Optimal Structure of Agency with Product Complementarity and Substitutability

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Abstract

The paper investigates a typical situation that arises in procurement, regulation or sub-contracting. A principal needs to obtain two different inputs which she can use in any positive amounts and in any proportion. Inputs are produced by agents who have private information about their costs. The paper studies a set of conditions determining the choice between a single-agent mechanism (one agent producing both inputs) and a two-agent mechanism (each agent producing a different input). An optimal mechanism is characterized in each of these cases. In the two-agent case, it is a combination of interdependent one-dimensional mechanisms, while in the single-agent case the mechanism is multi (two)-dimensional. It is shown that when the inputs are (weakly) complementary in the principal's benefit function, a single-agent mechanism is superior, unless the production costs of the two inputs are distributed over sufficiently asymmetric supports. If they are substitutes, then either mechanism can be optimal depending on the degree of substitutability (the appropriate measure of which is derived in the paper), and the frequency of efficient types in the population.

1 Introduction

Consider a situation commonly occurring in several different contexts, such as procurement, regulation, hiring or contracting. A central authority, which could be a

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government, a city council, a firm or a household, has to allocate several tasks to providers, e.g. firms or employees, whose true costs she does not know with certainty. The economic literature has typically focused on the analysis of optimal incentives and mechanisms to extract agents' information at the minimal cost. Yet there is another side to this problem which recently started to receive increasing attention (see e.g. [3], [7], [8], [10]). Before offering a mechanism, the central authority often has an opportunity to choose an institutional structure, a framework in which its relations with the agents are to be organized. In particular, it faces a dilemma: whether to centralize production in the hands of a single provider or decentralize it by using a different provider for each good or service.

A municipal council that wants to upgrade its facilities can either place a contract for the whole project with a large construction firm, or split it and allocate a number of smaller projects between several firms. A government that needs a new defense system consisting of several weapons has to decide whether to procure some or all of them from the same manufacturer, or to order each weapon from a different manufacturer. In the sphere of regulation, a government may opt to keep the components of a regulated product or service in the hands of a monopoly, or break it up into several firms.

A similar question regarding the optimal allocation of tasks arises in the context of outsourcing and purchasing. It has become particularly relevant in the 1990s, when the process of corporate downsizing and restructuring resulted in the outsourcing of numerous functions which were previously performed "in-house". For example, these days many large firms hire outside consultants for information technology projects such as set-up and maintenance of databases and networks. Start-up companies rely on outside contractors for the development and manufacturing of the vital components of their products. This strategy allows them to bring their products to market in a much shorter period of time than is usual.

In the business literature, we find the following observations: 'Best-practice companies reduce the number of suppliers, then choose a few as partners'. 'Reducing the supplier base is undoubtedly a sensible cost-saving strategy' (see [5]). Yet, how small should this number be? What is the optimal way to allocate tasks, organize the production of inputs or structure of a multiproduct industry?

Obviously, it may be optimal for a single entity to perform technologically related tasks. However, there are many situations when components of a single system are produced independently of each other. One important example is the electric power industry, where the production of electricity and the construction and the maintenance of the grid are technologically unrelated activities. In the context of deregulation of the electric power industry, an important issue which arose was whether the production of energy and the control over the transmission grid should remain in the hands of one firm, or whether the grid should be controlled by an independent entity.

In this paper I study the optimal choice between two alternatives: allocating two tasks to two different agents, or allocating them both to a single agent. A similar question has been previously analyzed, in chronological order, by Baron & Besanko (1992), Dana (1993), Gilbert & Riordan (1995), and Jansen (1997).

Dana considers the effects which the technology of production may have on the optimal number of agents. Accordingly, he studies the case when the benefit from the two inputs, or goods, is separable, but the costs of producing them may be correlated. It is shown that when the correlation is positive and sufficiently high, a two-agent mechanism dominates a single-agent one, because the 'yardstick' competition allows the principal to extract the agents' information at a lower cost. Dana's model is closest to the one analyzed here. However, Dana concentrates on the analysis of the cost side and assumes separability in the benefit function, while in this paper the focus is on the nature of the benefit function and on the degree of complementarity and substitutabilty between the goods or inputs.

Several authors have studied the situation with complementary components that have to be combined in fixed proportions, in which case the quantity assignment is the same for both goods. In Baron & Besanko (1992) and Gilbert & Riordan (1995) it is shown that, under quite general conditions, a single-agent mechanism is superior. However, Jansen (1997) demonstrates that a two-agent mechanism becomes more profitable for the principal, if the standard limited liability assumption is relaxed. Baron & Besanko also study whether an optimal mechanism can be decentralized via delegation, or subcontracting (on this topic, also see [14]).

Fixed proportions and separability in the benefit (production) function are limiting cases of the more general situation in which the inputs combined in a final product or integrated into a system may be either complements or substitutes. As Gilbert & Riordan point out, 'Our analysis also depends on the fixed proportions production technology for the final good. This is perhaps questionable even in the electricity example, because optimizing the transmission grid may reduce the need for the new generation capacity.' Similarly, two subcontracted goods or services may be such that in the final consumption the marginal utility of one good depends on the consumption of the other.

Consider an electric power industry example. From the standpoint of final consumption of electricity, the quality of the grid and the quantity of electricity produced at the power plants are substitutes: a suboptimal grid with high percentage of energy losses necessitates an extra quantity to be produced and transmitted through the grid. This relationship is likely to be non-linear, because the extra quantity may have to be produced at other plants and sent via other, suboptimal routes with higher transmission losses. At the same time, a higher quality of the grid, as reflected in the stability of transmission and a lower probability of outages, increases the demand for electricity. This complementarity may be quite important from the societal point of view incorporating environmental concerns: extra electricity consumption means less reliance on other sources of energy such as natural gas and local fuel-powered generators of electricity.

Complementarity and substitutability can also be found between elements of integrated defense systems and between different municipal projects. For the consumers, certain regulated products may be substitutes, such as gas and electricity, or express and regular mail. Alternatively, there are obvious complementarities between such services as long-distance calling or internet connection on the one hand, and local telephony on the other.

The main contribution of this paper is to analyze the issue of optimal organization for any degree of complementarity and substitutability between two inputs, which can be combined in any proportion. It is demonstrated that in the complementarity case a single-agent mechanism is superior, unless there is a sufficiently large asymmetry in the values which marginal costs of the two goods may take. In the substitutability case the conclusion is more complex. The optimal structure depends on the degree of substitutability, the appropriate measure of which is suggested in the paper, the frequency of efficient (low cost) types in the population, and the heterogeneity between the stochastic production technologies of the two inputs.

The conclusions of the paper are based on the comparison of profitability of the optimal single-agent and two-agent mechanisms in the situation when the agent(s) have private information about marginal costs. While in the two-agent case the optimal mechanism is a combination of two interdependent single-dimensional mechanisms, in the single-agent case the analysis moves into the domain of multidimensional mechanism design. Multidimensional mechanisms have been studied by McAfee & McMillan (1988), and more recently Rochet (1992), and Choné & Rochet (1997). Results of this paper illustrate some of the phenomena identified in this literature, such as the presence of binding upward or "horizontal" incentive constraints, the existence of a region where zero quantities are optimal. At the same time, a new phenomenon is identified: the zero quantity region can be disconnected and located away from the origin.

In the course of the analysis, I identify four factors which account for different profitability of the single-agent mechanism for the principal compared to the two-agent mechanism. The first factor has to do with a larger set of possible deviations in the single-agent mechanism. In this mechanism the information about the costs of both goods is centralized in the hands of a single agent. Consequently, this agent can manipulate her cost announcement in several ways. Specifically, an agent who has low costs in production of both goods may announce that both costs are high. This particular deviation is impossible in the two-agent mechanism and potentially causes the principal to bear a larger cost of information revelation in the single-agent mechanism, which increases the attractiveness of the two-agent mechanism. I will henceforth refer to the possibility of this deviation as 'an extra deviation factor'.

The second factor is also due to the differences in the informational structures.

In the single-agent case, because the information is centralized in the hands of a single agent, the principal can prevent several deviations by making a single payment: the principal only needs to pay enough to prevent the deviation which is the most profitable for the agent. This factor produces an economy of 'informational rent' in the single-agent mechanism and, by analogy with national defense which is used to protect many different individuals at the same time, it will be referred to as a 'public good' property of the informational rent. [1] In the two-agent mechanism the situation is different: one agent's deviation and the informational rent which she earns is independent of the other agent's deviation and informational rent.[2]

The third, 'non-standard constraints' factor comes from the extra cost of satisfying non-downwards incentive constraints (upwards or horizontal) which are never binding, or even relevant, in the two-agent mechanism, but may be binding in the single-agent mechanism because of its multidimensionality. Although this factor makes a single-agent mechanism less attractive, it always remains of second-order importance: it either reduces the principal's profit in a state which is relatively unimportant because of its low probability, or it is collinear with other, stronger factors.

