quartiles, which are only available for the NLSY97, tend to be even greater than differences by family income quartiles (see Belley and Lochner 2007).

[10]See Belley and Lochner (2007) for additional details on the sample and variables used in this analysis.

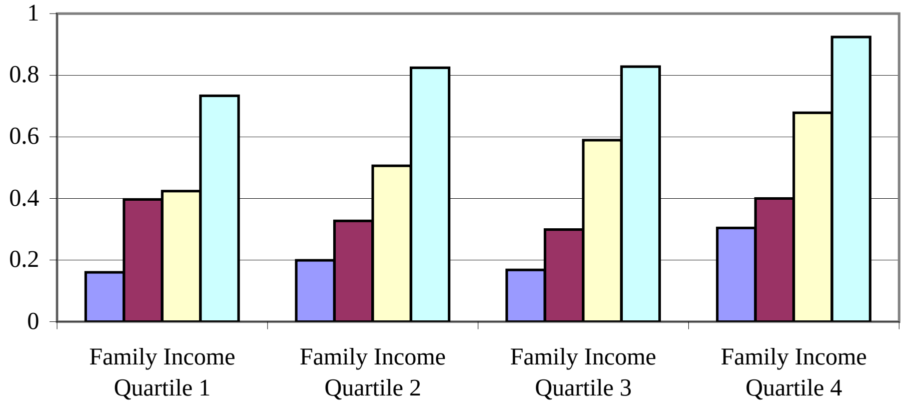
[11]Schooling attainment by age 22 is used if it is missing or unavailable at age 21 (fewer than 10% of all respondents in both surveys).

[12]Net wealth measures the net value of owned home, real estate, business and vehicles. Added to that is money kept in checking and savings bank accounts as well as Educational IRA accounts or other prepaid tuition savings accounts. Assets like bills, bonds, life insurance policies, pension savings, shares in publicly held corporations and mutual funds are included. Loans and credit card debt are subtracted.

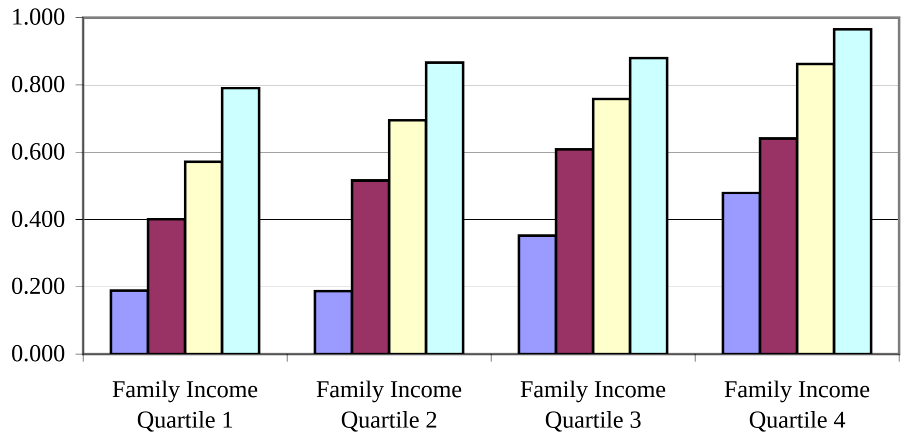
[13]Since AFQT percentile scores increase with age in the NLSY79, we determine an individual's quartile based on year of birth. (All NLSY79 respondents took the ASVAB tests in the summer and fall of 1980. See the *NLSY79 User's Guide* for details.) AFQT percentile scores in the NLSY97 have already been adjusted to account for age differences.

Figure 1: College Attendance by AFQT and Family Income or Wealth (NLSY79 and NLSY97)

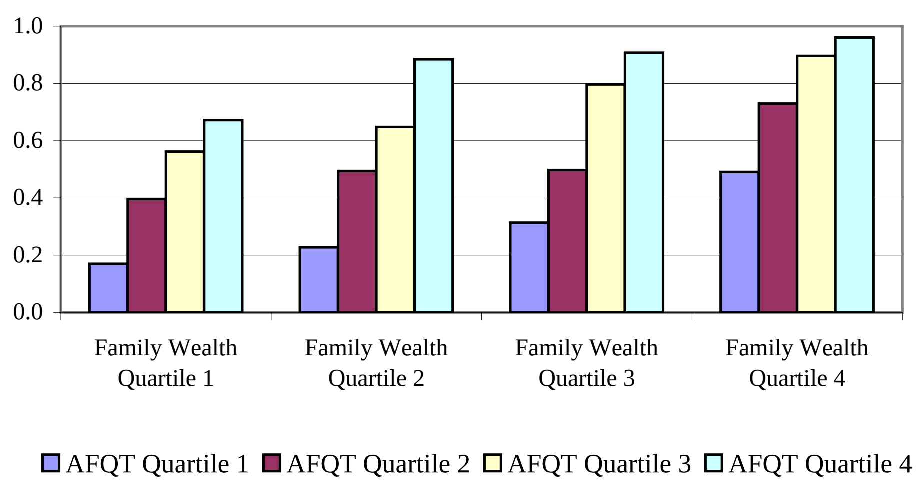
(a) Attendance by AFQT and Family Income (NLSY79)



(b) Attendance by AFQT and Family Income (NLSY97)



(c) Attendance by AFQT and Family Wealth (NLSY97)



Of course, AFQT scores may be correlated with other family background variables that influence college attendance decisions conditional on family resources. We, therefore, control for a host of other family background measures in addition to AFQT quartiles using ordinary least squares. Table 1 reports the estimated effects of AFQT (all estimates reflect the difference in attendance rates between the reported AFQT quartile and AFQT quartile 1) on college attendance after controlling for family background characteristics.[14] Results are reported for separate regressions by family income or wealth quartile. The estimates confirm the general patterns observed in Figure 1: ability has strong positive effects on college attendance for all family income and wealth quartiles. The estimated effects of ability are quite similar across all income categories in the NLSY79. Estimates for the NLSY97 suggest that the effects of ability may be smallest (though they are still quite large) at the top end of the income and wealth distributions rather than the bottom.

3 Available Sources of Credit

This section briefly reviews the primary sources of borrowing used for human capital investment in the U.S. We first describe key institutional features of GSL programs, which we incorporate in our endogenous constraint models below. Then, we discuss the rise of private lending for post-secondary schooling.

3.1 Government Student Loan Programs

Federal student loans are an important source of finance for higher education in the U.S., accounting for 71% of the federal student aid disbursed in 2003-04.[15] Most of these government-backed loans are provided through the Stafford Loan program, which awarded nearly $50 billion to students in the 2003-04 academic year, compared to the disbursement of $1.6 billion through the Perkins Loan program. Slightly more than $7 billion was awarded to parents of undergraduate students in the form of Parent Loans for Undergraduate Students (PLUS).[16]

GSL programs generally have three important features. First, lending is directly tied to investment. Students (or parents) can only borrow up to the total cost of college (including tuition, room, board, books, supplies, transportation, computers, and other expenses directly related to

[14] For both cohorts, we control for maternal education by categorizing mothers as high school dropouts, those who completed high school or more, and those who completed at least one year of college. We account for family structure in the NLSY79 by controlling for the number of siblings the youth reported in 1979 and whether both parents were present in the home when the respondent was age 14. In the NLSY97, we control for the number of household members under the age of 18 in 1997 (respondents are ages 13-17) and whether both parents are present in the home in 1997. Family residence in an urban (metropolitan) area at age 14 (age 12) is accounted for with the 1979 (1997) cohort. We control for the mother's age at birth as well as the respondent's gender and race (blacks, hispanics, and whites for the NLSY79; blacks, hispanics, other non-whites, and whites for the NLSY97 data). Finally, we allow for differences by year of birth.

[15] Many other countries have similar types of government student loan programs.

[16] See The College Board (2006) for further details about financial aid disbursements and their trends over time.

Table 1: Estimated Effects of AFQT on College Attendance at Age 21 by Family Income and Wealth (NLSY79 and NLSY97)

	Effects of AFQT by Family Income:				Effects of AFQT by Family Wealth:			
	Family Income Quartile 1	Family Income Quartile 2	Family Income Quartile 3	Family Income Quartile 4	Family Wealth Quartile 1	Family Wealth Quartile 2	Family Wealth Quartile 3	Family Wealth Quartile 4
a. NLSY79								
AFQT Quartile 2	0.2114	0.1098	0.1201	0.0629				
	(0.0441)	(0.0505)	(0.0593)	(0.0650)				
AFQT Quartile 3	0.2598	0.2762	0.3689	0.3373				
	(0.0576)	(0.0531)	(0.0600)	(0.0618)				
AFQT Quartile 4	0.5170	0.5151	0.5740	0.5374				
	(0.0677)	(0.0580)	(0.0613)	(0.0624)				
Sample Size	545	556	596	591				
b. NLSY97								
AFQT Quartile 2	0.1880	0.3477	0.2252	0.1299	0.2602	0.2480	0.1934	0.1777
	(0.0469)	(0.0499)	(0.0517)	(0.0514)	(0.0496)	(0.0486)	(0.0498)	(0.0524)
AFQT Quartile 3	0.3960	0.4745	0.3521	0.3353	0.3747	0.3761	0.4426	0.3412
	(0.0529)	(0.0510)	(0.0513)	(0.0496)	(0.0548)	(0.0519)	(0.0478)	(0.0529)
AFQT Quartile 4	0.5754	0.6624	0.4721	0.4029	0.4949	0.6428	0.5274	0.3703
	(0.0615)	(0.0546)	(0.0515)	(0.0492)	(0.0703)	(0.0556)	(0.0503)	(0.0513)
Sample Size	553	597	677	702	541	573	716	666

Notes: All regressions control for gender, race/ethnicity, mother's education (HS graduate, college attendance), intact family during adolescence, number of siblings/children under 18, mother's age at child's birth, urban/metropolitan area during adolescence, and year of birth. Education measured as of age 21 (age 22 if missing at age 21). Standard errors are in parentheses schooling) less any other financial aid they receive in the forms of grants or scholarships. Thus, students cannot borrow from GSL programs to finance non-schooling related consumption goods or activities. Second, student loan programs set fixed upper limits on the total amount of credit available for each student. Students face both cumulative and annual loan limits for U.S. federal loan programs.[17] Third, loans covered by GSL programs typically have extended enforcement rules compared to standard private loans.

Historically, private lenders have provided the capital to student borrowers (and their parents) under the Stafford and PLUS programs, the government guaranteeing those loans with a promise to cover any unpaid amounts. Since the 1994-95 academic year, the federal government has begun to directly provide these loans to some students under the same rules and terms.[18] While Stafford loans are disbursed to students, PLUS loans can be taken out by parents to help cover the costs of their children's schooling. The Perkins Loan Program provides an additional source of government funds to students most in need; however, its loan offerings depend the level of program funding at the post-secondary institution attended by a student.

Table 2 reports loan limits (based on on the dependency status and class level of each student) for Stafford and Perkins student loan programs for the period 1993-2007.[19] In recent years, dependent students could borrow up to $23,000 from the Stafford Loan Program over the course of their undergraduate careers. Independent students could borrow roughly twice that amount, although most traditional undergraduates would not fall into this category. Qualified undergraduates from low income families could receive as much as $20,000 in Perkins loans, depending on their need and post-secondary institution. It is important to note, however, that amounts offered through this program have typically been less than mandated limits.[20] Student borrowers can defer loan re-payments until six (Stafford) to nine (Perkins) months after leaving school.

Figure 2 shows how annual Stafford loan limits for dependent undergraduate students have evolved from 1980-81 to 2006-07 in year 2000 dollars.[21] In most years, the cumulative loan limit is equal to or slightly greater than the sum of all five annual loan limits. The jumps up reflect nominal adjustments to the limits in 1986-87 and 1993-94; otherwise, inflation has continuously eroded these

[17]Since 1993-94, the PLUS loan program no longer has a fixed maximum borrowing limit; however, parents still cannot borrow more than the total cost of college less other financial aid received by the student.

[18]The Stafford program offers both subsidized and unsubsidized loans. The government covers the interest on subsidized loans while students are enrolled. Unsubsidized loans accrue interest over this period; however, the student is not required to make any payments until after leaving school. To qualify for subsidized loans, students must demonstrate financial need on the basis of family income, dependency status, and the cost of the school attended. Most students under age 24 are considered dependent, and their parents' income is an important determinant of their financial need. Prior to the introduction of unsubsidized Stafford Loans in the early 1990s, Supplemental Loans to Students (SLS) were an alternative source of unsubsidized federal loans for independent students.

[19]Stafford loan limits for freshman, sophomores, and graduate students increased slightly in July, 2007.

[20]Parents that do not have an adverse credit rating can borrow up to the cost of schooling from the PLUS program, with repayment typically beginning within 60 days of loan disbursement. Dependent students whose parents do not qualify for PLUS loans (due to a bad credit rating) are able to borrow up to the independent student loan limits.

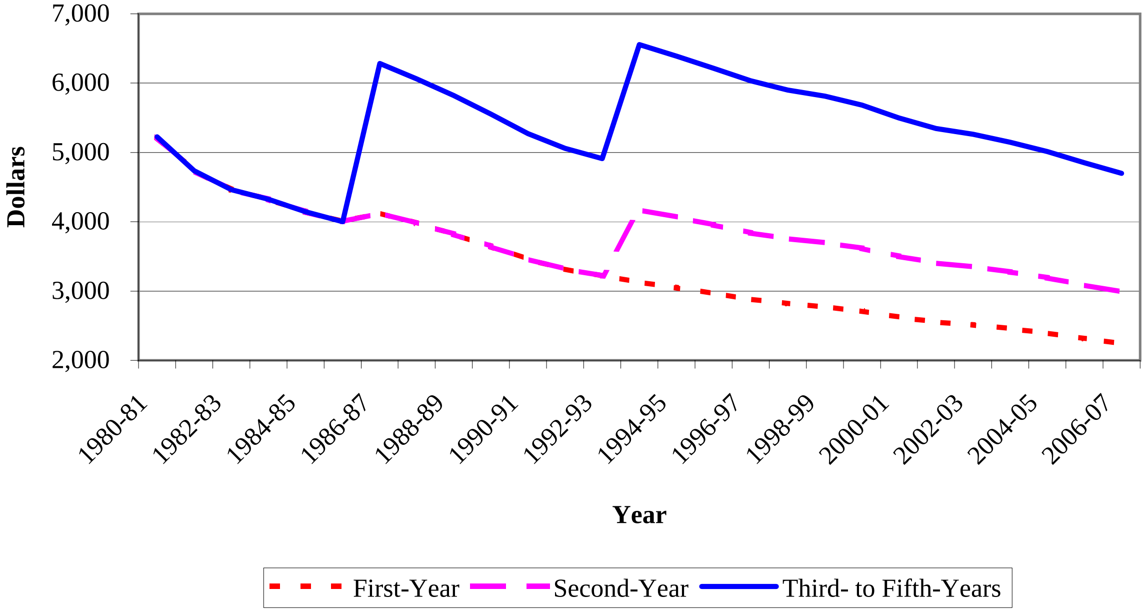
[21]The Consumer Price Index for All Urban Consumers (CPI-U) is used to adjust for inflation.

Table 2: Borrowing Limits for Stafford and Perkins Student Loan Programs (1993-2007)

	Stafford Loans		
	Dependent Students	Independent Students*	Perkins Loans
Eligibility Requirements	Subsidized: Financial Need** Unsubsidized: All Students		Financial Need
Undergraduate Limits:			
First Year	\$2,625	\$6,625	\$4,000
Second Year	\$3,500	\$7,500	\$4,000
Third-Fifth Years	\$4,000	\$8,000	\$4,000
Cum. Total	\$23,000	\$46,000	\$20,000
Graduate Limits:			
Annual	\$18,500		\$6,000
Cum. Total***	\$138,500		\$40,000

Notes:
* Students whose parents do not qualify for PLUS loans can borrow up to independent student limits from Stafford program.
** Subsidized Stafford loan amounts can be no greater than the borrowing limits for dependent students; independent students can also borrow unsubsidized Stafford loans provided that their total (subsidized and subsidized)loan amount is not greater than the independent student limits.
*** Cumulative graduate loan limits include loans from undergraduate loans.

Figure 2: Annual Stafford Student Loan Limits for Dependent Undergraduates from 1980-2006 (in Year 2000 Dollars)



limits. The nominal limit for first-year students rose $125 in 1987-88 and has remained at $2,625 through 2007. Thus, the entry year into college has seen the greatest erosion in real borrowing opportunities – a 44% decline from 1982-83 to 2002-2003.[22] Second-year undergraduates saw an additional nominal increase in the amount they can borrow in 1993-94 to $3,500, but this was more than offset by inflation. In real terms, the borrowing limit for second-year students declined by about 25% from 1982-83 to 2002-2003. By contrast, third- through fifth-year undergraduates were able to borrow nearly 20% more in 2002-03 than in 1982-83 due to more substantial nominal increases of $1,500 each in 1986-87 and 1993-94. Cumulative Stafford loan limits were almost identical in real terms in 1982-83 and 2002-03.[23]

Student loans covered by these federal programs have extended enforcement rules compared to typical private loans. Except in very special circumstances, these loans cannot generally be expunged through bankruptcy. If a suitable re-payment plan is not agreed upon with the lender once a borrower enters default, the default status will be reported to credit bureaus and collection costs (up to 25% of the balance due) may be added to the amount outstanding.[24] Up to 10% of the borrower's wages can also be garnished. This fraction increases to 15% if the Department of Education becomes involved in the collection process. Moreover, federal tax refunds can be seized and applied toward any outstanding balance. Other sanctions include a possible hold on college transcripts, ineligibility for further federal student loans, and ineligibility for future deferments or forbearances.[25]

3.2 The Emergence of Private Sources of Financing

Until the mid-1990s, few private lenders offered loans to students outside the GSL programs. In 1995-96, total non-federal student loans amounted to $1.3 billion. By 2005-06, that amount had risen to $17.3 billion.[26] Private student loans typically charge higher interest rates than Stafford or Perkins loans and are, therefore, taken after exhausting available credit from GSL programs. Thus, the rise in borrowing from private lenders outside the Stafford and Perkins Loan Programs suggests that the GSL limits are no longer enough to satisfy many students' demand for credit. Private loans are most prevalent among graduate students (especially in professional schools) and undergraduates at high-cost private universities.

[22]Our NLSY79 and NLSY97 respondents made their college attendance decisions around these two periods, respectively.

[23]Throughout most of this period, loan limits for independent undergraduates remained about twice the amounts available to dependent students. Stafford loan limits for graduate students declined by about 35% in real terms from 1986-87 to 2006-07, roughly the time our NLSY respondents would have began attending graduate school.

[24]Formally, a borrower is considered to be in default once a payment is 270 days late.

[25]Since the early 1990s, the government has also begun to punish educational institutions with high student default rates by making their students ineligible to borrow from federal lending programs.

[26]These figures do not include student borrowing on credit cards, which has also increased considerably over this period.

