

UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY

July 27, 2004

9:00 am - 1:00 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers,. To complete the label, write:

(1) your assigned number

(2) the number of the question you are answering

(3) the position of the page in the sequence of pages used to answer the questions.

Example: "#001"

Question 1, page 2 of 3

- Do not answer more than one question on the same page !

When you start a new question, start a new page.

- DO NOT write your name anywhere on your answer sheets!

After the examination, the question sheets and answer sheets will be collected.

- Please DO NOT WRITE on the question sheets.

- Please solve any four of the six problems.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- Please return unused portions of the yellow tablet.
- There are 7 pages in the exam – please make sure you have all of them.
- Good luck!

Question 1

Let G denote the normal form game below, and let $G^\infty(\delta)$ denote the infinite repetition of G with discount rate $\delta \in (0, 1)$.

		2		
		a	b	c
1	A	1, 1	0, -1	-1, 1
	B	-1, 0	0, 0	-1, 0
	C	1, -1	0, -1	-2, -2

(i) (4 points) Compute the set of Nash equilibria of G , and graph the set of Nash equilibrium payoffs.

(ii) (2 points) Is there a correlated equilibrium of G yielding a payoff of $(-\frac{1}{4}, -\frac{1}{4})$? If so, construct such an equilibrium; if not, prove that no such equilibrium exists.

(iii) (2 points) Show that if $\delta = \frac{1}{2}$, then the payoff vector $(-1, -1)$ can be attained in a subgame perfect equilibrium of $G^\infty(\delta)$.

(iv) (2 points) Show that if $\delta = 2^{-k-1}$ for some positive integer k , then the payoff vector $(-1, -1)$ can be attained in a subgame perfect equilibrium of $G^\infty(\delta)$.

Question 2

Consider a "Robinson Crusoe" production economy that produces two goods, Y_1 and Y_2 , using homogeneous of degree one (i.e. constant returns to scale), concave, increasing, production functions $Y_1 = F_1(L_1, K_{11}, K_{12})$, $Y_2 = F_2(L_2, K_{21}, K_{22})$ that each use labor L_j and two capital goods, K_{j1} and K_{j2} . Here $L_j, K_{js}, s = 1, 2$ denote labor input and capital input of type s into sector $j = 1, 2$.

The economy has a representative consumer with a strictly quasiconcave, increasing utility function $U(X_1, X_2)$ defined on two consumer goods, X_1 and X_2 . The consumer has an endowment of L_0 units of labor. Steady state interior equilibrium (denoted by starred quantities) is defined by the following conditions:

- (1) $(X_1^*, X_2^*) = \operatorname{argmax} U(X_1, X_2) \text{ s.t. } P_1^* X_1 + P_2^* X_2 = I^*$.
- (2) $I^* = W^* L_0$.
- (3) $X_1^* + d_1 K_1^* = Y_1^*$, $X_2^* + d_2 K_2^* = Y_2^*$.
- (4) $(L_j^*, K_{j1}^*, K_{j2}^*) = \operatorname{argmax}\{P_j^* F_j(L_j, K_{j1}, K_{j2}) - W^* L_j - R_1^* K_{j1} - R_2^* K_{j2}\} \text{ (} j = 1, 2\text{)}$.
- (5) $L_1^* + L_2^* = L_0$, $K_{11}^* + K_{21}^* = K_1^*$, $K_{12}^* + K_{22}^* = K_2^*$.
- (6) $P_i^* = R_i^* / d_i \text{ (} i = 1, 2\text{)}$.

Equations (1) and (2) define the consumer side. The consumer is a price taker who faces prices P_1^* , P_2^* and chooses X_1^* , X_2^* to maximize its utility subject to its budget constraint, where the steady state net equilibrium income I^* is given by wages from its labor endowment.

Equation (3) is a material balance constraint that follows from the assumption that there is a costless transformation technology available that can turn sector j output into consumption of type j or capital of type j (on a one for one basis). Hence, consumption of type j plus steady state replacement of capital of type j (which depreciates at rate $d_j > 0$, $j = 1, 2$) must add up to output of type j in the steady state.

