Do Credit Market Imperfections and Balance Sheet Effects Explain why Emerging Markets React so Sharply to Negative Shocks?

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Abstract

This paper develops a fully microfounded two-sector DGE model for a small open economy subject to credit market imperfections (CMIs) and balance sheet problems (due to the presence of a currency mismatch). Specifically, some agents in this economy face credit constraints to finance investment due to information asymmetries and, while their assets are denominated in the domestic currency, their liabilities are denominated in the foreign currency. The article investigates whether these two factors rationalize why emerging markets (EMs) react more sharply to negative shocks relative to developed economies and if, at the same time, these frictions may explain why EMs face a countercyclical credit spread to finance investment. The model's behavior is studied while considering an unexpected increase in the foreign interest rate under a *floating* and a *fixed* exchange rate regime. The paper shows that CMIs and balance sheet problems can generate amplification relative to a case in which financial markets work perfectly, but at most it is moderate. Nonetheless, the degree of amplification strongly depends on the exchange rate that prevails in the economy, being always larger under a *floating* regime. It is also shown that the model provides a good account of why it is more expensive to finance investment when a negative shock hits the economy, a fact observed in many EMs.

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1 Introduction

One of the most puzzling questions in international finance is why the business cycles of emerging markets (EMs) and developed economies (DEs) differ so much. To illustrate this idea, Neumeyer and Perri (2005) have recently compared time series data for EMs and small open DEs,[1] finding that the most striking differences between the two groups are: (i) output, investment, and net exports are *more* volatile in EMs, (ii) net exports are *more* countercyclical in EMs and (iii) real interest rates are *countercyclical* in EMs but *procyclical* in DEs.

The objective of the present article is to develop a theoretical two-sector dynamic general equilibrium (DGE) model, to rationalize these particular differences in the behavior of EMs and DEs. With this objective in mind, the DGE constructed here is designed for a small open economy and incorporates credit market imperfections (CMIs) and currency mismatches[2] as its two most noteworthy features. The decision to introduce these elements is twofold. First, since the works of Krugman (1999) and Aghion *et al.* (2000), a number of researchers has successfully included them to explain why EMs respond so sharply to negative shocks. Second, there is important evidence that credit constraints and currency misalignments are severe and widespread in EMs, especially in the nontradable sector, where unexpected changes in the exchange rate can be particularly harmful[3]. Due to better regulations and a relatively larger financial system, one would expect DEs to be less exposed to these frictions.

To see why this framework may provide insights that are absent in more conventional DGE setups, consider the following intuitive description of the model. Assume that the economy is populated by some agents (hereafter entrepreneurs) that invest to produce capital. To do this, they require external funding. This is constrained, however, due to the existence of an asymmetric information problem between the lender and the entrepreneur regarding the technology to produce capital. Entrepreneurs will also consume the final good in direct proportion to their net worth (i.e., the difference between the real value of their assets and liabilities). For exogenous reasons, lending is entirely in the foreign currency[4]. Due to this friction, the lender optimally charges a *spread* for lending funds, relatively to the risk-free interest rate. Once entrepreneurs produce capital, they rent it to the domestic nontradable sector, and hence the real value of their assets is fully indexed to the domestic currency.

Suppose that an adverse shock hits the economy, requiring a *real depreciation* of the currency as part of the adjustment process. Also assume that due to an exogenous reason, the price of the nontradable good is unable to adjust within the period of the shock, but it becomes fully flexible afterwards[5]. Given the value of entrepreneurs' assets, the depreciation raises the real value of their debt. Their net worth falls, and external

[1] The EMs are: Argentina, Brazil, Korea, Mexico and Philippines. The small open DEs are: Australia, Canada, Netherlands, New Zealand and Sweden. The paper considers quarterly data for the period 1983Q3-2001Q4; with the exceptions of Brazil, Korea, Mexico and Philippines, cases in which the period is 1994Q1-2001Q4.

[2] By a currency mismatch the paper refers to a situation in which the currency of denomination of assets and liabilities, for a particular agent in the economy, differs.

[3] Using 2001 intra-firm data, a recent study of the Interamerican Development Bank (see IADB, 2005) reports the share of dollar-debt in the nontradable sector for a number of Latin American countries. It ranges from around 0% in Brazil, Chile and Colombia to about 67% in the case of Argentina and more than 70% in the case of Uruguay. It is well known, moreover, that the large private (and public) currency mismatch of Argentina in 2001 highly contributed to the deepening of the financial crisis that lead to the 11% real GDP contraction in 2002; and also to the widespread private and public debt defaults on previously-contracted dollar-debt that followed the devaluation of January 2002.

[4] To see how sensitive results are to this assumption, the paper also considers the case in which entrepreneurs' debt is totally denominated in the domestic currency.

[5] This assumption will be explained in more detail later in the paper.

credit becomes even more restricted and expensive than before; as a result, the initial investment drop due to the adverse shock is magnified. This channel is not present when financial markets work perfectly. In such a case the *Modigliani-Miller* paradigm holds, implying that the evolution of entrepreneurs' net worth is irrelevant to determine financing opportunities: any profitable project is always funded. Current and future output fall, since there is less investment today but also a lower future supply of capital. The demand for tradable goods will shrink for the lower economic activity and the more depreciated real exchange rate (i.e., the current account improves); all these effects being amplified through credit markets.

More specifically, the paper studies how the model reacts to a temporary but unexpected increase in the foreign interest rate. The election of this shock facilitates the comparison with the related literature, since it has been widely addressed by other researchers. It is also justified on empirical grounds. Recent works document that changes in the foreign interest rate may account for 20% to 50% of the observed output volatility in EMs (Neumeyer and Perri, 2005; Uribe and Yue, 2006). Moreover, from a policy-oriented perspective, some well-recognized observers point out that the current financial turmoil affecting major industrialized countries may lead to a higher foreign interest faced by EMs[6]. This exogenous perturbation can then become a potential vehicle for *contagion*, worth exploring in this setup.

Since entrepreneurs may suffer from a debt burden as the exchange rate depreciates, the paper evaluates results considering both a pure *floating* and a pure *fixing* exchange rate regime. Intuitively, a *fixed* exchange rate might reduce their exposure to a negative balance sheet effect in the wake of the adverse shock, helping reduce amplification. To be more specific, the questions that guide the analysis herein can be summarized as follows:

(i) How do CMIs and balance sheet problems affect the economy and amplify shocks?

(ii) Do results depend on the exchange rate regime that prevails in the economy?

(iii) How different would results be if entrepreneurs' debt was instead in domestic currency?

Although the main motivation of this work relies on a practical and policy-oriented problem, its methodology follows a scholar approach. The DGE model developed here is fully microfounded yet extremely tractable, with the financial friction derived from first principles following Carlstrom and Fuerst (1997) and Bernanke *et al.* (1999). That is, the model incorporates the celebrated *financial accelerator mechanism*. Although they assume a closed economy environment, a number of researchers have successfully extended the approach to open economy settings (Céspedes *et al.,* 2004; Choi and Cook, 2004; Cook, 2004; Devereux *et al.*, 2006; Gertler *et al.,* 2007).

An important innovation of this paper is, however, to provide a well microfounded model in which the short and long-run implications of financial frictions can be analyzed *qualitatively.* Furthermore, its linearized version strongly resembles that of a standard Ramsey model for a small open economy (e.g., Blanchard and Fisher, 1989, pp. 58-69), with one important exception: entrepreneurs' net worth becomes a key additional state variable that provides additional dynamics. This very simple solution gives the possibility to understanding thoroughly the forces at work in the model, clearly identifying how credit spreads, balance sheet effects and the degree of CMIs are related to the fundamentals of the economy[7]. The main findings of the paper can be summarized as follows.

[6]See Calvo and Talvi (2006).

[7]The fact that the relation between credit spreads and the economy's fundamentals is clearly identified differentiates this paper from Neumeyer and Perri (2005) and Uribe and Yue (2006); models in which the evolution of credit spreads is *exogenously* postulated.

The first novel result is to show that negative balance sheet effects arise not only from unexpected changes in the real value of entrepreneurs' liabilities, but also from unanticipated fluctuations in the real value of their assets, an issue that has received little attention in the related literature. This latter channel strongly resembles the amplification mechanism emphasized in the closed-economy contribution of Kiyotaki and Moore (1997). Whether balance sheet effects arise from the assets or the liabilities side depends, moreover, on the *exchange rate regime* and the currency of *denomination* of entrepreneurs' debt.

To see this, assume first that the exchange rate is *floating* and that entrepreneurs' debt is in foreign currency. The immediate effect of the rise in the foreign interest rate is to give the incentive to domestic households to accumulate foreign bonds, which is only possible if the economy runs a current account surplus. The exchange rate depreciates and entrepreneurs suffer from a debt burden (i.e., the liabilities channel). As households increase total saving, however, their demand for current consumption falls. This effect is magnified since entrepreneurs also reduce their demand for consumption and investment. Lower economic activity implies lower demand for capital, and its equilibrium price decreases accordingly (note that its supply is given at each point in time). As a consequence of this, entrepreneurs' net worth suffers from an additional negative effect owing to the reduction in the real value of their assets (i.e., the assets channel). These results perhaps help explain formally why, as Calvo and Reinhart (2002) accurately point out, EMs exposed to large currency misalignments face 'fear of floating'.

Suppose now that the exchange rate is *pegged* before and after the shock. The real exchange rate is not affected and so is the liabilities side of entrepreneurs' balance sheets. The burden of the adjustment process lies, however, on real economic activity. The price of capital plummets, and so does the real value of entrepreneurs' assets. The paper shows, moreover, that their net worth decreases roughly as under *floating* for the same size of the shock. To put it simply, *pegging* the exchange rate alleviates one source of net worth fluctuations at the cost of exacerbating the other.

How do results change if debt is in domestic currency? Observe that negative balance effects do not arise anymore through the liabilities side. The paper shows, besides, that under *floating* the dynamics of the model are essentially the same as if financial frictions were absent. From a policy perspective, this finding highlights the relevance of preserving an adequate currency alignment in the denomination of assets and liabilities, especially in the nontradable sector. If the exchange rate is *pegged,* the behavior of the model is, however, independent of the currency in which liabilities are denominated. This result is somehow expected, since under *fixing* entrepreneurs' liabilities are equally unaffected in both cases.

A second key result is to show that credit market imperfections and balance sheet effects do *not necessarily* generate amplification. The shock might be magnified only if the initial level of entrepreneurs' debt relative to net worth (i.e., entrepreneurs' leverage) is large. That is, to have amplification, entrepreneurs' net worth should be very sensitive to the adverse shock, and this may occur if they are heavily leveraged. The leverage ratio is, nonetheless, determined by the structural parameters of the model, and so it is endogenous. A small-scale calibration exercise considering plausible parameter values shows that, even in those cases in which net worth reacts sharply to the shock, the overall degree of amplification generated by financial frictions is *moderate* under *floating* and essentially nil under *fixing.* The reason is that changes in net worth affect aggregate output on impact only through changes in entrepreneurs' consumption and investment. But these two components of aggregate demand are small relatively to households' consumption, its main determinant. This result differs notoriously from previous contributions (e.g., Devereux *et* *al.*, 2006; Gertler *et al.*, 2007), in which there is always a substantial degree of amplification after the shock when the *financial accelerator* is introduced.

Finally, this article shows that CMIs can explain the countercyclical behavior of credit spreads to finance investment observed in EMs. To see why, notice first that the probability that entrepreneurs default on their debt is endogenous. Additionally, it negatively depends on the state of their net worth. When the latter falls, entrepreneurs are relatively less 'solvent' and the probability of default increases. The opposite also holds. This is an important result since Céspedes *et al.* (2004) considers the *financial accelerator mechanism* in a very similar environment, but the authors find ambiguity in the evolution of the credit spread after considering exactly the same shock. That is, they claim that when the economy is in a situation of 'financial vulnerability' the risk premium rises after the shock whereas in a situation of 'financial robustness' the opposite happens. This puzzling result in which output and investment fall sharply with the shock while the risk premium decreases (see Céspedes *et al.* 2004, pp. 1189-90), completely disappears when the microfoundations of the financial friction are properly acknowledged[8].

2 The model

To better understand the model, Figure 1, below, shows how the different sectors are interrelated in this economy.

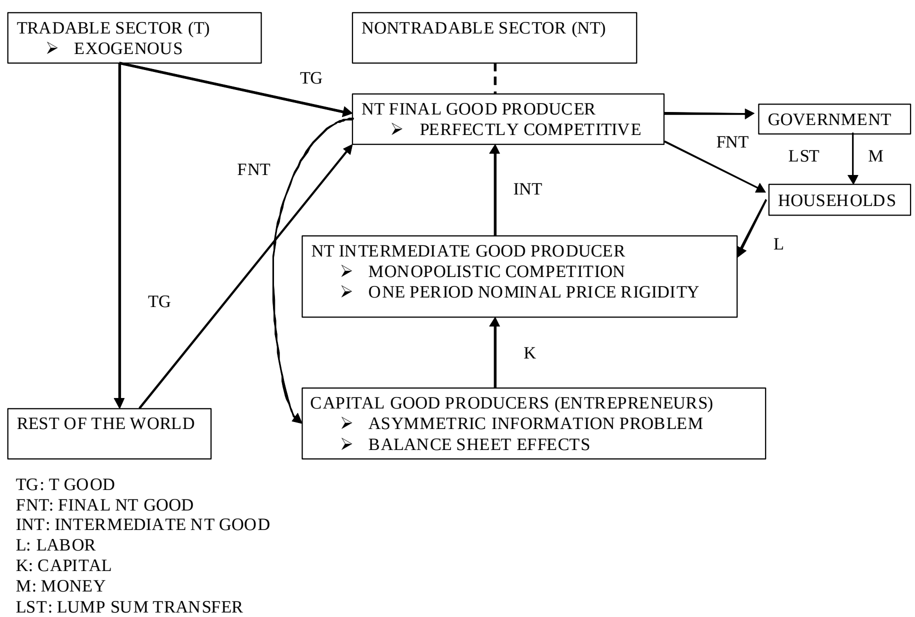

Figure 1. Flow of goods

As in the small open economy model of Obstfeld and Rogoff (1995), the output of the tradable sector is assumed to be exogenous. To justify this, think of a small economy in which there is a natural resource

[8]To be more concrete, the two key differences with the model of Céspedes *et al.* (2004) are: (a) the form in which financial frictions affect the economy is modeled from first principles and (b) current account dynamics are fully incorporated. These are two elements of central relevance to construct a fully consistent model. As already emphasized, a more *ad hoc* approach may lead to counter-intuitive and perhaps misleading results.

(e.g., oil), which is an endowment at each point in time. It follows, then, that the richness of the model is associated with the behavior of the nontradable sector. This is in turn composed of the final good sector, the intermediate goods sector and the capital good sector.

Producers of the final good are perfectly competitive and require a tradable and a combination of differentiated nontradable inputs to undertake production. The price of this good is perfectly flexible. This specification highlights that to produce the final good a firm requires different inputs such as transport, retailing and so forth; all of which are nontradable. Observe that the demand for the tradable input by the final firm is the only source of absorption of tradables. Hence, the evolution of the trade balance surplus in this economy is directly associated with this demand. The final good is then sold to households and government for consumption, or to entrepreneurs as an input to produce the capital good.

There are also a continuum of intermediate firms producing the differentiated inputs required to produce the final good. These goods are produced with labor (supplied by households) and capital (supplied by entrepreneurs). Price rigidities affect only this sector during period $t = 0$, when the shock hits the economy. This assumption introduces a potential role for monetary and exchange rate policy, but can also be justified on empirical grounds. That is, Burstein *et al.* (2005) note, analyzing 5 recent episodes of large devaluations[9], that the observed changes in the real exchange rate were mostly associated with the slow adjustment in the prices of nontradable goods. Finally, the role of households and government is somehow standard in the literature, the approach followed here being essentially that of Obstfeld and Rogoff (1995). With this general background in hand, the details of the model are now introduced.

2.1 Firms

2.1.1 Tradable sector

There is a single homogeneous tradable good whose supply is constant and exogenously given each period t and is denoted by $Y_{T,t} = \overline{Y}_T$. The price of the tradable good, $P_{T,t}$, is assumed to be perfectly flexible for all t.

2.1.2 Nontradable sector

There are a continuum of intermediate firms indexed by i producing differentiated inputs and a perfectly competitive producer of the final good. The intermediate output of firm i at period t is produced by combining capital and labor with a Cobb-Douglas production function as follows,

$$Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},\ i \in [0,1],\ 0 < \alpha < 1,$$

where $Z_{i,t}$ indicates the production of input i, A_t is a technology parameter, $K_{i,t}$ is the stock of capital rented from entrepreneurs at the beginning of period t, $L_{i,t}$ indicates labor services obtained from households and α is the share of capital in the nontradable intermediate input. The producer of the final good combines the inputs provided by intermediate firms and a tradable input with a Cobb-Douglas-type production function as follows,

$$Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma},\ \theta > 1,\ 0 < \gamma < 1,$$

[9] They consider the cases of Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994) and Thailand (1997).

where Y_t is the final nontradable good, θ is the elasticity of demand, γ is the share of nontradable inputs in the final good and $X_{T,t}$ is the tradable input used in production. We define $P_t = MC_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}(P_{T,t})^{1-\gamma}P_{N,t}^{\gamma}$ and $P_{N,t} = [\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{1}{1-\theta}}$, where P_t, $P_{i,t}$ and $P_{T,t}$ are the prices of the final good, the intermediate good and the tradable good, respectively. It follows, then, that each intermediate firm in the nontradable sector faces the following downward sloping demand curve[10],

$$Z_{i,t} = \gamma Y_t \frac{P_t}{P_{N,t}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta}, \tag{1}$$

and, similarly, the conditional demand function for the tradable input is given by:

$$X_{T,t} = (1-\gamma)Y_t \frac{P_t}{P_{T,t}}. \tag{2}$$

It will be assumed that the law of one price (LOOP) holds for tradable goods at all t, implying that,

$$P_{T,t} = S_t,$$

where S_t is the nominal exchange rate measured as the domestic price of foreign exchange. Note that the foreign price of tradables was normalized to one.