The fourth factor, which is called an 'extra distortion' factor, reflects the reduction in revenue caused by distorting quantities produced in the state of the world where both costs are high. This distortion is larger in the single-agent case, because decreasing these quantities leads to a reduction of the informational rents paid in two states of the world as opposed to one only in the two-agent mechanism.

The interaction of these four factors determines which mechanism is superior: a single agent or a two agent one. The above discussion reveals that only the 'public good' factor is favorable for the single-agent mechanism. However, as will be shown below, this factor is sufficiently strong to make single-agent mechanism superior in a broad range of cirsumstances. This is partly due to the fact that some of the factors are mutually exclusive: the first and the second factors are always mutually exclusive; the second and the third factors are mutually exclusive in the substitutability case.

When the products are complementary, an economy of informational rent produced by the 'public good' factor ensures the superiority of the single-agent mechanism. The only offsetting factor which may be present in this case is the 'non-standard constraints'. However, this factor normally has only a second-order effect and does not reverse the ' public good' factor. An exception is the case when the marginal costs of the two goods may take very different values, because they are distributed over

[1] This intuitive interpretation is also formally correct. When this factor is present in the only state of the world where it can possibly appear, the marginal cost of increasing the informational rent in this state is equal to the sum of the marginal benefits of increasing quantities in two different states (net of production costs), which is a well-known characteristic of public goods due to Samuelson.

[2] Dana (1993) uses a term 'informational economies of scope' to describe the same effect in his model. Gilbert & Riordan (1995) mention a similar 'double marginalization' effect which produces a negative externality in the two-agent case. This last interpretation is tied to the framework in which only a single unit of both inputs are required.

highly asymmetric supports. For example, the two goods may have very dissimilar high production costs, yet similar low production costs, or vice versa. In this case the distortion produced by the 'horizontal constraint' is sufficiently strong to eliminate economies generated by the 'public property' and make the two-agent mechanism superior.

In the case of substitutability, the pattern of interaction between offsetting forces is more complex. The results become sensitive to the degree of substitutability and the parameters of the model. A high degree of substitutability and a large probability that one of the marginal costs is high trigger an 'extra deviation' factor, which makes a two-agent mechanism more profitable. However, a single-agent mechanism dominates in the case when the degree of substitutability is small and/or the both inputs are very likely to have low costs, because the 'public good' factor eliminates the 'extra deviation' factor and offsets other factors as well.

The rest of the paper is organized as follows. In section 2 the basic model is laid out. In section 3 the optimal two-agent mechanism is derived, and the relaxed program in the single-agent case is solved. In section 4 we consider the complementarity case, and in section 5 the substitutability case. In section 6 the properties of the single-agent mechanisms are illustrated with the quadratic function example. Most proofs are relegated to the appendix.

2 Model

A central entity, or principal, which can be envisioned either as a firm or a government, needs to procure two different goods which it uses as inputs. The principal's benefit is measured by the function $v(q_1, q_2)$, where q_1 is the quantity of the first good that she procures and q_2 is the quantity of the second good. The function $v(.,.)$ can be interpreted as a production function in case of a profit-maximizing firm, or in the case of benevolent government, as a transformation function describing the technology of combining two private goods into a public good.

We assume that $v : R_+^2 \to R$ is symmetric, increasing in both arguments, twice continuously differentiable, and concave. Formally, letting the subscript i indicate the derivative with respect to the i^{th} argument, $v_i(q_1, q_2) \geq 0$, $v_{ii} < 0$ for $i \in \{1, 2\}$, and $v_{11}v_{22} - v_{12}^2 > 0$. The symmetry assumption is needed for technical purposes: it simplifies the consideration of which incentive constraints are binding at the optimum. We also assume that the cross-partial derivative $v_{12}(.,.)$ has a constant sign over the relevant domain, i.e. the goods are either substitutes or complements. To ensure an interior solution for all values of the parameters, an Inada-type boundary condition is maintained: $\lim_{q_1 \to 0} v_1(q_1, q_2) = \infty$, $\forall q_2 > 0$. If this condition does not hold, the principal may find it optimal to set some quantities to zero. I consider this case separately. However, if not indicated otherwise, the maintained assumption is that the Inada condition holds. The principal is risk-neutral, and her goal is to maximize the value of her benefit less payment for the inputs.

On the other side of the market there is a sufficiently large number of agents (employees or contractors), each of whom can produce any amount of either or both goods. An agent has constant marginal cost in production of each good, at the ex-ante stage the level of this cost is random and comes from a distribution which is common knoledge. Because of their large number, the agents do not have any bargaining power in their relations with the principal, and their reservation utility level is normalized to zero. Because of limited liability, ex-post an agent of each type earns a nonnegative profit. All agents are risk-neutral, and an agent's utility is equal to the payment that she gets for delivering the goods minus the cost incurred in production.

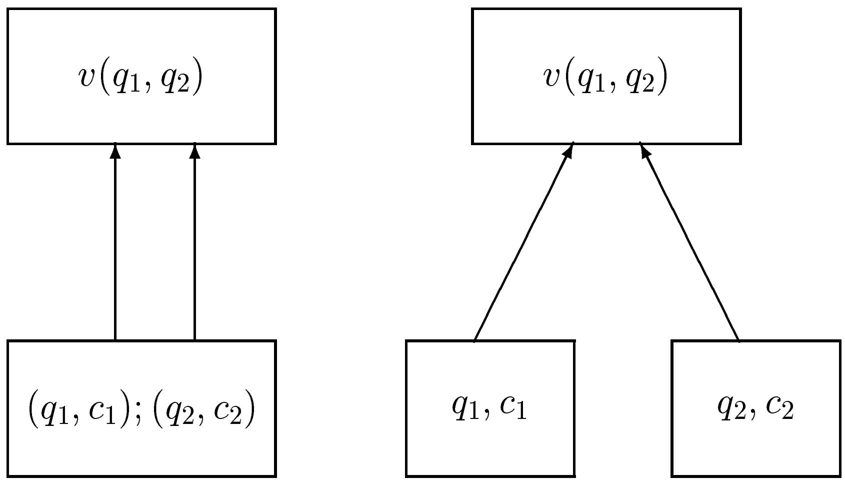
The process of procurement has two stages. In the first stage the principal chooses a supplier for each of the goods, and offers her a binding contingent contract specifying the payments and quantities in all states of the world. In the second stage the supplier(s) learn the production costs and report them to the principal. Then quantities are assigned according to the existing contracts, and production and payments takes place.

Equivalently, we may consider that in the first stage the principal commits to purchase all the supply of good i from a particular agent, while contingent contracts are signed in the second stage. An important assumption is that all supply of a particular good has to come from one source. Thus, in the first stage the principal chooses between a single-agent mechanism in which both goods are procured from the same source, and a two-agent mechanism in which each good is procured from a different source. This decision problem is illustrated in Figure 1.

Such commitment on the part of the principal may be necessary for a number of reasons. One of them is the presence of sufficiently large fixed costs which have to be incurred by an agent before she can learn the marginal cost(s) of producing the good(s). The fixed costs may take the form of R&D expenses, investments in equipment and machinery, such as generating capacity and transmission lines in the electric power industry example, or the costs of training, etc. The principal's commitment is then necessary for these costs to be expended. For example, when the government decides to develop a new defense system, it can choose among many defense contractors. Although bidding and comparison of proposals take place at the initial stage, only one firm is ultimately chosen as a supplier of each weapon, or each element of the system. Rogerson [17] points out 'economies of scale together with very small production runs render it economically infeasible to have two or more firms build fully functioning production lines.' Moreover, the final price is usually determined after the contracts have already been allocated. As Rogerson (1989) points out, '...the prices for all production runs may be left to be determined by future negotiations. Transaction costs together with constantly evolving technologies and requirements are thought to render long-term contracts infeasible.'

Thus, the first stage of this model reflects the decision of the the government

Figure 1: Optimal Choice of an Organizational Form.



to procure both elements of a defense system from the same manufacturer, or to decentralize their production between two manufacturers. Similarly, to implement its project a municipal council can hire a single contractor. Alternatively, it can divide the project into several smaller ones and allocate each of them to a different contractor. A regulator can either adopt a policy allowing the same firm to control the production of two related products, or bar a firm controling one product from dominating in the market for the second one. In the electric power industry the issue which the regulator has to resolve is whether the leading producer of electricity could also control the transmission grid, or the transmission grid should be under the control of a separate entity.

The contingent contracts implemented in the second stage are designed to elicit the information about the agent(s)' production costs. By the revelation principle, we can restrict attention to the set of direct mechanisms (see Baron (1989)) in which the agent(s) announce their types, i.e. their marginal costs, truthfully. For simplicity, I assume that the marginal cost can take only two values: it can either be low (c_L) or high (c_H), where $c_H > c_L > 0$. Marginal costs are uncorrelated across goods and across agents, and the probability that a given agent has a low marginal cost c_L in the production of good i is denoted by p_i. Although more general probability distributions can be considered (the case where marginal costs are correlated is explored in [7] and [10]), my primary goal is to investigate the effects of complementarity and substitutability in the principal's benefit function, and therefore I abstract from other effects.