Equations (4) and (5) are self evident. Equation (6) says that the price of capital of type i is the capitalized present value of returns, R_i^* , capitalized at the steady state "interest rate" of zero. Positive d_i imply the present value of returns is finite.

Answer the following questions.

- 1 (1). For any equilibrium, show that $R_i^* / d_i = C_i(W^*, R_1^*, R_2^*)$, $i = 1, 2$, where $C_i(W, R_1, R_2)$ is the unit cost function for sector i , $i = 1, 2$.
2. (3) Assume U is strictly concave and increasing in (X_1, X_2) . Investigate existence and uniqueness of equilibria. Assume extra conditions like Inada conditions on

production functions if you need them. But explain why you need any such assumptions.

3. (2) Suppose the utility function is given by $U(X_1, X_2) = (X_1)^a (X_2)^{1-a}$, $a \in (0, 1)$. Suppose the parameter a increases. What happens to the equilibrium quantities P_1^*/W^* , P_2^*/W^* ? Does your result depend upon the assumption that U is Cobb Douglas?

4. (2) Let A_{ij} denote the input of capital of type j into producing one unit of good Y_i . Find an expression for A_{ij} in equilibrium in terms of the unit cost function. Discuss the nature of the equilibrium dependence of A_{ij} on the parameter a in question 3 above.

5. (2) For the case where there is only one sector (instead of two sectors as above) show that the equation $R/d = C(W, R)$ has at most one positive solution $r = R/W$ under modest regularity conditions. Be precise in stating any assumptions you make.

Prelim Question

A decision maker, DM, chooses an action $a \in [0, 1]$, under an uncertain state of nature, s , which is drawn uniformly from $[0, 1]$. If DM chooses $a \in [0, 1]$ given a realized state $s \in [0, 1]$, her payoff is

$$-(a - s)^2.$$

She does not directly observe s , but an advisor, A, does with probability p . A's payoff is

$$-(a - b - s)^2,$$

when action a is chosen in state s , where $b \geq 0$. [Note that, in state s , DM prefers s whereas A prefers $\min\{s + b, 1\}$. In other words, A is biased toward a larger action by b .]

Consider the following simple communication/action game. In stage 0, Nature reveals the value of s to A with probability p ; DM cannot tell whether s was revealed to A (except when $p = 1$). In stage 1, A decides whether to disclose s if she observed it. (If she did not observe it, there is no disclosure.) In stage 2, DM picks her action. A's information is "hard" in the sense that he cannot fabricate or manipulate it once he observes it, but he can withhold it. (Hence, A's choice is either to reveal his information truthfully or not to disclose it.)

(a) Assume $p = 1$, so that DM always observes s . Consider a perfect Bayesian equilibrium in which A withholds s if and only if $s \in S$, for some $S \subset [0, 1]$. Characterize the equilibrium action DM will choose *when there is no disclosure*. Compute that action explicitly if $S = [s_1, s_2]$ (i.e., when S is an interval).

(b) Assume $p = 1$ and $b = 0$. Find a perfect Bayesian equilibrium of this game. In particular, characterize the equilibrium disclosure behavior by A and the action taken by DM in each state s .

(c) Assume $p = 1$ and $b > 0$. Find a perfect Bayesian equilibrium of this game. [Hint: Begin by supposing that DM chooses $\hat{a} \in [0, 1]$ when there is no disclosure. Next, characterize the equilibrium disclosure behavior by A, given \hat{a} . Finally, make sure that \hat{a} is consistent with that disclosure behavior.] Is the equilibrium unique?

(d) Assume $p = \frac{1}{2}$ and $b \in (0, \frac{1}{6})$. (That is, A observes s with probability $1/2$.) Find a perfect Bayesian equilibrium of this game.

Question 4

Suppose the age distribution of adults in the population is uniform between 20 and 60. All adults have the same concave von Neumann-Morgenstern utility function u and the same income y . However, the probability that an adult of age a will become ill is $a/100$. An adult who becomes ill incurs medical expenses of M . Suppose the insurance industry is competitive, but may only offer policies that fully insure. There are no overhead or transactions costs.