2.1.3 Demand for factors by intermediate firms

It is useful to analyze intermediate firms' problem in two stages. In the first stage, the conditional demand functions are obtained solving a cost minimization problem. Since these firms have monopoly power over their production, they also choose a price that maximizes their profits (second stage). The cost minimization problem (taking $Z_{i,t}$ as given) is then defined as:

$$\min_{\{K_{i,t},L_{i,t}\}} R_t^k K_{i,t} + W_t L_{i,t} \text{ s.t. } Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},$$

where R_t^k is the nominal rental price of capital and W_t denotes the nominal wage. Perfect competition is assumed in the factor markets and therefore each firm takes factor prices as given. The first-order conditions are,

$$K_{i,t}^* = (\frac{1-\alpha}{\alpha})^{\alpha-1}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{1-\alpha} \tag{3}$$

and

$$L_{i,t}^* = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}. \tag{4}$$

Note that the cost function evaluated at $K_{i,t}^*$ and $L_{i,t}^*$ takes the form,

$$C_{i,t}(Z_{i,t}, R_t^k, W_t) \equiv C_{i,t}^* = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}Z_{i,t}W_t^{1-\alpha}(R_t^k)^{\alpha}.$$

[10]The final good producer solves the following cost minimization problem:
$\min \int_0^1 Z_{i,t}P_{i,t}di + P_{T,t}X_{T,t}$ s.t. $Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}}di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma}$, giving the demand functions stated in Eqs. 1 and 2.

2.1.4 Profit maximization problem of intermediate firms

In the second stage, intermediate firms determine the price level $P_{i,t}$ and output $Z_{i,t}$ solving the following problem:

$$\max_{\{P_{i,t}\}} \pi_{i,t} = P_{i,t}Z_{i,t} - C^*_{i,t} \text{ s.t. } Z_{i,t} = \gamma Y_t \frac{P_t}{P_{N,t}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta},$$

whose solution is standard yielding,

$$P_{i,t} = \frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R_t^k)^{\alpha}, \tag{5}$$

where $\frac{\theta}{\theta-1}$ is a markup over marginal costs[11]. The model assumes perfect foresight. Therefore, the above expression will hold for all periods but $t=0$, when an unexpected shock hits the economy. During that period, the price $P_{i,0}$ differs from what firm i would have optimally chosen had it known the shock in advance. It is in this sense that the price level of the intermediate firm i is assumed to be preset at $t=0$.

2.2 Households

The representative household obtains utility from consumption of the final good C_t, real money balances $\frac{M_t}{P_t}$[12] and leisure. Lifetime utility of the representative agent is defined as,

$$U_t = \sum_{t=0}^{\infty}\beta^t[\log C_t + \chi\log(\frac{M_t}{P_t}) - \frac{\kappa}{2}(L_t)^2].$$

The budget constraint that the household faces when maximizing utility, in nominal terms, is given by,

$$P_tC_t + M_t + S_tD_{t+1} = P_{T,t}\overline{Y}_T + W_tL_t + \pi_t + S_tR_t^*D_t + M_{t-1} + P_tT_t. \tag{6}$$

Household's sources of funding are the endowment of the tradable good $P_{T,t}\overline{Y}_T$, wage earnings for working in the nontradable intermediate sector W_tL_t, dividends from owning intermediate firms π_t, nominal gross return from previous-period foreign currency denominated deposits $S_tR_t^*D_t$, holdings of previous-period money balances M_{t-1} and lump-sum government transfers P_tT_t. These resources are used to purchase consumption goods P_tC_t, to accumulate nominal money balances M_t or to acquire new interest-bearing deposits S_tD_{t+1}. The first-order conditions of this problem are:

$$C_{t+1} = \beta R^*_{t+1}\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}C_t \tag{7}$$

$$\frac{M_t}{P_t} = \chi C_t\frac{R_{t+1}}{R_{t+1}-1} \tag{8}$$

$$\frac{1}{\kappa}\frac{1}{C_t}\frac{W_t}{P_t} = L_t. \tag{9}$$

Eq. 7 is an Euler equation indicating that the marginal rate of substitution of consumption between two successive periods must be equal to the real interest rate. UIP holds in the paper, implying that $R_{t+1} = R^*_{t+1}\frac{S_{t+1}}{S_t}$, where R_{t+1} and R^*_{t+1} indicate the gross nominal risk-free domestic and foreign interest

[11] Note that in the perfectly competitive case, when $\theta \to \infty$, the price of the intermediate firm is equal to the marginal cost.

[12] The fact that households obtain utility from real balances is common in the Money-in-the-Utility function literature. We think that money generates utility owing to the services that it provides in facilitating transactions (see Walsh, 2003).

rates, respectively[13]. Note that the demand for real money balances stated in Eq. 8 is positively associated with consumption and negatively related to the gross nominal interest rate R_{t+1}. This is a standard result in models with Money-in-the-Utility function and infinitely-lived agents. Finally, the labor supply equation shown in Eq. 9 increases in the real wage, while it decreases in consumption.

2.3 Government

This is a model in which Ricardian Equivalence holds and therefore we abstract from government debt. In this simple setting, the only source of funding for the government's current spending and the lump-sum transfer that is made to households is real seigniorage. The government's budget constraint is expressed as,

$$G_t + T_t = \frac{(M_t - M_{t-1})}{P_t}, \tag{10}$$

where G_t indicates government's consumption and $\frac{(M_t - M_{t-1})}{P_t}$ is the real seigniorage obtained for issuing money between t and $t-1$. For simplicity it is assumed that $G_t = 0$, indicating that any revenue due to seigniorage is immediately rebated to households.

2.4 Entrepreneurs

Entrepreneurs will play a central role in the model. They will produce the capital good that is afterward rented to firms. In producing capital, however, they must obtain external funding, which is denominated in foreign currency and subject to frictions, as described below[14].

2.4.1 Partial equilibrium contracting problem

The analysis of the debt contracting problem under asymmetric information developed in this section closely follows Carlstrom and Fuerst (1997). The existence of a continuum of entrepreneurs indexed by j in the interval $[0, 1]$ producing a homogeneous capital good is assumed. Each entrepreneur has the following stochastic linear technology,

$$K_{j,t+1} = \omega_{j,t} I_{j,t}, \tag{11}$$

where $K_{j,t+1}$ is the capital good produced by entrepreneur j in period t; $I_{j,t}$ denotes the input utilized by entrepreneur j to produce the capital good, which is part of the final good; $\omega_{j,t}$ is a *iid* random variable with a common distribution across j, where the cumulative and density functions $\Phi(\cdot)$ and $\phi(\cdot)$, respectively, have positive supports. To simplify the analysis it is assumed that $E(\omega) = 1$.

When the entrepreneur decides how much to invest at period t, he or she faces the following constraint,

$$S_t B^*_{j,t+1} = P_t(I_{j,t} - N_{j,t+1}), \tag{12}$$

[13] Although we directly assumed UIP, it implicitly assumes that households can also accumulate bonds in the domestic currency. Arbitrage between domestic and foreign bonds will then imply $R_{t+1} = R^*_{t+1}\frac{S_{t+1}}{S_t}$.

[14] To study how sensitive results are to the assumption about debt denomination, later in the paper we will assume that entrepreneurs' debt is entirely denominated in domestic currency.

where $S_t B^*_{j,t+1}$ indicates the domestic value of the foreign currency debt contracted at period t to be repaid at period $t+1$ and $N_{j,t+1}$ is the net worth of entrepreneur j at period t. This constraint shows that the entrepreneur can purchase inputs beyond his or her net worth only by incurring foreign currency debt[15].

Following Townsend (1979) and Gale and Hellwig (1985) among others, the model assumes a costly state verification problem. In this context, the optimal contract between the borrower and the lender will take the form of a standard non-contingent debt contract. For simplicity, we assume that the debt contract lasts only one period and that the credit market is fully anonymous.

The contract specifies a fixed payment to the lender in all states where the project generates a nominal gross return above the fixed nominal value of the debt repayment. In contrast, when this condition is not satisfied, the entrepreneur defaults on the debt and the lender recoups as much as he or she can from the project, after paying a fixed monitoring cost. The monitoring cost is given by the payment of $\mu I_{j,t}$ units of the final capital good, where $0 \leq \mu \leq 1$[16]. The payment to observe $\omega_{j,t}$, however, is only made in case the entrepreneur defaults on the debt.

Note that there are two different forms of 'capital' in this model. On the one hand, the capital that is produced by entrepreneurs and is rented to intermediate firms (i.e., $K_{j,t+1}$). On the other hand, the input that entrepreneurs require to produce $K_{j,t+1}$ (i.e., $I_{j,t}$). We assume that both forms of capital fully depreciate in the period in which are used for production[17].

Let $\overline{\omega}_{j,t}$ denote the minimum value of $\omega_{j,t}$ at which default does not occur, and let $R^{nd}_{j,t+1}$ indicate the non-default gross nominal interest rate charged to entrepreneur j when contracting the loan at period t. $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$ therefore satisfy,

$$R^k_{t+1}\overline{\omega}_{j,t}I_{j,t} = R^{nd}_{j,t+1}S_t B^*_{j,t+1} = R^{nd}_{j,t+1}P_t(I_{j,t} - N_{j,t+1}). \tag{13}$$

Eq. 13 indicates that entrepreneur j produces $\overline{\omega}_{j,t}I_{j,t}$ units of the capital good that are rented to firms at the nominal rental price R^k_{t+1}. The term $R^k_{t+1}\overline{\omega}_{j,t}I_{j,t}$, therefore represents the minimum nominal gross return on capital required to repay the principal and interest on the debt, $R^{nd}_{j,t+1}S_t B^*_{j,t+1}$.

Observe that taking as given the market prices P_t and R^k_{t+1}, and entrepreneur $j's$ net worth $N_{j,t+1}$, the contracting problem is fully specified once we solve for either $R^{nd}_{j,t+1}$ and $I_{j,t}$, or $\overline{\omega}_{j,t}$ and $I_{j,t}$. For simplicity, we analyze the contracting problem only in terms of $\overline{\omega}_{j,t}$ and $I_{j,t}$.

2.4.2 Expected profits

We assume that both the entrepreneur and the lender are risk-neutral. The net expected profits of the entrepreneur in nominal terms can then be expressed as,

[15] We point out here that $N_{j,t+1}$ is not a predetermined variable at period $t=0$, since the unexpected shock can affect its value. A more detailed explanation of why this is the case is left to a future section of the paper, in which $N_{j,t+1}$ is appropriately defined.

[16] Our assumption about monitoring costs implies that there is a fixed cost $\mu I_{j,t}$, known ex-ante by the lender, for observing the true realization of the project. Note that this cost depends on the scale of the investment $I_{j,t}$, but is independent of the ex-post realization of $\omega_{j,t}$. A slightly different approach is taken in Bernanke *et al.* (1999), where the monitoring cost is a fraction of the ex-post realization of the project. The main results of the model remain the same, however, regardless of the form in which monitoring costs are defined.

[17] Although these assumptions can be seen as particularly strong, they are critical to obtain a model which can be handled analytically. Departing from these assumptions complicates the analysis and, in our view, is not important for a clear understanding of the role played by CMIs in a DGE model.

$$R_{t+1}^k \int_{\overline{\omega}_{j,t}}^{\infty} I_{j,t}\omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]R_{j,t+1}^{nd}P_t(I_{j,t}-N_{j,t+1}),$$

where the first term indicates the expected gross income from producing the capital good whenever $\omega_{j,t} > \overline{\omega}_{j,t}$, while the second term shows the expected cost of the debt repayment in case the debt contract is fulfilled (i.e., whenever $\omega_{j,t} > \overline{\omega}_{j,t}$). The term $[1-\Phi(\overline{\omega}_{j,t})]$ thus indicates the probability that the entrepreneur repays the debt. Observe that in case of default, which occurs with probability $\Phi(\overline{\omega}_{j,t})$, the entrepreneur receives nothing; and any remaining value of the project is completely seized by the lender.

Using Eq. 13 it is possible to rewrite the above expression as,

$$R_{t+1}^k I_{j,t} f(\overline{\omega}_{j,t}) = R_{t+1}^k I_{j,t}\{\int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\},$$

where $f(\overline{\omega}_{j,t}) \equiv \{\int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\}$ is the expected share of the investment that the entrepreneur keeps from a successful project. Following a similar reasoning, the net expected profit of the lender can be written as,

$$R_{t+1}^k I_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1}^k I_{j,t}\{\int_{0}^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\},$$

where $g(\overline{\omega}_{j,t}) \equiv \int_{0}^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$ is the lender's expected share. In this case $R_{t+1}^k \int_{0}^{\overline{\omega}_{j,t}} I_{j,t}\omega\phi(\omega)d\omega$ is the expected gross income seized by the lender whenever $\omega_{j,t} < \overline{\omega}_{j,t}$ and $R_{t+1}^k \mu I_{j,t}\Phi(\overline{\omega}_{j,t})$ denotes expected monitoring costs. When $\omega_{j,t} > \overline{\omega}_{j,t}$, the entrepreneur repays the loan, and thus the lender receives $[1-\Phi(\overline{\omega}_{j,t})]R_{t+1}^k I_{j,t}\overline{\omega}_{j,t}$. Considering the definitions of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ it then follows that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$[18]. This fact implies that a fraction $\mu\Phi(\overline{\omega}_{j,t})$ of the total investment is expected to be lost due to monitoring.

2.4.3 Optimal contract

The optimal contract is determined by the pair $(I_{j,t}, \overline{\omega}_{j,t})$ that maximizes entrepreneur's expected profits, subject to the lender receiving at least the opportunity cost of the loan. To ensure the optimality of the contract, entrepreneur's expected profits must be maximized. Otherwise, there might be another lender that provides a different contract under better conditions. It is assumed that the entrepreneur's participation constraint, given by $R_{t+1}^k I_{j,t} f(\overline{\omega}_{j,t}) > R_{t+1}P_t N_{j,t+1}$, holds. We will also assume the existence of a large number of lenders. Therefore, arbitrage ensures that the lenders' participation constraint,

[18] Noting that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = \int_{0}^{\infty} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t})$ and that $E(\omega) = \int_{0}^{\infty} \omega\phi(\omega)d\omega = 1$, immediately gives the result.

$R^k_{t+1} I_{j,t} g(\overline{\omega}_{j,t}) \geq R_{t+1} P_t (I_{j,t} - N_{j,t+1})$, always binds. The optimal contract is hence given by the solution of this problem:

$$\max_{\{I_{j,t}, \overline{\omega}_{j,t}\}} R^k_{t+1} I_{j,t} f(\overline{\omega}_{j,t}) \text{ s.t. } R^k_{t+1} I_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1} P_t (I_{j,t} - N_{j,t+1}).$$

The first-order conditions can be written as[19],

$$R^k_{t+1}\{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})} g'(\overline{\omega}_{j,t})\} = R_{t+1} P_t, \tag{14}$$

and

$$I_{j,t} = \frac{N_{j,t+1}}{1 - \frac{R^k_{t+1}}{R_{t+1} P_t} g(\overline{\omega}_{j,t})}. \tag{15}$$

Note first that Eq. 14 gives an implicit function of the form,[20]

$$\overline{\omega}_{j,t} = F(\frac{R^k_{t+1}}{R_{t+1} P_t}) = \overline{\omega}_t, \text{ where } \frac{d\overline{\omega}_t}{d\frac{R^k_{t+1}}{R_{t+1} P_t}} > 0, \tag{16}$$

implying that $\overline{\omega}_{j,t}$ is the same for all entrepreneurs. Also notice that Eq. 15 provides an investment-demand schedule (derived from the lenders' zero-profits condition) that linearly depends on $N_{j,t+1}$, a fact that facilitates aggregation. This result essentially follows from the linearity of both the production function of capital and the monitoring technology. It is easy to show that taking $N_{j,t+1}$ as given, there is a positive relation between $I_{j,t}$ and $\frac{R^k_{t+1}}{R_{t+1} P_t}$, provided that $g'(\overline{\omega}_t) > 0$.[21]

To better understand the model, it is helpful to ask now how CMIs affect investment decisions. To answer this, rewrite Eq. 14 using the fact that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$:

$$\frac{R^k_{t+1}}{P_{t+1}} = \{1 - \mu\Phi(\overline{\omega}_{j,t}) + \mu\phi(\overline{\omega}_{j,t}) \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}\}^{-1} \frac{R_{t+1} P_t}{P_{t+1}}.$$

When $\mu = 0$ the real rental rate $\frac{R^k_{t+1}}{P_{t+1}}$ exactly equals the risk-free (gross) real interest rate $\frac{R_{t+1} P_t}{P_{t+1}}$. The investment-demand schedule derived in such a case is essentially the same that would arise from the same model but without CMIs (i.e., when households instead of entrepreneurs undertake investment). Therefore, the case $\mu = 0$ roughly converges to a situation in which credit markets are prefect[22]. When $\mu > 0$, in contrast, there is a positive wedge between $\frac{R^k_{t+1}}{P_{t+1}}$ and $\frac{R_{t+1} P_t}{P_{t+1}}$. It is possible to show that this wedge is an increasing function of μ, therefore providing a good measure of how CMIs affect the economy[23]. For future reference we now rewrite Eq. 14 dropping the subindex j :

$$\frac{R^k_{t+1}}{R_{t+1} P_t} = \{g(\overline{\omega}_t) - \frac{f(\overline{\omega}_t)}{f'(\overline{\omega}_t)} g'(\overline{\omega}_t)\}^{-1}. \tag{17}$$

[19]See Appendix A for details.