A direct mechanism is a mapping from the set of possible types $\{c_L, c_H\} \times \{c_L, c_H\}$ (or states of the world) into the set of possible quantities and transfers: $R^2_+ \times R^2$ (in the two-agent mechanism), or $R^2_+ \times R$ (in a single agent mechanism). Depending on the realized costs combination, there are four possible states of the world which are denoted by LL, LH, HL and HH, where the first letter indicates the marginal cost in the production of the first good, and the second letter indicates the marginal cost in the production of the second good.

We denote by $\mathbf{q}^i = (q^i_{LL}, q^i_{LH}, q^i_{HL}, q^i_{HH})$ the vector of quantities of the good i assigned in a two-agent mechanism in different states of the world. We adopt the convention that the first letter of the subscript corresponds to the marginal cost of the good i. For example, in the state LH the mechanism assigns quantities q^1_{LH} and q^2_{HL}. Similarly, $\mathbf{g}^i = (g^i_{LL}, g^i_{LH}, g^i_{HL}, g^i_{HH})$ is the vector of quantities of good i assigned in a single-agent mechanism.

Let $t^i_{K_i}$ denote the (expected) transfer received by the agent producing good i in the two-agent mechanism, where K_i stands for the marginal cost announced by i. Without loss of generality we can take $t^i_{K_i}$ to be independent of the other agent's type, because both the principal and the agent are risk-neutral and care only about the expected transfer. Finally, T_{KF} denotes the transfer to the agent in the single-agent mechanism when she announces a pair of marginal costs (c_K, c_F) where $K, F \in \{H, L\}$.

In the two-agent mechanism agent i's type is either c_L or c_H. Since agents do not coordinate their actions, for agent i to reveal her type truthfully the following incentive constraints $IC^i(L)$ and $IC^i(H)$ have to be satisfied:

$$t^i_L - c_L \left(q^i_{LL} p_j + q^i_{LH}(1 - p_j)\right) \geq t^i_H - c_L \left(q^i_{HL} p_j + q^i_{HH}(1 - p_j)\right)$$

$$t^i_H - c_H \left(q^i_{HL} p_j + q^i_{HH}(1 - p_j)\right) \geq t^i_L - c_H \left(q^i_{LL} p_j + q^i_{LH}(1 - p_j)\right)$$

The second constraint is not binding in the optimal mechanism, while the first gives rise to an informational rent of the low cost type. It is equal to the surplus which a low-cost agent can obtain by misrepresenting her cost.

By the limited liability assumption, an agent has to earn nonnegative profits, if she is to participate in the mechanism. Therefore the following individual rationality constraints $IR^i(L)$ and $IR^i(H)$ for $i \in \{1, 2\}$ have to be satisfied in the two-agent mechanism:

$$t_L^i - c_L\left(q_{LL}^i p_j + q_{LH}^i(1-p_j)\right) \geq 0$$
$$t_H^i - c_H\left(q_{HL}^i p_j + q_{HH}^i(1-p_j)\right) \geq 0$$

In the single-agent case the set of constraints which have to be satisfied is different. An agent's type is a pair of marginal costs (c_1, c_2), and she can report any possible cost combination. Therefore, in each state of the world three incentive constraints have to hold, instead of compared two in the two-agent mechanism. The number of individual rationality constraints in the single-agent mechanism is reduced to one per state, instead of two in the two-agent case. The last fact has no effect on the results of this paper, because by risk-neutrality the wealth can be freely shifted from one state to another without any consequences for the utility level of either party.

The following list includes the downwards incentive constraints in the states LL, LH and HL, and the individual rationality constraint in the state HH.

$$\begin{array}{ll}
IC(LL-LH): & T_{LL} - c_L(g_{LL}^1 + g_{LL}^2) \geq T_{LH} - c_L(g_{LH}^1 + g_{HL}^2) \\
IC(LL-HL): & T_{LL} - c_L(g_{LL}^1 + g_{LL}^2) \geq T_{HL} - c_L(g_{HL}^1 + g_{LH}^2) \\
IC(LL-HH): & T_{LL} - c_L(g_{LL}^1 + g_{LL}^2) \geq T_{HH} - c_L(g_{HH}^1 + g_{HH}^2) \\
IC(LH-HH): & T_{LH} - c_L g_{LH}^1 - c_H g_{HL}^2 \geq T_{HH} - c_L g_{HH}^1 - c_H g_{HH}^2 \\
IC(HL-HH): & T_{HL} - c_H g_{HL}^1 - c_L g_{LH}^2 \geq T_{HH} - c_H g_{HH}^1 - c_L g_{HH}^2 \\
IR(HH): & T_{HH} - c_H(g_{HH}^1 + g_{HH}^2) \geq 0
\end{array}$$

The constraint $IC(LL-HH)$ has to be satisfied to prevent the agent from misrepresenting both marginal costs in the state LL. The cost of satisfying this constraint constitutes the 'extra deviation' factor described in the introduction. As we are going to show, when $IC(LL-HH)$ is binding, a single-agent mechanism is less profitable for the principal than a two-agent mechanism.

Because a single-agent mechanism is two-dimensional, other incentive constraints (upward or 'horizontal') may be binding in it, thereby reducing its profitability for the principal. This effect is described as a 'non-standard constraints' factor in the paper. Different subsets of these additional constraints are relevant in the complementarity and substitutability cases, and therefore we defer their discussion to the appropriate sections.

3 Optimal Mechanisms

In this section we characterize an optimal two-agent mechanism, and solve a relaxed program (to be defined below) in the single-agent case. As a benchmark, we will at first derive the first-best mechanism which maximizes the principal's profits when she has complete information about the marginal costs. The solution does not depend on the number of agents, and the vector of optimal quantities $\hat{\mathbf{q}}$ is the same for both goods. Its elements satisfy the following first-order conditions:

$$v_i(\hat{q}_{LL}, \hat{q}_{LL}) = c_L; \quad v_i(\hat{q}_{HH}, \hat{q}_{HH}) = c_H$$
$$v_2(\hat{q}_{LH}, \hat{q}_{HL}) = c_H; \quad v_1(\hat{q}_{LH}, \hat{q}_{HL}) = c_L$$

From the first-order conditions it is easy to derive the following ordering:

- In the complementarity case: ($v_{12} > 0$): $\hat{q}_{LL} > \hat{q}_{LH} > \hat{q}_{HL} > \hat{q}_{HH}$
- In the substitutability case: ($v_{12} < 0$): $\hat{q}_{LH} > \hat{q}_{LL} > \hat{q}_{HH} > \hat{q}_{HL}$

Next, we turn to an optimal two-agent mechanism. Essentially it consists of two one-dimensional mechanisms with two binding constraints in each - that is: *i)* an individual rationality constraint of the high cost type and *ii)* an incentive constraint of the low cost type. Although the sets of constraints in the two mechanisms are separate, the mechanisms are not independent because the complementarity or substitutability in the principal's benefit function create a link between the optimal quantities of the first and the second good. The complete characterization is provided in the following lemma:

Lemma 1 *The optimal quantity vectors $\boldsymbol{q}^i$ in the two-agent mechanism are uniquely determined by the following first-order conditions:*

$$v_1(q^1_{LL}, q^2_{LL}) = c_L; \quad v_2(q^1_{LL}, q^2_{LL}) = c_L$$
$$v_1(q^1_{LH}, q^2_{HL}) = c_L; \quad v_2(q^1_{HL}, q^2_{LH}) = c_L \tag{1}$$

$$v_1(q^1_{HL}, q^2_{LH}) = c_H + (c_H - c_L)\frac{p_1}{1-p_1} \tag{2}$$
$$v_1(q^2_{HL}, q^1_{LH}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2} \tag{3}$$
$$v_1(q^1_{HH}, q^2_{HH}) = c_H + (c_H - c_L)\frac{p_1}{1-p_1} \tag{4}$$
$$v_2(q^1_{HH}, q^2_{HH}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2} \tag{5}$$

The expected informational rent $EIR(2)$ paid by the principal is given by the following expression:

$$\frac{EIR(2)}{c_H - c_L} = p_1 p_2 (q^1_{HL} + q^2_{HL}) + p_1(1-p_2) q^1_{HH} + (1-p_1) p_2 q^2_{HH} \tag{6}$$

Proof: See Appendix.

Some of the results in this paper depend on the ordering of the elements of the optimal quantity vectors. As the following lemma shows, in the two-agent mechanism this ordering is unambiguously determined by the sign of the cross-partial derivative $v_{12}(.,.)$.