While many private student lending programs are loosely structured like the federal GSL programs (i.e. many limit borrowing to the cost of schooling less financial aid or a fixed upper limit on total borrowing), they vary substantially in their terms and eligibility requirements. Private lending programs typically use a broader concept of schooling costs than do GSL programs, often allowing students to borrow against previous educational expenses or expenses for study abroad. Specified maximum loan limits are generally quite high, especially for students in professional schools (e.g. law, medical, or business schools); however, actual amounts offered to students vary depending on their creditworthiness, institution attended, and area of study. A cosigner with a good credit history tends to improve the terms of any loans and can affect whether a loan is offered in the first place.

4 Basic Models of Borrowing Constraints

In this section, we use a simple two-period model economy to study the impact of alternative forms of credit constraints on the incentives to invest in human capital. We allow for some generality in preferences and skill production and derive the qualitative relationships between investment, ability, and initial wealth implied by the different constraints.

In all of our models, limits on credit can induce poorer individuals to reduce educational investments – an often-cited implication of borrowing constraints. More interestingly, we show that the implied relationship between ability and investment depends crucially on the nature of credit constraints. We evaluate the empirical relevance of alternative forms of constraints by comparing their implied investment-wealth and investment-ability relationships with the empirical patterns in Section 2.

4.1 The Model

Consider two-period-lived individuals who invest in human capital (i.e. schooling) in the first period and work in the second. Preferences are given by

$$U = u(c_0) + \beta u(c_1), \tag{1}$$

where c_t is the consumption level for periods $t = 0$ and $t = 1$, $\beta > 0$ is a discount factor and $u(\cdot)$ an utility function with the following properties:

Condition 1. *The function $u : R \to R$ is strictly increasing, strictly concave, twice continuously differentiable and satisfies* $\lim_{c \searrow 0} u'(c) = +\infty$.

Agents are endowed with initial financial assets $w \geq 0$ and ability $a > 0$. Initial assets can be viewed as transfers from parents and other family members. Ability captures innate factors, early parental investments and other characteristics that shape the returns to investment in human capital. For each individual we take (w,a) as fixed and exogenously given and focus on human capital investments individuals make largely on their own.

The production of skills is as follows. An individual with ability a that invests h units of human capital at $t=0$ will receive labor earnings at $t=1$ equal to:

$$y = af(h).$$

For the function $f(\cdot)$, we assume:

Condition 2. *The function $f: R \to R_+$ is non-negative, strictly increasing and concave, twice continuously differentiable and satisfies* $\lim_{h\searrow 0} f'(h) = +\infty$ *and* $\lim_{h\nearrow\infty} f'(h) = 0$.

Human capital investment is in terms of consumption goods.[27] In period $t=0$, individuals can borrow d units (or save, which is indicated by $d<0$) at an interest rate $R>1$. Consumption levels for the two periods are given by

$$c_0 = w + d - h, \tag{2}$$

$$c_1 = af(h) - Rd. \tag{3}$$

In the absence of other restrictions, these two sequential constraints are equivalent to the present-value lifetime budget constraint:

$$c_0 + \frac{c_1}{R} = w + \frac{af(h)}{R} - h. \tag{4}$$

Conditions 1 and 2 are standard and imply that optimal solutions are interior – in all models – and defined by first order conditions. We make use of this and other implications of Conditions 1 and 2 without further reference.

4.2 Unrestricted Allocations

In the absence of financial frictions young individuals choose $\{c_0, c_1, h\}$ to maximize utility (1) subject to (4). Such maximization can be broken down into two steps. First, choose investments to maximize $\left\{R^{-1}af(h) - h\right\}$, i.e. the present value of lifetime earnings minus investment costs. Optimal investment, $h^U(a)$ is defined by the equality between the marginal returns of investing in human capital and financial assets:

$$af'\left[h^U(a)\right] = R. \tag{5}$$

Optimal investment is positive and strictly increasing in ability a and independent of initial assets w. The indepedence of investment (or its returns) to the initial assets of the individual is the the basis for most empirical tests for borrowing constraints in the market for human capital.

[27] As long as $t=0$ earnings are independent of (w,a), our model is isomorphic to one in which investments in human capital require both consumption goods (tuition) and foregone earnings.

The second step is to choose consumption profiles to equate discounted marginal utilities. Then, given initial assets w and optimal investment $h^U(a)$, individuals borrow (or save) an amount $d^U(a,w)$ defined by

$$u'\left[w + d^U(a,w) - h^U(a)\right] = \beta R u'\left[af\left[h^U(a)\right] - Rd^U(a,w)\right]. \tag{6}$$

Optimal debt, $d^U(a,w)$, is strictly decreasing in w. For large enough w, individuals would save rather than borrow (savings imply $d^U(a,w) < 0$). Optimal debt is strictly increasing (savings decreasing) in ability a because of two key forces. First, more able individuals have higher $h^U(a)$; hence, they need more resources at $t = 0$ to invest. Second, more able individuals have higher net-lifetime earnings $\left\{R^{-1}af(h) - h\right\}$ and want to consume more in all periods. The second force implies that ability has a stronger positive effect on borrowing than on investment. We will make repeated reference to these results, which are summarized in the following lemma:

Lemma 1 *Let $h^U(a)$ and $d^U(a,w)$ be the unrestricted investment in human capital and borrowing. Then, $h^U(a)$ is strictly increasing in a, while $d^U(a,w)$ is strictly increasing in a and strictly decreasing in w. Moreover, $\frac{\partial d^U(a,w)}{\partial a} > \frac{dh^U(a)}{\partial a} > 0$ and $-1 < \frac{\partial d^U(a,w)}{\partial w} < 0$.*

See the Appendix for all proofs.

4.3 Exogenous Borrowing Constraints

At least since Becker (1975), economists have introduced financial market imperfections in models of human capital. Becker shows that with imperfect access to credit, youth from poor families will invest less (and have higher marginal returns on their investment) than otherwise identical youth from wealthier families. Such a simple and sharp contrast with unconstrained allocations provides the basis for nearly all empirical tests of borrowing constraints in the market for human capital.

Credit constraints are typically introduced in models of human capital investment with a fixed and exogenous upper bound on the amount of debt.[28] Following this approach, assume now that in addition to the consumption equations (2) and (3), borrowing is restricted by the exogenous constraint:

$$d \leq d_0, \tag{EXC}$$

where $0 < d_0 < \infty$ is fixed and uniform for all agents.

We first characterize the set of abilities and assets (a,w) for which (EXC) does not bind, and then describe the behavior of investment and consumption when it does. We use the superscript X to reference the allocations implied by this model.

[28] See, for example, Aiyagari, Greenwood, and Seshadri [1], Belley and Lochner (2007), Caucutt and Kumar [14], Hanushek, Yilmaz, and Leung, [21], and Keane and Wolpin (2001). Instead, Becker (1972) assumes that individuals face an increasing interest rate schedule as a function of their investment. Becker's formulation yields similar predictions to those discussed here.

For each ability, a, there is a threshold of assets, $w^X_{\min}(a)$, above which an agent is unconstrained and below which he is constrained. This threshold is the value of w for which $d^U(a,w) = d_0$. From Lemma 1, we can show that $w^X_{\min}(a)$ is strictly increasing in a. (Equivalently, for each w we can define an ability, $a^X_{\max}(w)$, below which an individual is unconstrained and above which he is constrained.) More able individuals are constrained (given any wealth level), since they desire more investment and consumption.[29] The threshold $w^X_{\min}(a)$ is increasing in ability at a faster rate than is unconstrained investment, since more able youth wish to consume some of the increased future earnings associated with higher ability and investment levels.[30] In Appendix A, we illustrate the function $w^X_{\min}(a)$ for two common closed-form cases.

Individuals with initial wealth $w \geq w^X_{\min}(a)$ attain the unconstrained allocations as described in the previous subsection. Their investment equals $h^U(a)$, and the trade-off between c_0 and c_1 is determined by the return on financial assets. In contrast, individuals with $w < w^X_{\min}(a)$ must reduce investment and/or early consumption to accomodate the credit constraint.

Constrained optimal investment levels, $h^X(a,w)$, will never exceed the unconstrained optimal amount $h^U(a)$. The marginal rate of return on human capital investment, $af'(h)$, must be greater than or equal to R; otherwise c_1 could be increased without changing c_0 by marginally reducing both investment and borrowing. This result holds for all forms of credit constraints considered in this paper.

A constrained individual borrows $d = d_0$ to bring as many resources as possible to the investment period. In this case, the marginal trade-off between early and late consumption is entirely driven by the choice of human capital – investing more increases late consumption but decreases early consumption. Thus, investment in human capital must strike a balance between two goals that are independent in the unrestricted case: maximizing lifetime earnings vs. smoothing consumption. This conflict lies at the heart of the key empirical prediction that investment is increasing in wealth for constrained agents. For these individuals, optimal investment $h^X(a,w)$ is uniquely determined by:

$$u'\left(w + d_0 - h^X(a,w)\right) = \beta a f'\left[h^X(a,w)\right] u'\left[af\left(h^X(a,w)\right) - Rd_0\right],$$

i.e. equality between the marginal cost of additional investment (left-hand side) with the marginal benefit (right-hand side).

The following proposition characterizes the implied relationship between investment and the ability and assets of an individual. The intertemporal elasticity of substitution (IES), defined as $-u'(c)/[cu''(c)]$, plays a crucial role for the implied investment - ability relationship when (EXC)

[29] Given the desire to smooth consumption, $w^X_{\min}(a)$ is strictly greater than $h^U(a) - d_0$, the minimum level of wealth needed to finance $h^U(a)$.

[30] Using the definition of $w^X_{\min}(a)$ and implicit differentiation, it is easy to verify that $\frac{dw^X_{\min}(a)}{da} = \frac{\partial d^U(a,w^X_{\min})}{\partial a} / \frac{\partial d^U(a,w^X_{\min})}{\partial w} > \frac{\partial d^U(a,w^X_{\min})}{\partial a} > \frac{dh^U(a)}{da} > 0$.

binds.

Proposition 1 *Let $h^X(a,w)$ and $h^U(a)$ denote, respectively, the optimal investment with and without the constraint (EXC). If (EXC) binds, then: (1) $h^X(a,w) < h^U(a)$; (2) $h^X(a,w)$ is strictly increasing in w; (3) $h^X(a,w)$ is **strictly decreasing** in a if the IES is less than one.*

This proposition is central to the empirical literature on credit constraints. Results (1) and (2) are well-known and discussed in Becker (1975). Cameron and Heckman (1998, 1999), Ellwood and Kane (2000), Carneiro and Heckman (2002), and Belley and Lochner (2007) empirically examine whether youth from lower income families acquire less schooling, conditional on family background and youth ability. These results also imply that the marginal return on investment for constrained individuals, $af'[h^X(a,w)]$, is greater than R and decreasing in w, which motivates an alternative empirical strategy to test for borrowing constraints based on differential rates of return (see, e.g., Lang 1994, Card 1995, and Cameron and Taber 2004).

More remarkable is result (3) of this proposition because it reveals a serious shortcoming of this model that has not been recognized in the literature. The model predicts a *negative* relationship between (observable factors of) ability and investment for any value of the IES below one.[31] Since most estimates of the IES in the literature are in that range (see Browning, Hansen, Heckman 1999), the model is strongly at odds with the observed positive relationship between ability and investment, perhaps the most robust empirical regularity in the data. This prediction is particularly troublesome for analyses of inequality in investment and earnings.

We next show that linking borrowing limits to investment and observable individual characteristics (as government loan programs and private lenders do) yields predictions consistent with both of the key empirical findings in Section 2. We start by considering government student loan programs.

4.4 Government Student Loan (GSL) Programs

As described in Section 3, lending in existing government-backed student loan programs is directly tied to the cost of investment (including tuition, room, board, books, supplies, transportation, computers, and other expenses directly related to schooling) net of other financial aid (e.g. grants or scholarships). These loans cannot be used to finance non-schooling related consumption goods or activities. GSL programs also usually set fixed upper limits on the total amount of credit available to any student. In this section, we incorporate these two restrictions on borrowing.

First, the fact that lending is *tied to investment* implies that

$$d \leq h. \tag{TIC}$$

[31] An IES less than one is only a sufficient condition for a negative ability - investment relationship. More generally, the model may predict a negative relationship for IES values greater than one.

This condition is equivalent to $c_0 \geq w$, since credit cannot finance first period consumption.

Second, GSL loans are subject to a *maximum amount* of credit, $d_{\max} < \infty$:

$$d \leq d_{\max}, \tag{7}$$

which is equivalent to the exogenous constraint (EXC) for $d_{max} = d_0$. The actual credit limit defined by the GSL program is, therefore,

$$d \leq \min\{h, d_{\max}\}. \tag{GSLC}$$

The only formal difference between the constraints embodied in GSL programs and the standard exogenous constraint model is the addition of (TIC). To understand the importance of this additional constraint, we first examine the behavior of human capital investment imposing the (TIC) constraint alone.[32]

Consider investment and borrowing decisions restricted to satisfy (TIC). Unconstrained allocations solve the problem as long as $h^U(a) \leq d^U(a, w)$, since (TIC) would not bind. Because $d^U(a, w)$ is strictly decreasing in w, there is a finite threshold wealth level for any ability level, $\tilde{w}_{\min}(a)$, above which the agent is unconstrained. As with the exogenous constraint model, this threshold is increasing in ability, but it increases at a slower rate than does $w^X_{\min}(a)$.[33] In general, $\tilde{w}_{\min}(a) > w^X_{\min}(a)$ for all ability levels satisfying $h^U(a) < d_0$; conversely, $\tilde{w}_{\min}(a) < w^X_{\min}(a)$ for all ability levels satisfying $h^U(a) > d_0$. In this sense, (TIC) is more stringent than (EXC) for low ability individuals, while the opposite is true for high ability individuals. Equivalently, for any w, we can define a threshold ability level $\tilde{a}_{\max}(w)$ below which individuals are unconstrained and above which they are constrained. This reflects the fact that desired borrowing increases at a faster rate in ability than does desired investment in human capital.

Consider the case when (TIC) holds with equality. With $d = h$, early consumption is restricted to initial wealth, w, so the optimal choice of human capital solves

$$\max_h \{u(w) + \beta u[af(h) - Rh]\}.$$

This problem is equivalent to maximizing late consumption, $c_1 = af(h) - Rh$, which clearly implies the optimal investment choice $h = h^U(a)$, the unconstrained optimal level of investment. Tying borrowing directly to investment removes the conflict between investment and consumption smoothing present in the exogenous constraint model. Yet, consumption allocation can be severely distorted even if investment decisions are not. Indeed, even if their investment is higher, low ability individuals may be much worse off if they face (TIC) instead of (EXC).

[32]This is the most appropriate model of constraints when upper borrowing limits are non-existent or set very high (e.g. PLUS program for students' parents).

[33]This threshold is defined by the condition $h^U(a) = d^U(a, \tilde{w}_{\min}(a))$. Implicit differentiation yields $\frac{d\tilde{w}_{\min}(a)}{da} = \left[\frac{\partial h^U}{\partial a} - \frac{\partial d^U}{\partial a}\right] / \frac{\partial d^U}{\partial w} = \frac{dw^X_{\min}(a)}{da} + \frac{\partial h^U}{\partial a} / \frac{\partial d^U}{\partial w} < \frac{dw^X_{\min}(a)}{da}$.

If (TIC) is the only restriction on borrowing, everyone will invest the unconstrained optimal amount, $h^U(a)$, regardless of their wealth. Empirical tests for differences in human capital investments (or in their marginal returns) by family resources would lead to the conclusion that there are no binding borrowing constraints even if many are constrained. Empirical tests based on consumption allocations over time are necessary to detect this type of constraint.

Now consider the full GSL constraint (GSLC). For ease of exposition and comparison, assume that $d_{\max} = d_0$, so there is no difference between (EXC) and (7). As before, we first characterize the set of (w, a) for which (GSLC) holds with equality and then characterize investment decisions in that set. We use the superscript G to reference variables pertaining to the full GSL model.

For each a, the threshold $w^G_{\min}(a) \equiv \max\{w^X_{\min}(a), \tilde{w}_{\min}(a)\}$, defines the level of assets below which an agent is constrained. Since both $w^X_{\min}(a)$ and $\tilde{w}_{\min}(a)$ are increasing in ability, so is this threshold. Now, define $\bar{a}$ to be the ability level for which an unconstrained individual would invest $d_{\max}$ (i.e. $h^U(\bar{a}) = d_{\max}$). At this ability level, $\tilde{w}_{\min}(\bar{a}) = w^X_{\min}(\bar{a}) = w^G_{\min}(\bar{a})$, and both (EXC) and (TIC) bind for the same set of wealth levels. Since $\tilde{w}_{\min}(a) > w^X_{\min}(a)$ for $a < \bar{a}$, the GSL constraint will be binding for some low ability persons that would be unconstrained under (EXC) alone. However, as we soon show, more individuals will invest the unconstrained optimal amount, $h^U(a)$, when facing the GSL constraint.

There are three potential categories of credit constrained individuals. First, (TIC) may be the only binding constraint. In this case, as discussed earlier, human capital investment is at the unconstrained optimal level and credit restrictions only affect the ability to smooth consumption. Second, (7) may be the only binding constraint, in which case the behavior of consumption and investment coincides with the exogenous constraint model. Third, both constraints may bind. In this case, investment and borrowing are both fixed at $d_{\max}$ and independent of ability and initial wealth (on the margin).

The GSL model implies a complex relationship between investment, ability, and initial wealth due to the different categories of constrained persons. For the following discussion, it is useful to define $\bar{w} \equiv w^G_{\min}(\bar{a})$, the wealth constraint threshold for an individual who would invest $d_{\max}$ if unconstrained. Among poor individuals with initial wealth $w \leq \bar{w}$, the upper limit on borrowing, $d_{\max}$, will never constrain behavior unless (TIC) also binds. As a result, low-wealth individuals (i.e. $w \leq \bar{w}$) with ability $a \leq \bar{a}$ will invest the unconstrained optimal amount $h^U(a)$, while those with ability $a > \bar{a}$ will invest (and borrow) the maximum allowable amount, $d_{\max}$.

For wealthier agents with $w > \bar{w}$, the relationship between investment and ability is multifaceted. The (TIC) constraint will not bind unless (7) also binds. Those of low enough ability will be completely unconstrained and will invest $h^G(a, w) = h^U(a)$. Those with higher ability will be constrained by (7). Some of these individuals will be unconstrained by (TIC), which implies that $h^G(a, w) = h^X(a, w) \in (d_{\max}, h^U(a))$. Those also constrained by (TIC) – only the most able – will borrow and invest $d_{\max}$.

Notice if $h^X(a,w)$ is always increasing in a, then human capital investment for wealthy individuals with $w > \bar{w}$ will always be increasing in ability. If instead $h^X(a,w)$ is decreasing for constrained individuals (e.g. if the $IES < 1$ – see Proposition 1), then investment for wealthy individuals will be increasing in ability for unconstrained persons with $a < a^X_{\min}(w)$, decreasing in ability for a mid-level ability group with $a \in \left(a^X_{\min}(w), \hat{a}(w)\right)$ that is constrained only by (7), and finally constant at $d_{\max}$ for those with ability above $\hat{a}(w)$ who are constrained by both (TIC) and (7). See Appendix A for details.