[20]See Appendix B for details.

[21]The fact that in equilibrium $g'(\overline{\omega}_t) > 0$, follows from the second order condition of the entrepreneur's maximization problem (see Appendix A).

[22]We will return to this point when discussing the long-run properties of the model.

[23]This is perhaps why some authors call this wedge as 'risk premium' (see, for example, Bernanke *et al.,* 1999). We prefer to call it, however, 'excess return' from producing capital. The concept of 'risk premium' here refers to the difference between the non-default and the risk-free interest rate, an interpretation that seems to be closer to the general understanding of its meaning.

We close this section pointing out that $R^{nd}_{j,t+1}$ can be expressed as:

$$R^{nd}_{j,t+1} = R_{t+1}\overline{\omega}_t g(\overline{\omega}_t)^{-1} = R^{nd}_{t+1}. \tag{18}$$

This equation indicates that the non-default interest rate is the same for all entrepreneurs. This result is somehow expected, since the same threshold $\overline{\omega}_t$ implies that in equilibrium all entrepreneurs are equally risky, and hence they are charged with the same interest rate.

3 Aggregation and equilibrium

Aggregate investment at period t is obtained by summing over j Eq. 15:

$$I_t = \frac{N_{t+1}}{1 - \frac{R^k_{t+1}}{R_{t+1}P_t} g(\overline{\omega}_t)}, \tag{19}$$

where N_{t+1} denotes aggregate net worth at period t (defined below). Turning to the aggregate supply of capital, recall that a fraction $\mu\Phi(\overline{\omega}_t)$ of total investment is expected to be lost due to monitoring at each period t. It is hence defined as,

$$K^s_{t+1} = (1 - \mu\Phi(\overline{\omega}_t))I_t. \tag{20}$$

The existence of an asymmetric information problem between lenders and entrepreneurs implies that K^s_{t+1} is a fraction $(1 - \mu\Phi(\overline{\omega}_t))$ of what would be supplied without CMIs (i.e., when $\mu = 0$).

3.1 Aggregate net worth

Entrepreneur's net worth is a key endogenous variable. It can be affected by policy shocks, thus incorporating an additional source of dynamics into the model. We now explain how it evolves over time. For simplicity we follow Carlstrom and Fuerst (1996) and Bernanke *et al.* (1999), assuming that a constant fraction of entrepreneurs υ dies each period, where death implies leaving the economy and consuming the net profits of the period. The size of the population remains constant, however, since for each entrepreneur that dies there is a newcomer entering the economy. This from of modeling the evolution of net worth eliminates the possibility that it reaches a level where entrepreneurs become fully self-financed.[24]

Recall that $R^k_{t+1} f(\overline{\omega}_{j,t})I_{j,t}$ denotes the net expected profits of entrepreneur j at period t. Lagging this expression one period yields $R^k_t f(\overline{\omega}_{j,t-1})I_{j,t-1}$. We can work out this definition to express net worth in a more intuitive way. We know that in equilibrium $\overline{\omega}_{j,t-1} = \overline{\omega}_{t-1}$ for all j. Aggregate net worth at period t can then be written as:

$$P_t N_{t+1} = (1 - \upsilon)R^k_t f(\overline{\omega}_{t-1})I_{t-1}. \tag{21}$$

Since there is a fraction υ of entrepreneurs that die or are 'out of business' at period t, entrepreneurs' consumption evolves according to:

[24] In Carlstrom and Fuerst (1997) a different approach is considered, however. They assume that entrepreneurs are more impatient than households. In a related contribution Carlstrom and Fuerst (2001) show, however, that the main results of the model are somehow invariant to the form in which the evolution of net worth is modeled.

$$P_tC^e_t = \upsilon R^k_t f(\overline{\omega}_{t-1})I_{t-1}.$$

Using the period $t-1$ (aggregate) constraint in the entrepreneurs' maximization problem, $R^k_t I_{t-1} g(\overline{\omega}_{t-1}) = R_t P_{t-1}(I_{t-1} - N_t)$, the period $t-1$ (aggregate) budget constraint of entrepreneurs, $S_{t-1}B^*_t = P_{t-1}(I_{t-1} - N_t)$, and the fact that $f(\overline{\omega}_{t-1}) = 1 - \mu\Phi(\overline{\omega}_{t-1}) - g(\overline{\omega}_{t-1})$ we finally obtain:

$$N_{t+1} = (1-\upsilon)[\frac{R^k_t K_t}{P_t} - \frac{S_t}{P_t} R^*_t B^*_t]. \tag{22}$$

To derive this equation we have also considered that in equilibrium $K^s_t = K^d_t = K_t$ and that UIP holds. This expression embeds the essential mechanism through which balance sheet effects may affect the economy. Other things equal, an unexpected real exchange rate depreciation at period t, measured as a rise in S_t/P_t, negatively affects N_{t+1}. Lower net worth reduces total investment at period t, thus reducing the total supply of capital that will be available at the beginning of period $t+1$.

Observe also that N_{t+1} can be affected by changes in the real value of entrepreneurs' assets. That is, although K_t is predetermined at period t, the real rental price of capital, $\frac{R^k_t}{P_t}$, is not. Changes in this relative price, therefore, can also have potential effects on entrepreneurs' creditworthiness. It is useful, then, to think of $\frac{R^k_t}{P_t}$ as an 'asset price'. Notice that this fact strongly resembles the idea emphasized by Kiyotaki and Moore (1997) that changes in asset prices are key to generate amplification when agents are credit-constrained.

The aggregate budget constraint of the entrepreneurial sector at period t can be written as,

$$S_t B^*_{t+1} = P_t(I_t - N_{t+1}). \tag{23}$$

Using this expression, the definition of aggregate net worth stated in Eq. 22 and its counterpart for entrepreneurs' consumption, we can rewrite entrepreneurs' budget constraint as:

$$P_t I_t + P_t C^e_t + S_t R^*_t B^*_t = R^k_t K_t + S_t B^*_{t+1}. \tag{24}$$

Each period t entrepreneurs invest $P_t I_t$ to produce capital, consume $P_t C^e_t$ and repay capital and interest on the debt contracted at period $t-1$, $S_t R^*_t B^*_t$. These expenditures are financed with the income from renting the capital produced at period $t-1$ to firms, $R^k_t K_t$, and issuing new debt $S_t B^*_{t+1}$.

3.1.1 Contracting problem in domestic currency

For further reference, we point out that if entrepreneurs' debt is instead in domestic currency, most of the equations discussed previously hold after some minor notational changes. In this case, however, we assume that households' financial assets in Eq. 6 are also denominated in the domestic currency. Note, therefore, that in this model there is always a 'matching' between the currency in which households accumulate bonds and that of entrepreneurs' debt.

With domestic-currency debt, there are two equations that have to be modified, however, which will affect some of the model's implications. Let B_{t+1} denote the (aggregate) nominal debt contracted at period t (to be repaid at period $t+1$), in domestic currency. Eqs. 22 and 23 should now be replaced with:

$$N_{t+1} = (1-\upsilon)[\frac{R^k_t K_t}{P_t} - \frac{R_t B_t}{P_t}], \tag{25}$$

and

$$B_{t+1} = P_t(I_t - N_{t+1}). \tag{26}$$

Note from Eq. 25 that changes in the exchange rate do not produce the negative balance-sheet effect highlighted previously anymore. On the contrary, other things being equal, a policy shock that produces unexpected inflation will tend to reduce the real value of debt repayments, positively affecting N_{t+1}. Observing the assets side of entrepreneurs' net worth, it is apparent that changes in $\frac{R_t^k}{P_t}$ affect their balance sheets in the same way as under foreign currency debt.

Although the equilibrium conditions and the steady state of the model coincide regardless of entrepreneurs' debt denomination, there will be important differences in the dynamic behavior of the model after the shock, as we shall see.

3.2 Equilibrium conditions

To define the equilibrium of the model it is still necessary to specify equilibrium conditions for: i. Nominal money balances, ii. Goods market, iii. Capital good market, iv. Labor market, v. Intertemporal balance of trade and vi. Domestic credit market. Note that under symmetry each firm i sets the same price for the intermediate good (i.e., $P_{i,t} = P_{N,t}$), implying that $Z_{i,t} = Z_t$ for all i.

3.2.1 Money market equilibrium

This condition is given by Eq. 8 under the assumption that aggregate supply equals aggregate demand for real money balances.

3.2.2 Goods market equilibrium

Since the only source of absorption of tradables is given by the demand for tradable inputs by the final producer firm, the (nominal) trade balance surplus at period t is given by,

$$P_{T,t}(\overline{Y}_T - X_{T,t}) = TB_t. \tag{27}$$

Turning to intermediate firms, in a symmetric equilibrium its production function is $Z_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. Note that their aggregate income equals the payment to the two factors of production plus any remaining profit or: $P_{N,t}Z_t = R_t^k K_t + W_t L_t + \pi_t$[25]. Regarding the final producer firm, observe that its production function takes the form $Y_t = Z_t^{\gamma} X_{T,t}^{1-\gamma}$. Cost minimization then implies: $P_t Y_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$. Finally, market clearing for the final nontradable good implies,

$$Y_t = C_t + C_t^e + I_t. \tag{28}$$

3.2.3 Capital good market equilibrium

Equilibrium in this market implies $K_t^s = K_t^d = K_t$. From Eqs. 3 and 5 we thus obtain,

$$R_t^k = \alpha \frac{\theta - 1}{\theta} \frac{P_{N,t}Z_t}{K_t}, \tag{29}$$

[25] The profits of intermediate firms are given by $\pi_t = \frac{1}{\theta} P_{N,t} Z_t$. There is then an inverse relation between π_t and θ, where the former converges to zero as the demand for intermediate inputs becomes perfectly elastic (i.e., $\theta \to \infty$).

where K_t is predetermined and K_{t+1} is given by Eq. 20.

3.2.4 Labor market equilibrium

Equilibrium in this market implies $L_t^s = L_t^d = L_t$. From Eqs. 4 and 5 it is possible to obtain,

$$W_t = (1-\alpha)\frac{\theta - 1}{\theta}\frac{P_{N,t}Z_t}{L_t}, \tag{30}$$

where L_t is given by Eq. 9.

3.2.5 Intertemporal balance of trade equilibrium

By adding the budget constraints of households, government and entrepreneurs, we obtain the budget constraint of the economy as a whole (i.e., the balance of payments):

$$\begin{aligned} & P_t C_t + P_t C_t^e + P_t I_t + S_t R_t^* B_t^* + S_t D_{t+1} \\ = \quad & P_{T,t}\overline{Y}_T + S_t B_{t+1}^* + R_t^k K_t + W_t L_t + \pi_t + S_t R_t^* D_t. \end{aligned}$$

Let $F_t = D_t - B_t^*$ and $F_{t+1} = D_{t+1} - B_{t+1}^*$ denote economy-wide net foreign assets at periods t and $t+1$, respectively, in foreign currency. After a small number of substitutions, the balance of payments can be written as,

$$F_{t+1} = \frac{TB_t}{S_t} + R_t^* F_t.$$

This expression implicitly defines an equilibrium condition between the domestic economy and the rest of the world, where goods and assets are cleared at the price $P_{T,t}$ $(= S_t)$ and the foreign risk-free interest rate R_t^*, respectively. We can iterate forward this equation to obtain, after imposing the 'no-Ponzi-game-condition':

$$-F_0 = B_0^* - D_0 = \sum_{s=0}^{\infty} \frac{TB_s/S_s}{\prod_{v=0}^{s} R_v^*}. \tag{31}$$

3.2.6 Domestic credit market equilibrium

By Walras' law, equilibrium in the domestic credit market is guaranteed whenever the remaining markets are in equilibrium.

4 Zero-inflation steady state

We briefly discuss now the solution in a zero-inflation steady state (ZISS)[26]. Consider first the non-conventional variable $\overline{\omega}_t$. Combining Eqs. 17, 19 and 21 in a ZISS we obtain:

[26] It would have been possible to choose a different steady state to undertake the analysis; for instance one in which the money supply growth rate and therefore the inflation rate are both constant but different from zero. For simplicity, and since the main results do not depend on the chosen steady state, we consider the analytically simpler zero-inflation case.

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1}. \tag{32}$$

There is only one endogenous variable in this equation, $\overline{\omega}$, whose solution can now be derived (call it $\overline{\omega}^*$). From Appendix A and B we know that $f'(\overline{\omega}) < 0$ and that $g'(\overline{\omega}) > 0$; and therefore the LHS of Eq. 32 will always be positive. With the value of $\overline{\omega}^*$ in hand, it is then possible to solve for a number of endogenous variables directly related to the presence of CMIs, such as r^k ($\equiv \frac{R^k}{P}$, call it r^{k*}). Since ω is a random variable, to proceed with the solution of the model we should specify a distribution function for it. From here onwards we let ω be uniformly distributed in the interval $[0, 2]$. Eq. 32 then yields:

$$\overline{\omega} = 2 + \frac{\mu\beta}{1-(\beta+\upsilon)} \equiv \overline{\omega}^*. \tag{33}$$

For $\overline{\omega}^*$ to be within the interval $[0, 2]$ it is also required that $\beta + \upsilon > 1$ (υ must be 'sufficiently' large) and that $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$; restrictions that we will assume to hold. Note that $\overline{\omega}^*$ is not affected by parameters related to production functions or preferences other than β. Having derived $\overline{\omega}^*$ we can now solve for the associated steady state levels of $f(\overline{\omega}^*)$ and $g(\overline{\omega}^*)$ and the remaining variables of the model. For brevity, we present here only a subset of variables in the ZISS, while leaving the derivations to Appendix C:

$$Y = \Lambda^{\frac{\gamma(\alpha-1)}{2(1-\alpha\gamma)}}\{A(\frac{\alpha}{r^{k*}})^{\alpha}(\frac{\theta-1}{\theta}\gamma)^{\frac{1+\alpha}{2}}(\frac{1-\alpha}{\kappa})^{\frac{1-\alpha}{2}}\}^{\frac{\gamma}{1-\alpha\gamma}}(\overline{Y}_T + \digamma\frac{1-\beta}{\beta})^{\frac{1-\gamma}{1-\alpha\gamma}}$$

$$C = \Lambda Y$$

$$K = \alpha\gamma\frac{\theta-1}{\theta}(r^{k*})^{-1}Y$$

$$N = \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)(1-\upsilon)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}Y$$

$$L = (\frac{\gamma}{\Lambda}\frac{\theta-1}{\theta}\frac{1-\alpha}{\kappa})^{1/2},$$

where $\Lambda \equiv 1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)} \in (0, 1)$.

Observe that $\digamma(\equiv D - B^*)$ denotes the steady state level of net foreign of the economy as a whole. Notice, however, that although endogenous, $\digamma$ cannot be determined from the steady state conditions examined here. As in other open economy DGE models with infinitely-lived agents, there is path-dependence in net foreign assets (e.g., Correia *et al.,* 1995; Fender and Rankin, 2003). In this steady state analysis, we then treat $\digamma$ as exogenous.

We can see that Y is increasing in $\digamma$, and through it there will be a positive effect on C, K, and N but L remains unaffected, since its solution is independent of $\digamma$. This is a direct consequence of households' preferences over leisure: income and substitution effects exactly cancel out to leave L unaffected by wealth.

Why is it the case that Y increases in $\digamma$? As $\digamma$ rises, the relative price of tradable inputs (i.e., the real exchange rate $s(\equiv \frac{S}{P})$) decreases, thus raising the demand for X_T. This boosts the production of final output Y. There is also an increase in the relative price of nontradable inputs, p_N ($\equiv \frac{P_N}{P}$), which offsets, but only partially, this expansion. With labor unaffected intermediate output Z expands. To increase Z a higher level of capital K is required; which is the result obtained here. The larger demand for capital rises entrepreneurs' profits and thus net worth N rises. Finally, note that with labor unchanged real wages increase, and thereby households' consumption C must also rise.

4.1 Long-run implications of financial frictions

To close this section we present the solutions for a number of variables relative to K. Note that these solutions are independent of net foreign assets F. To better understand the implications of financial frictions, we also consider the case in which $\mu = 0$.

Table 1. ZISS solutions

Variable	$\mu \in (0,1]$	$\mu = 0$
$\frac{Y}{K}$	$r^{k*}\frac{1}{\alpha\gamma}\frac{\theta}{\theta-1}$	$\frac{1}{\beta}\frac{1}{\alpha\gamma}\frac{\theta}{\theta-1}$
$\frac{I}{K}$	$\frac{1}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{sB^*}{K}$	$r^{k*}\frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{N}{K}$	$r^{k*}\frac{(1-\upsilon)f(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	0

It is useful to note, first, that when $\mu = 0$ two results follow: $f(\overline{\omega}^*) = 0$ and $g(\overline{\omega}^*) = 1$. Hence, the ZISS solution of the model converges to that of a 'fairly standard' RBC model for an open economy (i.e., a model with the same structure but developed without CMIs from the outset). A subtle but important difference between these two cases remain, however.