- a) What will be the premium charged to an agent of age a ?
- b) Suppose the government offers an alternative policy that does not discriminate by age. However, the government requires that the premium be adjusted so that the policy breaks even. Which adults will it attract, and what premium will it charge?
- c) Suppose the private market is not permitted to discriminate by age. Which agents will insure, and what will the premium be?

Prelim Question

Consider a Cournot duopoly. The market price is given by $1 - x_1 - x_2$, where x_1 and x_2 are the quantities of output produced by the two firms. There are no costs.

1. Find the (Nash) equilibrium quantities of output.
2. Suppose now that firm 1's owner first hires a manager, after which the manager of firm 1 and owner of firm two simultaneously choose outputs x_1 and x_2 . The manager of firm 1 is paid $\lambda x_1 + \pi(x_1, x_2) - F$, where x_1 is the quantity chosen by the manager, $\pi(x_1, x_2)$ is the profit earned in the duopoly game (given outputs x_1 , and x_2), and λ and F are parameters chosen by the owner of firm 1. Notice that if $\lambda > 0$, then the firm-1 owner is inducing the manager to be more aggressive in her output choices. Assume that the firm-1 owner sets F so that, in equilibrium, the manager's remuneration is zero. Assume that firm 2 cannot observe the contract chosen by the owner of firm 1, i.e., cannot observe λ and F , nor the output x_1 , when choosing x_2 . What are the equilibrium values of λ , x_1 and x_2 ? Explain your answer.
3. Repeat your answer to the previous question, assuming now that firm 2 observes the values of λ and F before the two firms choose their outputs. Compare the result to the outcome of the Stackelberg model (without managers). Provide a graphical representation, using best-response and isoprofit functions, for the resulting equilibrium output.
4. Now suppose that both owners hire managers, simultaneously making public the terms $((\lambda_1, F_1)$ and $(\lambda_2, F_2))$ of the managers' contracts, after which the managers simultaneously choose outputs. Solve for the equilibrium values of λ_1 and λ_2 .
5. How do firms' outputs and profits in the previous part compare to the those of the Nash equilibrium without output subsidies? Explain your answer. Notice that your answer to the previous two parts should allow you to make these comparisons without calculating equilibrium outputs and profits.
6. Suppose that a law is proposed making it illegal to disclose the compensation contracts of managers of firms. Given that the owners always have the option of not disclosing such contracts, why would such a law have any effect? Would you expect the owners of the two firms to support this law?

QUESTION

- A) From the Slutsky equation, express the own-price elasticity of the uncompensated demand for good ℓ in terms of the own-price elasticity of the compensated demand, good ℓ 's wealth elasticity, and good ℓ 's budget share. What has to be true if good ℓ is a Giffen good?

For the rest of the question, we consider the consumption set

$$X = \{(x_1, x_2) \in \mathfrak{R}_{++}^2 : x_1 > 1, x_2 < 2\}$$

and the utility function

$$u : X \rightarrow \mathfrak{R} : u(x_1, x_2) = \ln(x_1 - 1) - 2 \ln(2 - x_2),$$

and restrict ourselves to the price-wealth domain

$$A = \{(p_1, p_2, w) \in \mathfrak{R}_{++}^3 : p_1 + 2p_2 > w > p_1 + p_2\}.$$

- B) Prove that u is strictly increasing and strictly quasiconcave.
 C) Find the Walrasian demand functions (on A).
 D) Is good 1 normal? Inferior? Giffen? Is Good 2 normal? Inferior? Giffen? If necessary, specify the subsets of A on which each of these properties hold.
 E) What is the budget share of good 1 as a function of (p_1, p_2, w) ? What is the budget share of good 2 as a function of (p_1, p_2, w) ?
 F) Consider the price-wealth sequence

$$(p_1^n, p_2^n, w^n) = \left(\frac{4}{n}, \frac{1}{2} - \frac{1}{n}, 1\right).$$

Check that this sequence does not leave the set A (for large n). Compute the limits of the budget shares of goods 1 and 2 as $n \rightarrow \infty$.

- G) It is sometimes claimed that for a good to be Giffen it must take a large share of the consumer's budget. Discuss this claim in light of your previous answers.