Under the assumption that the IES < 1, Figures 3 and 4 graphically show how optimal investment, $h^G(a,w)$, depends on initial wealth and ability under the GSL program. These figures also graph unconstrained investment, $h^U(a)$, investment under exogenous constraints, $h^X(a,w)$, and desired unconstrained borrowing, $d^U(a,w)$ for comparison. Figure 3 displays investment and unconstrained borrowing behavior as functions of ability for a low level of initial wealth, $w_L < \bar{w}$, and for initial wealth equal to $\bar{w}$. (Recall that $\bar{w}$ reflects the GSL wealth constraint threshold for an individual who would invest $d_{\max}$ if unconstrained.) The figure shows that low wealth individuals (i.e. $w \leq \bar{w}$) under the GSL will invest $h^G(a,w) = h^U(a)$ for all ability levels below $\bar{a}$ and will invest $h^G(a,w) = d_{\max}$ for all ability levels above $\bar{a}$ (recall $\bar{a}$ satisfies $h^U(\bar{a}) = d_{\max}$). Among low wealth agents (i.e. $w \leq \bar{w}$), marginal changes in wealth have no effect on investment behavior and investment is increasing or constant in ability. The latter is not true under exogenous constraints, as we observe that $h^X(a,w_L)$ is decreasing in ability above a_2 (the point at which $d^U(a,w_L) = d_0$. Note that investment for individuals with wealth w_L and ability $a \in (a_2, \bar{a})$ is higher under the GSL than under exogenous constraints.

Figure 4 shows optimal investment, $h^G(a,w)$ and $h^X(a,w)$, for individuals with wealth, $w_h \geq \bar{w}$. Optimal investment as a function of ability is the same under the GSL and exogenous constraints up to ability level $\hat{a}(w_H)$, first increasing along the $h^U(a)$ curve until $d^U(a,w_H)$ reaches $d_{\max}$ at ability a_3, then decreasing until ability $\hat{a}(w_H)$. At this point, more able individuals constrained from borrowing more than $d_{\max}$ would like to invest less than $d_{\max}$; without the (TIC) constraint, investment continues to decline with ability as observed for $h^X(a,w_H)$. The GSL prevents borrowing from exceeding investment, so that $h^G(a,w_H) = d_{\max} > h^X(a,w_H)$ for all ability levels above $\hat{a}(w_H)$.

The following results summarize investment behavior under the GSL:

Lemma 2 *Impose $d_{\max} = d_0$ and let $h^G(a,w)$, $h^X(a,w)$, and $h^U(a)$ denote, respectively, optimal investment in the presence of GSL constraints, exogenous credit constraints, and in the unrestricted*

Figure 3: d^U, h^U, h^X, and h^G for low wealth individuals ($w \leq \bar{w}$)

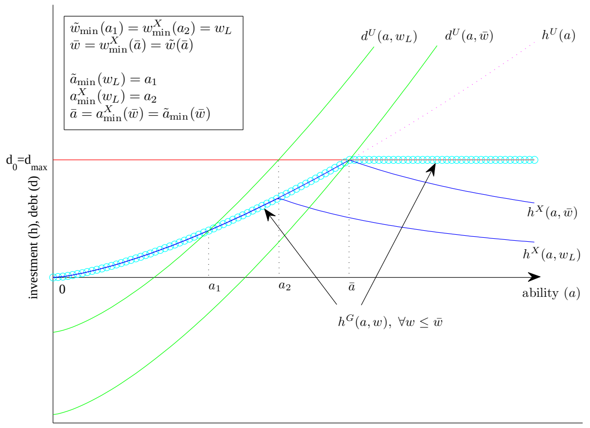
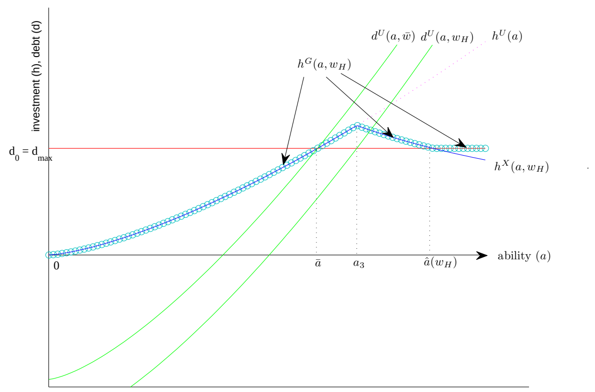


Figure 4: d^U, h^U, h^X, and h^G for high wealth individuals ($w \geq \bar{w}$)



allocation. Then, for $w \leq \bar{w}$, $h^G(a,w) = h^U(a)$ *if* $a \leq \bar{a}$ *and* $h^G(a,w) = d_{\max}$ *if* $a > \bar{a}$. *For* $w > \bar{w}$

$$h^G(a,w) = \begin{cases} h^U(a) & \text{if } a \leq a^X_{\min}(w) \\ h^X(a,w) & \text{if } a \in (a^X_{\min}(w), \hat{a}(w)) \\ d_{\max} & \text{otherwise.} \end{cases}$$

Proposition 2 *Let* $d_{\max} = d_0$ *and let* $h^G(a,w)$ *denote optimal investment under the GSL. Then, for* $w \leq \bar{w}$, $h^G(a,w)$ *is weakly increasing in* a *and independent of* w. *For* $w > \bar{w}$ *and the IES less than one, investment is strictly decreasing in* a *and strictly increasing in* w *if* $a \in (a^X_{\min}(w), \hat{a}(w))$; *otherwise, investment is weakly increasing in* a *and independent of* w.

The additional restriction that borrowing can only finance investment and not consumption drastically alters the behavior of constrained individuals and implies a more complex relationship of investment with ability and with assets. It is worth highlighting a number of key lessons. First, investment under the GSL will equal the unconstrained optimal amount for a larger range of middle-ability and middle-wealth individuals than would be predicted by the exogenous constraint model. While the addition of (TIC) increases the number of constrained agents, it does not reduce their investment. In fact, it increases investment for some individuals (i.e. those with $h^U(a) < d_{\max} < d^U(a,w)$) to the unconstrained optimal amount. This directly implies that investment is increasing in ability and independent of initial wealth for a broader range of ability and wealth levels. Second, among high ability, wealthy individuals, the restriction that investment cannot fall below borrowing levels shrinks the range of abilities (compared with the exogenous constraint model) for which the ability - investment relationship can be negative. When the IES is less than one, the exogenous constraint model predicts that investment is decreasing in ability for all individuals constrained by $d_{\max}$. However, (TIC) ensures that investment never falls below $d_{\max}$ for the most able. For the very able who are constrained by both (TIC) and (7), investment is independent (at the margin) of ability. Third, among individuals whose ability is high enough that they would like to invest more than $d_{\max}$, investment is a weakly increasing function of initial assets. Interestingly, investment among the least wealthy of these high ability individuals is fixed at $d_{\max}$ and, therefore, unaffected by marginal changes in wealth. Overall, the GSL produces a weakly positive wealth - investment relationship, with investment constant at $d_{\max}$ for those of high ability and with low initial assets.

It is important to remember, however, that while investment under GSL programs may be closer to the unrestricted optimum for some agents, utility is lower than with the exogenous constraint alone. The resulting distortions in consumption from (TIC) can be quite costly in terms of utility as we show in Section 5.

Proposition 3 *Impose* $d_0 = d_{\max}$ *and fix* $(a,w) \in \mathbb{R}^2_+$. *Let* $\{h^G(a,w), c^G_0(a,w), c^G_1(a,w), U^G(a,w)\}$, $\{h^X(a,w), c^X_0(a,w), c^X_1(a,w), U^X(a,w)\}$, *and* $\{h^U(a), c^U_0(a,w), c^U_1(a,w), U^U(a,w)\}$ *denote the op-*

timal allocations and attained utilities with the GSL, exogenous credit constraints, and in the unrestricted problem. Then, the following inequalities hold:

$$h^U(a) \geq h^G(a,w) \geq h^X(a,w),$$
$$c_0^U(a,w) \geq c_0^X(a,w) \geq c_0^G(a,w),$$
$$c_1^G(a,w) \geq c_1^U(a,w) \geq c_1^X(a,w),$$
$$U^U(a,w) \geq U^X(a,w) \geq U^G(a,w).$$

Moreover, an inequality is strict if the additional constraint between the respective pair of models is binding.

Unlike the standard exogenous constraint model that motivates prior empirical tests for constraints in the market for human capital, a GSL program only distorts the intertemporal allocation of consumption for a broad range of ability and wealth levels. With high enough maximum loan limits, many individuals may be constrained with severely distorted consumption profiles even though they appear to be investing like they are unconstrained. Empirical tests focusing on distortions in investment behavior identify just that – they do not identify the full extent to which borrowing constraints may affect individual behavior and welfare.

4.5 Endogenous Constraints under Limited Commitment

The inalienability of human capital and the lack of other collateralizeable assets are the standard arguments for introducing borrowing constraints in the analysis of human capital accumulation. Most researchers, however, use the arguments to introduce an ad-hoc constraint such as (EXC) and do not explore the nature of constraints that would effectively arise from these incentive problems. In this subsection, we derive the credit constraints that endogenously arise in private markets from the limited capacity of borrowers to commit to repay their loans. Depending on ability and wealth levels, default incentive problems may restrict borrowing opportunities and affect investment in human capital. We first consider an economy with only private lending and limited commitment. In the following subsection, we consider the co-existence of private lenders with a GSL program as we currently see in the U.S.

When borrowers cannot fully commit to repay debts, lenders will naturally limit the amount of credit.[34] In equilibrium, the maximum credit a borrower can receive is determined by the penalties and other costs he would face in the event of default. Penalties that lenders can inflict on defaulting borrowers include restricting further access to financial markets (by reporting them to credit bureaus), repossessing some of their assets, and garnishing their earnings. Even when borrowers

[34]See, e.g. Gropp, Scholz, and White (1997) for evidence supporting this form of response by private lenders.

can avoid direct sanctions (e.g. by re-locating, working in the informal economy, borrowing from loan sharks, or renting instead of buying a home), avoidance actions are likely to be costly.

As we show below, the maximum amount of credit an individual can receive will be increasing in his earnings potential. The monetary cost of punishments and avoidance actions increase with earnings, since only so much can be taken from someone with little to take. Earnings potential, and therefore a person's credit limit, is determined exogenously through ability and endogenously through investment in human capital. The nature of credit limits affects investment decisions, which feeds back into the equilibrium determination of those limits.

In this section, we consider two simple punishments lenders can use to enforce repayment of loans. (The next section considers additional punishments in a more realistic lifecycle economy.) First, lenders can repossess any physical assets that a borrower has saved from a previous period. Second, lenders can garnish a fraction $\tilde{\kappa} \in (0,1)$ of their post-school earnings. In this environment, individuals will repay any liabilities (principal plus interest) that are less than the cost of punishment (i.e. $Rd \leq \tilde{\kappa} a f(h)$). Recognizing this, private lenders will limit borrowing so that

$$d \leq \kappa a f(h), \tag{8}$$

where $\kappa = R^{-1}\tilde{\kappa}$.[35]

As with the previous models, we first characterize the set of assets and abilities for which individuals are constrained and then characterize the optimal investments when constraint (8) binds. We use the superscript L to reference allocations pertaining to this model.

For each ability level a, there is a threshold level of assets, $w^L_{\min}(a)$ defined by $d^U(a, w^L_{\min}(a)) = \kappa a f\left[h^U(a)\right]$. Individuals with w above (below) $w^L_{\min}(a)$ are unconstrained (constrained). This minimum wealth threshold increases less with ability than in the exogenous constraint model.[36] The fact that borrowing limits respond to ability can even cause $w^L_{\min}(a)$ to be decreasing in a for high enough κ.[37] Indeed, $w^L_{\min}(a)$ can be negative for high enough ability levels. In such cases, contrary to exogenous constraint models, individuals with high abilities are less likely to be constrained. In Appendix A we illustrate $w^L_{\min}(a)$ with two leading cases in closed-form.

When $w \geq w^L_{\min}(a)$, the optimal allocation is the unconstrained solution discussed earlier. Consider, now, individuals for which $w < w^L_{\min}(a)$. Since the constraint (8) holds with equality, $c_0(h) = w + \kappa a f(h) - h$ and $c_1(h) = a f(h)(1 - \kappa R)$, so the intertemporal trade-off for consumption is entirely determined by investment in human capital. In this case, c_1 is always increasing in h, since

[35] In Section 5, we show that this form of constraint arises in a multi-period life-cycle economy with time separable homothetic preferences, exogenous growth of earnings and finitely lived punishments.

[36] Implicit differentiation yields $\frac{dw^L_{\min}}{da} = \left[\kappa\left(f(h^U) + R\frac{\partial h^U}{\partial a}\right) - \frac{\partial d^U}{\partial a}\right] / \frac{\partial d^U}{\partial w} = \frac{dw^X_{\min}}{da} + \left[\kappa\left(f(h^U) + R\frac{\partial h^U}{\partial a}\right)\right] / \frac{\partial d^U}{\partial w} < \frac{dw^X_{\min}}{da}$.

[37] However, $w^L_{\min}(a)$ is greater than $h^U(a) - \kappa a f\left[h^U(a)\right]$, the minimum level of w for which it is feasible to finance $h^U(a)$.

$0 < \kappa R < 1$ and $f(\cdot)$ is increasing. Interestingly, c_0 is initially increasing but then decreasing in h. Obviously, optimal investment will lie in the latter region where early consumption is decreasing in investment. See Appendix A for a detailed discussion.

Optimal investment in human capital, $h^L(a, w)$, equates the marginal cost of investing (the value of foregone early consumption) with the marginal benefit (the value of increased late consumption):

$$\left\{1 - \kappa a f'\left[h^L(a, w)\right]\right\} u'\left[c_0\left(h^L(a, w)\right)\right] = \beta a f'\left[h^L(a, w)\right](1 - \kappa R) u'\left[c_1\left(h^L(a, w)\right)\right].$$

It is easy to verify that (given our assumptions on $u(\cdot)$, $f(\cdot)$, and κ) this condition is sufficient and necessary and that it uniquely determines a positive and finite $h^L(a, w)$. From this expression, we can characterize the implied ability - investment and wealth - investment relationships under private lending with limited commitment.

Proposition 4 *Let $h^L(a, w)$ and $h^U(a)$ denote, respectively, optimal investment in human capital with credit constraints driven by limited commitment to repay loans and in the unrestricted allocation. If constraint (8) binds, then: (1) $h^L(a, w) < h^U(a)$, (2) $h^L(a, w)$ is strictly increasing in w; (3) a sufficient condition for $h^L(a, w)$ to be strictly increasing in a is that the IES is uniformly bounded below by $(1 - \kappa R)$; (4) Moreover, if the utility function $u(\cdot)$ exhibits non-decreasing IES and $\beta R \geq 1$, then $h^L(a, w)$ is strictly increasing in a if either (i) $\kappa(1 + R) > 1$ for any IES or (ii) $\kappa(1 + R) < 1$ and the IES is uniformly bounded below by $[1 - \kappa(1 + R)]$.*

Unlike exogenous constraints, the responsiveness of endogenous credit constraints to ability and investment creates a tendency for higher ability individuals to be unconstrained. This proposition further shows that this responsiveness makes it more likely that investment is increasing in ability among those who are constrained. The proposition also shows a similarity between binding endogenous constraints from limited commitment and exogenous constraints: both cause investment to increase in wealth. As we illustrate below, empirically plausible values of κ imply a strong enough effect of ability and investment on endogenously determined credit limits to ensure that investment is increasing in ability, even for intertermporal preference parameters that imply a negative ability - investment relationship under the exogenous constraint model. This model of credit constraints is, therefore, consistent with the two key cross-sectional investment patterns reported in Section 2, while the exogenous constraint model is not.

Intuitively, the responsiveness of constraints should imply higher investment when credit constraints are endogenous rather than exogenous. Such intuition, however, requires a valid comparison of available credit in both models. The following proposition provides such a comparison and shows that the intuition is indeed correct.

Proposition 5 *Fix any (a, w) such that $w < w^L_{\min}(a)$. Let $h^L(a, w)$ and $d^L(a, w)$ denote, respectively, optimal investment and borrowing in the limited commitment model. Consider the* *allocations in an exogenous constraint model, where* $d_0 = d^L(a,w)$. *Then,* $w < w^X_{\min}(a)$ *and* $h^X(a,w) < h^L(a,w)$.

At the same level of credit, the responsiveness of the constraint reduces the marginal cost of investing from 1 to $1-\kappa a f'(h)$ units of current consumption, encouraging more investment.

4.6 GSL *Plus* Private Markets

In Section 3 we documented an enormous increase in the participation of private lenders in financing post-secondary education as well as a large rise in the fraction of individuals borrowing the maximum amount from the GSL program. The co-existence of both sources of financing shapes the credit available to individuals and affects investments in human capital for a large fraction of the U.S. population.

We now study the interaction of these two sources of financing by considering two extreme cases. First, we assume that both types of lenders (GSL and private) have the same limited mechanisms to punish default. Then, we consider the case where loans from the GSL are fully enforced, while private lenders face limited repayment incentives among borrowers.

4.6.1 GSL with Limited Enforcement

Assume that individuals can borrow from two sources, the GSL program or from private lenders. Assume also that these two sources are not mutually exclusive and that both face the same problem of limited incentives to repay from borrowers. We abstract from issues of private information and assume that private lenders observe an individual's borrowing from the GSL program and other private lenders. Private lenders will not lend amounts for which they foresee default. Yet, the lending terms of the GSL need not prevent default.

The vulnerability of the GSL to default would impact the decision to invest in human capital. An individual that plans to default would find it optimal to borrow the maximum amount $d_{\max}$ and not repay anything. Partial defaults are suboptimal when lenders can exercise the full extent of the punishment. Since borrowing is tied to investment, borrowers who default would invest at least $d_{\max}$. Therefore, the possibility of default can cause some low ability individuals with $a < \bar{a}$ to overinvest.

The temptation of default on GSL loans is decreasing in the ability of the borrower, since the maximum benefit of default ($Rd_{\max}$) is fixed but the punishment is increasing labor earnings.[38] Obviously, the temptation to default rises with the value of $d_{\max}$ and falls with the punishment $\tilde{\kappa}$. The following result identifies sufficient conditions on $d_{\max}$ and $\tilde{\kappa}$ that guarantee that the GSL program is default-proof.

[38] Even if $d_{\max} = +\infty$, this result would hold when $af\left[h^U(a)\right]/h^U(a)$ is strictly increasing in a, because the cost of punishments would grow faster than debt.

Proposition 6 *The GSL program is default-proof if the minimum ability a in the population is above* $a_L \equiv \frac{Rd_{\max}}{\tilde{\kappa} f(d_{\max})}$ *and the elasticity of the skill production function* $f(\cdot)$, $\alpha(h) \equiv f'(h)\, h / f(h)$, *is uniformly bounded above by the punishment* $\tilde{\kappa}$.