Without CMIs investment and therefore the supply of capital is *directly* undertaken by households. With CMIs but setting $\mu = 0$ capital is still supplied by households, but in an *indirect* process. To see this, note that when $\mu = 0$ entrepreneurs' failure rate is equal to 1 (since $\Phi(\overline{\omega}^*) = \frac{1}{2}\overline{\omega}^* = 1$). This result is somehow puzzling at first glance. The reason is that in the model with CMIs entrepreneurs exist only if there is an 'excess return' or profit from producing capital. When this 'excess return' goes to zero, entrepreneurs' profits also go to zero (net worth goes to zero as well); meaning that entrepreneurs become 'insolvent'. This is why the probability that they default is equal to 1. This is also the reason behind the real debt left by entrepreneurs when $\mu = 0$ (i.e., defaulted debt). In such a case, lenders' assets (in the form of defaulted debt) are exactly matched with the capital recouped at a zero cost (i.e., $\frac{sB^*}{K} = 1$). Therefore, with $\mu = 0$ it follows that the lenders (i.e., households) supply the capital to the market, but after this indirect process.

Consider now the case in which $\mu > 0$. Note that $I/K > 1$: there is always a fraction of investment lost due to monitoring. Since r^{k*} increases in μ[27], Y/K rises in μ as well. Both Y and K are lower due to the presence of monitoring, but K falls more sharply since it is more sensitive than Y to changes in μ. It is also possible to show that $\frac{sB^*}{K}$ decreases while $\frac{N}{K}$ increases in μ[28]. Entrepreneurs' profits increase in μ (net worth rises), and thereby it is intuitive to find that $\frac{N}{K}$ rises in μ. Also, as entrepreneurs' profits increase, it becomes easier to substitute external *vis a vis* internal resources to produce capital, and therefore $\frac{sB^*}{K}$ falls with μ.

[27] In this ZISS where ω is uniformly distributed in the interval $[0, 2]$ it follows that: $r^{k*} = \beta^{-1}(1 - \frac{1}{4}\mu\overline{\omega}^* - \frac{1}{2}\mu)^{-1}$. Therefore, $\frac{dr^{k*}}{d\mu} > 0$ will hold provided that $\mu\beta < 2(\beta + \upsilon - 1)$; condition that will be satisfied since $\beta \in (0, 1)$.

[28] We have proved this statement considering a numerical exercise setting $\mu = 0.12$, $\beta = 0.99$ and $\upsilon = 0.07$.

Define entrepreneurs' leverage as total debt relative to net worth. In steady state this measure is hence given by $\frac{sB^*}{N} = \frac{\beta g(\overline{\omega}^*)}{(1-\upsilon)f(\overline{\omega}^*)}$. It follows, from the previous discussion, that leverage decreases in μ. This variable, moreover, will help us in understanding under which conditions negative balance sheet effects tend to amplify shocks. Intuitively, amplification through this channel is likely to occur when entrepreneurs' leverage is initially large (when μ is low), a result that will clearly arise when studying the model's dynamics.

5 Linearized model

Since the model is highly non-linear, we analyze its dynamic properties undertaking a linear approximation of it about a reference steady state in which inflation and the balance of trade (and thus net foreign assets) are both zero (we henceforth denote this steady state as RSS). From here onwards a lower-case variable denotes a percentage deviation of the original variable from the RSS[29]. For instance, for any variable X_t we define $x_t \equiv \frac{X_t - X^{RSS}}{X^{RSS}}$ $\left(\approx \log \frac{X_t}{X^{RSS}}\right)$, where X^{RSS} is the value of X_t in the RSS. To facilitate the exposition, we present below a list with the approximations of the key equations of the model.

$$c_{t+1} = r_{t+1} + p_t - p_{t+1} + c_t \tag{34}$$

$$m_t - p_t = c_t - \frac{\beta}{1-\beta} r_{t+1} \tag{35}$$

$$z_t = a_t + \alpha k_t + (1-\alpha) l_t \tag{36}$$

$$y_t = \gamma z_t + (1-\gamma) x_{T,t} \tag{37}$$

$$\overline{y}_{T,t} = 0 \tag{38}$$

$$x_{T,t} = y_t - (s_t - p_t) \tag{39}$$

$$z_t = y_t - (p_{N,t} - p_t) \tag{40}$$

$$p_t = (1-\gamma) s_t + \gamma p_{N,t} \tag{41}$$

$$p_{N,t} = -a_t + (1-\alpha) w_t + \alpha r_t^k \tag{42}$$

$$r_{t+1}^k - p_t - r_{t+1} = \sigma_1 \widehat{\omega}_t \tag{43}$$

$$i_t = \sigma_2 \widehat{\omega}_t + \sigma_3 (r_{t+1}^k - p_t - r_{t+1}) + n_{t+1} \tag{44}$$

$$k_{t+1} = i_t - \sigma_4 \widehat{\omega}_t \tag{45}$$

$$n_{t+1} = c_t^e = \sigma_5 (r_t^k - p_t + k_t) - \sigma_6 (s_t - p_t + r_t^* + b_t^*) \tag{46}$$

$$b_{t+1}^* = p_t - s_t + \sigma_7 i_t - \sigma_8 n_{t+1} \tag{47}$$

$$r_t^k - p_{N,t} = z_t - k_t \tag{48}$$

[29] The only exception is $\overline{\omega}_t$, whose linearized version is denoted $\widehat{\omega}_t$.

$$w_t - p_{N,t} = z_t - l_t \tag{49}$$

$$l_t = w_t - p_t - c_t \tag{50}$$

$$y_t = \sigma_9 c_t + \sigma_{10} c_t^e + \sigma_{11} i_t \tag{51}$$

$$-x_{T,t} = \tau_t \tag{52}$$

$$(b_0^* - d_0)\sigma_{12} = \beta \sum_{t=0}^{\infty} \beta^t \tau_t, \tag{53}$$

where the coefficients $\sigma_i > 0$, $i = 1, ..., 12$, are functions of the structural parameters of the model evaluated in the RSS (see Appendix D for full expressions). Eq. 34 is the linearized version of the Euler equation for consumption. The approximation of the money-demand equation (i.e., Eq. 8) is given by Eq. 35. The supply-side of the model is essentially described by Eqs. 36-38. The linearized production function of intermediate firms is given by Eq. 36, while that of the final firm is stated in Eq. 37. Due to the assumption that the supply of tradable goods is constant over time $\overline{y}_{T,t}$, its linearized version, is equal to zero. Eqs. 39 and 40 are linearized versions of the input demand functions stated in Eqs. 2 and 1, respectively. We have used the fact that the LOOP holds and thus $p_{T,t} = s_t$. The linearized price index of the economy is given by Eq. 41; while that of nontradable goods is given by Eq. 42.

The presence of CMIs is essentially reflected in Eqs. 43-47. Eq. 43 is the linear approximation of Eq. 17. Similarly, aggregate investment (i.e., Eq. 19), is approximated in Eq. 44. The log-linear version of Eq. 20, the aggregate supply of capital, is given by Eq. 45. Entrepreneurs' net worth, as defined in Eq. 22, is approximated in Eq. 46. Since entrepreneurs' net worth and consumption are constant fractions of profits ($(1 - \upsilon)$ and υ, respectively, while $C^e = \frac{\upsilon}{1-\upsilon} N$ in steady state), the log-deviations of these two variables are the same. Finally, note that Eq. 47 is the linearized version of Eq. 23: once net worth, investment and the real exchange rate at period t are determined, this equation gives the total amount of debt contracted by entrepreneurs at period t.

Eqs. 48-50 are linearized versions of Eqs. 29, 30 and 9, respectively. Similarly, Eqs. 51 and 52 are the linear approximations of the clearing condition for the final good and the definition of the trade balance surplus. Since the trade balance is zero in the RSS, however, we defined τ_t ($\equiv \frac{TB_t}{(S\overline{Y}_T)^{RSS}}$) as the absolute deviation of the trade balance surplus at period t deflated by the value of tradable output. The approximation of the intertemporal national budget constraint defined in Eq. 31 gives Eq. 53. Notice that in this expression we defined $b_0^* (\equiv \frac{B_0^* - B^{*RSS}}{B^{*RSS}})$ and d_0 ($\equiv \frac{D_0 - D^{RSS}}{D^{RSS}}$), where both B_0^* and D_0 are given by the previous history of the model. Finally, observe that since in this RSS the net foreign assets of the economy as a whole are zero, $B^{*RSS} = D^{RSS}$.

5.1 Solution of the monetary side

To study the dynamics of the model it is useful to take advantage of the dichotomy which exists between the monetary and the real side of the economy, due to the households' assumed logarithmic preferences. A similar approach for solving their respective models is taken, for instance, in Benassy (1995) and in Fender and Rankin (2003). To do this, it is helpful to consider the following set of equations:

$$x_t \equiv m_t - p_t - c_t \tag{54}$$

$$h_t \equiv m_{t+1} - m_t \tag{55}$$

$$x_{t+1} = \beta^{-1} x_t + h_t \tag{56}$$

$$x_t = -\frac{\beta}{1-\beta} r_{t+1} \tag{57}$$

$$s_{t+1} - s_t = r_{t+1} - r^*_{t+1}. \tag{58}$$

Defining the demand for real money balances per unit of consumption as X_t ($\equiv \frac{M_t}{P_t C_t}$) and the (gross) growth rate of money supply between $t+1$ and t as H_t ($\equiv \frac{M_{t+1}}{M_t}$), it is easy to see that Eqs. 54 and 55 are their linearized versions. Eqs. 34, 35 and 55 can be combined to obtain Eq. 56. The money-demand equation stated in Eq. 35 is rewritten in Eq. 57 (using the definition of x_t). UIP does not require approximation, and its linearized version is given by Eq. 58. Observe that Eqs. 54-58 hold even when prices are preset at $t = 0$. Although our objective is to analyze how the model reacts to a temporary but unexpected increase in the foreign interest rate at $t = 0$, it is useful to abstract for now from this shock. For the time being we simply assume that $r^*_{t+1} = 0$ for all t. Later in the paper we will return to this assumption. It is instructive to start analyzing this subset of equations depending on the exchange rate regime that prevails in the economy. Suppose, first, that there is *a pure floating regime.* The policy variable is then the money supply. Note that Eq. 56 defines a first-order linear difference equation in the non-predetermined variable x_t. Since $\beta < 1$, this difference equation is unstable in its forward dynamics. Consider now a case in which the money supply is set permanently and unexpectedly at a level $\overline{m}$ at $t = 0$. Saddle point stability then requires that x_t immediately jumps to the steady state value $\frac{h}{1-\beta^{-1}} = 0$. It follows that $r_{t+1} = 0$ and therefore $c_t = \overline{m} - p_t$: the domestic interest rate is unaffected by the shock, implying that consumption and real money balances move together over time. Under the assumption that $r^*_{t+1} = 0$ for all t, UIP gives the result that the nominal exchange rate jumps immediately to its steady state level and remains there forever (i.e., $s_t = s_{t+1} \equiv \overline{s}$)[30].

Assume now that the economy has *a fully credible fixed exchange rate.* Consider also a case in which the economic authority sets unexpectedly though permanently the nominal exchange rate at a level $\overline{s}$ in period $t = 0$; it is easy to see that $r_{t+1} = 0$ provided that $r^*_{t+1} = 0$ for all t. The now endogenous money supply will be constant at some level, say, $\overline{m}$.

This dichotomy between the monetary and the real side of the model is not complete, however, since $\overline{s}$ (or $\overline{m}$ under a fixed exchange rate) still remains to be determined. To do this, it becomes necessary to also study the real side of the model.

6 Dynamic properties under flexible prices and no shocks

The time paths for the different endogenous variables are particularly difficult to obtain. The reason is, essentially, that we have net worth and entrepreneurs' debt as additional state variables. Although this

[30] This is an important implication of the model, implying that the nominal exchange rate does not show non-trivial dynamics as in the well-known overshooting model of Dornbusch (1976). With a more general specification of preferences, such as $U_t = \sum_{t=0}^{\infty} \beta^t [\log C_t + \frac{\chi}{1-\epsilon} \log(\frac{M_t}{P_t})^{1-\epsilon} - \frac{\kappa}{2}(L_t)^2]$, where the consumption-elasticity of money demand ϵ is different from 1, it is possible to recover the overshooting result.

fact makes the computation of the solution cumbersome, it adds interesting elements to the analysis. As we shall see, net worth plays a key role in determining both the amplification and the degree of persistence of the shock.

Recall that the increase in the foreign interest rate is temporary, occurring at $t = 0$ while prices of intermediate firms are preset. For all $t > 0$, when prices are fully flexible, we still keep the assumption that $r^*_{t+1} = 0$. The solution discussed in this section holds for all $t > 0$ (flexible prices and no shocks).

We keep here the assumption that either the money supply is set permanently at $\overline{m}$ under a pure floating regime or that the nominal exchange rate is set permanently to $\overline{s}$ under a pure fixing regime at $t = 0$. In this sense, the solution discussed here is 'general'. As we shall see, whether the exchange rate is floating or pegged reduces to an adequate identification of the policy variable (either $\overline{m}$ or $\overline{s}$) at the time of solving the model. It is useful to emphasize that net worth and capital at period $t = 0$, n_1 and k_1, can be affected by the shock. From the viewpoint of period $t = 1$, however, both n_1 and k_1 are predetermined. Conditional on these variables we derive now the time paths of the model. To complete the solution we will later turn to $t = 0$, where both n_1 and k_1 are determined.

6.1 Minimum state-space representation

In what follows we assume that the exchange rate is floating (the same logic applies to a fixed exchange rate, however). Since we are not interested in productivity shocks we set here $a_t = 0 \ \forall \ t$[31]. Let $\overline{e} \equiv \overline{s} - \overline{m}$ denote the difference between the log-deviation of the exchange rate and the money supply, respectively. In Appendix D we show that Eqs. 34-51 can be reduced to the following system[32]:

$$\begin{bmatrix} k_{t+1} \\ n_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} k_t \\ n_t \\ c_t \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \overline{e} \ \forall t > 0, \tag{59}$$

where the coefficients $a_{i,j}$ and b_j, $i, j = 1, 2, 3$, are complicated expressions of the structural parameters of the model (defined in this Appendix). Observe that Eq. 59 defines a nonhomogeneous first-order system of linear difference equations, in which the capital stock k_t and net worth n_t are predetermined variables at each period t; while c_t is a non-predetermined or 'jump' variable. Note that in this representation b^*_{t+1}, which is also a state variable, does not appear. The reason is that it can be 'eliminated' for all $t > 0$ introducing Eq. 47 into Eq. 46. Its initial level b^*_0, however, will affect the solution of the model through its effect on net worth at period $t = 0, n_1$.

This representation takes the value of the endogenous variable $\overline{s}$ as given. This variable does not affect the matrix of coefficients that pre-multiplies the vector of endogenous variables, however, and thus it will not affect the speed of convergence to the new steady state. But we then have to solve for $\overline{s}$. The strategy for doing this follows a number of steps.

We first obtain the time paths for k_t, n_t and c_t $\forall t > 0$ conditional on n_1, k_1 and $\overline{e}$. We can then obtain the time path for τ_t $\forall t > 0$, also conditional on n_1, k_1 and $\overline{e}$. We will later move to period $t = 0$ to express n_1, k_1 and τ_0 as functions of $\overline{e}$ and r^*_1.[33] With these results in hand, τ_t can be written as a function of

[31]The model can be extended to also consider technology shocks in the intermediate sector. It will then be required to relax the assumption that $a_t = 0 \ \forall t$ and to define a law of motion for this variable.

[32]To derive this system we have considered the result that $r_{t+1} = 0$ under the the assumption that $r^*_{t+1} = 0$ for all $t > 0$.

[33]Note that r^*_1 is the value of the foreign interest rate at $t = 0$. Also, since we will assume that before the shock the system is located in the RSS, initial conditions such as b^*_0 are not part of the final solution (i.e., are equal to zero).

only one endogenous variable, $\overline{s}$, $\forall t$. Substituting the obtained expression for τ_t into Eq. 53, we obtain one equation in one unknown, from which the nominal exchange rate is determined (recall that $\overline{m}$ is the policy variable). There will be a unique value of $\overline{s}$ that guarantees the solvency of the economy from an intertemporal perspective.

To study the dynamic behavior of the model $\forall t > 0$ it is necessary to first evaluate the roots of the above system of difference equations. Notice that satisfying the saddle point stability property will require us to find two roots lying inside and one root lying outside the unit circle in absolute value. Since the coefficients $a_{i,j}$ and b_j, $i, j = 1, 2, 3$ are highly complicated functions of the underlying parameters of the model, to proceed with its solution it becomes necessary to undertake a calibration exercise.

6.2 Parameterization

The following table defines the baseline parameter values of the paper:

Table 2. Baseline parameter values

υ	μ	β	α	θ	γ
0.07	0.12	0.99	0.33	10	0.74

Regarding υ, the fraction of entrepreneurs dying each period, we set $\upsilon = 0.07$. This value implies that in steady state entrepreneurs consume 7% of their profits. Turning to monitoring costs, we set $\mu = 0.12$. This value is the same as in Bernanke *et al.* (1999), but below of that considered in Carlstrom and Fuerst ($\mu = 0.25$). Here we set $\beta = 0.99$, a standard value in the RBC literature[34]. From Chari *et al.* (2002) we take: $\alpha = 0.33$ (share of capital in the intermediate good) and $\theta = 10$ (elasticity of demand for intermediate goods). Finally, we calibrate the value of γ, the share of nontradable inputs in the final good, considering Argentine data for the period $1993 - 2007$, giving $\gamma = 0.74$[35].