Lemma 2 *In the optimal-two agent mechanism the elements of the optimal quantity vectors* $\boldsymbol{q}^i$ *are ordered in the following way:*

i) if $v_{12} \geq 0$ *(complementarity)*

$$q^i_{LL} > \max\{q^i_{LH}, q^i_{HL}\} \geq \min\{q^i_{LH}, q^i_{HL}\} > q^i_{HH}$$

ii) if $v_{12} \leq 0$ *(substitutability)*

$$q^i_{LH} > \max\{q^i_{LL}, q^i_{HH}\} \geq \min\{q^i_{LL}, q^i_{HH}\} > q^i_{HL}$$

Proof: See Appendix.

Compared to the first-best mechanism, there are no distortions at the 'top' - i.e. in the state LL. All other quantities are distorted downwards in order to decrease the informational costs, except the quantity q^i_{LH} which in the substitutability case is set above its first-best level. Notice that the ordering in the two-agent mechanism coincides with the ordering in the first-best solution. This need not be the case in the single-agent mechanism under substitutability, which turns out to be quite important.

In the single-agent case, the mechanism design problem is more difficult because of the complex structure of incentive constraints. Figure 2 illustrates the differences in incentive structures of the two mechanisms. To begin with, we solve a relaxed program $RP(1)$ in which the constraints set includes only the downward incentive constraints and individual rationality constraints of the most inefficient type $IR(HH)$ (see page 2). Solving this problem is useful for two reasons. First, in many cases it provides a solution for the complete problem. Second, the solution to the relaxed program helps to identify other constraints which may be binding. Third, in the case of substitutes solving the relaxed program is sufficient, because if it fails to deliver the solution to the complete problem, then a two-agent mechanism is superior.

Lemma 3 *There is a unique direct mechanism solving the relaxed program* $RP(1)$*. In this mechanism the quantity vectors* $\boldsymbol{g}^i$*,* $i \in \{1, 2\}$*, satisfy the following first-order conditions for some* α *and* $\beta \in [0, 1]$*,* $\alpha + \beta \leq 1$

$$v_1(g^1_{LL}, g^2_{LL}) = c_L; \quad v_2(g^1_{LL}, g^2_{LL}) = c_L \tag{7}$$

$$v_1(g^1_{LH}, g^2_{HL}) = c_L; \quad v_2(g^1_{HL}, g^2_{LH}) = c_L \tag{8}$$

$$v_1(g^1_{HL}, g^2_{LH}) = c_H + (c_H - c_L)\frac{p_1}{1-p_1}\alpha \tag{9}$$

$$v_2(g^1_{LH}, g^2_{HL}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2}(1-\alpha-\beta) \tag{10}$$

$$v_1(g^1_{HH}, g^2_{HH}) = c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}(1-\alpha)\right) \tag{11}$$

$$v_2(g^1_{HH}, g^2_{HH}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2}\left(1 + \frac{p_1}{1-p_1}(\alpha+\beta)\right) \tag{12}$$

The expected informational rent $EIR(1)$ is :

$$\frac{EIR(1)}{c_H - c_L} = p_1 p_2 \max\left\{g^1_{HL} + g^2_{HH}, g^1_{HH} + g^2_{HL}, g^1_{HH} + g^2_{HH}\right\} + p_1(1-p_2)g^1_{HH} + (1-p_1)p_2 g^2_{HH}$$

Proof: See Appendix.

An optimal mechanism is given by the solution to the relaxed program if this solution satisfies all omitted incentive constraints. As we are going to see, this is not always the case. The endogenous coefficients α and β can be interpreted either as normalized Lagrange multipliers or factors determining the proportions in which q^i_{HL} and q^j_{HH} change, when the informational rent in the state LL is changed by 1 unit. If in the state LL all three incentive constraints are binding, then $q^1_{HL} + q^2_{HH} = q^2_{HL} + q^1_{HH} = q^1_{HH} + q^2_{HH}$. If not all three incentive constraints are binding, then one or both of the above equalities do not hold, and some of the associated coefficients α, β or $\alpha+\beta$ are at their boundary values of 0 or 1. Thus, $\alpha = 0$ if $IC(LL-HL)$ is not binding. $\alpha+\beta = 1$ if $IC(LL-LH)$ is not binding. $\beta = 0$ if $IC(LL-HH)$ is non-binding, and $\beta = 1$ if $IC(LL-HL)$ and $IC(LL-LH)$ are non-binding.

In the single-agent mechanism incentive constraints in any particular state are interdependent. An agent can only make the best of all possible deviations, and one informational rent prevents all of them. This generates what we characterize as a 'public good' effect. In our model this effect appears in the state LL when $IC(LL-HH)$ is not binding. In this case the agent's informational rent is equal to the maximum of $q^1_{HL} + q^2_{HH}$ and $q^2_{HL} + q^1_{HH}$. If the principal decides to increase this informational rent by p_1p_2, both q^1_{HL} and q^2_{HL} can be increased by 1 unit without violating $IC(LL-HL)$ or $IC(LL-LH)$. Thus, the principal is able to 'act on several margins' simultaneously. Consequently, q^1_{HL} and q^2_{HL} are distorted by less relative to the first-best levels. The 'public good' factor generates an economy of informational rent in the single-agent mechanism and makes a single-agent mechanism superior in a variety of circumstances. Since it appears only when $IC(LL-HH)$ is not binding, the 'public good' and 'extra deviation' factors are mutually exclusive.

This 'public good' property can be formally demonstrated by combining the first-order conditions (9) and (10) to obtain:

$$(1-p_1)p_2\left(v_1(g^1_{HL},g^2_{LH})-c_H\right)+p_1(1-p_2)\left(v_2(g^1_{LH},g^2_{HL})-c_H\right)=p_1p_2(c_H-c_L)$$

The left hand side of this equation is the sum of the 'excess' marginal benefits of q^1_{HL} and q^2_{HL} over their true marginal costs. The right-hand side is equal to the marginal cost of the informational rent in the state LL. In other words, we obtain a modified Samuelson characterization of the public goods.

Further, if we increase q^i_{HL} by some amount and decrease q^j_{HH} by an equal amount, the informational rent in the state LL remains the same, but decreases in the states LH and HL. This magnifies the economies generated by the 'public good' factor, but at the same time the quantity distortions in the state HH are increased, which is the 'distortion' factor discussed in the introduction. The 'public' food and 'extra distortion' factors are illustrated in figure 3.

The ordering of the optimal quantities in the single-agent mechanism depends on the sign of the cross-partial derivative, and also on the parameters of the model in the substitutability case. We defer the derivation of the ordering until the next two sections where these cases are considered.

4 Complementarity

In this section we compare the profitability of the optimal single-agent and two-agent mechanisms under complementarity. The optimal two-agent mechanism was derived in the previous section. In the following lemma the optimal single-agent mechanism is characterized:

Lemma 4 *When the goods are complementary ($v_{12}(.,.)>0$), the optimal mechanism for the single-agent problem is unique and either:*
(i) is given by the solution to the relaxed program $RP(1)$ and satisfies the following ordering:

$$g^i_{LL}>g^i_{LH}>g^i_{HH}$$
$$g^i_{LL}>g^i_{HL}>g^i_{HH}$$

The sufficient conditions for this case are:
a)$-v_{22}(g_1,g_2)\geq -v_{11}(g_1,g_2)\ \forall g_1\geq g_2$.[3] *b)*$\frac{p_i}{1-p_i}\leq\frac{p_j}{1-p_j}+1$.
(ii) satisfies an additional pooling condition $g^i_{HL}=g^i_{LH}$ for $p_i<p_j$

[3] This property is satisfied by most functional forms that economists operate with, such as CES, Cobb-Douglas, quadratic.

(or, in the case of a corner solution $g^i_{LH} + g^j_{HH} = g^j_{HL} + g^i_{HH}$ for $p_i < p_j$).
In this case the ordering is:

$$g^i_{LL} > g^i_{LH} \geq g^i_{HL} > g^i_{HH}$$

Proof: See appendix.

The two sufficient conditions in the lemma guarantee that horizontal incentive constraints $IC(HL-LH)$ and $IC(HL-LH)$ are not binding. Actually, when p_1 and p_2 are sufficiently close to each other, the condition on the second partials is no longer necessary. When these conditions do not hold, it is possible that one of the 'horizontal' incentive constraints is not satisfied by the solution to the relaxed program. This constraint is $IC(HL-LH)$ when $p_1 < p_2$, and $IC(LH-HL)$ when $p_2 < p_1$ (see figure 2). In this case an extra pooling constraint is imposed which produces an additional quantity distortion. As shown in the appendix, pooling causes quantity distortions in a low probability state: when $p_1 < p_2$ ($p_1 < p_2$), this state is LH (HL) and it occurs with probability $p_1(1-p_2)$ ($p_2(1-p_1)$. Therefore, this factor, which we call a 'non-standard' constraint factor, has only a second-order effect under complementarity.

No matter which case of the lemma applies, the ordering satisfies $g^i_{HL} > g^i_{HH}$, which implies that the 'public good' factor delivers economies of informational rent in the single-agent mechanism. Since $IC(LL-HH)$ is not binding, the 'extra deviation' factor is irrelevant. The 'extra distortion' factor does not appear also: the quantities g^1_{HH} and g^2_{HH} do not have to be distorted additionally to ensure $g^i_{HL} > g^j_{HH}$. Thus, the 'public good' factor ensures that a single-agent mechanism is superior under complementarity.