If the conditions are such that the GSL is default-proof, then its presence has no impact whatsoever on observed investments and consumption profiles *relative* to an economy in which there are only private lenders. If, given the same penalties, no borrower chooses to default on the GSL, then, private lenders could have granted the same amount of credit without concern for default. Private lenders would, therefore, offer at least as much as the GSL program in total borrowing opportunities, reducing any private loan amounts by the amount borrowers take from the GSL program. In this sense, GSL programs are completely redundant.

4.6.2 GSL with Full Enforcement

Consider now the case in which the GSL has full enforcement but private lenders face limited enforcement as described earlier. Under these conditions, there is no default in equilibrium. GSL loans are fully enforced, and private lenders restrict loans to amounts that borrowers will repay in equilibrium.

An individual chooses human capital investments h, borrowing from the GSL d_g, and borrowing from private lenders d_p to maximize utility (1) subject to the sequential budget constraints

$$c_0 = w + d_g + d_p - h,$$
$$c_1 = af[h] - Rd_g - Rd_p,$$

the GSL lending guidelines

$$d_g \leq \min\{h, d_{\max}\},$$

and the repayment enforcement constraint for private lending

$$d_p \leq \kappa a f[h].$$

We refer to this case with the superscript $G + L$.

For each ability level a, we can define a threshold level of initial assets $w_{\min}^{G+L}(a)$ above which an individual is unconstrained. In this model, the threshold is defined by the equation $d^U\left(a, w_{\min}^{G+L}(a)\right) = \kappa a f\left[h^U(a)\right] + \min\left\{h^U(a), d_{\max}\right\}$. The threshold can be decreasing in ability and even negative (as in the private lenders only case). With both sources of credit, $w_{\min}^{G+L}(a) < \min\left\{w_{\min}^{G}(a), w_{\min}^{L}(a)\right\}$, so more individuals are unconstrained than with either the GSL or private lenders alone.

We now compare the $G + L$ allocations with the L and the G allocations. First, consider agents with ability $a < \bar{a}$, i.e those for which $h^U(a) < d_{\max}$. The GSL program provides these individuals with enough credit to finance the unrestricted level of education. For them, limits on credit only impact the intertemporal allocation of consumption. The availability of private credit has no impact on human capital investment decisions. However, relative to the GSL only, private lenders would provide up to an amount $\kappa af\left[h^U(a)\right]$ for consumption smoothing, allowing individuals with $w < w^G_{\min}(a)$ to attain a higher level of utility. Consider now individuals with $a > \bar{a}$. These individuals would like to invest above $d_{\max}$, but the the GSL program does not provide sufficient funds for those with wealth below $w^G_{\min}(a)$. At minimum, private lenders could provide an extra amount of credit equal to $\kappa af\left[d_{\max}\right]$. The amount of credit offered grows as the agent invests above $d_{\max}$. Thus, investment by these agents would be larger when both sources of credit are present than when only the GSL is available.

The presence of a fully-enforced GSL leads to larger investments than with private lending alone. With respect to private markets alone, the introduction of a fully enforced GSL program leads to unrestricted investment levels for lower ability levels $a \leq \bar{a}$. Those with $w < w^L_{\min}(a)$ will invest more than if they did not have access to government loans. For abilities $a > \bar{a}$, the GSL ensures a minimum investment of $d_{\max}$, which ensures that these individuals will repay at least $\kappa af\left[d_{\max}\right]$ in private loans. The availability of extra resources during the schooling period allows for further investment, which increases the credit available from private sources even more.

Proposition 7 *Let $h^L(a,w)$, $h^G(a,w)$ and $h^{L+G}(a,w)$ denote, respectively, optimal investment in human capital of an individual with ability and wealth (a,w) under private markets with limited commitment, fully enforced GSL, and the two sources combined. Then $h^L(a,w) \leq h^{L+G}(a,w)$ and $h^G(a,w) \leq h^{L+G}(a,w)$. The first inequality is strict if $w < w^L_{\min}(a)$ (i.e. 8 binds), and the second is strict if $a > \bar{a}$ and $w < w^G_{\min}(a)$. Finally, letting $h^{L+G}(a,w;d_{\max})$ denote the dependence of optimal investment on the credit limit $d_{\max}$, then $h^{L+G}(a,w;d_{\max})$ is strictly increasing in $d_{\max}$ when $a > \bar{a}$*

Finally, consider the implied empirical relationship between human capital investment, ability, and wealth. If ability is low (i.e. $a < \bar{a}$), investment equals the unconstrained amount for any w. Among more able agents with $a \geq \bar{a}$, someone is unconstrained if $w \geq w^{G+L}_{\min}(a)$; otherwise he is constrained with $h^{L+G}(a,w)$, which is less than $h^U(a)$ and increasing in w. Under the conditions described in Proposition 4, $h^{L+G}(a,w)$ is increasing in a. Thus, for reasonable parameterizations, this model with a fully enforced GSL program and private lending with limited commitment reproduces the two key empirical findings of Section 2.

5 A Quantitative Model

We now explore the quantitative implications of alternative forms of borrowing constraints. To this end, we extend our basic model to a continuous time life-cycle model with standard preferences, demographics and production functions. We also explicitly introduce government education subsidies. Then, we calibrate the model with U.S. data on schooling, abilities, subsidies and earnings, comparing the implied relationship of ability and wealth with investment under alternative forms of credit constraints. The results are compared with the patterns observed in NLSY data as discussed in Section 2. We also examine the impact of an increase in the returns and costs of investment (as witnessed in the 1980s and 1990s) on investment outcomes and on the amount and composition of borrowing.

5.1 The Environment

All individuals have a lifespan equal to the interval $[0, T]$, where $T > 1$. There are three stages in life. In the first stage, the interval $[0, 1]$, the individual is "young." He attends school and receives no labor earnings. In the second stage, the interval $[1, P]$ (where $0 < P < T$), the individual is "mature," does not attend school and earns labor earnings. In the third stage, the interval $[P, T]$, individuals are "retired," receive zero earnings and finance consumption with accumulated savings.

Preferences are standard. As of any $t_0 \in [0, T]$, the utility of an individual is

$$U(t_0) = \int_{t_0}^{T} e^{-\rho(t-t_0)} \left[\frac{c(t)^{1-\sigma}}{1-\sigma} \right] dt, \tag{9}$$

where $c(t)$ is the level of consumption as of time t, $\sigma > 0$ is the inverse of the IES, and $\rho > 0$ is the discount rate.

At the beginning of their lives, $t = 0$, individuals are endowed with initial financial assets $w \geq 0$ and an ability level $a > 0$.[39] Then, they decide on the investments in schooling and borrowing or savings in all three stages.

During youth, individuals attend school. Schooling is full-time but individuals choose the effective level of investment. Schooling investments during youth determine the level of human capital with which the person enters the labor market at $t = 1$.[40] Let $i(t)$ be the flow of schooling investment for $t \in [0, 1]$. Then the accumulated stock of human capital as of $t = 1$ is

$$h = \mu \int_{0}^{1} e^{g_s(1-t)} i(t)\, dt, \tag{10}$$

where $g_s > 0$ is a growth factor that implies a higher productivity of earlier investments and $\mu \equiv \rho / [e^{\rho} - 1]$ is a normalization. We assume that $g_s = \rho$ to simplify the analysis.

We assume that the government subsidizes schooling. In particular, young persons receive – free of charge – a constant investment flow $i_{pub} \geq 0$ for all $t \in [0, 1]$. Furthermore, the government

[39] The value of w may reflect the present value of the flow of transfers, $b(t)$, from parents over this period (i.e. $w = \int_0^1 e^{-\rho t} b(t)\, dt$).

[40] This setting is isomorphic to one in which young individuals also choose how much time to allocate to investment vs. work, assuming their wages during $[0, 1]$ do not depend on their ability.

matches each unit that the individual invest above i_{pub} with additional $s \geq 0$ units. Therefore, if an individual privately invests $x(t) \geq 0$, then his total investment for the period is

$$i(t) = i_{pub} + (1+s)\, x(t)\,. \tag{11}$$

During maturity, an individual obtains labor earnings $y(t)$ determined by his ability a, his investment h and his labor market experience $t-1$. Specifically, for all $t \in [1, P]$,

$$y(t) = \mu^{-1} a h^{\alpha} e^{g(t-1)}, \tag{12}$$

where g is the rate of return to experience (i.e. the constant growth rate in the earnings with age).[41]

Finally, we assume that the market interest rate equals ρ.

5.2 Unrestricted Allocations

With frictionless and fully enforceable financial markets, an individual with ability a and initial assets w maximizes the $t_0 = 0$ value (9) subject to the budget constraint:

$$\int_0^T e^{-\rho t} c(t)\, dt + \int_0^1 e^{-\rho t} x(t)\, dt \leq w + \int_1^P e^{-\rho t} y(t)\, dt. \tag{13}$$

Since the interest rate equals the discount rate, the optimal consumption path constant over time. Also, since $g_s = \rho$, the optimal timing of investment is indeterminate. However, total investment, h, is uniquely determined. Without loss of generality, we can impose $x(t) = x \geq 0$ for all $t \in [0,1]$ and then solve for the optimal constant level of out-of-pocket investments, x, using the fact that $h = i_{pub} + (1+s)\,x$ and $y(t) = \mu^{-1} a e^{g(t-1)} h^{\alpha}$.

With these two results, the budget constraint (13) simplifies to:

$$c\left[\frac{1-e^{-\rho T}}{\rho}\right] \leq w + \frac{e^{-\rho}}{\mu}\left[a\Phi\left[i_{pub} + (1+s)\,x\right]^{\alpha} - x\right] \tag{14}$$

where the constant Φ (which depends on P, g and ρ) converts initial earnings into the present value of life-time earnings. The formula for Φ and other technical details of the model are in Appendix B.

Optimal investment maximizes the right-hand side of (14). In this maximization, a person with ability $a \leq a_0 \equiv \frac{[i_{pub}]^{1-\alpha}}{\alpha(1+s)\Phi}$ does not find it worthwhile to invest more than the publicly provided amount, so $h = i_{pub}$. A person with $a > a_0$ would add investment until its marginal return equals the (private) marginal cost. In either case, investment is independent of wealth and solely determined by ability:

$$h^U(a) = \max\left\{i_{pub}, \left[\alpha(1+s)\, a\Phi\right]^{\frac{1}{1-\alpha}}\right\}. \tag{15}$$

[41] As shown in the appendix, our results generalize to the case in which g is increasing in a.

As in the two-period model, optimal investment in human capital is completely independent of consumption decisions and initial assets.

Using (14) and (15), in Appendix B we give the formula (in closed-form) for $d^U(a,w)$, which here represents the amount of debt that the individual has at age $t=1$, the time when he stops investing and enters labor markets. The function $d^U(a,w)$ satisfies the properties derived in Lemma 1 for the two period model (i.e. it is decreasing in w and increasing and steeper than $h^U(a)$ as a function of a).

5.3 Exogenous Borrowing Constraints

We now consider exogenous credit constraints. As in the two-period model assume that loans are fully enforceable but that there is an upper bound d_0 on the amount of credit that an individual can accumulate. To restrict the attention to limitations on financing of schooling, we assume that there are no credit constraints after $t=1$ (i.e. once the agent starts working).

Assume that there is a level $0 \leq d_0 < \infty$ such that,

$$d \leq d_0, \tag{16}$$

where d is the accumulated amount of debt as of $t=1$.

Own financial resources w plus debt d finance the flows of investment x and consumption $c(t)$ for $t \in [0,1]$. The budget constraint for the period is $\int_0^1 e^{-\rho t}\left[c(t)+x\right]dt \leq w + e^{-\rho}d$. Moreover, since the borrowing constraint is only on cumulative debt at $t=1$, it does not distort consumption within periods of youth. Since the interest and the discount rates are equal, optimal consumption during youth is a constant, c_0. The budget constraint for the investment period simplifies to

$$c_0 + x \leq \mu e^{\rho}\left[w + e^{-\rho}d\right]. \tag{17}$$

With perfect post-schooling financial markets, individuals are able to fully smooth consumption once they begin working. Optimal consumption during $t \in [1,T]$ is equal to another constant c_1 that depends only on the difference between the present value of earnings $\mu^{-1}a\Phi h^{\alpha}$ and the financial liabilities d accumulated during the investment period. Appendix B contains the details and derives the utility as of $t=1$, which equals

$$\Theta\frac{\left[\mu^{-1}a\Phi_a h^{\alpha} - d\right]^{1-\sigma}}{1-\sigma},$$

where $\Theta \equiv \left(\left[1-e^{-\rho T}\right]/\rho\right)^{\sigma} > 0$. Using this time $t=1$ utility and the formula for h in terms of x, time $t=0$ utility is

$$U(c_0,x,d;a) \equiv \frac{e^{-\rho}}{\mu}\frac{c_0^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta\frac{\left[\mu^{-1}a\Phi\left[i_{pub}+(1+s)\,x\right]^{\alpha} - d\right]^{1-\sigma}}{1-\sigma}. \tag{18}$$

Individuals choose $x \geq 0$, $c_0 \geq 0$ and d to maximize $U(c_0, x, d; a)$ subject to (16) and (17). Aside from the possibility of $x = 0$, this problem is analytically identical to the two-period case and the equivalent to Proposition 1 holds.[42]

5.4 Government Student Loan Programs

It is straightforward to analyze a GSL program in this environment. Instead of (16), cumulative debt as of $t = 1$, d, is restricted to satisfy:

$$d \leq \min\{x, d_{\max}\}.$$

The only important change with respect to the version in the two-period model is that, with government subsidies, borrowing is tied to out-of-pocket investments x and not to total investments h. Otherwise, the models are virtually identical.[43]

5.5 Private Lending with Limited Commitment

Consider now an environment in which loans for schooling cannot be directly enforced. As in the two-period model, loans are repaid only if the cost of default – from punishments and/or avoidance actions – is higher than the cost of repaying. The incentives to repay after $t \geq 1$ define the amount of credit lenders are willing to supply during $0 \leq t \leq 1$.

We consider two forms of punishments. First, defaulting borrowers are reported to credit bureaus, an action that blocks all access to further borrowing and lending. This penalty does not reduce earnings but disrupts the ability to smooth consumption. It can be quite costly if earnings grow very quickly with experience and if the IES is low. Second, the lender can garnish a fraction $\gamma \in [0, 1]$ of the labor earnings of defaulting borrowers. Both punishments apply immediately after a default and last for only a finite time interval of length $\pi \leq (0, P - 1]$.

To simplify the analysis we make two additional assumptions. First, we abstract from post-default bargaining. After the punishment period, individuals have a fresh start with zero debt – regardless of (d, h, a, w).[44] Second, loans contracted after schooling are fully enforceable. The temptation to default is only on loans contracted before the person starts working. As in the previous models, credit constraints will only affect the financing of human capital and consumption levels during the investment period.[45]

Consider a person with ability a and human capital h that defaults at $t = 1$ on debt d. Since punishments are not reduced by partial re-payment, an individual that defaults would default on his entire debt with all creditors. The inaccessibility of financial markets and the wage garnishment pins

[42] Appendix B discusses the formula for $w^X_{\min}(a)$ and other details of this model.
[43] Appendix B discusses the formula for $w^L_{\min}(a)$ and other details of this model.
[44] See Yue (2007) for a model in which with post-default bargaining would allow a, h affect borrowing constraints.
[45] Monge-Naranjo (2007) considers a continuous time model in which the agent can default in any period.

down consumption at $c(t) = (1-\gamma) y(t)$ for $t \in [1, 1+\pi]$. At time $t = 1+\pi$ the punishment ends, and the individual is allowed to attain the optimal constant consumption flow over the remaining period $[1+\pi, T]$.

The highest discounted utility of an individual that defaults at $t = 1$ is given by

$$\hat{\Theta}_{\gamma,\pi} \frac{\left[\mu^{-1} a \Phi h^{\alpha}\right]^{1-\sigma}}{1-\sigma},$$

where $0 < \hat{\Theta}_{\gamma,\pi} \leq \Theta$, a constant whose formula is shown in Appendix B.

It can be verified that $\hat{\Theta}_{\gamma,\pi}$ is decreasing in γ (the option of default is less tempting with higher garnishments) and increasing in π (the option of default is less tempting with longer punishments). As long as $\pi > 0$, $\hat{\Theta}_{\gamma,\pi} < \Theta$ even if $\gamma = 0$, because the exclusion from financial markets is costly in terms of consumption smoothing. If $\pi \to 0$, then $\hat{\Theta}_{\gamma,\pi}$ converges to Θ for any $\gamma \in (0,1)$. Penalties that only apply for an interval of measure zero would not prevent default on any positive stock of debt. Finally, for low values of IES (i.e. $\sigma > 1$), if $\pi \to P-1$, then individuals would always repay any debt below $\mu^{-1} a \Phi h^{\alpha}$ (Aiyagari's "natural limit"), because not saving for retirement would push utility to $-\infty$.

The option to default limits the amount of debt that borrowers can credibly commit to repay. Rational lenders foresee repayment incentives and restrict their credit to avoid triggering default. Given penalties (π, γ) a borrower with ability a and human capital investment h is better off repaying a debt d when $\hat{\Theta}_{\gamma,\pi} \frac{\left[\mu^{-1} a \Phi h^{\alpha}\right]^{1-\sigma}}{1-\sigma} \leq \Theta \frac{\left[\mu^{-1} a \Phi h^{\alpha} - d\right]^{1-\sigma}}{1-\sigma}$, or, equivalently, when

$$d \leq \kappa \mu^{-1} a \Phi \left[i_{pub} + (1+s) x\right]^{\alpha}, \tag{19}$$

where $\kappa \equiv 1 - \left[\hat{\Theta}_{\gamma,\pi}/\Theta\right]^{\frac{1}{1-\sigma}} \geq 0$. As in the two-period model of Section 4.5, the maximum amount of credit that an individual can obtain is proportional to his post-school labor earnings. The difference is that preferences parameters (ρ, σ) and institutions (γ, π) endogenously determine the value of κ. In the two-period model, we interpret κ as garnishments, but here κ is positive and can be quite high even if garnishments are zero.

With credit limits endogenously determined by limited commitment, the optimization problem at $t = 0$ is choosing $x \geq 0$, $c_0 \geq 0$ and d to maximize $U(c_0, x, d; a)$ subject to (17) and (19).

As in the two-period model, the value $w^L_{\min}(a)$ is the threshold of wealth above which an agent is unconstrained and is characterized in Appendix B. A person with $w \geq w^L_{\min}(a)$ attains the unrestricted allocations. For those with $w < w^L_{\min}(a)$, a similar result to the two-period model holds:

Proposition 8 *Let the ability and financial assets of a young individual be given by (a, w), and let $h^L(a, w)$ and $h^U(a)$ indicate, respectively, the optimal investments in human capital with private lenders with limited commitment and in the unrestricted allocation. If $a \leq a_0$, then $h^L(a, w) =$*

$h^U(a) = i_{pub}$. *If instead* $a > a_0$ *and constraint (19) binds, then: (1)* $h^L(a,w) < h^U(a)$*; (2)* $h^L(a,w)$ *is strictly increasing in* w*; and (3)* $h^L(a,w)$ *is strictly increasing in* a *if either (i)* $\kappa > \left(\frac{e^{\rho}-e^{-\rho T}}{e^{\rho}-1}\right)^{-1}$ *and* $\sigma \geq 0$ *or (ii)* $\kappa \leq \left(\frac{e^{\rho}-e^{-\rho T}}{e^{\rho}-1}\right)^{-1}$ *and* $\sigma < \left[1-\kappa\left(\frac{1-e^{-\rho(T-1)}}{1-e^{-\rho}}\right)\right]^{-1}$.