Table 3. Steady state implications

$\Phi(\overline{\omega})$	$r^k - r$	$r^{nd} - r$	$\frac{sB^*}{N}$	$\frac{I}{N}$	$\frac{K}{N}$
0.010	0.065	0.070	0.020	1.020	1.019

Under the baseline specification the steady state probability of default $\Phi(\overline{\omega})$ is equal to 1% (the steady state value of $\overline{\omega}$ is 0.02), the 'excess return' from producing capital $r^k - r$ is 6.5% and our measure of risk premium $r^{nd} - r$ is 7% (see Table 3)[36]. Note that the degree of leverage ($= \frac{sB^*}{N}$) is particularly low; and so are the ratios $\frac{I}{N}$ and $\frac{K}{N}$. This can be thought of as a case in which entrepreneurs heavily rely on internal resources to undertake production. This fact seems to be consistent with developing economies, however. It is well known that in less developed financial markets self-finance takes a large stake in total investment.

[34] Note that υ, β and μ must satisfy the steady state restrictions $\beta + \upsilon > 1$ and $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. If β $(= 0.99)$ and υ $(= 0.07)$, it follows that $\mu = 0.12$ is the highest value of μ such that these restrictions are satisfied.

[35] We take the data from the Argentine Ministry of Economy for GDP by sectors (at 1993 prices). To obtain the shares of tradables and nontradables over GDP, we take simple averages for the period 1993-2007 following the methodology stated in the Appendix of Canzoneri *et al.* (1999).

[36] Observe that r and r^{nd} denote the gross real domestic risk-free and non-default interest rate, respectively, in the zero-inflation steady state.

6.3 Stability properties of the model

Under the baseline parameterization the saddle point stability property is satisfied: there are two roots inside, λ_1 and λ_2, and one root outside, λ_3, the unit circle (see Table 4). The stable solution (conditional on $\overline{s}$) can then be written as:

$$\begin{bmatrix} k_t \\ n_t \\ c_t \end{bmatrix} = \kappa_1 \begin{bmatrix} v_{11} \\ v_{21} \\ v_{31} \end{bmatrix} (\lambda_1)^t + \kappa_2 \begin{bmatrix} v_{12} \\ v_{22} \\ v_{32} \end{bmatrix} (\lambda_2)^t + \begin{bmatrix} ss_1 \\ ss_2 \\ ss_3 \end{bmatrix} \overline{e}, \tag{60}$$

where we have set to zero the constant of integration associated with the unstable root[37],[38]. Letting $i = 1,2,3$ and $j = 1,2$ we have that: $v_{i,j}$ are components of the associated eigenvectors of each eigenvalue λ_j; κ_j are constants of integration to be determined and ss_i are steady state coefficients. Under the baseline parameterization the solution of the system is completed with the components of the RHS of Eq. 60 taking the following values:

Table 4. Solution under baseline parameter values

λ_1	λ_2	λ_3	v_{11}	v_{21}	v_{31}	v_{12}	v_{22}	v_{32}	ss_1	ss_2	ss_3
0.24	0.97	4.75	0.70	0.70	0.17	-0.05	-1.00	-0.01	-0.34	-0.34	-0.34

It is easier to analyze the solution of the model in terms of the recursive equilibrium law of motion of the three endogenous variables conditional on $\overline{s}$ $\forall t > 0$:

$$\begin{aligned} c_t &= \eta_{c,k} k_t + \eta_{c,n} n_t + \eta_{c,e} \overline{e} \\ k_{t+1} &= \eta_{k,k} k_t + \eta_{k,n} n_t + \eta_{k,e} \overline{e} \\ n_{t+1} &= \eta_{n,k} k_t + \eta_{n,n} n_t + \eta_{n,e} \overline{e}, \end{aligned} \tag{61}$$

where the coefficients $\eta_{p,q}$ are the elasticities of each variable p with respect to variable q. To further understand the properties of the model, we show in the table below the elasticities $\eta_{p,q}$ for a subset of endogenous variables. We address here three cases: (i) $\mu = 0.01$, a case in which agency costs are negligible, (ii) $\mu = 0.07$, a 'half way' value and finally (iii) $\mu = 0.12$, the benchmark case. The remaining parameters are set according to the baseline specification.

Table 5. Equilibrium law of motion under flexible prices

	$\mu = 0.01$			$\mu = 0.07$			$\mu = 0.12$		
	k_t	n_t	$\overline{e}$	k_t	n_t	$\overline{e}$	k_t	n_t	$\overline{e}$
k_{t+1}	0.244	0.000	-0.260	0.232	0.012	-0.260	0.206	0.038	-0.260
n_{t+1}	-0.725	0.969	-0.260	-0.736	0.980	-0.260	-0.761	1.005	-0.260
c_t	0.244	0.000	-0.260	0.246	-0.002	-0.260	0.250	-0.006	-0.260
y_t	0.244	0.000	-0.260	0.242	0.002	-0.260	0.238	0.006	-0.260
i_t	0.244	0.000	-0.260	0.252	-0.008	-0.260	0.267	-0.023	-0.260
$\widehat{\omega}_t$	0.042	-0.042	0.000	0.661	-0.661	0.000	50.87	-50.87	0.000

[37] This approach is extensively used in Sargent (1987).

[38] To be more precise, we are imposing the following set of conditions: (i) initial conditions on the predetermined variables k_t and n_t, indicating that their initial values k_1 and n_1 are taken as given and (ii) a final condition on the non-predetermined variable c_t, $\lim_{T\to\infty} \lambda_3^{-T} c_T = 0$, implying that consumption must grow at a rate lower that λ_3.

When $\mu = 0.01$ the elasticities of c_t, k_{t+1}, y_t and i_t coincide. This result is also obtained when the same model is developed without CMIs from the outset. Note also that the elasticities with respect to $\overline{e}$ remain unaffected by changes in μ. This is a direct consequence of the fact that the steady state of the linearized model is independent of those variables associated with CMIs. This can be seen observing that the steady state solutions of the variables defined in Eq. 60 coincide, thus yielding $k = n = c = -\frac{(1-\gamma)}{1-\alpha\gamma}\overline{e}$ (equal to $-0.34\overline{e}$ under our specification of parameters). This result is not completely surprising since we have considered a large number of simplifications. Our perception is that the linearity of the production function of capital and the monitoring technology are driving this result. A more detailed study of this property is left for further research.

Note also that the elasticity of c_t with respect to n_t is negative when $\mu = 0.07$ or $\mu = 0.12$. This follows from the fact that higher values of n_t also imply higher entrepreneurial consumption, c_t^e (through n_{t+1}), which other things being equal reduces the availability of the final good for households' consumption. It is somehow puzzling the fact that the elasticity of i_t with respect to n_t is negative when $\mu = 0.07$ or $\mu = 0.12$. To explain this result it is useful to combine Eqs. 43 and 44: $i_t = (\sigma_1\sigma_3 + \sigma_2)\widehat{\omega}_t + n_{t+1}$. So, as n_t rises there is a positive effect on i_t through n_{t+1}. The probability that entrepreneurs default on their debt, however, decreases ($\widehat{\omega}_t$ becomes negative) more than proportionately. This latter effect has a negative impact on i_t, and overcomes the first positive effect. Putting it another way, when net worth rises there is a gain in efficiency, implying a reduction in expected monitoring costs and therefore in the total investment required to produce capital.

7 Results

We now study the dynamics of the model including $t = 0$, the period in which intermediate firms do not adjust prices and the foreign interest rises unexpectedly. In what follows the subindex 0 indicates a short-run value for any given variable. By assumption, then, $p_{N,0} = 0$. In this case intermediate output will be demand-determined by Eq. 40 and Eq. 42 will not hold.

7.1 Monetary side at $t = 0$

It is helpful to start describing the shock and its effects on the monetary side. At time $t = 0$ the foreign interest rate unexpectedly rises say, from 0 to r_1^*. From $t = 1$ onwards r_{t+1}^* returns to its pre-shock level $(= 0)$, and stays there forever. No further shocks affect the economy. Suppose that the economy has *a pure floating regime*. We keep for the moment the assumption that money supply is also set permanently to $\overline{m}$ at $t = 0$. It follows then that the domestic interest rate is unaffected by the shock even at $t = 0$ (i.e., $r_{t+1} = 0 \ \forall t$). The nominal exchange rate takes, however, two different values (s_0 and $\overline{s}$):

$$s_0 = s_1 + r_1^* = \overline{s} + r_1^*,$$

where we have considered the fact that the shock is transitory, implying that $s_t = s_{t+1} = \overline{s} \ \forall t > 0$. It follows that the short-run level of the nominal exchange rate, s_0, will always be above (i.e., more depreciated) than its long-run level $\overline{s}$.

Assume now that the exchange rate is *fixed.* We also keep the assumption that the nominal exchange rate is set permanently to $\overline{s}$ at $t = 0$. Since $r_{t+1}^* = 0 \ \forall t > 0$, we know that $r_{t+1} = 0 \ \forall t > 0$; implying that the demand for nominal money balances is constant over time ($m_t = m_{t+1} = \overline{m}\ \forall t > 0$). At $t = 0$, however, UIP yields $r_1 = r_1^*$, and therefore the domestic risk-free interest rate is now affected by the shock on impact. Combining Eqs. 34 and 35 we obtain:

$$m_0 = \overline{m} - \frac{r_1^*}{1-\beta}.$$

There is a short-run level of nominal money balances below its long-run level. We can now turn to the analysis of the rest of the model. For a better understanding of the effects of the shock and the implications of CMIs, it is useful to first study the model without entrepreneurs; a case that can be fully solved by paper and pencil.

7.2 Solution without CMIs

This section provides only a brief discussion of the main effects of the shock. A more detailed analysis is left to the next subsections, where CMIs are reintroduced. Appendix D shows that recursive equilibrium law of motion without CMIs for all $t > 0$ is given by:

$$\begin{aligned} c_t &= y_t = i_t (= k_{t+1}) = \alpha\gamma k_t - (1-\gamma)\overline{e} \\ \tau_t &= \overline{e}, \end{aligned}$$

where $\overline{e} \equiv (\overline{s} - \overline{m})$ as before. Conditional on k_1, which must still be determined, the dynamics of consumption, output and investment (equal to next-period capital stock under full depreciation) coincide. The trade balance surplus is in turn constant for all $t > 0$. To solve for the full time path of these variables, it is necessary to also consider the behavior of the model at $t = 0$.

Pure floating. It is not difficult to show, assuming that the system is initially in the RSS, that the solution for the nominal exchange rate is $\overline{s} = \overline{m} - (1-\beta)r_1^*$, while that of the trade balance surplus takes the form:

$$\tau_0 = \beta r_1^*$$

and

$$\tau_t = -(1-\beta)r_1^*\ \forall t > 0.$$

The rise in the return of foreign relative to domestic bonds, encourages domestic households to accumulate a larger fraction of foreign assets at $t = 0$. Without entrepreneurs, households' net claims on foreigners are also the net foreign assets of the economy as a whole. Therefore, to accumulate a larger amount of foreign assets the economy must run a short-run trade balance surplus. This is what we observe here. After $t = 0$ the economy runs a permanent deficit, which is essentially financed with the return on the foreign bonds accumulated on impact. Provided that monetary policy remains inactive ($\overline{m} = 0$), there will be a contraction in output, consumption and investment at $t = 0$:

$$y_0 = c_0 = i_0 (= k_1) = \gamma\overline{m} - (1-\gamma)\beta r_1^*$$

and

$$y_t = c_t = i_t (= k_{t+1}) = \alpha\gamma k_t + (1-\gamma)(1-\beta)r_1^*\ \forall t > 0.$$

Things start to change when $t > 0$. The economy will eventually recover, reaching a new steady state in which $y = c = i = k > 0$: the short-run accumulation of net foreign assets implies that the economy is richer in the long-run. To further understand the effects of the shock, consider the evolution of the real exchange rate:

$$s_0 - p_0 = \gamma\overline{m} + \gamma\beta r_1^*$$

and

$$s_t - p_t = \alpha\gamma k_t - \gamma(1-\beta)r_1^* \ \forall t > 0.$$

The trade balance surplus at $t = 0$ is accompanied by a real exchange rate depreciation. However, in the new long-run equilibrium the real exchange rate will be more appreciated. Although it is beyond the scope of the present paper, notice that there is room for monetary policy to cushion the negative shock in the short-run (i.e., by setting $\overline{m} > 0$). The long-run solution of the model, however, is not affected by monetary policy.

How do results change if the *exchange rate is instead fixed*? First observe that the degree of nominal price rigidities is *de facto* larger: the general price index p_0 $(= (1-\gamma)\overline{s})$ becomes now essentially preset. It is not difficult to show that the trade balance surplus still remains the same, but output, consumption and investment are given by:

$$y_0 = c_0 = i_0 (= k_1) = \gamma\overline{s} - \beta r_1^*$$

and

$$y_t = c_t = i_t (= k_{t+1}) = \alpha\gamma k_t + (1-\gamma)(1-\beta)r_1^* \ \forall t > 0.$$

Letting exchange rate policy be inactive ($\overline{s} = 0$), the adjustment in real economic activity is larger under a peg. The reason is that the real exchange rate does not adjust at $t = 0$:

$$\overline{s} - p_0 = \gamma\overline{s},$$

and

$$s_t - p_t = \alpha\gamma k_t - \gamma(1-\beta)r_1^* \ \forall t > 0.$$

Since the relative price of tradable goods is unaffected when $\overline{s} = 0$, to generate the same accumulation of net foreign assets as under floating, the adjustment in economic activity must be larger. Nevertheless, in the long-run the economy converges to the same steady state as under floating. Although we are abstracting from exchange rate policy, note that it can also play a role in cushioning the adverse shock. As we shall see, these general intuitions about the transmission mechanism of the shock hold even when CMIs are reintroduced.

7.3 Solution with CMIs and foreign currency debt

In this subsection we discuss how the response of the model changes when CMIs are present. Since results strongly depend on the exchange regime and the currency in which entrepreneurs' debt is denominated, we study each case separately.

7.3.1 Floating exchange rate

Assume first that the exchange rate is *floating.* It is helpful to start with a brief intuitive discussion of how results might be affected by the introduction of financial frictions. To do this, consider Eq. 46 at $t = 0$:

$$n_1(=c_0^e) = \sigma_5(r_0^k - p_0 + k_0) - \sigma_6(s_0 - p_0 + r_0^* - b_0^*) = \sigma_5 y_0 - \sigma_6(\overline{s} + r_1^* - p_0),$$

where the last equality follows from the facts that $y_t = r_t^k - p_t + k_t$ (from Eqs. 40 and 48) and that the system is initially located in the *RSS* (implying that $r_0^* = b_0^* = k_0 = 0$).[39] For future reference, notice that $\sigma_5 \equiv \frac{(1-\upsilon)R^k K}{PN} = 1 + \sigma_6 > 0$ and that $\sigma_6 \equiv \frac{(1-\upsilon)SB^*}{\beta PN} > 0$. Other things equal, the rise in the foreign interest rate directly reduces net worth through its effect on the liabilities side of entrepreneurs' balance sheet. The intuition is simple: the shock depreciates the real exchange rate, therefore increasing the burden of inherited foreign-currency debt. Note that there might be an additional negative effect on net worth through the *assets side*, if the real return on previously produced capital, $r_0^k - p_0 + k_0$ $(= y_0)$ also falls after the shock.

How do these endogenous changes in net worth affect real economic activity? There are two main channels: (i) lower net worth reduces entrepreneurs' consumption, negatively affecting the demand for the final good and (ii) lower net worth raises the probability that entrepreneurs default on the debt, thereby rising monitoring costs and reducing the future supply of capital. As we shall see, these endogenous mechanisms are present when studying the dynamics of the model in full.

We now present the impulse responses to a 10% increase in the foreign interest rate at $t = 0$ (i.e., $r_1^* = 0.1$), while keeping the assumption that $r_{t+1}^* = 0 \; \forall t > 0$[40]. Three cases are considered in the analysis: (i) no credit market imperfections (no CMIs), (ii) low monitoring costs, $\mu = 0.01$ and (iii) high monitoring costs, $\mu = 0.12$ (baseline). The first case is, essentially, the model without CMIs introduced previously. All remaining parameters are set according to the baseline specification (see Table 2)[41].

Again, in the period of the shock the economy runs a trade balance surplus and the real exchange rate depreciates (see Figure 2). It is noteworthy to observe that besides a few exceptions, the responses of the variables are about the same regardless of whether financial frictions are present or not. A somehow puzzling result arises, moreover, observing that to some extent there is more amplification and persistence when $\mu = 0.01$ rather than when $\mu = 0.12$ (recall that higher values of μ are associated with a higher degree of CMIs). This fact is particularly apparent when observing the behavior of intermediate output and labor.

