

University of Wisconsin. Department of  
Economics. Macroeconomic Theory Preliminary  
Exam.

July 28, 2006, from 9:00 AM to 12:30 PM.

- On the top of EACH yellow sheet, write your ASSIGNED NUMBER, date, and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the questions sheets and yellow pads will be collected. Do not write on the question sheets.
- This is a closed book exam.
- Please solve any *two* (2) of the *three* (3) problems. The time assigned to each is one hour and fifteen minutes. Thus, you have an extra hour to read the questions and revise your answers.
- The total time allotted is three and one half (3 and 1/2) hours.
- Each problem receives the same weight (100 points). The points allocated to each subsection are indicated in each problem.
- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
- Read the problems carefully and completely before you begin your answer. The problems will not be explained. If you think that a problem is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.
- Please return unused portion of the yellow tablets.
- There are 5 pages in this exam (including this cover page). Please make sure that you got all of them.
- Good luck!

## 1 Problem I: Stock Prices (100 points)

Suppose that the stock prices are determined by the present discounted value of future dividends, i.e.

$$P_t = \sum_{j=1}^{\infty} \beta^j E_t D_{t+j}$$

Note that in this model a share of stock purchased at  $t$  provides the owner with dividends starting at time  $t + 1$ ; the sale of a share occurs after the dividend is paid.

1. (40 points) Assume that the dividend series contains a unit root, but is difference stationary. Assume there are no deterministic components. Evaluate the validity of each of the following claims:
  - (a) (10 points) Stock prices and dividends are cointegrated with cointegration vector  $(1,-1)$ .
  - (b) (10 points) Stock prices obey a random walk.
  - (c) (10 points) Excess holding returns are unpredictable.
  - (d) (10 points) If  $\Delta D_t$  lies in the space  $H_t(\lambda)$ , the Hilbert space generated by current and lagged productivity shocks  $\lambda_t$ , one can recover those shocks from  $H_t(\Delta D)$ .
2. (60 points) Suppose that the fundamental MA process for dividends is

$$D_t = \eta_t + \rho\eta_{t-1}$$

(Note that the dividend series is now assumed to be stationary in levels.)

- (a) (30 points) Derive the vector autoregressive representation of the price and the dividend series under the assumption that the information set used to form expectations is  $H_t(D)$ . What are the Granger-causal relationships between dividends and prices in this formulation? Interpret.
- (b) (30 points) Suppose that the observed (to the econometrician) dividend process is subject to measurement error, i.e. the econometrician observes

$$D_t^* = D_t + \xi_t$$

where  $\xi_t$  is white noise; assume  $E(\eta_t \xi_{t+j}) = 0, \forall j$ . Derive the vector autoregressive representation of the price series and the observed dividend series. What are the Granger-causal relationships between prices and observed dividends in this formulation? Interpret.

## 2 Problem 2: The Neoclassical Growth Model and Gains from Financial Integration (100 points)

1. **Neoclassical Growth.** Think of a country that does not trade with the outside world. Time is discrete and there is no uncertainty. The population  $N_t$  grows at the rate of  $n$ :  $N_t = n^t N_0$ ,  $n \geq 1$ . The population can be thought of as a representative dynasty that maximizes the welfare function:

$$U_t = \sum_{s=0}^{\infty} \beta^s N_{t+s} u(c_{t+s}),$$

where  $c_t$  is consumption per capita and  $u(c) \equiv c^{1-\gamma}/(1-\gamma)$  with  $\gamma > 0$ . The representative firms in the economy produce homogeneous goods according to a Cobb-Douglas production function  $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$ , where  $K_t$  denotes the capital that firms rent from the households, and  $A_t$  is labor-augmenting technology. Labor ( $L_t$ ) is exogenously supplied by the households, and  $L_t = N_t$ . Goods and factor markets are perfectly competitive. The technology advances at a constant rate:  $g = A_t/A_{t-1}$ ,  $g \geq 1$ . Capital stock depreciates at the rate of  $\delta$ ,  $0 < \delta < 1$ . For notational convenience, let's use lower case letters and tildes, respectively, to denote variables normalized by population and by efficiency unit of labor. For example,  $k_t = K_t/N_t$  and  $\tilde{k}_t = k_t/A_t = K_t/(A_t N_t)$ .