We show below that empirically plausible parameter values imply a positive relationship between investment and ability.

5.6 GSL Programs *Plus* Private Lending

We now introduce a GSL program in an environment in which private lending also operates. As in the two-period model, we consider two extremes case: First, the GSL has the same limited enforcement as private lending. Second, GSL has full enforcement, i.e. its loans are much better enforced than private unsecured loans. In practice, GSL programs in the U.S. appear closer to the latter extreme. First, GSL loans cannot be cleared by bankruptcy proceedings. This implies an effective punishment period π much longer for GSL than in bankruptcy procedures such as Chapter 7 or Chapter 13. Second, wage garnishments of up to 15% are explicitly incorporated in the GSL system. This implies an effective garnishment rate higher than in private unsecured loans which are defined by bankruptcy regulations and, therefore, almost certainly lower.

As before, we provide sufficient conditions under which the GSL is default-proof, even with the limited enforcement of private loans. The equivalent of Proposition 6 in this environment is as follows:

Proposition 9 *The GSL program is default-proof if* $a \geq \frac{\mu d_{\max}}{\kappa\Phi\left[i_{pub}+(1+s)d_{\max}\right]^{\alpha}}$ *and either (i)* $\kappa \geq \alpha\mu$ *or (ii)* $\kappa < \alpha\mu$ *and* $i_{pub} \geq (1+s)\left\{\frac{\alpha\mu-\kappa}{\kappa}\right\} d_{\max}$.

That is, we would not observe default in equilibrium if the lowest ability level is high enough (relative to the loan size) and the penalties are severe enough. The elasticity of the skill production function, α, plays the same role as in Proposition 6. This proposition also highlights the impact of government subsidies on repayment incentives. A higher lump sum transfer, i_{pub}, unambiguously reduces incentives to default, while a higher matching subsidy rate, s, may increase them.

Under the conditions of Proposition 9, a GSL with the same limited enforcement as private lenders would be completely redundant. Consumption and investment allocations would coincide with the allocations under private lenders alone.

Consider now the case in which, regardless of default on private loans, borrowers must repay GSL loans. Private lending is endogenously limited to ensure repayment. At $t = 0$, a young person with (a, w) chooses out-of-pocket investment x, consumption during youth c_0, borrowing from GSL

d_g, and borrowing from private lenders d_p to maximize $U(c_0, x, d; a)$ subject (17) and

$$d \leq d_p + d_g \tag{20}$$

$$d_p \leq \kappa\mu^{-1} a \Phi \left[i_{pub} + (1+s)\, x \right]^{\alpha}, \tag{21}$$

$$d_g \leq \min\left\{x, d_{\max}\right\}. \tag{22}$$

The threshold level of initial assets $w_{\min}^{G+L}(a)$ above which individuals with ability a are unconstrained satisfies

$$d^U\left(a, w_{\min}^{G+L}(a)\right) = \kappa\mu^{-1} a \Phi \left[h^U(a)\right]^{\alpha} + \min\left\{d_{\max}, \max\left\{0, \left(h^U(a) - i_{pub}\right)/(1+s)\right\}\right\},$$

where $d^U(a, w)$ is given by expression (24) and $h^U(a)$ by expression (15). Since borrowers combine both sources of credit, then $w_{\min}^{G+L}(a) < \min\left\{w_{\min}^{G}(a), w_{\min}^{L}(a)\right\}$, where $w_{\min}^{G}(a)$ is the threshold of a GSL alone as given by $d^U\left(a, w_{\min}^{G+L}(a)\right) = \min\left\{d_{\max}, \max\left\{0, \left(h^U(a) - i_{pub}\right)/(1+s)\right\}\right\}$.

All the conclusions obtained for the $G+L$ model in the two-period economy, including a variant of Proposition 7, go through.

5.7 Calibration

We now set parameter values to explore quantitative implications of the model. Some of the parameters are estimated using data on earnings and educational attainment from the random sample of males in the NLSY79. We use AFQT quartiles as a measure of ability. Other parameter values are calibrated to replicate features of the U.S. economy. All dollar amounts are deflated using the CPI to year 1999 dollars.

We assume that youth (investment period) begins when the individual is 16 years of age and ends when he is 24. Maturity (labor market participation period) runs from age 24 until aget 65. Retirement runs from age 65 until death, at age 80. Given the normalization for the length of youth in the model, each time interval with length equal to one corresponds to 8 years of life. Therefore, $P = 6.125$, $T = 7$.

To match an annual interest rate of 4% we set $\rho = 8 \times ln(1.04) = 0.3138$ as the discount rate in the model. We set $\sigma = 2.5$ as our baseline value, which implies an IES of 0.4. This is an intermediate value within the range of reported IES estimates in Browning, Hansen, and Heckman (1999). We have verified that values of σ that lie inside the interval $[1.5, 3.5]$ yield similar results.

Penalties for default are set to last for a length of $\pi = 0.875$, which corresponds to the 7-year rule in U.S. bankruptcy regulations. We set $\gamma = 0.1$ for the earnings garnishment rate faced by individuals who default. Under the GSL program guidelines, defaulting borrowers face an explicit 15% wage garnishment. For private loans, an explicit garnishment rule does not exist. However, actual costs of default — either via direct penalties or via avoidance actions — extend beyond simple garnishments, (e.g. individuals may end up suboptimally employed, renting instead of owing a house, and paying subprime interest rates for short-term transactions, etc.). Our results are not very sensitive to using other reasonable values for γ.

We assume all investment up until age 16 is publicly provided for free. After age 16, schooling entails direct costs (i.e. tuition and public expenditures on primary, secondary, and post-secondary schooling) and indirect costs (i.e. foregone earnings). To compute direct costs we use an annual government expenditure of \$5,928 for primary and secondary schooling. Annual direct expenditures for college and graduate education are assumed to equal \$16,838.[46] (College expenditures include government expenditures as well as tuition and fees paid by students.) We set $i_{pub} = \$65,239$, which is equal to the discounted value of all direct schooling expenditures through grade nine.[47] This is consistent with our focus on investments made at age 16 onwards.

We ignore foregone earnings up until grade nine because of compulsory schooling attendance and minimum age work laws, but we include foregone earnings from grade ten onwards. To estimate foregone earnings, we use data from the NLSY79 to regress log earnings on indicators for each of the grades 10-18, indicators for AFQT quartiles, total years of potential work experience and experience-squared.[48] With this regression we compute foregone earnings for $S \geq 10$ years of schooling using the predicted earnings of someone with nine years of completed schooling, $S - 10$ years of potential work experience, and the desired AFQT quartile. These foregone earnings estimates are included in our estimates of total schooling expenditures and are reported in Table C1 of the Appendix.

We calibrate the government subsidy rate s as follows. We assume that the private costs of investments are foregone earnings plus a fraction of post-secondary tuition and other direct costs. From Table 333 of the *Digest of Education Statistics (2003)* the ratio of tuition and fees to total current-fund revenue for degree-granting higher education institutions varies from 0.20 in 1979-80 to 0.24 in 1988-89. Assuming a ratio of 0.20 of private to total direct expenditures for college, the subsidy rates for investment education beyond i_{pub} ranges from 0.47 to 0.6 for completed schooling levels 12-16 depending on the AFQT quartile and completed years of schooling. Taking a government subsidy rate of 50% as our baseline implies $s = 1$.

We estimate the parameters g and α of the earnings equation using data from the NLSY79. From the model, the wage earnings of someone with ability a who invested h and has been working τ periods is $w(\tilde{a}, h, \tau) = \tilde{a}h^{\alpha}e^{g\tau}$ where $\tilde{a} \equiv a/\mu$. Since this equation is additively separable in logs,

[46] Annual expenditure for education through grade twelve (for college and graduate education) is the average of annual current expenditures per pupil for public primary and secondary schools (for all two and four year colleges) over the academic years 1979-80 through 1988-89 as reported in Table 170 (Table 342) of the *Digest of Education Statistics, 1999.* These years roughly reflect the years our NLSY79 sample respondents made their final schooling decisions.

[47] We use a 4% annual discount rate, reporting the value discounted to the end of grade nine. Less than 0.2% of our NLSY79 sample acquired less than 10 years of school.

[48] This regression uses all available earnings observations for male respondents with at least nine years of completed schooling when they were ages 16-24 and no longer enrolled in school. Potential work experience is measured as age - years of completed schooling - 6. The estimates (available upon request) suggest that earnings for these young workers are generally increasing in years of completed schooling and increasing and concave in potential work experience.

Table 3: Baseline Model Parameters

Calibrated Parameters			Estimated Parameters (from Log Earnings)		
Parameter	Value	To match:	Parameter	Value	Estimates for:
P	6.125	Retirement at 65	g	0.369	Experience
T	7	Lifespan of 80	α	0.432	Schooling investment
π	0.875	7 yrs., U.S. Bankruptcy			
ρ	0.3138	Annual rate = 4%			Ability Levels:
σ	2.5	IES = 0.4	$\tilde{a}_1$	106.70	AFQT quartile 1
γ	0.1	Garnishment & other costs	$\tilde{a}_2$	137.83	AFQT quartile 2
i_{pub}	65,239	Educ. costs through grade 9	$\tilde{a}_3$	157.38	AFQT quartile 3
s	1	Educ. subsidy grades 10+ = 50%	$\tilde{a}_4$	158.29	AFQT quartile 4

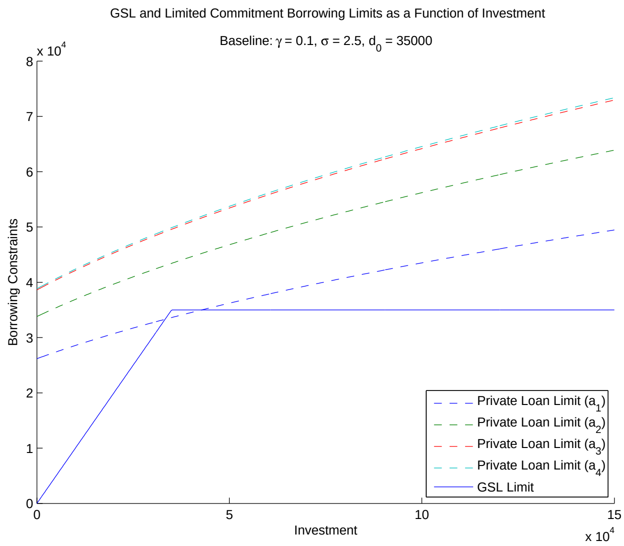
we estimate this equation by regressing log earnings on AFQT quartile indicators (represented by the vector A_i), estimated total schooling expenditures (h_i) as reported in Table C1, and years of experience ($\tau = age - 24$):

$$ln[w_{i\tau}] = \beta_0' A_i + \beta_1 ln(h_i) + \beta_2 \tau + \nu_{i\tau},$$

where $\nu_{i\tau}$ is a mean zero idiosyncratic earnings shock, i.e. $E(\nu_{i\tau}|A_i, h_i, \tau) = 0$. The implied estimates for α and g are $\hat{\alpha} = \hat{\beta}_1$ and $\hat{g} = 8\hat{\beta}_2$, where the latter accounts for the fact that a unit time interval in the model corresponds to a calendar time of 8 years. Notice that even though $\nu_{i\tau}$ is mean zero, $E[e^{\nu_{i\tau}}] > 1$. To match average earnings levels in the data (i.e. $E(w|\tilde{a}, h, \tau)$), we normalize the coefficient on AFQT quartiles by the sample average $\overline{e^{\hat{\nu}_{i\tau}}}$. Doing so, we obtain ability measures $\tilde{a}_q = e^{\hat{\beta}_{0q}} \overline{e^{\hat{\nu}_{i\tau}}}$ for quartile q. These estimated ability measures range from 106.70 for the least able to 158.29 for the most able.

Table 3 reports the value of all parameters used in our baseline simulations.

Figure 5: GSL and Private Lending Constraints (Baseline Economy)

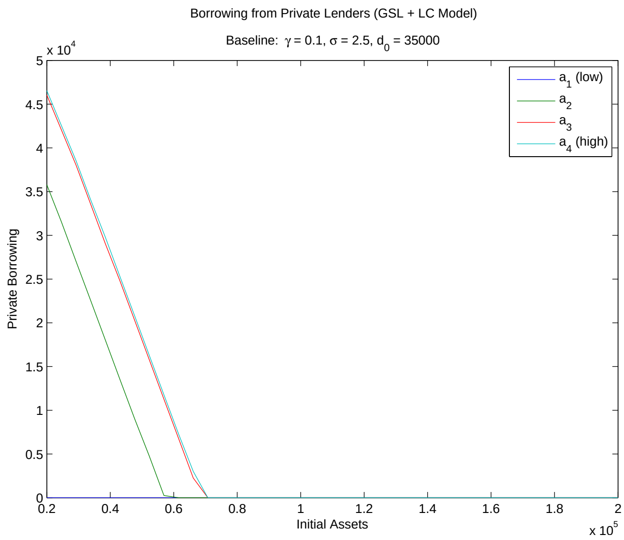


5.8 Baseline Simulations

Figure 5 shows the implied borrowing constraints as a function of investment for the GSL program. The figure also shows the private lending constraints as a function of investment for all four ability groups. For all but the least able, the private lending market would offer more than the GSL program at any level of investment. Only for investment levels very close to the GSL upper loan amount, $35,000, does the GSL limit exceed the private lending limit for the least able. At any level of investment, the difference in private lending limits between the most and least able is sizeable, ranging from around $10,000 to nearly $20,000. The limits are also strongly increasing in investment.

Figure 6 reveals the amount of borrowing from private lenders as a function of ability and initial assets in our baseline economy with both private lending and a GSL program. Here, we assume individuals borrow first from the GSL, then private lenders after exhausting all government credit. It is difficult to know the exact range of initial assets, since this depends on parental resources and their willingness to transfer funds to their children. Since investment expenditures include foregone earnings, one should assume that all individuals have at least the total amount of foregone earnings for someone taking the maximum amount of schooling at their disposal. As Table C1 shows, foregone earnings vary from roughly $50,000-80,000 (depending on ability) for the highest level of schooling, so it is reasonable to assume that individuals with few other resources have at least $50,000 in ‘initial assets’; although, we show borrowing for initial assets of $20,000-200,000. As one can see, private borrowing should be negligible in the baseline period, since only those with initial assets less than foregone earnings would borrow from private lenders.

Figure 6: Private Borrowing (Baseline Economy)



Next, Figure 7 shows total investment as a function of ability and initial assets under the GSL (with full enforcement) and private lending. As can be seen from the figure, the combined GSL program and private lending market environment produces efficient investment for all levels of initial assets above \$30,000. As a result, investment is increasing in ability and independent of initial assets, consistent with reported schooling patterns in the NLSY79 data. Despite the fact that our calibration did not target investment levels, total investment amounts shown in this figure are quite reasonable and close to average total expenditures in the NLSY79 data.[49] We leave a comparison of different credit market environments to the following subsection, which considers a rise in returns and costs of schooling and yields more 'interesting' investment patterns since constraints are binding for a wider set of ability and initial asset levels.[50]

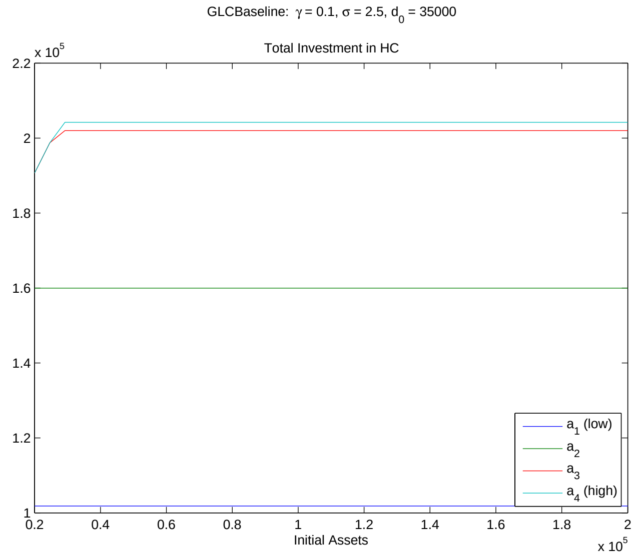
5.9 Rising Costs and Returns to Investment and the Rise in Private Lending

Now, consider an increase in both the private returns and costs to schooling — two major economic changes affecting schooling decisions that took place from the early 1980s to the early 2000s. We model an increase in the wage returns to education by assuming that α increases to 0.46. This produces a modest increase in the college - high school log wage differential of a couple percentage points. To capture the rise in net tuition costs taking place over the 1980s and 1990s, we reduce

[49] Combining the total costs by AFQT quartile and schooling level reported in Table C1 with the distribution of educational attainment by AFQT in the NLSY79, we obtain average total investment amounts ranging from \$88,000 for the least able to \$178,000 for the most able.

[50] We note that with private lending markets alone, the most able are constrained up through initial asset levels of nearly \$80,000. With the GSL alone or with exogenous constraints, the most able are constrained (and under-investing) through initial asset levels of around \$120,000.

Figure 7: Total Investment (Baseline Economy) under GSL and Private Lending



the government subsidy rate, s, to 0.8. This reduction reflects the increased importance of tuition and fees as a fraction of total current-fund revenue for public and private universities in the U.S. Consistent with the fact that GSL maximum loan limits changed little over this period, we assume that $d_{\max}$ remains at $35,000. We explore how these economic changes affect investment in human capital, consumption while in school, and the extent of private lending.

Figure 8 graphs the new private lending limits along with the GSL limits used in our baseline calibration. Private lending limits increase substantially for all investment and ability levels. For youth investing $100,000, the least able are offered nearly $65,000 in credit while the most able are offered about $85,000. This increase in private lending is entirely due to the increased returns to investment.

Youth wish to borrow more in response to both the increased costs and returns to investment. Borrowing from private lending increases substantially as seen in Figure 9, which assumes that youth borrow the maximum allowable amount from the GSL before borrowing from private lenders. While low ability youth with $80,000 would not borrow from private lenders, the most able borrow on the order of $20,000.