If $\frac{v_{12}(g_1,g_2)}{v_{11}(g_1,g_2)} \leq 1$, for all $g_1 \leq g_2$, the proof of the superiority of a single-agent mechanism is straightforward: the vector of quantities optimal in the two-agent case can be implemented in a single-agent mechanism with a lower informational rent. This restriction on the second partials is satisfied by all functional forms normally used in the economic analysis, such as quadratic and CES. (In fact, I could not find examples where this property does not hold). If this property does not hold, then a more complex proof is given in the appendix. Notice also that for concave functions this property is strictly weaker than the condition $-v_{22}(g_1,g_2) < -v_{11}(g_1,g_2)$ for $g_1 < g_2$, which is one of the sufficient conditions in lemma 4.

Proposition 5 *When the goods are complements ($v_{12} > 0$), then an optimal single-agent mechanism is superior to a two-agent mechanism.*

Proof: Here we will only prove the proposition in the case $\frac{v_{12}(g_1,g_2)}{v_{11}(g_1,g_2)} \leq 1 \; \forall g_1 \leq g_2$. The proof for the other case is relegated to the appendix.

Consider the quantity vectors $\mathbf{q}^i$ ($i \in \{1,2\}$) optimal in the two-agent mechanism. The ordering of its elements in characterized in lemma 2. Let the principal assign these quantity vectors in the single-agent mechanism and offer the following transfers:

$T'_{HH} = c_H(q^1_{HH} + q^2_{HH})$
$T'_{HL} = c_H q^1_{HL} + c_L q^2_{LH} + (c_H - c_L)q^2_{HH}$
$T'_{LH} = c_L q^1_{LH} + c_H q^2_{HL} + (c_H - c_L)q^1_{HH}$
$T'_{LL} = c_L(q^1_{LL} + q^2_{LL}) + (c_H - c_L)\max\{q^1_{HL} + q^2_{HH}, q^2_{HL} + q^1_{HH}\}.$

Then individual rationality and all downwards incentive compatibility constraints are satisfied. The upward incentive constraints are not binding, because $q^i_{HH} < q^i_{LH} < q^i_{LL}$. It remains to show that the 'horizontal' incentive constraints $IC(HL-LH)$ and $IC(HL-LH)$ are satisfied. Without loss of generality assume that $p_2 \geq p_1$. Then $q^1_{HH} \geq q^2_{HH}$, and $IC(LH-HL)$ holds. $IC(HL-LH)$ is satisfied, if $q^2_{HH} \geq q^1_{HH} + q^2_{HL} - q^1_{LH}$. It can be shown that this inequality holds by applying the Property 2 derived in the appendix (see the proof of lemma 2) to the two pairs of the first-order conditions (1), (3), and (4),(5). Thus, the mechanism is incentive-compatible. The expected informational rent is equal to:

$$\frac{EIR(1)(\mathbf{q}^1,\mathbf{q}^2)}{c_H - c_L} = p_1 p_2 \max\left\{q^1_{HL} + q^2_{HH}, q^2_{HL} + q^1_{HH}\right\} + p_1(1-p_2)q^1_{HH} + (1-p_1)p_2 q^2_{HH}$$

By lemma 2 the difference between the informational rents in the two-agent and single-agent mechanisms is:

$$EIR(2)(\mathbf{q}^1,\mathbf{q}^2) - EIR(1)(\mathbf{q}^1,\mathbf{q}^2) \geq (c_H - c_L)\min\{q^1_{HL} - q^1_{HH}, q^2_{HL} - q^2_{HH}\} > 0$$

Thus, the quantity allocations optimal in the two-agent mechanism can be implemented at a lower cost in the single-agent mechanism. Therefore, a single-agent mechanism is superior for the principal. **QED.**

The last inequality in the proof illustrates how the 'public good' factor generates informational rent economies, making a single-agent mechanism more profitable. In the state LL the agent can only get the larger of the two quantities $g^i_{HL} + g^j_{HH}$. This creates an interdependence of the rents earned, so to speak, on the information about the cost of the first good and the cost of the second good. As figure 4 illustrates, these economies are reinforced by the complementarity effect. The use of the optimal two-agent allocation in the proof also confirms our earlier assertion, that the fourth factor, additional quantity distortions in the state HH, is irrelevant under complementarity.

When one of the 'horizontal' constraints is binding, we have to show that this does not create a distortion overpowering the 'public good' effect. This is done in the appendix by using a homotopy technique.

The borderline case when the benefit function is separable in the first and the second goods, i.e. $v(q_1, q_2) = u(q_1) + u(q_2)$, is studied by Dana [7] under a more general assumption of correlated types (costs). It is demonstrated that when types are distributed independently, a single agent mechanism gives higher profits to the principal. This conclusion also follows from our lemma 3. It is interesting to note that in the separability case the optimal quantity vectors in the two-agent mechanism $\mathbf{q}^i$ satisfy $q^i_{HL} = q^i_{HH}$, while in the single-agent mechanism $g^i_{HL} > g^i_{HH}$.

The discussion of the complementarity case proceeded so far under the assumption that the marginal costs are distributed with identical supports, i.e. $c^1_H = c^2_H$ and $c^1_L = c^2_L$. Relaxing this assumption makes the analysis quite complicated. However, in the special case of a quadratic benefit function, it is possible to demonstrate that if the two marginal costs have sufficiently asymmetric supports the above result may be reversed and a two-agent mechanism becomes optimal. This happens because the 'horizontal' constraint in the single-agent mechanism imposes sufficiently strong distortion outweighing the economies generated by the 'public good' factor. This result is proved in the appendix as *Asymmetric Supports Case*.

5 Substitutability

In the substitutability case ($v_{12} < 0$) the pattern of interaction between the identified forces becomes more complex. Consequently, the results are not uniform. Which regime is optimal depends on the degree of substitutability in the principal's benefit function, as well as the probability distribution parameters p_1 and p_2 reflecting either the difficulty of the tasks or the frequency of the efficient types in the population. A larger degree of substitutability, the appropriate measure of which is derived in the paper, and a higher frequency of the high cost realizations both strengthen the factors making the two-agent mechanism more profitable. Under the opposite conditions a single-agent mechanism is more profitable.

Before presenting these results formally, let us first consider how the four factors discussed in the introduction behave under substitutability. Among them, only the 'public good' property of the informational rent makes the single-agent mechanism more profitable. As figure 5 illustrates, the 'public good' factor is weakened by the substitutability effect. The 'public good' factor appears only in the state LL, and only if $g^i_{HL} > g^i_{HH}$ for $i \in \{1, 2\}$. When the substitutability effect is sufficiently strong, this inequality can go the other way. In this case, we expect the two-agent mechanism to be superior, because of the 'extra deviation' factor. This is formally established in the next lemma, which shows that the 'public good' and 'extra deviation' factors are mutually exclusive.

Yet, even if the 'public good' factor is present, it can still be offset by the 'distortion' and the 'non-standard constraints' factors. The 'non-standard constraints' factor under the substitutability can appear in the form of binding 'horizontal' and 'upwards' incentive constraints (see Figure 2).

Lemma 6 *Let $v_{12} < 0$ and consider the relaxed single-agent problem $RP(1)$ in lemma 3. i) If in the solution to the relaxed program the constraint $IC(LL - HH)$ is not* *binding, then this solution gives a unique optimal single-agent mechanism. ii) If $IC(LL-HH)$ is binding, then a two-agent mechanism is superior.*

Proof:
i) Suppose that $IC(LL-HH)$ is not binding. Then $g^i_{HL} \geq g^i_{HH}$, and from the first-order conditions in lemma 3 it follows that:

$$g^i_{LH} > g^i_{LL} = g^j_{LL} > g^j_{HL}$$

Then, $g^i_{LH} > g^i_{HL}$, and because $IC(LL-HH)$ is assumed to be non-binding we have:
$q^i_{LH} > q^i_{HH}$, therefore $IC(HH-LH)$ and $IC(HH-HL)$ hold.
$q^i_{LL} > q^i_{HH}$, therefore $IC(HH-LL)$, $IC(HL-LL)$, $IC(LH-LL)$ hold.

We will show now that both $IC(LL-HL)$ and $IC(LL-LH)$ must be binding. Since $IC(LL-HH)$ is not binding, $\beta = 0$ in the first-order conditions of lemma 3. Suppose without loss of generality that $IC(LL-HL)$ is not binding. Then $\alpha = 0$, and

$$g^1_{HL} + g^2_{HH} < g^2_{HL} + g^1_{HH}$$

Using $\alpha = 0$ in the first-order conditions (10) and (12), it is easy to see that $g^2_{HH} > g^2_{HL}$, which implies that $IC(LL-HH)$ is binding. Contradiction.