Why do lower values of μ imply higher amplification and persistence? The answer is given in the last two graphs. Net worth becomes more volatile when μ takes low values[42], and thereby its impact on the rest of the economy is higher. Intuitively, low values of μ imply a large leverage ratio in the initial steady state (i.e., high values of debt over net worth, $\frac{sB^*}{N}$, hence implying that σ_5 and σ_6 become very

[39] Note that this assumption also implies that $b_0^* = d_0 = 0$, and therefore Eq. 53 is simplified to $0 = \beta \sum_{t=0}^{\infty} \beta^t \tau_t$.

[40] Here we set the policy variable (either $\overline{m}$ or $\overline{s}$) to zero. We leave, then, either monetary or exchange rate policy to be inactive. Other interesting exercises (not shown) can be easily computed in this framework. For instance, setting $r_1^* = 0$ and $\overline{m} > 0$ under a floating exchange rate gives a permanent monetary expansion. Similarly, $r_1^* = 0$ and $\overline{s} > 0$ gives a permanent devaluation under a fixed exchange rate. Interesting results arise in these cases, which are not discussed here for brevity.

[41] Appendix E briefly outlines the required steps to combine the short and the long-run solutions of the model to obtain the impulse response functions discussed here.

[42] Note that we considered the evolution of entrepreneurs' consumption instead of net worth. The evolution of both variables is, however, the same since $n_{t+1} = c_t^e$. Observe also that we weighted n_{t+1} by its contribution in total output, σ_{10}. The reason is that as $\mu \to 0$, $N^{RSS} \to 0$. Therefore, the percentage deviation of net worth from its steady state level $n_{t+1}(\equiv \frac{N_{t+1} - N^{RSS}}{N^{RSS}})$ takes very large values whenever $N_{t+1} - N^{RSS} \neq 0$. The greater amplification of the shock when $\mu \to 0$ is essentially driven by this fact. Large values of n_{t+1} may not have important effects on the economy, however. The values of n_{t+1} weighted by σ_{10} $(\equiv C^e/Y)$, then provide a better account of the effects of net worth on real economic activity.

significant). In such cases, net worth is very sensitive to the unexpected currency depreciation, the key element that provides amplification. Recall that we imposed a simple rule for entrepreneurs' consumption: a constant fraction of each period's profits. This fact explains the slow adjustment of net worth, thus adding persistence. Note also that as net worth falls so does entrepreneurs' consumption, magnifying the fall in output when $\mu = 0.01$. Consider now the behavior of investment. As net worth decreases more when $\mu = 0.01$, expected agency costs also rise more in this case; thereby implying that the future supply of capital and hence intermediate and final output, tend to be lower in the transition towards the new steady state.

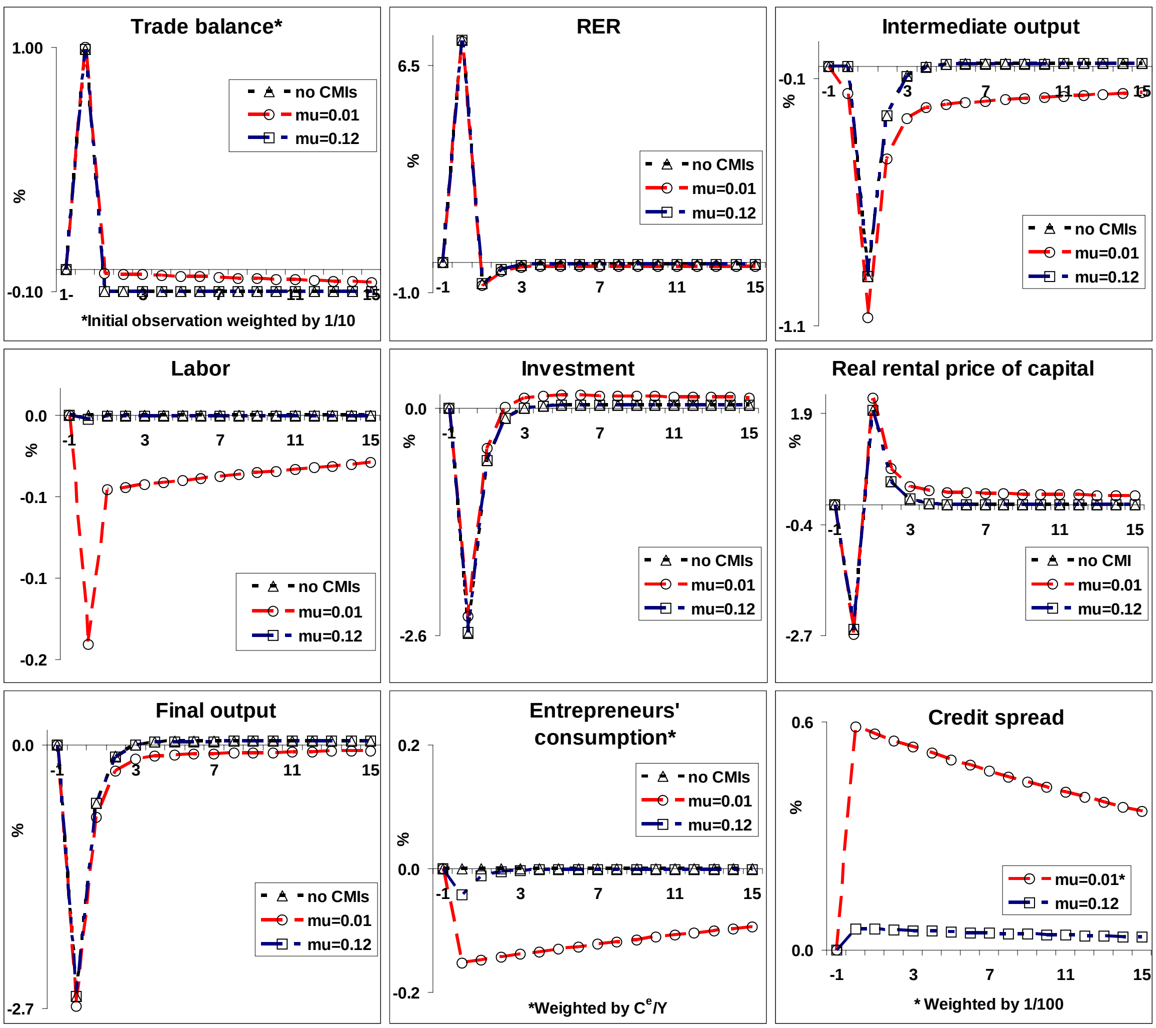

Figure 2. Responses to a rise in the foreign interest rate (floating exchange rate)

The last graph measures the credit spread between the interest rate at which entrepreneurs obtain loans, r^{nd}_{t+1}, and the risk-free interest rate of the economy, r_{t+1}, in log-linear terms: $r^{nd}_{t+1} - r_{t+1} = \frac{1}{4}\frac{(\overline{\omega}^*)^2}{g(\overline{\omega}^*)}\widehat{\omega}_t$[43] (where $\frac{1}{4}\frac{(\overline{\omega}^*)^2}{g(\overline{\omega}^*)}$ is a positive coefficient evaluated in the steady state). It clearly shows a countercyclical pattern: it goes up when net worth goes down and vice versa. This is an important result of the paper, which helps in capturing the countercyclical behavior of credit spreads to finance investment often observed in emerging

[43] This log-linear expression is derived from Eq. 18.

economies. Note also that in the related contribution of Céspedes *et al.*, (2004), the authors denote 'risk premium' to what we have called here excess return in producing capital: $r_{t+1}^k - p_t - r_{t+1} = \sigma_1 \widehat{\omega}_t$, $\sigma_1 > 0$ (i.e., Eq. 43). It is clear, however, that both measures will essentially behave in the same manner[44].

Another important result arises while observing the behavior of intermediate output at period $t = 0$. When prices are preset, this can be written as $z_0 = y_0 - (p_{N,0} - p_0) = y_0 + p_0$ (see Eq. 40). Output falls with the shock, but so does the relative price of intermediate nontradable goods. Without CMIs these two effects exactly cancel each other at $t = 0$, implying that $z_0 = 0$. Essentially the same happens when $\mu = 0.12$. As we have seen, when agency costs are set to $\mu = 0.01$ the fall in output is even larger, and thus z_0 falls (and so does l_0). This fact further amplifies the negative effect on net worth, but this time through the *assets side* of entrepreneurs' balance sheet. To see why, notice that with preset prices and $k_0 = 0$ Eq. 48 gives $z_0 = r_0^k$. Whenever $z_0 < 0$, r_0^k must be < 0: the change in this *asset price* produces additional amplification, a mechanism emphasized by Kiyotaki and Moore (1997).

To summarize, under *floating* and foreign currency debt CMIs and balance sheet effects endogenously amplify the shock - although moderately - through a fall in entrepreneurs' net worth. There are two mechanisms for this: first, the depreciation of the currency rises the real value of inherited foreign currency debt (the liabilities channel). Second, the fall in capital demand reduces its rental price, therefore reducing the real value of entrepreneurs' assets (the assets channel). Both effects are larger as $\mu \to 0$, since entrepreneurs are initially more leveraged in those cases, and thus their net worth becomes more sensitive to the shock.

We now revisit the question of amplification to see how it is affected by changes in μ or υ in more detail. To do this, consider how the components of aggregate demand change with the shock at $t = 0$, when either μ or υ varies, leaving the remaining parameters unaffected.

Table 6. Change in aggregate demand and its components (in %)

	$\mu = 0.01$	$\mu = 0.05$	$\mu = 0.12$	$\upsilon = 0.1$	$\upsilon = 0.15$	$\upsilon = 0.25$
$\sigma_9 c_0$	-2.008	-2.008	-2.008	-2.008	-2.008	-2.008
$\sigma_{10} c_0^e$	-0.153	-0.134	-0.042	-0.146	-0.285	-0.537
$\sigma_{11} i_0$	-0.519	-0.521	-0.526	-0.505	-0.470	-0.407
y_0	-2.680	-2.663	-2.576	-2.659	-2.763	-2.952

The most important determinant of y_0 is households' consumption, with its contribution being independent of μ and υ. The relative importance of entrepreneurs' consumption decreases with μ; while that of investment increases with μ (although at a much lower pace). This is essentially why lower values of μ amplify the shock. We emphasize this result since the importance of entrepreneurs' consumption in generating amplification through aggregate demand is a channel considered of low relevance by Bernanke *et al.* (1999) and is not discussed in Carlstrom and Fuerst (1997, 2001). Turning to υ, we would expect to find that larger values of υ increase amplification through the larger share of profits that entrepreneurs consume. This is essentially the message of the table. We now address the question of persistence in more detail (see Table 7).

[44] Céspedes *et al.*, (2004) show, however, that the evolution of the risk premium is *ambiguous* after considering the same shock in a model with a similar structure. We suspect that this ambiguity arises since the financial friction, which follows the approach developed by Carlstrom and Fuerst (1997) and Bernanke *et al.* (1999), is not derived from first principles as it is done here.

Table 7. Persistence (different values of μ and υ)

	$\mu = 0.01$	$\mu = 0.05$	$\mu = 0.12$	$\upsilon = 0.1$	$\upsilon = 0.15$	$\upsilon = 0.2$
λ_2	0.9687	0.9685	0.9672	0.9508	0.9223	0.8922

Note first that λ_1 (=$\alpha\gamma$, not shown) is not affected by changes in μ or υ. In contrast, the root associated with n_t, λ_2, decreases in μ (although moderately) and in υ. Higher values of these two parameters will then reduce persistence, with changes in υ being relatively more significant. To summarize: larger values of μ reduce both amplification and persistence while larger values of υ increase amplification but reduce persistence[45].

7.3.2 Fixed exchange rate

To explain the intuition of the adjustment process when the exchange rate is *fixed,* consider again Eq. 46 at $t = 0$:

$$n_1(= c_t^e) = \sigma_5(r_0^k - p_0 + k_0) - \sigma_6(\overline{s} - p_0 + r_0^* + b_0^*) = \sigma_5 y_0,$$

where the last equality follows from the fact that exchange rate policy is inactive (i.e., $\overline{s} = 0$, and hence $p_0 = (1-\gamma)\overline{s} = 0$), that $r_0^k - p_0 + k_0 = y_0$ and that the system is initially in the RSS. Since the real exchange rate is not affected by the shock on impact, balance sheet effects can only occur through the assets side of entrepreneurs' balance sheets. We present below the same impulse response functions discussed previously, but under the assumption that the exchange rate is *pegged* before and after the shock.

The trade balance surplus behaves exactly like before (see Figure 3). The real exchange rate, however, does not adjust on impact; therefore, a larger contraction in economic activity follows the shock. A first interesting observation is that, although the liabilities side of the balance sheet remains unaffected, the rental price of capital plummets; thereby generating a contraction in net worth similar to that under *floating* when $\mu = 0.01$, but even larger when $\mu = 0.12$. This can be seen observing that the real return on capital, $r_0^k - p_0 = z_0 - p_0 = y_0$, markedly falls on impact, thus reducing unexpectedly the real value of entrepreneurs' assets. This analysis raises an issue sometimes overlooked in the literature on balance sheet effects: a large fall in asset prices can be as damaging as the debt burden generated by the currency depreciation when entrepreneurs' debt is in the foreign currency.

Although the fall in net worth is still important, and this is why the credit spread still rises, its contribution as a source of amplification essentially disappears. The reason is that this time the fall in output (and households' consumption) is already very important even in the absence of financial frictions. With the change in net worth essentially as in the case of a *floating* regime*,* its relative importance in explaining the contraction of intermediate and final output is hence wiped away.

[45] Carlstrom and Fuerst (2001) also discuss at length how CMIs affect both amplification and persistence. They find that CMIs generate more persistence due to the slow adjustment of net worth over time. They do not find more amplification, however. We suspect that two main reasons can explain their result: (i) the fact that they set a relatively large value of $\mu(= 0.15)$ and (ii) the fact that their contracting problem is entirely real; thus reducing the influence of changes in nominal variables on net worth.

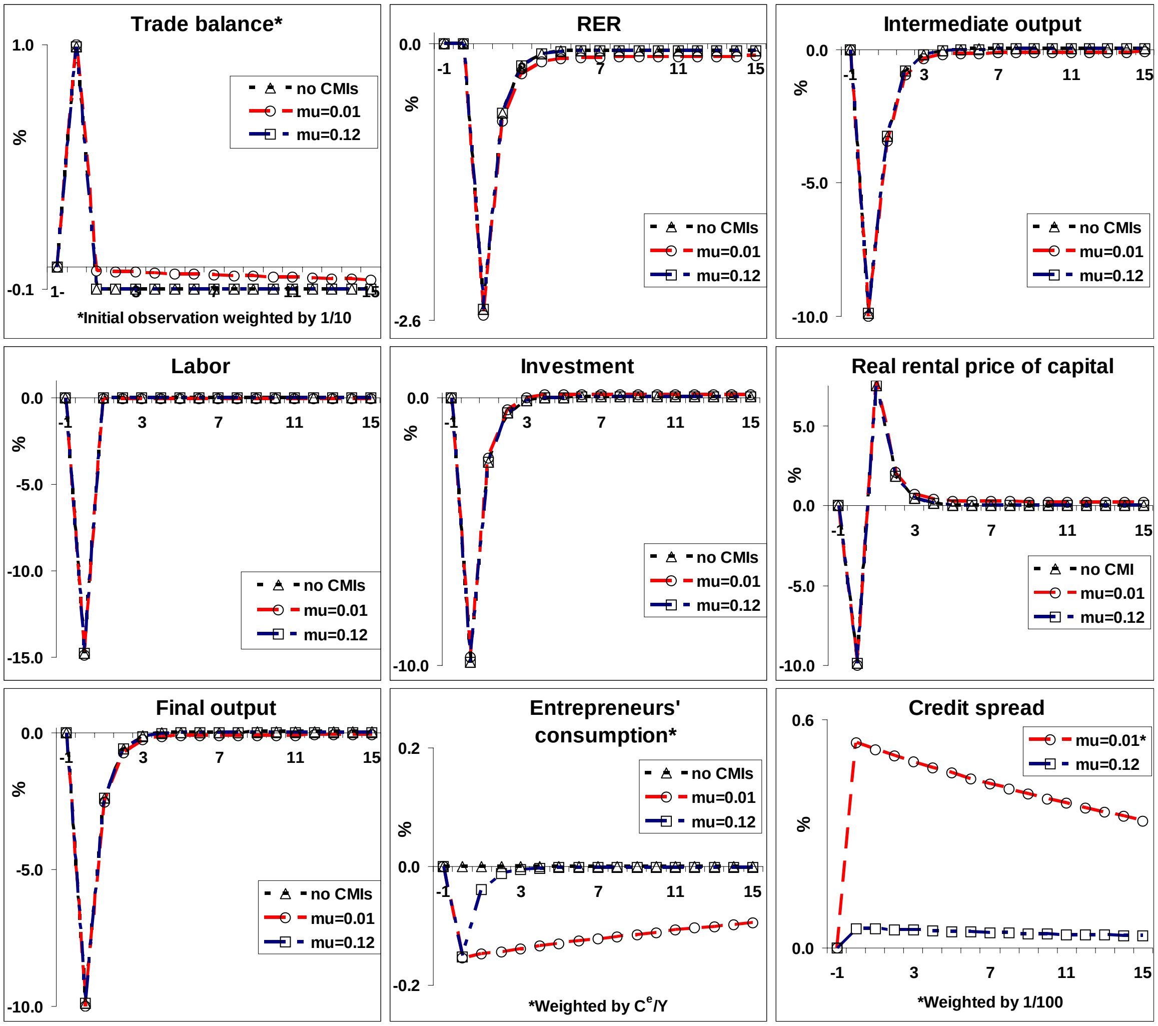

Figure 3. Responses to a rise in the foreign interest rate (fixed exchange rate)

To finish, notice that the change in the credit spread is about the same as under a pure *floating* regime when either $\mu = 0.01$ or $\mu = 0.12$. It is somehow puzzling that when $\mu = 0.12$ net worth falls more than under *floating*, whereas the credit spread rises roughly in the same proportion. To understand this, recall that the credit spread at $t = 0$ depends on $\widehat{\omega}_0$. But $\widehat{\omega}_0$ increases in k_1 while it decreases in n_1 (see Appendix D). For a given n_1, a larger level of k_1 can be supplied only if entrepreneurs increase the total amount of debt contracted at period $t = 0$. As they increase the degree of leverage to produce more capital, they become relatively riskier (i.e., $\widehat{\omega}_0$ rises). When $\mu = 0.12$, n_1 falls more than under *floating* but so does k_1, leaving the change in $\widehat{\omega}_0$ (and thus in the credit spread) essentially the same as when the exchange rate is flexible.