Next, we show optimal total investment under all four credit market assumptions (GSL with private lending, private lending only, GSL only, and exogenous borrowing constraints with $d_0 = d_{\max}$) in Figure 10. First, consider investment with private lenders and the GSL (panel a). Optimal total investment is higher here than in the baseline simulation. The increased returns dominate the rise in tuition costs. In general, optimal unconstrained investment is quite sensitive to changes in α. Borrowing constraints appear to be binding for a broad range of initial asset levels among the

Figure 8: 'Year 2000' GSL and Private Lending Constraints

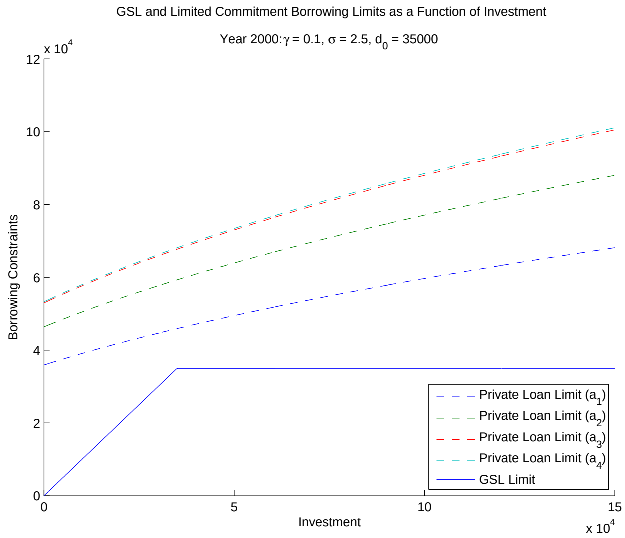
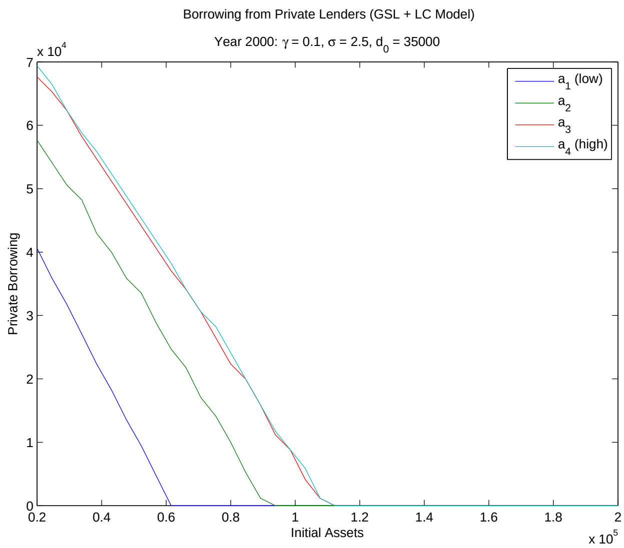


Figure 9: Private Borrowing ('Year 2000')



higher ability types. Among the most able, those with initial assets below $130,000 are constrained. Even those from the second lowest ability are constrained if they have initial assets below $80,000. Among constrained individuals, investment is increasing in both ability and initial assets, as is observed in the NLSY97 data.

Now, compare total investment with both the GSL and private lenders with either private lending alone (panel b) or the GSL alone (panel c). Clearly, investment is higher for nearly all initial asset levels shown when individuals have access to both the GSL and private lending markets. Individuals from a much broader range of initial asset levels would be constrained if either private lending markets or the GSL were shut down. For most initial asset and ability levels, the private lending market tends to yield greater investment than under the GSL; however, this is not true for the least able with very low initial assets (amounts below foregone earnings). Under the GSL, these youth invest the maximum amount they can borrow, $35,000, above the publicly provided amount. Investing less does not provide them with any more consumption while in school, since they would also be required to borrow less. Private lenders do not impose this tight restriction, so very poor youth of low ability invest less even though they are able to borrow more than under the GSL. Investment is increasing in ability for all levels of initial assets under private lending; however, this is not true for the GSL since the upper limit on borrowing is the binding constraint for a broad range of initial asset levels and ability types. The perverse relationship between ability and optimal investment is even worse for the exogenous constraint model as shown in panel d.

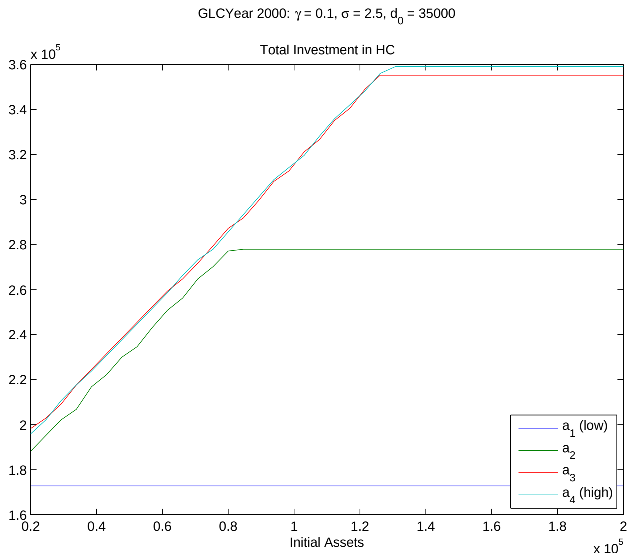
Finally, we show consumption during the investment period under all four credit market assumptions in Figure 11. Consumption is substantially higher when both the GSL and private lending markets are available than when either is not. As expected from our discussion of the GSL program in Section 4, consumption while in school is quite low for those with low initial assets. The fact that borrowing cannot be used to finance consumption while in school under the GSL would is quite costly for the poor in the absence of private lending. All other forms of credit constraints allow for more intertemporal consumption smoothing conditional on investment.

6 Conclusions

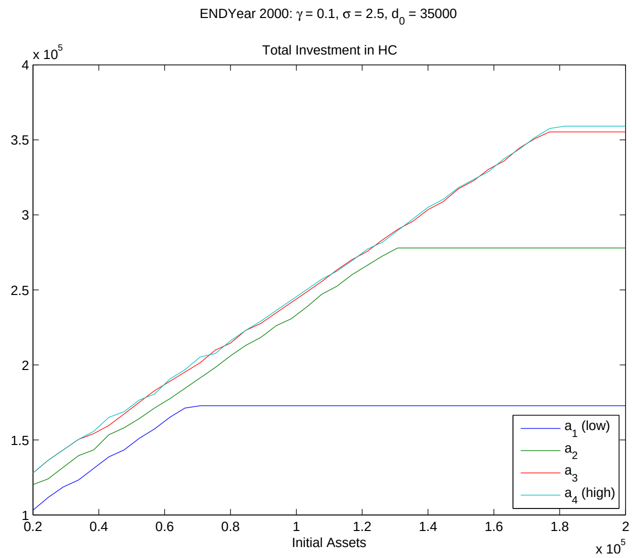
To be written.

Figure 10: Total Investment ('Year 2000') with Different Credit Market Assumptions

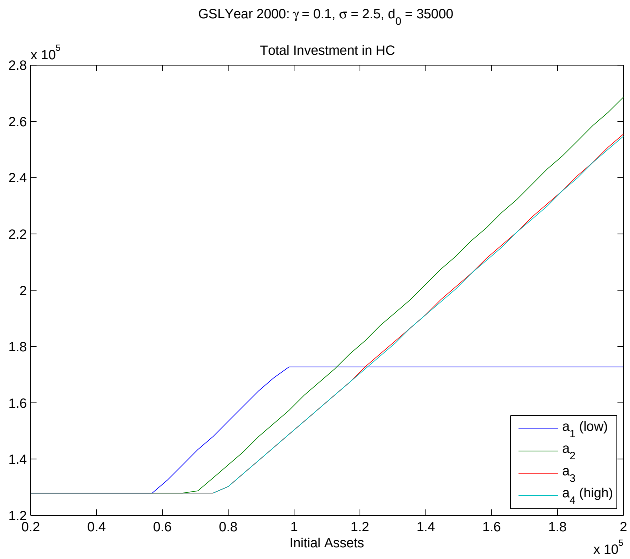
(a) GSL and private lending



(b) Private Lending



(c) GSL



(d) Exogenous borrowing limits

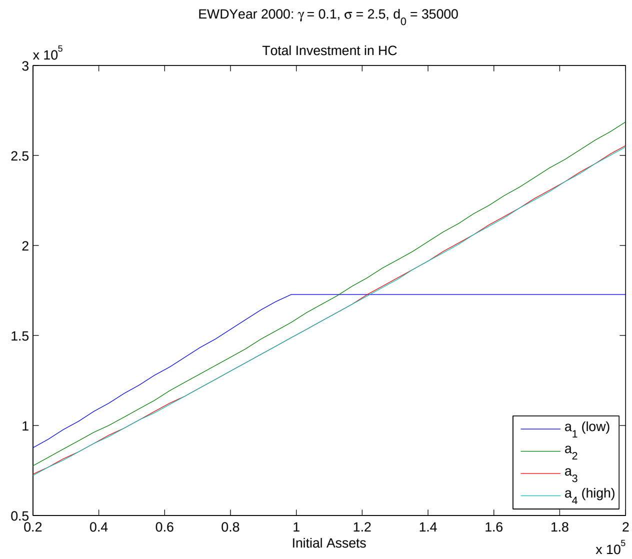
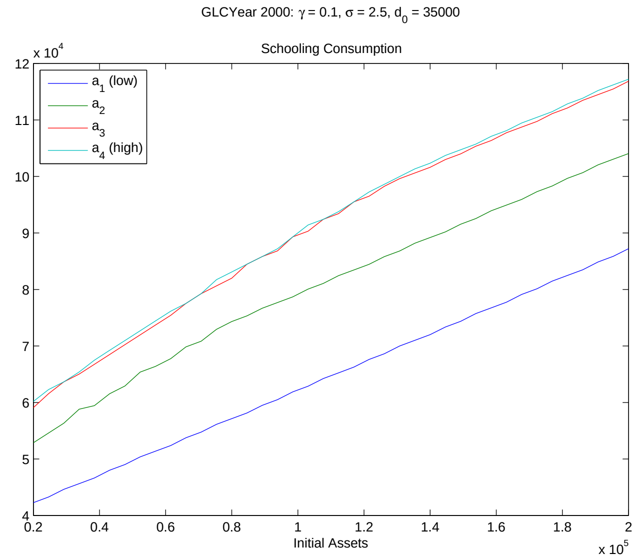
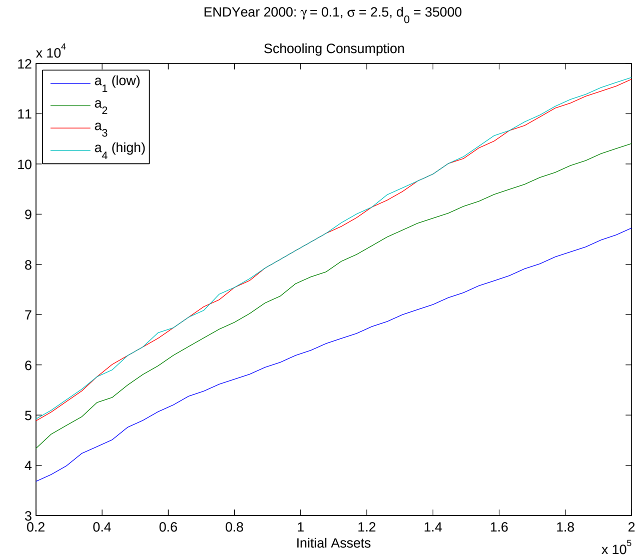


Figure 11: Consumption during Investment Period ('Year 2000') with Different Credit Market Assumptions

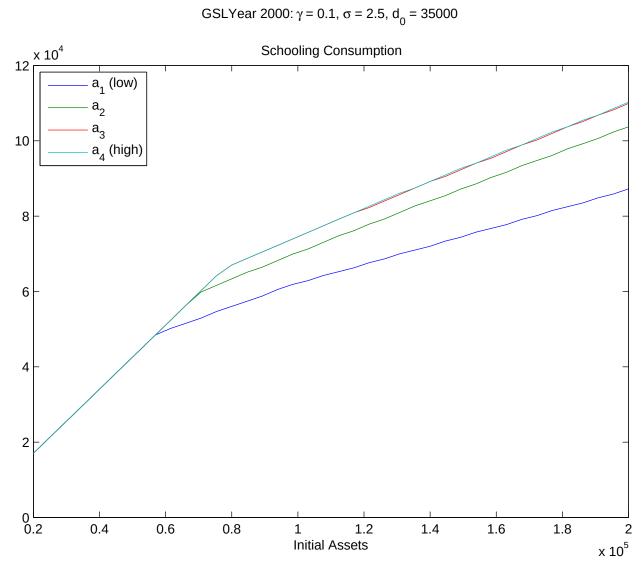
(a) GSL and private lending



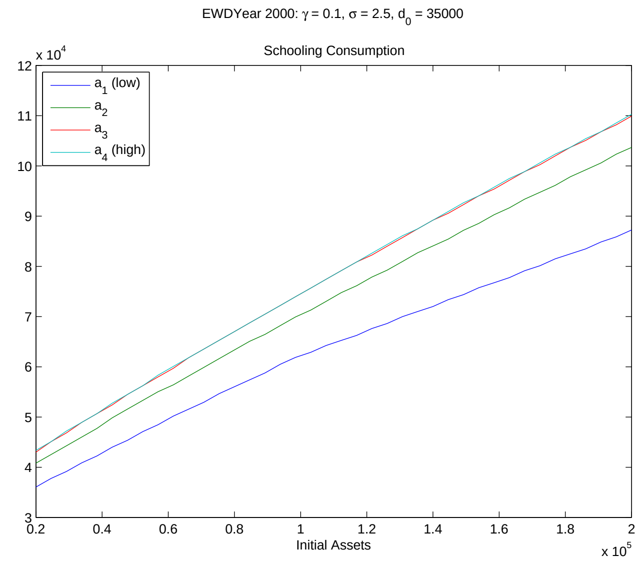
(b) Private Lending



(c) GSL



(d) Exogenous borrowing limits



Appendices

A Specifics of the Two-Period Model

Here, we provide the proofs and other analytical details relevant for the two period model that are omitted in the text.

A.1 Unrestricted Allocations

Proof of Lemma 1 Using expression (6), define

$$F \equiv u' \left[w + d - h^U(a)\right] - \beta R u' \left[af\left[h^U(a)\right] - Rd\right] = 0.$$

From the implicit function theorem

$$\frac{\partial d^U(a,w)}{\partial a} = -\left[\frac{\partial F}{\partial a} / \frac{\partial F}{\partial d}\right].$$

The following are readibly obtained:

$$\frac{\partial F}{\partial d} = u'' \left[w + d - h^U(a)\right] + \beta R^2 u'' \left[af\left[h^U(a)\right] - Rd\right],$$

and

$$\begin{aligned}\frac{\partial F}{\partial a} &= \frac{\partial h^U(a)}{\partial a}\left\{-u''\left[w + d^U(a,w) - h^U(a)\right]\right\} - \beta R u''\left[af\left[h^U(a)\right] - Rd\right]\left\{f\left[h^U(a)\right] + af'\left[h^U(a)\right]\frac{\partial h^U(a)}{\partial a}\right\} \\ &= (-1)\left\{\begin{array}{c}\frac{\partial h^U(a)}{\partial a}\left(u''\left[w + d^U(a,w) - h^U(a)\right] + \beta R\left(af'\left[h^U(a)\right]\right)u''\left[af\left[h^U(a)\right] - Rd\right]\right) \\ +\beta R u''\left[af\left[h^U(a)\right] - Rd\right] f\left[h^U(a)\right]\end{array}\right\},\end{aligned}$$

where the second line follows from re-arranging and using $af'\left[h^U(a)\right] = R$. Using these three expressions:

$$\frac{\partial d^U(a,w)}{\partial a} = \frac{\partial h^U(a)}{\partial a} + \beta R \frac{u''\left[af\left[h^U(a)\right] - Rd\right] f\left[h^U(a)\right]}{u''\left[w + d - h^U(a)\right] + \beta R^2 u''\left[af\left[h^U(a)\right] - Rd\right]} > \frac{\partial h^U(a)}{\partial a} > 0,$$

as claimed. Also,

$$\frac{\partial F}{\partial w} = u''\left[w + d - h^U(a)\right],$$

and, therefore,

$$\frac{\partial d^U(a,w)}{\partial w} = -\frac{u''\left[w + d - h^U(a)\right]}{u''\left[w + d - h^U(a)\right] + \beta R^2 u''\left[af\left[h^U(a)\right] - Rd\right]} = -\left(\frac{1}{1 + \beta R^2 \frac{u''\left[af\left[h^U(a)\right] - Rd\right]}{u''\left[w + d - h^U(a)\right]}}\right).$$

Since $\left[1 + \beta R^2 \frac{u''\left[af\left[h^U(a)\right] - Rd\right]}{u''\left[w + d - h^U(a)\right]}\right] \in (0,1)$, the argument is complete.■

A.2 Exogenous Borrowing Constraints

Closed-form expressions for $w^X_{\min}(a)$ can be determined in two cases. First, if $\beta R = 1$, then the unrestricted optimum calls for $c_0 = c_1$ implying

$$w^X_{\min}(a) = af\left[h^U(a)\right] + h^U(a) - d_0(1 + R).$$

Second, if $u(c) = c^{1-\sigma}/(1-\sigma)$, then

$$w^X_{\min}(a) = (\beta R)^{\frac{-1}{\sigma}} af\left[h^U(a)\right] + h^U(a) - d_0\left(1 + R(\beta R)^{\frac{-1}{\sigma}}\right).$$

Proof of Proposition 1. From the FOC define

$$F \equiv -u'(w + d_0 - h) + \beta a f'[h]\, u'[af(h) - Rd_0] = 0$$

By the second order condition for maximization: $\frac{dF}{dh} < 0$. By the implicit function theorem:

$$\frac{\partial h}{dw} = -\frac{dF}{dw} / \frac{dF}{dh} \text{ and } \frac{\partial h}{da} = -\frac{dF}{da} / \frac{dF}{dh}.$$

Therefore,

$$sign\left\{\frac{\partial h}{dw}\right\} = sign\left\{\frac{dF}{dw}\right\} \text{ and } sign\left\{\frac{\partial h}{da}\right\} = sign\left\{\frac{dF}{da}\right\}$$

Solving for these derivatives:

$$\frac{dF}{dw} = -u''(w + d_0 - h) > 0,$$

and

$$\begin{aligned}
\frac{dF}{da} &= \beta f'[h]\, u'[af(h) - Rd_0] + \beta a f'[h]\, f(h)\, u''[af(h) - Rd_0] \\
&= \beta f'[h]\, u'[af(h) - Rd_0] \left\{1 + af(h) \frac{u''[af(h) - Rd_0]}{u'[af(h) - Rd_0]}\right\} \\
&\leq \beta f'[h]\, u'[af(h) - Rd_0] \left\{1 + [af(h) - Rd_0] \frac{u''[af(h) - Rd_0]}{u'[af(h) - Rd_0]}\right\} \\
&\leq \beta f'[h]\, u'[af(h) - Rd_0] \{1 - 1/\eta[af(h) - Rd_0]\},
\end{aligned}$$

where the first equality is direct derivation; the second equality reflects simple factorization; the first inequality holds from the fact that $u'' < 0$ and that $f' > 0$, $u' > 0$, and $d_0 \geq 0$; and the last inequality uses the definition of $\eta(\cdot)$. If for any $c > 0$, $\eta(c) < 1$ the right-hand-side (RHS) is negative and therefore $\frac{dF}{da} < 0$.■

A.3 Government Student Loan (GSL) Programs

The threshold level of wealth for which an agent is unconstrained is $w^G_{\min}(a) \equiv \max\{w^X_{\min}(a), \tilde{w}_{\min}(a)\}$. Thefore, if $\beta R = 1$, the unrestricted optimum calls for $c_0 = c_1$ and

$$w^G_{\min}(a) = af\left[h^U(a)\right] + h^U(a) - \min\{h^U(a), d_0\}(1 + R).$$

Second, if $u(c) = c^{1-\sigma}/(1-\sigma)$, then

$$w^G_{\min}(a) = (\beta R)^{\frac{-1}{\sigma}} af\left[h^U(a)\right] + h^U(a) - \min\{h^U(a), d_0\}\left(1 + R(\beta R)^{\frac{-1}{\sigma}}\right).$$

Proof of Lemma 2 . Direct upon examination of optimality condition under the three different cases.■

The function $\hat{a}(w)$ is defined by $h^X(\hat{a}(w), w) = d_{\max}$. Using the FOC of the exogenous constraint model, $\hat{a}(w) \equiv \sup\{\hat{a} : u'(w) \geq \beta \hat{a} f'[d_{\max}]\, u'[\hat{a} f(d_{\max}) - Rd_{\max}]\}$. Hence, in principle it could be $+\infty$. If $u(c) = c^{1-\sigma}/(1-\sigma)$, then a finite $\hat{a}(w)$ would be given by

$$\hat{a} : w(\beta f'[d_{\max}])^{\frac{1}{\sigma}} = \left[(\hat{a})^{\frac{\sigma-1}{\sigma}} f(d_{\max}) - Rd_{\max}(\hat{a})^{\frac{-1}{\sigma}}\right].$$

If $\sigma > 1$ (IES< 1), the RHS is strictly increasing and unbounded and, hence, $\hat{a}(w)$ is finite.