Because $IC(LL-HL)$ and $IC(LL-LH)$ are binding, we have: $g^i_{HH} + g^j_{HL} = g^i_{HL} + g^j_{HH}$. We can now use $g^j_{LH} > g^j_{HL}$ to obtain:

$$g^i_{HH} + g^j_{LH} > g^i_{HL} + g^j_{HH}$$

This implies that $IC(LH-HL)$ and $IC(HL-LH)$ are satisfied. Since $IR(HH)$ is satisfied, all other individual rationality constraints also hold. Thus, the solution to the relaxed program satisfies all the constraints in the single-agent problem, and therefore it produces a unique optimal single-agent mechanism.

ii) Assume that $IC(LL-HH)$ is binding. In this case the quantity vectors $\mathbf{g}^i$ solving the relaxed program are such that $g^i_{HH} \geq g^i_{HL}$ for $i \in \{1,2\}$, and the expected informational rent paid by the principal is equal to

$$(c_H - c_L)\left(p_1 p_2 (g^1_{HH} + g^2_{HH}) + p_1(1-p_2) g^1_{HH} + (1-p_1) p_2 g^2_{HH}\right)$$

If the principal implements $\mathbf{g}^i$ in the two-agent mechanism, she pays an informational rent equal to:

$$(c_H - c_L)\left(p_1 p_2 (g^1_{HL} + g^2_{HL}) + p_1(1-p_2) g^1_{HH} + (1-p_1) p_2) g^2_{HH}\right)$$

This is strictly less than the informational rent in the relaxed program, if for some $i \in \{1,2\}$ $g^i_{HH} > g^i_{HL}$. But even if both inequalities are non-strict, note that the optimal quantity vector $\mathbf{q}^i$ in the two-agent mechanism is such that $q^i_{HH} > q^i_{HL}$. Then, by a revealed preference argument, the optimal two-agent mechanism gives strictly greater profits than the solution to the relaxed program, and the latter is at least as large as the profit from the optimal single-agent mechanism. **QED.**

This lemma provides a rule for comparing single-agent and two-agent mechanisms in the substitutability case: solve the relaxed program and establish whether $IC(LL-HH)$ is binding. If it is, then two-agent mechanism is optimal. If it is not, then the solution to the relaxed program is the optimal single-agent mechanism, and its payoff has to be compared with that from the optimal two-agent mechanism. Thus, we do not have to derive the optimal single-agent mechanism when the incentive constraints omitted in the relaxed program are binding. This significantly simplifies our task.

As the lemma shows, the question whether $IC(LL-HH)$ is binding or not is quite important. Because the ordering $g^j_{LH} > g^j_{HH}$ always holds, the nature of the interaction between inputs in the substitutability case is likely to produce $g^i_{HH} > g^i_{HL}$, in which case $IC(LL-HH)$ is binding. This ordering applies in the first-best and in the optimal two-agent mechanisms. But, as we are going to show, this is true in the single-agent mechanism only under certain conditions. Because of the informational rent economies generated by the 'public good' factor in the single-agent mechanism, it is sometimes more profitable for the principal to set $g^i_{HL} > g^i_{HH}$. The negative side of this is the reduction in revenue caused by distorting the quantities g^1_{HH} and g^2_{HH} downwards, the so-called 'distortion' factor.

Whether the 'public good' or the distortion effect dominates, can be determined by applying a homotopy technique, i.e. a continuous transformation between a single-agent and a two-agent optimal mechanisms. Essentially it measures the relative distance to the first-best of the revenues from the single-agent and two-agent mechanisms.

At first, we solve the relaxed program and find the normalized Lagrange multiplier α (remember that in the case under consideration $\beta = 0$). Then we define $V(t)$ in such a way that $V(0)$ ($V(1)$) is equal to the principal's profit from the optimal single-agent (two-agent) mechanism. This is done in the appendix on page 32. There it is shown that a two-agent mechanism is superior iff

$$\frac{V(1)-V(0)}{(c_H-c_L)p_1p_2} = \int_0^1 (q^2_{HH}(t) - q^2_{HL}(t))\alpha + (q^1_{HH}(t) - q^1_{HL}(t))(1-\alpha)dt \geq 0 \qquad (13)$$

In order to use this method of comparison we need the following lemma:

Lemma 7 $(1-\alpha)q^1_{HH}(t) + \alpha q^2_{HH}(t)$ *is increasing in* t. *For* $i \in \{1,2\}$, $q^i_{HL}(t)$ *is decreasing in* t.

Proof: See appendix.

Based on lemma 7 we can illustrate the method of comparison in equation 13 graphically. This is done in figure 6. As follows from figure 7, this method can also be applied to prove that the two-agent mechanism is superior when $IC(LL-HH)$ is binding, which we have proved by other methods in lemma 6.

Now we are in a position to compare the profitability of the two regimes. This is done in a series of five propositions. As it turns out, the appropriate measure of substitutability is given by the following ratio, (essentially a normalized cross-partial derivative):

$$\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}$$

Sometimes a restriction of the form $q_1 \leq q_2$ (or $q_1 \geq q_2$) applies. In any case, most functions used in economics have the property that $|v_{11}(q_1,q_2)| > |v_{22}(q_1,q_2)|$ $\forall q_1 < q_2$. Then we are able to provide tighter sufficient conditions.

It is easy to see why the value of the cross-partial derivative by itself does not provide a good measure of substitutability. If the benefit function $v(.,.)$ and unit costs c_L and c_H are multiplied by some factor, the relative profitability of the two mechanisms does not change. The ratio $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}$ also remains the same, while the cross-partial derivative changes by the same factor,

At first we establish the conditions for the superiority of the two-agent regime under substitutability. These results are easier to prove, because by lemma 6 it is sufficient to show that $IC(LL-HH)$ is binding, i.e. the 'extra deviation' factor overpowers the 'public good' factor. This happens when the substitutability effect is large enough, or p_1 and p_2 are low enough so that the state HH occurs quite frequently.

Proposition 8 *There exists $\delta(p) \in (0,1)$ s.t. if $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} > 1-\delta(p_1,p_2)$ for $\forall q_1 \geq q_2$, then the two-agent mechanism is superior.*

Proof: See Appendix.

When the two inputs are easily substitutable, it becomes optimal to decrease the quantity of the high-cost input (g^i_{HL}) and, instead, increase the quantity of the low cost input (q^j_{LH}) in the states LH and HL. In the state HH there is no such incentive, because both quantities (g^i_{HH} and q^j_{HH}), are produced at high cost. Then $g^i_{HL} > g^i_{HH}$, if the substitutability effect is large enough. Consequently, the informational rent economy generated by the 'public good' factor disappears, and the 'extra deviation' factor drives the result.

As the following proposition shows, a similar thing happens when one of p_1 or p_2 is sufficiently small, but the reason is different.

Proposition 9 *Suppose that $v_{12} < 0$. There is a $\theta > 0$ such that if $\min\{p_1,p_2\} \leq \theta$, then the two-agent mechanism is superior.*

Proof: See Appendix.

To understand this result, suppose that p_1 is small enough. Then the state HH occurs with much greater frequency than the state LH, and it becomes suboptimal to distort g^1_{HH} downwards and trade-off lower revenue in the state HH for a higher revenue in the state LH. The distortion factor becomes more important and offsets potential informational rent economies generated by the 'public good' factor.

The next proposition shows that a two-agent mechanism is superior even when both p_1 and p_2 are bounded away from 0, but are sufficiently different. Here extra regularity assumptions are needed. I assume that i) either an Inada-type condition on the behavior of $v(.,.)$ near the origin is satisfied, or the ratio $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}$ is bounded away from zero. One of the two conditions is satisfied by most functional forms which are used in the economic literature.

Proposition 10 *Let the principal's utility function* $v(q_1, q_2)$ *be such that* $v_{12}(q_1, q_2) < 0$*, and suppose that at least one of the following conditions hold:*

a)$\forall q_1 > 0 \lim_{q_2 \to 0} v_1(q_1, q_2) = \infty$

b)$\exists r > 0$ *s.t.* $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \geq r \ \forall(q_1, q_2)$.

Then for any $u = \min\{p_1, p_2\}$ *there exists* $w < 1$ *s.t. if* $\max\{p_1, p_2\} \geq w$ *then a two-agent mechanism is superior.*

Proof: See Appendix.

This result can be explained in terms of trading off the informational rents across the states in the single-agent mechanism. Suppose that p_1 is much smaller than p_2. Then the state HL is much more likely than the state LH, and the principal would like to trade-off the informational rents by paying less in the state HL and more in the state LH. Since inputs are substitutable, this trade-off leads to an increasing asymmetry of allocation in the state HH: q^1_{HH} is increased, while q^2_{HH} is decreased. In turn, the asymmetry of allocation implies that the 'public good' factor becomes less powerful, and it becomes more profitable to use a two-agent mechanism because of the 'extra deviation' factor

The above three propositions provide sufficient conditions for the two-agent mechanism to be superior under substitutability. Now we turn to the sufficient conditions for the optimality of the single- agent mechanism. These conditions are the opposite to the ones which ensure the superiority of the two-agent mechanism. In short, a smaller substitutability effect and higher frequency of the states other than HH make the 'public good' factor more powerful than the 'extra deviation' and 'distortion' factors.