7.4 Solution with CMIs and domestic currency debt

Most of the recent literature on balance sheet effects has left aside the question of how results change if debt is instead denominated in the domestic currency. We explore here this issue in detail. Notice that in this case Eqs. 46 and 47 must be replaced with the linearized versions of Eqs. 25 and 26, respectively, which take the form:

$$n_{t+1} = c_t^e = \sigma_5(r_t^k - p_t + k_t) - \sigma_6(r_t + b_t - p_t), \tag{62}$$

and

$$b_{t+1} = p_t + \sigma_7 i_t - \sigma_8 n_{t+1}, \tag{63}$$

where σ_5, σ_6, σ_7 and σ_8 are defined as in Appendix D. The rest of the model remains unaffected. As we shall see, non trivial implications emerge.

7.4.1 Floating exchange rate

If the exchange rate is *floating* and the system is initially in the RSS, Eq. 62 at $t = 0$ gives:

$$n_1 (= c_0^e) = \sigma_5(r_0^k - p_0 + k_0) - \sigma_6(r_0 + b_0 - p_0) = \sigma_5 y_0 + \sigma_6 p_0.$$

The foreign interest rate r_1^* does not affect directly the liabilities side of entrepreneurs' balance sheet anymore. Using the fact that $\sigma_5 = (1 + \sigma_6)$ net worth can be further rewritten as:

$$n_1 (= c_0^e) = y_0 + \sigma_6 z_0,$$

since $z_0 = y_0 + p_0$ from Eq. 40 (recall that $p_{N,0} = 0$). If $z_0 = 0$, it follows that $n_1 (= c_0^e) = y_0$. In such a case, Eq. 35 gives $c_0 = y_0$ (recall that monetary policy is inactive, $\overline{m} = 0$, implying that $r_1 = 0$); a fact that in turn implies from Eq. 51 that $i_0 = y_0$. This is the result if debt is denominated in domestic currency. The fall in output exactly matches the reduction in the relative price of nontradable inputs, and then $z_0 = r_0^k = 0$. It follows that $n_{t+1} = k_{t+1} = y_t = c_t = i_t$ for all t, and hence the behavior of the system is essentially the same as if CMIs were absent. Hence, those parameters associated with CMIs will not have any particular role affecting the dynamics of the model (since μ and υ affect net worth at $t = 0$ only through σ_6). To put it simply, with the liabilities side of the balance sheet remaining unaffected, the period $t = 0$ return on capital r_0^k must fall to provide amplification. This result is not obtained when entrepreneurs' debt is in domestic currency, since final output and the relative price of nontradable intermediate inputs decrease in the same proportion, leaving z_0 and hence r_0^k unchanged[46].

7.4.2 Fixed exchange rate

We finally ask how results are affected if entrepreneurs' debt is in domestic currency, but the exchange rate is *fixed* before and after the shock. Evaluating Eq. 62 at $t = 0$ yields,

$$n_1 (= c_t^e) = \sigma_5(r_0^k - p_0 + k_0) - \sigma_6(r_0 + b_0 - p_0) = \sigma_5 y_0,$$

where the last equality follows from assuming inactive exchange rate policy ($p_0 = (1 - \gamma)\overline{s} = 0$), and that the system is initially in the RSS. Interestingly, the solution of n_1 is the same as under foreign currency debt. Since in this simple model the key difference between domestic and foreign debt is essentially the determination of n_1 at $t = 0$, which is the same in both cases, one would expect the dynamics of the model to be the same as well. This is the result obtained here. Under *fixing*, the model behaves in the same way regardless of the currency in which entrepreneurs' debt is denominated.

[46] The linear homogeneity of the production function of the final good is a key technical assumption driving this result. For future research, it could be interesting to study how the model behaves when the intratemporal elasticity of substitution between the two inputs required to produce the final good is different from 1.

8 Concluding remarks

This article studies how CMIs and balance sheet problems affect a small open economy. To evaluate the model's dynamics, a temporary though unanticipated rise in the foreign interest rate has been considered. To obtain amplification, the paper shows that entrepreneurs' net worth should be very sensitive to the shock, and for this to occur they must be heavily leveraged. Results strongly depend, however, on the exchange rate regime and the currency in which their liabilities are denominated. Nonetheless, even with their net worth reacting sharply to the shock and for plausible parameter values, the extra amplification due to financial frictions is at best *moderate*. An important conclusion is, however, that a policymaker willing to protect the real value of entrepreneurs liabilities by *fixing* the exchange rate in the wake of an adverse shock, effectively does so but at the cost of damaging even more the real value of entrepreneurs' assets (relative to a floating regime). Yet, from a policy-oriented perspective is always more desirable: first, prevent currency mismatches and second, allow the exchange rate to *float* when a negative shock hits the economy.

Further questions and issues can be explored in this setup. One that naturally follows is: how do simple monetary and exchange rate policies affect the performance of the economy? It is easy to explore, for instance, how a permanent monetary expansion affects the model under a floating exchange rate. Some preliminary results show that the final outcome depends on the currency in which entrepreneurs' debt is denominated. While the shock reduces the relative price of domestically-produced inputs, thus boosting output, distributive effects between lenders and borrowers may arise and operate in opposite direction. Note that this latter effect strongly resembles the amplification mechanism through credit markets emphasized by Fisher (1933). Another exercise of independent interest is to consider the effects of a permanent devaluation. This exercise may help us understand whether this framework can shed further light on the contractionary-devaluation debate, as originally summarized in Agénor and Montiel (1999). Initial explorations suggest that non-trivial results also arise in this case.

Appendix A

The entrepreneur's maximization problem can be written in terms of the following Lagrangean,

$$\max_{\{I_{j,t},\overline{\omega}_{j,t},\lambda\}} L = R^k_{t+1} I_{j,t} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} I_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (I_{j,t} - N_{j,t+1})], \tag{AA1}$$

giving the following first-order conditions:

$$\frac{\partial L}{\partial I_{j,t}} = R^k_{t+1} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t] = 0, \tag{AA2}$$

$$\frac{\partial L}{\partial \overline{\omega}_{j,t}} = R^k_{t+1} I_{j,t} f'(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} I_{j,t} g'(\overline{\omega}_{j,t})] = 0, \tag{AA3}$$

and

$$\frac{\partial L}{\partial \lambda} = R^k_{t+1} I_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (I_{j,t} - N_{j,t+1}) = 0. \tag{AA4}$$

Note that Eq. AA3 implies $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$. Replacing this expression into Eq. AA2 and rearranging gives Eq. 14. Solving Eq. AA4 for $I_{j,t}$ gives Eq. 15. To compute the second order condition of the entrepreneur's maximization problem we construct the following bordered Hessian:

$$H = \begin{bmatrix} 0 & R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t & R^k_{t+1} I_{j,t} g'(\overline{\omega}_{j,t}) \\ R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t & 0 & 0 \\ R^k_{t+1} I_{j,t} g'(\overline{\omega}_{j,t}) & 0 & R^k_{t+1} I_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})] \end{bmatrix}.$$

For a maximum the determinant of H must be greater or equal to zero (see Simon and Blume 1994, p. 461). This determinant is reduced to,

$$|H| = -R^k_{t+1} I_{j,t} [R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t]^2 [f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})],$$

and therefore the second order condition for a maximum simplifies to $[f''(\overline{\omega}_{j,t}) - \frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} g''(\overline{\omega}_{j,t})] \leq 0$. Assuming that $\overline{\omega}_{j,t}$ is within the support of $\omega_{j,t}$ we have $f''(\overline{\omega}_{j,t}) > 0$, $g''(\overline{\omega}_{j,t}) < 0$ and $f'(\overline{\omega}_{j,t}) < 0$ (See Appendix B); and therefore satisfying the second order condition also requires that $g'(\overline{\omega}_{j,t}) > 0$.

Appendix B

From the main text we have: $f(\overline{\omega}_{j,t}) = \int_{\overline{\omega}_{j,t}}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. $f(\overline{\omega}_{j,t})$ can then be written as $f(\overline{\omega}_{j,t}) = \int_0^{\infty} \omega \phi(\omega) d\omega - \int_0^{\overline{\omega}_{j,t}} \omega \phi(\omega) d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Recalling that $E(\omega) = \int_0^{\infty} \omega \phi(\omega) d\omega = 1$ we obtain,

$$f(\overline{\omega}_{j,t}) = 1 - \int_0^{\overline{\omega}_{j,t}} \omega \phi(\omega) d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$f'(\overline{\omega}_{j,t}) = -[1 - \Phi(\overline{\omega}_{j,t})], \tag{AB1}$$

and

$$f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t}), \tag{AB2}$$

implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$. Similarly, from the main text we have,

$$g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})], \tag{AB3}$$

and

$$g''(\overline{\omega}_{j,t}) = -[\mu\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]. \tag{AB4}$$

Assume now that ω is uniformly distributed in the interval $[0,2]$; therefore the mean of ω is 1. It follows that: $\phi(\overline{\omega}) = \frac{1}{2}$, $\frac{\partial\phi(\overline{\omega})}{\partial\overline{\omega}} = 0$, $\Phi(\overline{\omega}) = \frac{1}{2}\overline{\omega}$ and $1-\ \Phi(\overline{\omega}) = 1 - \frac{1}{2}\overline{\omega}$. It is easy to see that $f(\overline{\omega}) = \frac{1}{4}\overline{\omega}^2 - \overline{\omega} + 1$, $g(\overline{\omega}) = -\frac{1}{4}\overline{\omega}^2 + \overline{\omega}(1-\frac{\mu}{2})$ and therefore Eqs. AB1-AB4 are reduced to:

$$f'(\overline{\omega}) = -(1-\frac{1}{2}\overline{\omega}),$$

$$f''(\overline{\omega}) = \frac{1}{2},$$

$$g'(\overline{\omega}) = 1 - \frac{1}{2}(\mu + \overline{\omega}),$$

$$g''(\overline{\omega}) = -\frac{1}{2}.$$

Let us consider now Eq. 14 in the main text:

$$\frac{R^k_{t+1}}{R_{t+1}P_t} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}.$$

Considering the second order condition discussed in Appendix A it is possible to show that the RHS of the above expression increases in $\overline{\omega}_{j,t}$. This fact implies that $\frac{d\overline{\omega}_{j,t}}{d\frac{R^k_{t+1}}{R_{t+1}P_t}} > 0$.

Appendix C

This Appendix explains how to derive the ZISS of the model. We define here nominal variables in real terms as follows: $r^k \equiv \frac{R^k}{P}$, $R^{nd} \equiv r^{nd}$, $p_N \equiv \frac{P_N}{P}$, $s = p_T \equiv \frac{S}{P}$, $w \equiv \frac{W}{P}$, $m \equiv \frac{M}{P}$ and $tb \equiv \frac{TB}{P}$. Note that we introduced the result that $R = R^* = \beta^{-1}$ (from the Euler equation for consumption and the UIP condition).

$$C = m\frac{(1-\beta)}{\chi} \tag{AC1}$$

$$Z = AK^{\alpha}L^{1-\alpha} \tag{AC2}$$

$$Y = Z^{\gamma}X_T^{1-\gamma} \tag{AC3}$$

$$sX_T = (1-\gamma)Y \tag{AC4}$$

$$p_N Z = \gamma Y \tag{AC5}$$

$$1 = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)} s^{1-\gamma} p_N^{\gamma} \tag{AC6}$$

$$p_N = \frac{\theta}{\theta - 1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A^{-1}w^{1-\alpha}(r^k)^{\alpha} \tag{AC7}$$

$$r^k = \beta^{-1}(g(\overline{\omega}) - \frac{f(\overline{\omega})}{f'(\overline{\omega})}g'(\overline{\omega}))^{-1} \tag{AC8}$$

$$N = (1-\upsilon)r^k f(\overline{\omega})I \tag{AC9}$$

$$C^e = \frac{\upsilon}{1-\upsilon}N \tag{AC10}$$

$$I = (1 - \beta r^k g(\overline{\omega}))^{-1}N \tag{AC11}$$

$$sB^* = I - N \tag{AC12}$$

$$K = I(1 - \mu\Phi(\overline{\omega})) \tag{AC13}$$

$$r^k = \alpha p_N \frac{\theta - 1}{\theta}\frac{Z}{K} \tag{AC14}$$

$$w = (1-\alpha)p_N \frac{\theta - 1}{\theta}\frac{Z}{L} \tag{AC15}$$

$$L = \frac{1}{\kappa}\frac{1}{C}w \tag{AC16}$$

$$Y = C + C^e + I \tag{AC17}$$

$$s(\overline{Y}_T - X_T) = tb \tag{AC18}$$

$$-s\digamma \equiv s(B^* - D) = \frac{\beta}{1-\beta}tb. \tag{AC19}$$

Note first that Eqs. AC9 and AC11 give the relation:

$$r^k = [(1-\upsilon)f(\overline{\omega}) + \beta g(\overline{\omega})]^{-1}. \tag{AC20}$$

Introducing this expression into Eq. AC8 gives, after rearranging:

$$-\frac{f'(\overline{\omega})}{g'(\overline{\omega})} = \frac{\beta}{1-\upsilon}. \tag{AC21}$$

This equation pins down the steady state value of $\overline{\omega}$. From here onwards we denote its value $\overline{\omega}^*$. The steady state value of r^k is also tied down from Eq. AC20, call it r^{k*}. Turning to the remaining variables of the model, we combine Eqs. AC18 and AC19 to obtain:

$$X_T = \overline{Y}_T + \digamma\frac{1-\beta}{\beta}. \tag{AC22}$$

The real exchange rate is then written as (using Eqs. AC4 and AC22):

$$s = (1-\gamma)(\overline{Y}_T + \digamma\frac{1-\beta}{\beta})^{-1}Y. \tag{AC23}$$

Turning to the supply side of the model, Eqs. AC5, AC15 and AC16 give:

$$w = [\gamma(1-\alpha)\frac{\theta-1}{\theta}\kappa YC]^{\frac{1}{2}}.$$

Therefore, labor is reduced to:

$$L = [\gamma(1-\alpha)\frac{\theta-1}{\theta}\frac{Y}{\kappa C}]^{\frac{1}{2}}. \tag{AC24}$$

Similarly, Eqs. AC5 and AC14 yield:

$$K = \frac{\alpha\gamma}{r^{k*}}\frac{\theta-1}{\theta}Y. \tag{AC25}$$

It is easy to also derive similar expressions for I and N (from Eqs. AC9 and AC13):

$$I = \frac{\alpha\gamma}{r^{k*}}\frac{\theta-1}{\theta}\frac{1}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}Y, \tag{AC26}$$

and

$$N = \alpha\gamma\frac{\theta-1}{\theta}\frac{(1-\upsilon)f(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}Y. \tag{AC27}$$

The supply of the final nontradable good can then be written as (from Eqs. AC2, AC3, AC24 and AC25):

$$Y^{\frac{2-\gamma(1+\alpha)}{2}} = \{A(\frac{\alpha}{r^{k*}})^{\alpha}(\frac{\theta-1}{\theta}\gamma)^{\frac{1+\alpha}{2}}[\frac{1-\alpha}{\kappa}\frac{1}{C}]^{\frac{1-\alpha}{2}}\}^{\gamma}(\overline{Y}_T + F\frac{1-\beta}{\beta})^{1-\gamma}. \tag{AC28}$$

We now turn to the demand for the final good. From Eqs. AC10 and AC17 this demand takes the form:

$$Y = C + \frac{\upsilon}{1-\upsilon}N + I, \tag{AC29}$$

Households' consumption, C, can then be expressed as:

$$C = (1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)})Y.$$

We are now ready to solve for Y introducing this expression into Eq. AC28:

$$Y = \Lambda^{\frac{\gamma(\alpha-1)}{2(1-\alpha\gamma)}}\{A(\frac{\alpha}{r^{k*}})^{\alpha}(\frac{\theta-1}{\theta}\gamma)^{\frac{1+\alpha}{2}}(\frac{1-\alpha}{\kappa})^{\frac{1-\alpha}{2}}\}^{\frac{\gamma}{1-\alpha\gamma}}(\overline{Y}_T + F\frac{1-\beta}{\beta})^{\frac{1-\gamma}{1-\alpha\gamma}},$$

where $\Lambda \equiv 1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)} \in (0,1)$. Having solved for Y we can obtain the solutions for all the remaining variables. In closing this appendix notice that entrepreneurs' debt can be derived from Eq. AC12:

$$sB^* = \alpha\gamma\frac{\theta-1}{\theta}\frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}Y,$$

which can be rewritten using Eq. AC23 as:

$$B^* = \frac{\alpha\gamma}{1-\gamma}\frac{\theta-1}{\theta}\frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}(\overline{Y}_T + F\frac{1-\beta}{\beta}).$$

Appendix D

We first define here the full expressions of the coefficients $\sigma_i > 0$, $i = 1, ..., 12$ introduced in the main text. We also show the values when $\mu = 0$ (an expression $\sigma_{/\mu=0} \rightarrow n/d$ indicates that the parameter is not defined).