A.4 Endogenous Constraints under Limited Commitment

As in the X model, if $\beta R = 1$, then optimal consumption levels are equal in both periods and

$$w^L_{\min}(a) = af\left[h^U(a)\right](1-\kappa[1+R]) + h^U(a).$$

Next, if $u(c) = c^{1-\sigma}/(1-\sigma)$, then

$$w^L_{\min}(a) = af\left[h^U(a)\right]\left([1-\kappa R](\beta R)^{\frac{-1}{\sigma}} - \kappa\right) + h^U(a).$$

With respect to the non-monotonicity of $c_0(h)$: when (8) holds with equality, the Inada conditions on $f(\cdot)$, implies that c_0 is initially increasing in h as increments in investment increase more the amount of credit that the cost of investing. Given ability a, let $h^O(a)$ reflect the amount of h that maximizes $c_0(h)$ is determined by $\kappa af'\left[h^O(a)\right] = 1$. Constrained individuals will never invest less than this amount. Moreover, since $\kappa < R^{-1}$, then it is obviously the case that $h^O(a) < h^U(a)$.

Proof Proposition 2. The maximization is

$$\max_h \{u[w + \kappa af(h) - h] + \beta u[af(h)(1-\kappa R)]\}$$

From the first order condition, define the expression

$$F \equiv (\kappa af'(h) - 1)u'(w + \kappa af(h) - h) + \beta af'(h)(1-\kappa R)u'[af(h)(1-\kappa R)] = 0.$$

Part (a) follows from the fact that if the constraint binds, then

$$(1-\kappa af'(h))u'(c_0) = \beta af'(h)(1-\kappa R)u'(c_1).$$

If $h^L(a,w) > h^U(a)$, then $af\left[h^L(a,w)\right] < R$. Therefore,

$$\begin{aligned}\beta R(1-\kappa R)u'[c_1] &> \beta af'(h)(1-\kappa R)u'[c_1] \\ &= (1-\kappa af'(h))u'(c_0) \\ &> (1-\kappa R)u'(c_0).\end{aligned}$$

Therefore, $\beta Ru'[c_1] > u'(c_0)$, which contradicts the hypothesis that the agent is constrained since he can increase his utility by marginal reducing $c_0$and increasing c_1.

We now prove part (b). By the second order condition for a maximum, $\partial F/\partial h < 0$. Therefore, the sign of $\partial h/\partial a$is equal the sign of $\partial F/\partial a$. Direct derivation implies that:

$$\begin{aligned}\frac{\partial F}{\partial a} &= (\kappa af'(h) - 1)\kappa f(h)u''(w + \kappa af(h) - h) \\ &+ (\kappa f'(h))u'(w + \kappa af(h) - h) \\ &+ \beta f'(h)(1-\kappa R)u'[af(h)(1-\kappa R)] \\ &+ \beta af'(h)(1-\kappa R)f(h)(1-\kappa R)u'[af(h)(1-\kappa R)].\end{aligned}$$

Re-grouping terms, taking common factors and using the expressions for $c_0$and $c_1$when the constraint binds, the previous equation becomes

$$\frac{\partial F}{\partial a} = \kappa f'(h)u'(c_0) + (1-\kappa af'(h))\kappa f(h)[-u''(c_0)] + \beta(1-\kappa R)f'(h)\{u'[c_1] + c_1u''[c_1]\}.$$

The first two terms of the RHS are always positive, while the third term can be either positive or negative depending on the value of the IES. The first order condition implies that $u'(c_1) = \frac{(1-\kappa a f'(h))}{\beta(1-\kappa R) a f'(h)} u'(c_0)$. Therefore, once $\partial F/\partial a$ is multiplied and divided by $u'(c_1) > 0$, we obtain

$$\begin{aligned}
\frac{\partial F}{\partial a} &= u'(c_1)\left[\left(\frac{u'(c_0)}{u'(c_1)}\right)\kappa f'(h) + (1-\kappa a f'(h))\kappa\left(\frac{[-u''(c_0)]}{\frac{(1-\kappa a f'(h))}{\beta(1-\kappa R) a f'(h)} u'(c_0)}\right) f(h) + \beta(1-\kappa R) f'(h)[1-\sigma(c_1)]\right], \\
&= u'(c_1)\left[\begin{array}{c} \left(\frac{u'(c_0)}{u'(c_1)}\right)\kappa f'(h) + \kappa\beta f'(h) c_1\left(\frac{[-u''(c_0)]}{u'(c_0)}\right) \\ +\beta(1-\kappa R) f'(h)\left[1-\frac{c_1 u''(c_1)}{u'(c_1)}\right]\end{array}\right], \\
&= u'(c_1) f'(h)\left[\left(\frac{u'(c_0)}{u'(c_1)}\right)\kappa + \kappa\beta\frac{c_1}{c_0}\frac{1}{\eta(c_0)} + \beta(1-\kappa R)\left[1-\frac{1}{\eta(c_1)}\right]\right], \\
&\geq u'(c_1) f'(h)\left[(\beta R)\kappa + \kappa\beta\frac{c_1}{c_0}\frac{1}{\eta(c_0)} + \beta(1-\kappa R)\left[1-\frac{1}{\eta(c_1)}\right]\right], \\
&= \beta u'(c_1) f'(h)\left[\kappa\frac{c_1}{c_0}\frac{1}{\eta(c_0)} + 1 - \frac{1}{\eta(c_1)}(1-\kappa R)\right],
\end{aligned}$$

where the second line follows from simplification and $c_1 = (1-\kappa R) a f(h)$. The third line results from multiplying and dividing the second term inside brackets by $c_0$and using the definition of $\eta(\cdot)$. The fourth line results from using $u'(c_0)/u'(c_1) \geq \beta R$. The last line results from simplification. Since $\kappa\frac{c_1}{c_0}\eta(c_0)$is non-negative, the condition $\eta(c_1) > 1-\kappa R$suffices for $\partial F/\partial a > 0$as stated. This completes the proof of part (b).

Obviously, this is only a sufficient condition that is overly restrictive since we drop a non-negative term. To see that $\partial h^L/\partial a > 0$holds more generally, we now prove part (c). Assume that $\beta R \geq 1$and that $\eta(\cdot)$is non-decreasing. From $\beta R \geq 1$it is the case that $c_1 \geq c_0$, which implies that $\eta(c_1) \geq \eta(c_0)$, yielding

$$\frac{\partial F}{\partial a} \geq \beta u'(c_1) f'(h)\left\{1-\frac{1}{\eta(c_0)}[1-\kappa(1+R)]\right\},$$

which is strictly positive if either $\kappa(1+R) > 1$for any $\eta(\cdot) \geq 0$or if $\kappa(1+R) < 1$and $\inf_{c\geq}\{\eta(c)\} > [1-\kappa(1+R)]$as stated in the proposition.■

Proof of Proposition 5. The first part is trivial since $w < w^L_{\min}(a)$implies that $d^U(a,w) > d^L(a,w)$and, since $d_0 = d^L(a,w)$, $d^U(a,w) > d_0$, then $w < w^X_{\min}(a)$To shorten notation, we now suppress the dependence of the endogenous variable on (a,w). Contrary to the statement, assume that $h^L \leq h^X$. Then $c_0^X = w + d^X - h^X \leq w + d_0 - h^X \leq w + d^L - h^L \leq c_0^L$, which implies that $u'(c_0^X) \geq u'(c_0^L)$. Similarly, $c_1^X = a[h^X]^\alpha - Rd^X \geq a[h^L]^\alpha - Rd^L = c_1^L$, which implies that $u'(c_1^X) \leq u'(c_1^L)$.

From the FOC of the L and Xmodels:

$$\left(1-\kappa\alpha a[h^L]^{\alpha-1}\right)u'(c_0^L) = \beta\alpha a[h^L]^{\alpha-1}[1-\kappa R]u'(c_1^L).$$

Then,

$$\begin{aligned}
u'(c_0^L) &> \beta\left[\alpha a[h^L]^{\alpha-1}\right]u'(c_1^L) \\
&\geq \beta\left[\alpha a[h^X]^{\alpha-1}\right]u'(c_1^L) \\
&\geq \beta\left[\alpha a[h^X]^{\alpha-1}\right]u'(c_1^X) \\
&= u'(c_0^X),
\end{aligned}$$

a contradiction. The first inequality follows from the fact that $h^L < h^U$, which implies that $\left(1-\kappa\alpha a\left[h^L\right]^{\alpha-1}\right) < (1-\kappa R)$. The second inequality follows from the hypothesis that $h^L \leq h^X$and the third from $u'\left(c_1^X\right) \leq u'\left(c_1^L\right)$. The last equality follows from the FOC of the Xmodel. ■

A.5 GSL Plus Private Markets

Proof of Proposition 6. For the GSL to be default-proof for all (a,w), two conditions must be satisfied. First, that at the time of choosing borrowing and investment, the discounted utility $U^G(a,w)$attained by the allocation $\left\{h^G(a,w), d^G(a,w)\right\}$dominates the maximum level of utility by defaulting, $U^{D,GSL}(a,w)$. This is, for any (a,w), the following inequality holds:

$$U^G(a,w) \geq U^{D,GSL}(a,w) \equiv \max_{\{h_d \geq d_{\max}\}} \left\{u\left[w+d_{\max}-h_d\right]+\beta u\left[af\left(h_d\right)(1-\tilde{\kappa})\right]\right\}$$

The second condition is that the repayment of debt at period two, given allocation $\left\{h^G(a,w), d^G(a,w)\right\}$, is time-consistent in the sense that, as of the second period, the agent is better-off repaying than defaulting. This is,

$$u\left[af\left(h^G(a,w)\right)-Rd^G(a,w)\right] \geq u\left[af\left(h^G(a,w)\right)(1-\tilde{\kappa})\right]$$

To verify the first condition, let $\{h_d(a,w), d_d(a,w)\}$be the generitc solution of this maximization. Since $h_d(a,w)$is feasible under the GSL guidelines, then $h_d(a,w) \leq w+d_{\max}$and, since the agent foresees that the will default, then $d_d(a,w)=d_{\max}$. First, consider the feasibility (consumption levels in both periods are non-negative) of this strategy if the agent were to take it and repay. First period consumption is clearly non-negative. Second period consumption would be $c_1 = af\left(h_d(a,w)\right)-Rd_{\max}$and it is guaranteed to be non-negative if $af\left(d_{\max}\right) \geq Rd_{\max}$, i.e. if $a \geq a_{LL} \equiv Rd_{\max}/f\left(d_{\max}\right)$. Now, since $\{h_d(a,w), d_d(a,w)\}$is feasible, then

$$U^G(a,w) \geq u\left[w+d_{\max}-h_d(a,w)\right]+\beta u\left[af\left[h_d(a,w)\right]-Rd_{\max}\right],$$

so a sufficient condition for $U^G(a,w) \geq U^{D,GSL}(a,w)$is that $u\left[w+d_{\max}-h_d(a,w)\right]+\beta u\left[af\left[h_d(a,w)\right]-Rd_{\max}\right] \geq u\left[w+d_{\max}-h_d(a,w)\right]+\beta u\left[af\left[h_d(a,w)\right](1-\tilde{\kappa})\right]$, which, of course, boils down to $af\left[h_d(a,w)\right]-Rd_{\max} \geq af\left[h_d(a,w)\right](1-\tilde{\kappa})$. Obviously, this last inequality holds for all $a \geq a_L \equiv \frac{Rd_{\max}}{\tilde{\kappa}f(d_{\max})}$. Under this condition, this strategy is always feasible under repayment and therefore, weakly dominated by $\left\{h^G(a,w), d^G(a,w)\right\}$. Notice that this condition is only sufficient as even if $a < a_L$, the maximum utility under defaulting can be easily dominated by the utility under repaying. To verify the second condition first notice that it boils down to $\tilde{\kappa}af\left(h^G(a,w)\right) \geq Rd^G(a,w)$. Recall that $h^G(a,w) \leq h^U(a)$implying that $af'\left[h^G(a,w)\right] \geq R$. Since $d^G(a,w) \leq h^G(a,w)$, then a sufficient condition for the second period repayment condition is that $\tilde{\kappa}af\left(h^G(a,w)\right) \geq \left[af'\left[h^G(a,w)\right]\right]h^G(a,w)$, i.e.

$$\tilde{\kappa} \geq \left[\frac{f'\left[h^G(a,w)\right]}{f\left(h^G(a,w)\right)}\right]h^G(a,w),$$

which holds if for any $h$the elasticity of the skill production function with respect to h, $[f'(h)h/f(h)]$is below the garnishment fraction $\tilde{\kappa}$.■

Proof of Proposition 7. The fact that $h^{L+G}(a,w;d_{\max}) \leq h^U(a)$follows from the fact that, as with other forms of borrowing constraints, it will never be optimal to over-invest in human capital. Now, define $F(h,d_{\max})$as

$$F \equiv \left(\kappa af'(h)-1\right)u'\left[w+d_{\max}+\kappa af[h]-h\right]+\beta af'(h)(1-\kappa R)u'\left[af(h)(1-\kappa R)-Rd_{\max}\right].$$

The first order condition that determines $h^{L+G}(a,w;d_{\max})$is $F\left[h^{L+G}(a,w;d_{\max}), d_{\max}\right]=0$. From the implicit function theorem, we have $\partial h^{L+G}(a,w;d_{\max})/\partial d_{\max} = -\left[\partial F/\partial d_{\max}\right]/\left[\partial F/\partial h\right]$. Since $h$is optimally chosen, $\partial F/\partial h < 0$at $h = h^{L+G}(a,w;d_{\max})$. Therefore, $sign\left\{\partial h^{L+G}(a,w;d_{\max})/\partial d_{\max}\right\} =$

$sign\left\{\partial F/\partial d_{\max}\right\}$. Notice

$$\frac{\partial F}{\partial d_{\max}} = \left[1-\kappa a f'(h)\right]\left[-u''(c_0)\right] + \beta a f'[h](1-\kappa R) R\left[-u''(c_1)\right] > 0,$$

which proves (b). This result also implies that, for any $d_{\max} > 0$, $h^{L+G}(a, w; d_{\max}) > h^{L+G}(a, w; 0) = h^{L}(a, w)$which completes the proof for (a).■

B Specifics of the Quantitative Model

B.1 Unrestricted Allocations

We formulate the problem in sequence form. Given (a, w) an individual maximizes the $t_0 = 0$ value of utility (9) subject to

$$\int_0^T e^{-\rho t} c(t)\, dt + \int_0^1 e^{-\rho t} x(t)\, dt \leq w + \int_1^P e^{-\rho t} y(t)\, dt. \tag{23}$$

With equal interest and discount rates, the optimal path of consumption is a constant over time. Since $g_s = \rho$, optimal total investment h is attainable with a constant flow $x(t) = x \geq 0$ for all $t \in [0, 1]$. Define

$$\Phi \equiv \begin{cases} \left[e^{(g-\rho)(P-1)} - 1\right]/\left[g-\rho\right] & \text{if } g \neq \rho \\ P-1 & g = \rho \end{cases}.$$

With this parameter, the optimal out-of-pocket investment solves

$$x^U(a) = \arg\max_{x \geq 0} \left\{ w + \frac{e^{-\rho}}{\mu}\left[a\Phi\left[i_{pub} + (1+s)x\right]^{\alpha} - x\right]\right\}.$$

Total investment is

$$h^U(a) = \max\left\{i_{pub}, \left[\alpha(1+s)a\Phi\right]^{\frac{1}{1-\alpha}}\right\}.$$

It is easy to see that individuals with abilities below $a_0 \equiv \frac{[i_{pub}]^{1-\alpha}}{\alpha(1+s)\Phi}$ would not invest from their own pockets and their human capital attainment would be i_{pub}.

For all $t \in [0, T]$, the optimal consumption path is

$$c^U(t, a, w) = \frac{\rho}{1-e^{-\rho T}}\left\{w + \frac{e^{-\rho}}{\mu}\left[a\Phi\left[i_{pub} + (1+s)x^U(a)\right]^{\alpha} - x^U(a)\right]\right\}.$$

Then, at age $t = 1$, i.e. at the time when labor market participation begins, the debt of the individual would be given by

$$d^U(a, w) = \left(\frac{1-e^{-\rho}}{\mu\left(1-e^{-\rho T}\right)}\right) a\Phi\left[h^U(a)\right]^{\alpha} + \left(\frac{e^{-\rho} - e^{-\rho T}}{\mu\left(1-e^{-\rho T}\right)}\right)\left(\frac{h^U(a) - i_{pub}}{1+s}\right) - \left(\frac{1-e^{-\rho(T-1)}}{1-e^{-\rho T}}\right) w, \tag{24}$$

a function that is decreasing in w and increasing in a. It is straigthforward to verify that $\frac{\partial d^U(a,w)}{\partial a} > \frac{\partial h^U(a)}{\partial a}$.

It is also straigthforward to verify that the solutions of this sequence problem are identical to the ones in the main body of the paper.