Proposition 11 *For any* p_1, p_2 *there exists an* $\epsilon(p_1, p_2) > 0$ *such that if* $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2}} < \epsilon(p_1, p_2)$, $\forall q_1, q_2 \geq 0$, *then a single-agent mechanism is superior.*

Proof: See Appendix.

The result of this proposition is graphically illustrated in figure 8. When the degree of substitutability is low, the optimal quantities of the two inputs in any particular state become almost independent of each other. Notably, the 'negative correlation' between the inputs in the states LH and HL is small. In the absence of this downward pressure, the quantities q^i_{HL} increase, and the 'public good' factor then provides substantial informational rent economies. To prove that 'public good' factor offsets the 'distortion' factor we use an argument based on the homotopy construction.

The limiting case when the measure of substitutability is equal to zero corresponds to the separability of the objective function in q_1 and q_2, when, as we already know, the principal prefers to hire a single agent. Thus, our results satisfy a certain continuity.

Since $v_{12}(q_1, q_2) = v_{12}(q_2, q_1)$, the condition of the lemma, in fact, requires the following ratio to be small enough:

$$\frac{v_{12}(q_1, q_2)}{\min\{v_{11}(q_1, q_2), v_{22}(q_1, q_2)\}}$$

At the same time, most functional forms which one encounters in economics have the property that $|v_{11}(q_1, q_2)| < |v_{22}(q_1, q_2)|\ \forall q_1 > q_2$. In this case the sufficient condition reduces to $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} < \epsilon(p)\ \forall q_1 \geq q_2$.

For the single-agent mechanism to be superior, this ratio has to be small uniformly over the whole domain, which is a rather strong requirement. On the other hand, Proposition 8 required the substitutability measure to be uniformly large over the whole domain. These are quite strong requirements, but some price has to paid for the generality of the benefit functions. Moreover, when specific functional forms are considered, these conditions translates into simple restrictions on the parameters, as demonstrated in the following examples.

Example 1. Quadratic: $v(q_1, q_2) = k + d(q_1 + q_2) - \frac{b}{2}(q_1^2 + q_2^2) - cq_1q_2$ where k, d, b, c are positive constants. According to the results of this section, a single-agent mechanism is optimal if $\frac{c}{b}$ is small enough, while a single-agent mechanism is optimal when $\frac{c}{b}$ is close enough to 1. If $c < 0$ (complementarity), then a single-agent mechanism is always superior.

Example 2. CES: $v(q_1, q_2) = -\left(q_1^\rho + q_1^\rho\right)^{-\frac{m}{\rho}}$ where $0 < \rho < 1$, $0 < m < 1$. We have $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} = \frac{m+\rho}{(m+1)(\frac{q_1}{q_2})^{\rho-1}+(1-\rho)\frac{q_1}{y_2}}$. The ratio $\frac{q_1}{q_2}$ is bounded from above and below by constant depending only on the cost and probability distribution parameters. Then the substitutability measure $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)}$ can be made arbitrarily small on the whole domain if m and ρ are chosen small enough, while this measure can be made arbitrarily close to 1 by choosing ρ close enough to 1.

Example 3. Cobb-Douglas: $v(q_1, q_2) = k - \frac{1}{\rho q_1^\rho q_2^\rho}$ where $\rho > 0$. In this case $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} = \frac{\rho q_1}{(\rho+1)q_2}$. Since the ratio $\frac{q_2}{q_1}$ is bounded from above and below by constants depending only on cost and probability distribution parameters, by taking ρ small enough, we can ensure that our substitutability measure is as small as necessary for the optimality of a single-agent mechanism. If ρ is large enough, our substitutability measure is close to 1, and thus a two-agent mechanism is optimal.

As the next proposition demonstrates, a single-agent mechanism becomes superior when HH occurs less frequently than both LH and HL. To rule out irregular behavior near the origin, we need the following assumption on the behavior of the second-partial $v_{11}(q, q)$ in the neighborhood of zero: $\exists k, \eta_1, \eta_2 > 0$, s.t. for all $q \in (0, \rho)$ for some $\rho > 0$ $\eta_1 q^{-k} < -v_{11}(q, q) \leq \eta_2 q^{-k}$.

Proposition 12 *Let* $v_{12}(q_1, q_2) < 0$ *and suppose that* $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \leq 1-\gamma$ *for some* $\gamma > 0$ $\forall q_1 \geq q_2$ *. Assume that the above regularity condition on the behavior of* $v_{11}(q, q)$ *near the origin holds.*

Then there exists $1 > \mu > 0$*, such that if* $p_1 > \mu$ *and* p_2 *is in some neighborhood* $U(p_1)$ *of* p_1*, then a single-agent mechanism is superior to a two-agent mechanism.*

Proof: See Appendix

This proposition is illustrated in figure 9. The intuition for this result is as follows. The principal can trade off a lower profit in the state HH for higher profits in the states LH and HL for. When p_1 and p_2 are both large, HH is much less frequent than any of LH and HL. Consequently, the loss of expected revenue from distorting the quantities in the state HH downwards becomes quite small, i.e. the 'distortion' factor is insignificant. On the other hand, the principal's profits are then magnified by the informational rent economies generated by the 'public good' factor in the relatively more frequent states HL and LH. Again, to establish that the distortion factor is small, a homotopy-based argument is required

6 Quadratic Case

So far we have only considered interior solutions: by the Inada assumption strictly positive quantities are assigned in all states of the world. However, this assumption does not hold for many functional forms, and thus some quantities can be optimally set to zero. To highlight this case consider a quadratic benefit (production) function:

$$v(q_1, q_2) = k + d(q_1 + q_2) - \frac{b}{2}(q_1^2 + q_2^2) + cq_1 q_2$$

where k, l, b are positive constants, while $c > 0$ ($c < 0$) in the complementarity (substitutability) case, and $\frac{|c|}{b} < 1$. For the sake of simplicity, symmetry is assumed, i.e. $p_1 = p_2 = p$. In the symmetric case non-standard incentive constraints are never binding in the single-agent mechanism, and the relaxed program always provides an optimal solution. The first-order condition for an optimal quantity q^1 in any state of the world can be written as follows:

$$d - bq^1 - cq^2 \leq C(p, c_L, c_H)$$

where $C(p, c_L, c_H)$ is an appropriate virtual cost and strict inequality holds only if $q_1 = 0$.

Under complementarity ($c > 0$), it is easy to obtain a uniform conclusion that a single-agent mechanism is superior. As p is increased, it becomes optimal to set some quantities to zero. We set $g_{HH} = 0$ when p satisfies $d \leq c_H + (c_H - c_L)\left(\frac{p}{1-p} + \frac{p^2}{2(1-p)^2}\right)$, and we set $q_{HL} = 0$ when p satisfies $d + (d - c_L)\frac{c}{b} < c_H + (c_H - c_L)\frac{p}{2(1-p)}$.

Under substitutability ($c < 0$), as we know from the previous section, the outcome depends on the values of p and the ratio $\frac{|c|}{b}$. When p is large (small) and/or $\frac{|c|}{b}$ is small (large), a single-agent (two-agent) mechanism is superior.

If both $\frac{|c|}{b}$ and p are sufficiently large, then a two-agent mechanism is preferred and the optimal quantity allocation is such that $g_{HH} > g_{HL} = 0$. This happens when:

$$(1)\ d + (d - c_L)\frac{c}{b} > c_H$$

$$(2)\ d < c_H + (c_H - c_L)\left(\frac{p}{1-p} + \frac{p^2}{2(1-p)^2}\right)$$

Interestingly, when $\frac{|c|}{b}$ is small and p is large enough, a single-agent mechanism is superior and we have $g_{HL} > g_{HH} = 0$. This is true under the following conditions:

$$1) d + (d - c_L)\frac{c}{b} < c_H + (c_H - c_L)\frac{p}{2(1-p)}$$

$$2) d > c_H + (c_H - c_L)\left(\frac{p}{1-p} + \frac{p^2}{2(1-p)^2}\right)$$

Thus, not producing the high-cost good can be optimal in the state LH and HL with positive production in the state HH. This result demonstrates a possible non-monotonicity of the 'zero quantity region' in the multidimensional mechanism design problems. As Armstrong (1996) has shown (see Proposition 1 in [1]), in the context of a non-linear pricing model, the zero quantity region lies is a connected neighborhood of the most inefficient type, which in our case is HH. In the case of complementarity and low substitutability our results are consistent with Armstrong's. Yet, when substitutability effect is large enough the example suggests that the zero quantity region can be disconnected and does not include the most inefficient type.