- (a) (14 points) Define a competitive equilibrium for this economy.
  - (b) (22 points) Formulate the associated Pareto planner's problem as a dynamic programming problem. Be explicit about your choice of state variable(s). There are more than one ways of "Bellmanizing" this problem. Choose the one that is the most practical for numerical computation.
  - (c) (10 points) Compute the steady-state (gross) interest rate of this economy. Denote it with  $R^*$ . Show your derivation.
  - (d) (10 points) Compute the capital stock per efficiency unit of labor ( $\tilde{k}$ ) in the steady state. Denote it with  $\tilde{k}^*$ .
2. **Gains from International Financial Integration.** At  $t = 0$ , this country now decides to open up the economy. In particular, domestic consumers can now lend or borrow at the world interest rate. The only restriction is that they cannot run a Ponzi scheme. Assume that the world interest rate is same as the  $R^*$  that you computed in 1.c. This country is small enough that its actions do not affect the world interest rate. The initial  $\tilde{k}$  is much lower than its steady-state value.
- (a) (24 points) Compare the competitive equilibrium allocations under financial integration with those of Part 1. (1) Is the new steady-state value of  $\tilde{k}$  equal to  $\tilde{k}^*$  of 1.d? (2) How does the transition speed of

$\tilde{k}$  under financial integration compare with that of Part 1? (3) Is the steady-state per-capita consumption under financial integration equal to that of Part 1? For all three questions, answer with as much detail as possible.

- (b) (10 points) Construct a measure that can quantify the welfare gains from international financial integration for this country. Your measure must have a straightforward economic interpretation.
- (c) (10 points) Given reasonable parameter values, do you think the gains from financial integration for this country will be big or small? Why?

### 3 Problem 3: Real Exchange Rates and the Current Account (100 points)

In the last few years, China's saving rate has been fairly high by international standards. At the same time, China enjoys a large trade (and current account) surplus. Recently, U.S. government officials have claimed that a devaluation of the Chinese currency will reduce China's trade surplus. In this problem you are asked to consider different models of the high saving rate/trade surplus that have characterized the Chinese economy in recent years, and to evaluate the devaluation proposal.

Consider an economy (China) populated by identical agents. Each individual has preferences defined over infinite sequences of consumption of two goods,  $c$  and  $z$ , given by

$$U = \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\theta}}{1-\theta} + \frac{z_t^{1-\theta}}{1-\theta} \right],$$

where  $0 < \beta < 1$ , and  $0 < \theta < 1$ . In the context of this model, we interpret  $c_t$  as a domestically produced (and exported) good, and  $z_t$  as an imported good. Let the international price of good  $z$  be denoted by  $q$ . Denote the real exchange rate at time  $t$  by  $e_t$ , and import taxes by  $\tau_t$ . Thus, the domestic price of good  $z$  measured in units of the (domestically produced and exported) good  $c$  is

$$p_t = qe_t(1 + \tau_t).$$

Output (of good  $c$ ) at time  $t$  is given by  $y_t$ . The aggregate feasibility constraint for this economy is

$$c_t + qe_t z_t + s_{t+1} \leq s_t(1 + r) + y_t,$$

where  $s_t$  indicates net holding of foreign assets (liabilities if negative). Thus, we are assuming that this economy can borrow and lend internationally at a fixed rate. To simplify the computations, assume that  $\beta(1 + r) = 1$ .

The individual budget constraint is such that the relevant price of good  $z$  is  $p_t = qe_t(1 + \tau_t)$  rather than  $qe_t$ . Throughout assume that government revenue (if any) is rebated lump sum to consumers.

1. (10 points) Assume that  $e_t = e$ ,  $\tau_t = 0$  and the sequence  $\{y_t\}$  satisfies

$$y_t = \begin{cases} y(1 + \gamma) & t = 0, \dots, T - 1, \\ y & t \geq T \end{cases} \quad \gamma > 0 \quad .$$

In this view of the Chinese economy, output in the short run is unusually high. Define an equilibrium that rules out Ponzi games.

2. (30 points) Go as far as you can describing the implications of this view for the time path of trade surpluses defined as

$$x_t \equiv y_t - c_t - qe_t z_t.$$

Explain your findings.

3. (10 points) What is the effect of a devaluation (interpreted as a permanent increase in  $e$ ) on the trade deficit. Explain your result.
4. (10 points) Assume that  $e_t = e$ ,  $y_t = y$  and the sequence  $\{\tau_t\}$  satisfies

$$\tau_t = \begin{cases} \tau & t = 0, \dots, T - 1, \\ 0 & t \geq T \end{cases} \quad \tau > 0 \quad .$$

In this view, at time 0, it is expected that restrictions on the importation of foreign goods will be relaxed at time  $T$ . Define an equilibrium that rules out Ponzi games.

5. (30 points) Go as far as you can describing the implications of this view for the time path of trade surpluses defined as

$$x_t \equiv y_t - c_t - qe_t z_t.$$

Explain your findings.

6. (10 points) What is the effect of a devaluation (interpreted as a permanent increase in  $e$ ) on the trade deficit. Explain your result.

**Extra Credit:** For both “views” of the reasons why this country runs a trade surplus, go as far as you can analyzing the effect of a temporary devaluation.