$\sigma_1 \equiv \frac{1}{4}\mu\beta\overline{\omega}^* r^{k*}$; $\sigma_{1/\mu=0} \equiv 0$

$\sigma_2 \equiv (g(\overline{\omega}^*) - \frac{1}{4}(\overline{\omega}^*)^2)\frac{I}{N}\frac{R^k}{RP} = \frac{\beta(g(\overline{\omega}^*)-\frac{1}{4}(\overline{\omega}^*)^2)}{(1-\upsilon)f(\overline{\omega}^*)}$; $\sigma_{2/\mu=0} \to n/d$

$\sigma_3 \equiv (\frac{I}{N} - 1) = \frac{\beta g(\overline{\omega}^*)}{(1-\upsilon)f(\overline{\omega}^*)}$; $\sigma_{3/\mu=0} \to n/d$

$\sigma_4 \equiv \frac{1}{2}\mu\overline{\omega}^*\frac{I}{K} = \frac{\frac{1}{2}\mu\overline{\omega}^*}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$; $\sigma_{4/\mu=0} \equiv 0$

$\sigma_5 \equiv \frac{(1-\upsilon)R^k K}{PN} = \frac{f(\overline{\omega}^*)+g(\overline{\omega}^*)}{f(\overline{\omega}^*)}$; $\sigma_{5/\mu=0} \to n/d$

$\sigma_6 \equiv \frac{(1-\upsilon)R^* SB^*}{PN} = \frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)}$; $\sigma_{6/\mu=0} \to n/d$

$\sigma_7 \equiv \frac{IP}{SB^*} = \frac{f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*)}{\beta g(\overline{\omega}^*)}$; $\sigma_{7/\mu=0} \to 1$

$\sigma_8 \equiv \frac{NP}{SB^*} = \frac{f(\overline{\omega}^*)(1-\upsilon)}{\beta g(\overline{\omega}^*)}$; $\sigma_{8/\mu=0} \to 0$

$\sigma_9 \equiv \frac{C}{Y} = 1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$; $\sigma_{9/\mu=0} \equiv 1 - \alpha\gamma\beta\frac{\theta-1}{\theta}$

$\sigma_{10} \equiv \frac{C^e}{Y} = \alpha\gamma\upsilon\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$; $\sigma_{10/\mu=0} \equiv 0$

$\sigma_{11} \equiv \frac{I}{Y} = \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$; $\sigma_{11/\mu=0} \equiv \alpha\gamma\beta\frac{\theta-1}{\theta}$

$\sigma_{12} \equiv \frac{B^*}{\overline{Y}_T} = \frac{D}{\overline{Y}_T} = \frac{\alpha\gamma}{1-\gamma}\frac{\theta-1}{\theta}\frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$; $\sigma_{12/\mu=0} \equiv \frac{\alpha\beta\gamma}{1-\gamma}\frac{\theta-1}{\theta}$.

We now proceed to explain how to derive the minimum state-space system described in Eq. 59. We set $a_t = 0 \;\forall\; t$ and we define $\overline{e} \equiv \overline{s} - \overline{m}$. Recall that this representation holds when $r^*_{t+1} = r_{t+1} = 0$. To express the system in terms of the endogenous variables k_{t+1}, n_{t+1} and c_{t+1}, as a first order linear system of difference equations, we need the following intermediate results: (i) $p_{t+1} - p_t$ as a function of y_{t+1}, k_{t+1} and n_{t+1} ; (ii) n_{t+1} as a function of y_t, k_t and n_t; (iii) y_t as a function of k_t, c_t and $\overline{e}$; (iv) n_{t+1} as a function of k_t, n_t, c_t and $\overline{e}$; (v) y_t as a function of k_{t+1}, n_{t+1} and c_t; (vi) k_{t+1} as a function of k_t, n_t, c_t and $\overline{e}$; and finally (vii) c_{t+1} as a function of k_t, n_t, c_t and $\overline{e}$.

(i) Observe that Eq. 43 gives:

$$p_t - p_{t+1} = y_{t+1} - k_{t+1} - \sigma_1\widehat{\omega}_t, \tag{AD1}$$

after using the fact that $y_{t+1} - k_{t+1} = r^k_{t+1} - p_{t+1}$ (from Eqs. 40 and 48). Note also that $\widehat{\omega}_t$ can be expressed as (from Eqs. 43, 44 and 45):

$$\widehat{\omega}_t = \frac{k_{t+1} - n_{t+1}}{\sigma_1\sigma_3 + \sigma_2 - \sigma_4}. \tag{AD2}$$

Investment, therefore, is given by:

$$i_t = \frac{\sigma_1\sigma_3 + \sigma_2}{\sigma_1\sigma_3 + \sigma_2 - \sigma_4}k_{t+1} - \frac{\sigma_4}{\sigma_1\sigma_3 + \sigma_2 - \sigma_4}n_{t+1}. \tag{AD3}$$

Therefore $p_t - p_{t+1}$ can be written as:

$$p_t - p_{t+1} = y_{t+1} - k_{t+1}(1 + \Delta_1) + \Delta_1 n_{t+1}, \tag{AD4}$$

where $\Delta_1 \equiv \frac{\sigma_1}{\sigma_1\sigma_3+\sigma_2-\sigma_4}$.

(ii) From Eqs. 46 and 47 we write n_{t+2} as:

$$n_{t+2} = \sigma_5 y_{t+1} - \sigma_6(p_t - p_{t+1} + \sigma_7 i_t - \sigma_8 n_{t+1}),$$

Using Eq. AD4 we can further rewrite net worth as:

$$n_{t+1} = y_t(\sigma_5 - \sigma_6) + \sigma_6(1+\Delta_2)k_t + \sigma_6\Delta_3 n_t \tag{AD5}$$

where
$\Delta_2 \equiv \frac{\sigma_1(1-\sigma_3\sigma_7)-\sigma_2\sigma_7}{\sigma_1\sigma_3+\sigma_2-\sigma_4}$
$\Delta_3 \equiv \sigma_8 - \frac{\sigma_1-\sigma_4\sigma_7}{\sigma_1\sigma_3+\sigma_2-\sigma_4}$.

(iii) Observe that Eqs. 40, 49 and 50 give:

$$w_t - p_t = \frac{1}{2}(y_t + c_t),$$

and therefore,

$$l_t = \frac{1}{2}(y_t - c_t).$$

Introducing Eqs. 35 and 37 in 39 the demand for tradable inputs, $x_{T,t}$, can be written as:

$$x_{T,t} = z_t - \gamma^{-1}(\bar{e} + c_t). \tag{AD6}$$

The aggregate supply of the final nontradable good is thus given by:

$$y_t = \frac{2\alpha}{1+\alpha}k_t + (1 - \frac{2}{1+\alpha}\frac{1}{\gamma})c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\bar{e}. \tag{AD7}$$

(iv) Combining this expression for y_t with Eq. AD5 we obtain:

$$\begin{aligned} n_{t+1} &= [\frac{2\alpha}{1+\alpha}(\sigma_5 - \sigma_6) + \sigma_6(1+\Delta_2)]k_t + \sigma_6\Delta_3 n_t \\ &\quad +(1 - \frac{2}{1+\alpha}\frac{1}{\gamma})(\sigma_5 - \sigma_6)c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}(\sigma_5 - \sigma_6)\bar{e}. \end{aligned} \tag{AD8}$$

(v) The combination of Eqs. 51 and AD3 gives, after some manipulations, the aggregate demand for the final nontradable good:

$$y_t = \Delta_4 k_{t+1} + \Delta_5 n_{t+1} + \sigma_9 c_t, \tag{AD9}$$

where
$\Delta_4 \equiv \frac{\sigma_{11}(\sigma_1\sigma_3+\sigma_2)}{\sigma_1\sigma_3+\sigma_2-\sigma_4}$
$\Delta_5 \equiv \sigma_{10} - \frac{\sigma_4\sigma_{11}}{\sigma_1\sigma_3+\sigma_2-\sigma_4}$.

(vi) Combining Eqs. AD7, AD8 and AD9 we arrive at:

$$\begin{aligned} k_{t+1} &= (\frac{2\alpha}{1+\alpha}\Delta_6 - \Delta_7)k_t - \Delta_8 n_t + [(1 - \frac{2}{1+\alpha}\frac{1}{\gamma})\Delta_6 - \frac{\sigma_9}{\Delta_4}]c_t \\ &\quad + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\Delta_6\bar{e}, \end{aligned} \tag{AD10}$$

where
$\Delta_6 \equiv \frac{1}{\Delta_4}[1 - \Delta_5(\sigma_5 - \sigma_6)]$
$\Delta_7 \equiv \sigma_6\frac{\Delta_5(1+\Delta_2)}{\Delta_4}$
$\Delta_8 \equiv \sigma_6\frac{\Delta_3\Delta_5}{\Delta_4}$.

(vii) Finally, introducing Eqs. AD7, AD8 and AD10 into Eq. AD4 and using the Euler equation for consumption gives, after a small number of substitutions:

$$c_{t+1} = \{\alpha\gamma\Delta_9 - \frac{\gamma(1+\alpha)}{2}[\Delta_7\Delta_{10} - \sigma_6\Delta_1(1+\Delta_2)]\}k_t + \frac{\gamma(1+\alpha)}{2}[-\Delta_8\Delta_{10} + \sigma_6\Delta_1\Delta_3]n_t \quad \text{(AD11)}$$
$$+\frac{\gamma(1+\alpha)}{2}[1 + (1 - \frac{2}{\gamma}\frac{1}{1+\alpha})\Delta_9 - \sigma_9\frac{\Delta_{10}}{\Delta_4}]c_t + (\gamma-1)(1+\Delta_9)\overline{e},$$

where:
$\Delta_9 \equiv \Delta_6[\frac{2\alpha}{(1+\alpha)} - (1+\Delta_1)] + \Delta_1(\sigma_5 - \sigma_6)$
$\Delta_{10} \equiv \frac{2\alpha}{(1+\alpha)} - (1+\Delta_1)$.

Collecting Eqs. AD8, AD10 and AD11 we can finally obtain the following system:

$$\begin{bmatrix} k_{t+1} \\ n_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} k_t \\ n_t \\ c_t \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \overline{e},$$

where:

$a_{11} \equiv \frac{2\alpha}{1+\alpha}\Delta_6 - \Delta_7$
$a_{12} \equiv -\Delta_8$
$a_{13} \equiv (1 - \frac{2}{1+\alpha}\frac{1}{\gamma})\Delta_6 - \frac{\sigma_9}{\Delta_4}$
$a_{21} \equiv \frac{2\alpha}{1+\alpha}(\sigma_5 - \sigma_6) + \sigma_6(1+\Delta_2)$
$a_{22} \equiv \sigma_6\Delta_3$
$a_{23} \equiv (1 - \frac{2}{1+\alpha}\frac{1}{\gamma})(\sigma_5 - \sigma_6)$
$a_{31} \equiv \alpha\gamma\Delta_9 - \frac{\gamma(1+\alpha)}{2}[\Delta_7\Delta_{10} - \sigma_6\Delta_1(1+\Delta_2)]$
$a_{32} \equiv \frac{\gamma(1+\alpha)}{2}[-\Delta_8\Delta_{10} + \sigma_6\Delta_1\Delta_3]$
$a_{33} \equiv \frac{\gamma(1+\alpha)}{2}[1 + (1 - \frac{2}{\gamma}\frac{1}{1+\alpha})\Delta_9 - \sigma_9\frac{\Delta_{10}}{\Delta_4}]$
$b_1 \equiv \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\Delta_6$
$b_2 \equiv \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}(\sigma_5 - \sigma_6)$
$b_3 \equiv (\gamma - 1)(1+\Delta_9)$.

The minimum state-space form of the model without entrepreneurs is reduced to the following system:

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{2}{1+\alpha}\frac{\alpha}{\rho} & 1 - \frac{2}{1+\alpha}\frac{1}{\rho\gamma} \\ -\frac{1-\alpha}{1+\alpha}\frac{\alpha\gamma}{\rho} & \alpha\gamma + \frac{1-\alpha}{1+\alpha}\frac{1}{\rho} \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix} + (\gamma-1) \begin{bmatrix} \frac{2}{1+\alpha}\frac{1}{\gamma\rho} \\ 1 - \frac{1-\alpha}{1+\alpha}\frac{1}{\rho} \end{bmatrix} \overline{e},$$

where $\overline{e}$ is defined as before and $\rho \equiv \alpha\gamma\beta\frac{\theta-1}{\theta}$ (< 1). It is not difficult to show that the saddle-point property is satisfied, with one root lying inside and one root lying outside the unit circle. Calling these roots λ_1 and λ_2 respectively, we find: $\lambda_1 = \alpha\gamma < 1$ and $\lambda_2 = \frac{1}{\rho} > 1$. We can then write the equilibrium law of motion of the system as follows:

$$c_t = \alpha\gamma k_t - (1-\gamma)\overline{e},$$
$$k_{t+1} = \alpha\gamma k_t - (1-\gamma)\overline{e}.$$

After a few steps it is possible to show that these solutions imply $\tau_t = \overline{e}$ (use Eq. AD6, the fact that $l_t = 0$ and the approximation of the trade balance surplus stated in Eq. 52).

Appendix E

This Appendix explains how to derive $k_1 = k_1(\overline{s}, \overline{m}, r_1^*)$ and $n_1 = n_1(\overline{s}, \overline{m}, r_1^*)$ when prices are preset at $t = 0$. There are three main steps in doing this, which we now outline.

(i) We consider here households' consumption at $t = 0$. If the exchange rate is flexible we have $c_0 = \overline{m} - (1 - \gamma)(\overline{s} + r_1^*)$, while if it is fixed yields $c_0 = \overline{m} - (1 - \gamma)\overline{s} - r_1^*$. In either case we can express c_0 as:

$$c_0 = c_0(\overline{s}, \overline{m}, r_1^*). \tag{AE1}$$

(ii) This stage involves equilibrium conditions in the market for the capital good. At $t = 0$ Eq. 43 can be rearranged to give:

$$r_1^k = r_1 + p_0 + \Delta_1(k_1 - n_1),$$

where
$\Delta_1 \equiv \frac{\sigma_1}{\sigma_1\sigma_3 + \sigma_2 - \sigma_4}$.
This expression determines the investment-demand schedule at $t = 0$. Putting it another way, it indicates how much capital entrepreneurs are willing to produce at $t = 0$, thus determining the stock of capital available at the beginning of $t = 1$. The aggregate demand for capital at $t = 1$ can be written as:

$$r_1^k = p_1 + y_1 - k_1.$$

The combination of these two expressions with the Euler equation for consumption yields:

$$c_1 = y_1 + c_0 + \Delta_1 n_1 - (1 + \Delta_1)k_1.$$

This equation involves c_1 and y_1, which must satisfy the equilibrium law of motion at $t = 1$ (conditional on k_1 and n_1). These are given by $c_1 = \eta_{c,k}k_1 + \eta_{c,n}n_1 + \eta_{c,e}\overline{e}$ and $y_1 = \eta_{y,k}k_1 + \eta_{y,n}n_1 + \eta_{y,e}\overline{e}$, respectively. We then obtain an expression of the form:

$$k_1 = k_1(c_0, n_1, \overline{s}, \overline{m}). \tag{AE2}$$

(iii) This is the final step, in which balance sheet effects may take place. First assume a floating exchange rate. Eq. 46 evaluated at period $t = 0$ yields,

$$n_1 = c_0^e = \sigma_5 y_0 - \sigma_6(\overline{s} + r_1^* - p_0 + r_0^* + b_0^*).$$

We assume that the system is, before the shock, in steady state. This implies that $r_0^* = b_0^* = 0$. Evaluating Eq. AD9 at $t = 0$ (i.e., $y_0 = \Delta_4 k_1 + \Delta_5 n_1 + \sigma_9 c_0$), and using the fact that $\overline{s} - p_0 = \overline{s} - \overline{m} + c_0$ we obtain:

$$n_1 = n_1(c_0, k_1, \overline{s}, \overline{m}, r_1^*). \tag{AE3}$$

A similar procedure shows that the same expression is obtained when the exchange rate is fixed. These intermediate steps allow us to obtain $k_1 = k_1(\overline{s}, \overline{m}, r_1^*)$ and $n_1 = n_1(\overline{s}, \overline{m}, r_1^*)$ (combining Eqs. AE1, AE2 and AE3). The trade balance surplus at period $t = 0$, τ_0, can then be written as: $\tau_0 = \tau_0(\overline{s}, \overline{m}, r_1^*)$. It is then possible to express the trade balance surplus τ_t also as a function of $\overline{s}, \overline{m}$ and r_1^*. Nonetheless, to fully solve the model we ought to obtain its full time-path, which also evolves over time. A few extra manipulations are required to finally derive the time path for the trade balance surplus, which also depends on t through Eq. 60. The final solution for either $\overline{s}$ or $\overline{m}$ is obtained using Eq. 53.

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