B.2 Exogenous and GSL Constraints

The expression for $w^X(a)$ is given by

$$w^X(a) = \left(\frac{1-e^{-\rho}}{\mu\left(1-e^{-\rho(T-1)}\right)}\right) a\Phi\left[h^U(a)\right]^{\alpha} + \left(\frac{h^U(a) - i_{pub}}{\mu(1+s)}\right) - d_0\left(\frac{1-e^{-\rho T}}{1-e^{-\rho(T-1)}}\right).$$

B.3 Private Lending with Limited Commitment

The highest discounted utility that can be attained by an individual that defaults at $t = 1$ is

$$V^D(a,h) = \hat{\Theta}_{\gamma,\pi} \frac{\left[\mu^{-1} a \Phi h^{\alpha}\right]^{1-\sigma}}{1-\sigma},$$

where

$$\hat{\Theta}_{\gamma,\pi} \equiv \left(\frac{1-\gamma}{\Phi}\right)^{1-\sigma} \left(\frac{1-e^{[g(1-\sigma)-\rho]\pi}}{\rho - g(1-\sigma)}\right) + e^{[g(1-\sigma)-\rho]\pi} \left(\frac{1-e^{-\rho(T-1-\pi)}}{\rho}\right)^{\sigma} \left(\frac{e^{(g-\rho)(P-1-\pi)}-1}{e^{(g-\rho)(P-1)}-1}\right)^{1-\sigma}.$$

As claimed in the text, direct inspection implies that: (a) $\hat{\Theta}_{\gamma,\pi} < \Theta$, (b) $\hat{\Theta}_{\gamma,\pi}$ is decreasing in γ, (c) for all $\gamma \in (0,1)$, $\hat{\Theta}_{\gamma,\pi}$ converges to Θ as $\pi \to 0$.

The borrowing limit is

$$d \leq \kappa\left[\mu^{-1} a \Phi h^{\alpha}\right], \tag{25}$$

where

$$\kappa \equiv 1 - \left[\hat{\Theta}_{\gamma,\pi}/\Theta\right]^{\frac{1}{1-\sigma}} \geq 0$$

Therefore, as of $t = 0$, the maximization problem consists in choosing a consumption c_0 for all $t \in [0,1]$, and investment and borrowing levels (x, d), such that

$$[BC]: \frac{e^{-\rho}}{\mu}\left[c_0 + x\right] \leq w + e^{-\rho} d \tag{26}$$

$$[CC]: d \leq \kappa\left[\mu^{-1} a \Phi\left[i_{pub} + (1+s)x\right]\right]^{\alpha}. \tag{27}$$

Aside from the government subsidies (s, i_{pub}) and the determination of Θ, Φ, and κ, this problem is equivalent to the two-period model of Section 4.5.

The value $w^L_{\min}(a)$ defined by $d^U\left(a, w^L_{\min}(a)\right) = \kappa\mu^{-1} a \Phi_a \left[h^U(a)\right]^{\alpha}$ is the threshold of wealth above which an agent is unconstrained. It is equal to

$$w^L_{\min}(a) = \begin{cases} a\Phi\left[i_{pub}\right]^{\alpha}\left[\frac{\left(1-e^{-\rho}\right)-\kappa\left(1-e^{-\rho T}\right)}{\mu\left(1-e^{-\rho(T-1)}\right)}\right] & \text{for } a \leq a_0 \\ h^U(a)\left[\frac{1-\kappa-(1-\alpha)e^{-\rho}+(\kappa-\alpha)e^{-\rho T}}{\mu\alpha(1+s)\left(1-e^{-\rho(T-1)}\right)}\right] - \frac{e^{-\rho}}{\mu}\left(\frac{i_{pub}}{1+s}\right) & \text{for } a > a_0. \end{cases}$$

Individuals with $w \geq w^L_{\min}(a)$ attain the unrestricted allocations. For those with $w < w^L_{\min}(a)$, constraint (25) holds with equality and we can use it to eliminate d. With this, the problem becomes

$$\max_{\{x: x \geq 0\}} \left\{\frac{e^{-\rho}}{\mu} \frac{\left[e^{\rho}\mu w + \kappa a \Phi_a \left[i_{pub} + (1+s)x\right]^{\alpha} - x\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta \frac{\left[(1-\kappa)\mu^{-1} a \Phi_a \left[i_{pub} + (1+s)x\right]^{\alpha}\right]^{1-\sigma}}{1-\sigma}\right\}.$$

Proof of Proposition 8. We allow for $\Phi_a$to depend on a. To shorten notation define the following variables:

$$\begin{aligned} h &\equiv i_{pub} + (1+s)x, \\ A &\equiv a\Phi_a, \\ c_0 &\equiv e^{\rho}\mu w + \kappa A h^{\alpha} - x, \\ m_1 &\equiv (1-\kappa)\mu^{-1} A h^{\alpha} \\ \delta &\equiv \alpha A h^{\alpha-1}(1+s). \end{aligned}$$

With those variables, the problem of the agent when the constraint binds is:

$$\max_{\{x\}}\left\{\frac{e^{-\rho}}{\mu}\frac{[c_0]^{1-\sigma}}{1-\sigma}+e^{-\rho}\Theta\frac{[m_1]^{1-\sigma}}{1-\sigma}\right\}$$

Using the FOC of this maximization, define

$$F\equiv[\kappa\delta-1][c_0]^{-\sigma}+\Theta[m_1]^{-\sigma}(1-\kappa)\delta.$$

Optimality requires that either $F<0$and $x=0$(i.e. $h=i_{pub}$) or that $F=0$and $x>0$(i.e. $h>i_{pub}$).

We now prove part (a). If the credit constraint binds, then $[c_0]^{-\sigma}>\Theta[m_1]^{-\sigma}$. If $F<0$, then $h^L(a,w)=i_{pub}$, and the result is trivial. If $F=0$, then $[1-\kappa\delta]<(1-\kappa)\delta$, implying that $\delta>1$. For the unconstrained case define

$$\begin{aligned}c_0^U(a,w)&=\mu e^{\rho}w+\mu d^U(a,w)-x^U(a),\\ m_1^U(a,w)&\equiv\mu^{-1}A\left[h^U(a)\right]^{\alpha}-d^U(a,w),\\ \delta^U(a)&\equiv\alpha A\left[h^U(a)\right]^{\alpha-1}(1+s).\end{aligned}$$

Given that $\left[c_0^U(a,w)\right]^{-\sigma}=\Theta\left[m_1^U(a,w)\right]^{-\sigma}$, the first order condition implies that $\delta^U(a)\leq1$. Thus, $\delta>\delta^U(a)$and hence $h^L(a,w)<h^U(a)$. We now prove part (b). For a maximization, we have the condition $\frac{\partial^2F}{\partial h^2}<0$and therefore $sign\left\{\frac{\partial h}{\partial A}\right\}=sign\left\{\frac{\partial F}{\partial A}\right\}$. The latter derivative is

$$\begin{aligned}\frac{\partial F}{\partial A}=&\left[\alpha\kappa Ah^{\alpha-1}(1+s)-1\right]\left\{-\sigma[c_0]^{-\sigma-1}\frac{\partial c_0}{\partial A}\right\}+\alpha\kappa h^{\alpha-1}(1+s)[c_0]^{-\sigma}\\ &+\Theta[m_1]^{-\sigma}\alpha(1-\kappa)h^{\alpha-1}(1+s)+\alpha(1-\kappa)Ah^{\alpha-1}(1+s)\left\{-\sigma\Theta[m_1]^{-\sigma-1}\frac{\partial m_1}{\partial A}\right\}\end{aligned}$$

First, from the first order condition we can use the equality

$$\left[\alpha\kappa Ah^{\alpha-1}(1+s)-1\right][c_0]^{-\sigma}=-\Theta[m_1]^{-\sigma}\alpha(1-\kappa)Ah^{\alpha-1}(1+s)$$

in the first term to get

$$\begin{aligned}\frac{\partial F}{\partial A}=&\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\sigma\kappa\frac{(1-\kappa)Ah^{\alpha}}{c_0}+\alpha\kappa h^{\alpha-1}(1+s)[c_0]^{-\sigma}\\ &+\Theta[m_1]^{-\sigma}\alpha(1-\kappa)h^{\alpha-1}(1+s)-\alpha(1-\kappa)Ah^{\alpha-1}(1+s)\sigma\Theta[m_1]^{-\sigma}\frac{(1-\kappa)\mu^{-1}h^{\alpha}}{m_1}\end{aligned}$$

Then, take $\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)>0$as a common factor:

$$\begin{aligned}\frac{\partial F}{\partial A}&=\left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\frac{(1-\kappa)Ah^{\alpha}}{c_0}+\kappa\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}}+(1-\kappa)-\sigma(1-\kappa)\frac{(1-\kappa)\mu^{-1}Ah^{\alpha}}{m_1}\right\}\\ &=\left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\mu\frac{m_1}{c_0}+\kappa\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}}+(1-\kappa)-\sigma(1-\kappa)\right\}\end{aligned}$$

where the second line follows by multiplying and dividing by $\mu$and then using the definition of m_1.Now, we have that $\frac{[c_0]^{-\sigma}}{\Theta[m_1]^{-\sigma}}\geq1$, and $\frac{m_1}{c_0}\geq\Theta^{\frac{1}{\sigma}}$. With these inequalities we can find a lower bound to $\partial F/\partial A$:

$$\begin{aligned}\frac{\partial F}{\partial A}&\geq\left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\mu\Theta^{\frac{1}{\sigma}}+\kappa+(1-\kappa)-\sigma(1-\kappa)\right\}\\ &=\left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{\sigma\kappa\frac{\rho}{e^{\rho}-1}\left(\frac{1-e^{-\rho T}}{\rho}\right)+1-\sigma(1-\kappa)\right\}\\ &=\left\{\Theta[m_1]^{-\sigma}\alpha h^{\alpha-1}(1+s)\right\}\left\{1-\sigma\left[1-\kappa\left(\frac{1-e^{-\rho(T-1)}}{1-e^{-\rho}}\right)\right]\right\}\end{aligned}$$

where in the second line we have used $\Theta \equiv \left(\frac{1-e^{-\rho T}}{\rho}\right)^{\sigma}$ and $\mu = \rho/\left[e^{\rho}-1\right]$and the third line results from re-grouping and simplification. Therefore, as claimed in the text, the derivative $\frac{\partial F}{\partial A}$is positive if either $\kappa > \left(\frac{e^{\rho}-e^{-\rho T}}{e^{\rho}-1}\right)^{-1}$ with any value of $\sigma \geq 0$or if $\kappa \leq \left(\frac{e^{\rho}-e^{-\rho T}}{e^{\rho}-1}\right)^{-1}$ but $\sigma \leq \left[1-\kappa\left(\frac{1-e^{-\rho(T-1)}}{1-e^{-\rho}}\right)\right]^{-1}$ ■

B.4 GSL Programs Plus Private Lending

Proof of Proposition 9. As in the two-period model, two conditions must be satisfied for all (a, w). The first is that at the time of choosing borrowing and investment, the discounted utility $U^{G}(a, w)$attained by the allocation $\left\{h^{G}(a, w), x^{G}(a, w), d^{G}(a, w)\right\}$(where the reundant term $x^{G}(\cdot)$is added for future reference) dominates the maximum level of utility by defaulting, $U^{D,GSL}(a, w)$. In this model,

$$U^{G}(a, w) = \left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^{\rho}\mu w + d^{G}(a, w) - x^{G}(a, w)\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta\frac{\left[\mu^{-1}a\Phi h^{G}(a, w)^{\alpha} - d^{G}(a, w)\right]^{1-\sigma}}{1-\sigma}\right\}$$

$$U^{D,GSL}(a, w) \equiv \max_{\{x \geq d_{\max}\}}\left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^{\rho}\mu w + d_{\max} - x\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\hat{\Theta}_{\gamma,\pi}\frac{\left[\mu^{-1}a\Phi\left[i_{pub} + (1+s)x\right]^{\alpha}\right]^{1-\sigma}}{1-\sigma}\right\}.$$

The second condition is that the repayment of debt, given allocation $\left\{h^{G}(a, w), d^{G}(a, w)\right\}$, is time-consistent in the sense that at period two the agent is better-off repaying than defaulting. This is,

$$\Theta\frac{\left[\mu^{-1}a\Phi h^{G}(a, w)^{\alpha} - d^{G}(a, w)\right]^{1-\sigma}}{1-\sigma} \geq \hat{\Theta}_{\gamma,\pi}\frac{\left[\mu^{-1}a\Phi\left[h^{G}(a, w)\right]^{\alpha}\right]^{1-\sigma}}{1-\sigma}.$$

The proof is in parallel to the two period case. For the first condition, let $\left\{x^{d}(a, w), d_{\max}\right\}$denote the optimal investment and borrowing of an individual who plans to default. This strategy implies a non-negative consumption during youth. The pair $\left\{x^{d}(a, w), d_{\max}\right\}$also implies a non-negative post-schooling consumption for an agent who repays if

$$\mu^{-1}a\Phi\left[i_{pub} + (1+s)x^{d}(a, w)\right]^{\alpha} - d_{\max} \geq 0,$$

Since $x^{d}(a, w) \geq d_{\max}$, the previous inequality always holds if

$$a \geq a_{LL}^{Q} \equiv \frac{\mu d_{\max}}{\Phi\left[i_{pub} + (1+s)d_{\max}\right]^{\alpha}}.$$

In such circumstances, $\left\{x^{d}(a, w), d_{\max}\right\}$is always a feasible, and therefore, weakly dominated for a repaying individual, i.e.

$$U^{G}(a, w) \geq \left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^{\rho}\mu w + d_{\max} - x^{d}(a, w)\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta\frac{\left[\mu^{-1}a\Phi h^{d}(a, w)^{\alpha} - d_{\max}\right]^{1-\sigma}}{1-\sigma}\right\}.$$

Therefore, a sufficient condition for the first period dominance of default is

$$\left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^{\rho}\mu w + d_{\max} - x^{d}(a, w)\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\Theta\frac{\left[\mu^{-1}a\Phi h^{d}(a, w)^{\alpha} - d_{\max}\right]^{1-\sigma}}{1-\sigma}\right\}$$
$$\geq \left\{\frac{e^{-\rho}}{\mu}\frac{\left[e^{\rho}\mu w + d_{\max} - x^{d}(a, w)\right]^{1-\sigma}}{1-\sigma} + e^{-\rho}\hat{\Theta}_{\gamma,\pi}\frac{\left[\mu^{-1}a\Phi h^{d}(a, w)^{\alpha}\right]^{1-\sigma}}{1-\sigma}\right\},$$

which boils down

$$\Theta\frac{\left[\mu^{-1}a\Phi h^{d}(a, w)^{\alpha} - d_{\max}\right]^{1-\sigma}}{1-\sigma} \geq \hat{\Theta}_{\gamma,\pi}\frac{\left[\mu^{-1}a\Phi h^{d}(a, w)^{\alpha}\right]^{1-\sigma}}{1-\sigma},$$

or, equivalently

$$\kappa\mu^{-1}a\Phi h^{d}(a, w)^{\alpha} \geq d_{\max}$$

where we recalled the definition $\kappa \equiv 1 - \left[\hat{\Theta}_{\gamma,\pi}/\Theta\right]^{\frac{1}{1-\sigma}}$. Since $h^{d}(a,w) \geq i_{pub} + (1+s)\,d_{\max}$, this condition always holds if

$$a \geq a_L^Q \equiv \frac{\mu d_{\max}}{\kappa \Phi \left[i_{pub} + (1+s)\,d_{\max}\right]^{\alpha}} = a_{LL}^Q/\kappa > a_{LL}^Q.$$

As before, with the two-period model, we note that this is only a sufficient condition and that default can be easily dominated for $a < a_L^Q$.

We now consider the optimality of repaying $d^G(a,w)$given the option of defaulting. The condition $\Theta \frac{\left[\mu^{-1} a \Phi h^G(a,w)^{\alpha} - d^G(a,w)\right]^{1-\sigma}}{1-\sigma} \geq \hat{\Theta}_{\gamma,\pi} \frac{\left[\mu^{-1} a \Phi \left[h^G(a,w)\right]^{\alpha}\right]^{1-\sigma}}{1-\sigma}$, boils down to

$$\kappa \mu^{-1} a \Phi h^G(a,w)^{\alpha} \geq d^G(a,w).$$

Notice that this inequality trivially holds if $h^G(a,w) = i_{pub}$, since $x^G(a,w) = d^G(a,w) = 0$. Now, assume $x^G(a,w) > 0$.Since $h^G(a,w) \leq h^U(a) \geq$, then $\alpha a \Phi \left[i_{pub} + (1+s)\,x^G(a,w)\right]^{\alpha-1}(1+s) \geq 1$(with equality iff $h^U(a) \geq h^G(a,w)$). From this inequality we get $a\Phi \geq \frac{\left[i_{pub}+(1+s)x^G(a,w)\right]^{1-\alpha}}{\alpha(1+s)}$, which used to replace $a\Phi$, and after some rearranging, implies the sufficient condition

$$i_{pub} \geq (1+s)\left\{\frac{\alpha\mu}{\kappa} d^G(a,w) - x^G(a,w)\right\}.$$

Since the GSL imposes $d^G(a,w) \leq x^G(a,w)$, this inequality always holds if $\kappa \geq \alpha\mu$. If instead, $\kappa < \alpha\mu$, then the inequality holds if

$$i_{pub} \geq (1+s)\left\{\frac{\alpha\mu - \kappa}{\kappa}\right\} d_{\max},$$

as claimed in the proposition.■

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Table C1: Educational Expenditures by Year of Schooling and AFQT Quartile (1999 Dollars)

Years of School	Direct Expenditures	Foregone Earnings by AFQT Quartile:				Total Costs by AFQT Quartile:			
		Quart. 1	Quart. 2	Quart. 3	Quart. 4	Quart. 1	Quart. 2	Quart. 3	Quart. 4
8	59,075	0	0	0	0	59,075	59,075	59,075	59,075
9	65,239	0	0	0	0	65,239	65,239	65,239	65,239
10	71,167	2,197	3,080	3,526	3,353	73,364	74,246	74,693	74,520
11	76,866	5,058	7,088	8,116	7,717	81,924	83,955	84,982	84,584
12	82,347	8,638	12,106	13,862	13,181	90,985	94,453	96,208	95,527
13	97,315	12,948	18,147	20,778	19,757	110,263	115,462	118,093	117,072
14	111,708	17,936	25,138	28,783	27,369	129,645	136,846	140,491	139,077
15	125,548	23,489	32,919	37,692	35,841	149,036	158,467	163,240	161,388
16	138,855	29,431	41,247	47,228	44,908	168,286	180,102	186,083	183,763
17	151,650	35,547	49,818	57,042	54,240	187,197	201,468	208,692	205,890
18	163,953	41,599	58,301	66,754	63,475	205,552	222,254	230,707	227,428
19	175,783	47,359	66,373	75,996	72,263	223,142	242,156	251,780	248,046
20	187,158	52,629	73,759	84,454	80,306	239,787	260,918	271,612	267,464

Notes:

1) Direct expenditures assume average expenditure per pupil in primary and secondary schooling through grade 12. Additional expenditures for higher grades are taken from average expenditures per student in all colleges and universities. Expenditures based on averages for school years 1979-80 to 1988-89. (Source: Tables 170 and 342, Digest of Education Statistics, 1999.)

2) Foregone earnings are calculated from regression of log(earnings) on AFQT quartile, education indicators, experience and experience-squared. Foregone earnings are based on someone with 9 years of schooling and the corresponding level of experience. Sample includes not enrolled youth ages 16-24.

3) Expenditures are discounted at a 4% annual interest rate to grade 10.