7 Concluding remarks

The results obtained in this paper demonstrate that the optimal organizational structure depends, among other factors, on the complementarity and substitutability between the inputs in the principal's benefit (production) function. Four different factors determining the optimal choice between the single-agent and two-agent mechanisms were identified. It was shown how the relative strength of these factors depends on the degree of substitutability or complementarity and the frequency of efficient cost realizations (efficient types in the population).

One of the contributions of this paper is in extending the multidimensional mechanism design to the case where the objective function is inseparable in the two goods. Our results illustrate some of the general properties established by other authors (e.g. [1], [6], [16]), but we also point out a new phenomenon: the the zero quantity region may be disconnected and not contain the most inefficient type.

We have considered a situation where at the ex-ante stage the principal chooses a supplier for each of the inputs. This may be reasonable in some contexts such as defense procurement, but is less likely in other situations. An interesting extension would be to consider the design of an optimal procurement auction for several goods with complementarity or substitutability between them. Another possible extension is to consider a continuous distribution of the marginal cost, although this might be tractable only in a few special cases (see [1]).

8 Appendix

First, we establish a simple property of concave functions. It is used in several arguments in this appendix, so I have decided to prove it separately.

Property 1
Let $v(.,.)$ be an increasing C^2-concave function, i.e. $v_{11} < 0$, $v_{22} < 0$, $v_{11}v_{22} - v_{12}^2 > 0$. Consider the following system of two equations with $d, c \in (0, \infty)$:

$$\begin{cases} v_1(q_1, q_2) = c \\ v_2(q_1, q_2) = d \end{cases}$$

Then, holding d constant, we obtain:

$$\frac{dq_1}{dc} < 0 \quad \text{and} \quad \frac{dq_2}{dc} v_{12} < 0$$

Proof:
1. Totally differentiating the second equation gives:

$$v_{12} dq_1 = -v_{22} dq_2$$

2. Totally differentiating the first equation and using step 1:

$$v_{11}dq_1 + v_{12}dq_2 = \frac{1}{v_{22}}\left(v_{11}v_{22} - v_{12}^2\right)dq_1 = dc$$

Since $v(.,.)$ is concave, its Hessian is negative definite. Therefore, $\frac{dq_1}{dc} < 0$. 3. Then by step 1 we can conclude that $\frac{dq_2}{dc}$ and $v_{12}(.,.)$ have the opposite signs. **QED.**

Proof of lemma 1:

We solve the principal's maximization problem assuming that the only binding constraints are $IR^i(H)$ and $IC^i(H)$, and then verify that the other constraints are satisfied by the solution. If $IR^i(H)$ is binding:

$$t_H^i = c_H\left(q_{HL}^i p_j + q_{HH}^i(1-p_j)\right) \tag{14}$$

Combining this equation with the assumption that $IC^i(L)$ is binding, obtain:

$$t_L^i = c_L\left(q_{LL}^i p_j + q_{LH}^i(1-p_j)\right) + (c_H - c_L)\left(q_{HL}^i p_j + q_{HH}^i(1-p_j)\right) \tag{15}$$

Thus, the low cost agent producing input i gets an informational rent $(c_H - c_L)\left(q_{HL}^i p_j + q_{HH}^i(1-p_j)\right)$. Then the expected informational rent paid by the principal can easily be computed. The result is given in the statement of the lemma. Using the two above expressions, we can rewrite the principal's objective function as follows:

$$\begin{aligned}
\max_{(\mathbf{q}^1,\mathbf{q}^2)} & p_1p_2\left(v(q_{LL}^1,q_{LL}^2) - c_L(q_{LL}^1 + q_{LL}^2)\right) \\
& + p_1(1-p_2)\left(v(q_{LH}^1,q_{HL}^2) - c_Lq_{LH}^1 - c_Hq_{HL}^2\right) \\
& + (1-p_1)p_2\left(v(q_{HL}^1,q_{LH}^2) - c_Hq_{HL}^1 - c_Lq_{LH}^2\right) \\
& + (1-p_1)(1-p_2)\left(v(q_{HH}^1,q_{HH}^2) - c_H(q_{HH}^1 + q_{HH}^2)\right) \\
& - p_1(c_H - c_L)\left(q_{HL}^1 p_2 + q_{HH}^1(1-p_2)\right) - p_2(c_H - c_L)\left(q_{HL}^2 p_1 + q_{HH}^2(1-p_1)\right)
\end{aligned}$$

Since the function $v(.,.)$ is concave, the optimal quantity vector q^i ($i \in \{1,2\}$) solving this maximization problem is uniquely determined by eight first-order conditions given in the statement of the lemma.

Because the low-cost type can imitate the high-cost type, her individual rationality constraint is satisfied. In the next lemma it will be shown that under all possible assumptions $q_{LL}^i > q_{HL}^i$ and $q_{LH}^i > q_{HH}^i$. Thus, a high-cost type would make negative profits if she imitated a low-cost type - i.e. her incentive constraint is satisfied. **QED.**

Proof of lemma 2:

Consider the first-order conditions in lemma 1. By symmetry, $q^1_{LL} = q^2_{LL}$. Applying Property 1 to the following two pairs of the first-order conditions:

$$\begin{aligned}
&\text{(i)} \quad v_1(q^1_{LL}, q^2_{LL}) = v_2(q^1_{LL}, q^2_{LL}) = c_L \\
&\text{(ii)} \quad v_1(q^i_{LH}, q^j_{HL}) = c_L; \quad v_2(q^i_{LH}, q^j_{HL}) = c_H + (c_H - c_L)\frac{p_j}{1 - p^j}
\end{aligned}$$

we obtain:

a) if $v_{12} > 0$, then $q^i_{LL} > q^i_{HL}$ and $q^i_{LL} > q^i_{LH}$.

b) if $v_{12} < 0$, then $q^i_{LH} > q^i_{LL} > q^i_{HL}$.

Now apply Property 1 to the following pairs of the first-order conditions:

$$\begin{aligned}
&\text{(i)} \; v_1(q^i_{HL}, q^j_{LH}) = c_H + (c_H - c_L)\frac{p_i}{1 - p_i}; \;\; v_2(q^i_{HL}, q^j_{LH}) = c_L \\
&\text{(ii)} \; v_1(q^i_{HH}, q^j_{HH}) = c_H + (c_H - c_L)\frac{p_i}{1 - p_i}; \;\; v_2(q^i_{HH}, q^j_{HH}) = c_H + (c_H - c_L)\frac{p_j}{1 - p_j}
\end{aligned}$$

From $c_L < c_H + (c_H - c_L)\frac{p_i}{1-p_i}$, it follows that $\forall i \in \{1, 2\}$ $q^i_{LH} > q^i_{HH}$. In the complementarity case we also obtain $q^i_{HL} > q^i_{HH}$; while in the substitutability case $q^i_{HL} < q^i_{HH}$.

Combining all the results together, we get the orderings given in the lemma. **QED.**

Proof of lemma 3

In the relaxed problem the vectors of transfers $\mathbf{T}^i$ and output vector $\mathbf{g}^i$ have to satisfy the incentive compatibility constraints $IC(LL-HL)$, $IC(LL-LH)$, $IC(LH-HH)$, $IC(HL-HH)$ and the individual rationality constraint $IR(HH)$ which are set on page 10. This is a strictly concave programming problem with linear constraints. Therefore, it has a unique solution, which can be obtained by using the Lagrangian:

$$\begin{aligned}
\mathcal{L} =& p_1 p_2 \left(v(q^1_{LL}, q^2_{LL}) - T_{LL}\right) + p_1(1 - p_2)\left(v(q^1_{LH}, q^2_{HL}) - T_{LH}\right) \\
&+ (1 - p_1)p_2 \left(v(q^1_{HL}, q^2_{LH}) - T_{HL}\right) + (1 - p_1)(1 - p_2)\left(v(q^1_{HH}, q^2_{HH}) - T_{HH}\right) \\
&+ \lambda_{LH}\left(T_{LL} - c_L(g^1_{LL} + g^2_{LL}) - T_{LH} + c_L(g^1_{LH} + g^2_{HL})\right) \\
&+ \lambda_{HL}\left(T_{LL} - c_L(g^1_{LL} + g^2_{LL}) - T_{HL} + c_L(g^1_{HL} + g^2_{LH})\right) \\
&+ \lambda_{HH}\left(T_{LL} - c_L(g^1_{LL} + g^2_{LL}) - T_{HH} + c_L(g^1_{HH} + g^2_{HH})\right) \\
&+ \delta^1_{HH}\left(T_{LH} - c_L g^1_{LH} - c_H g^2_{HL} - T_{HH} + c_L g^1_{HH} + c_H g^2_{HH}\right) \\
&+ \delta^2_{HH}\left(T_{HL} - c_H g^1_{HL} - c_L g^2_{LH} - T_{HH} + c_H g^1_{HH} + c_L g^2_{HH}\right) \\
&+ \eta\left(T_{HH} - c_H(g^1_{HH} + g^2_{HH})\right) \qquad (16)
\end{aligned}$$

Differentiating with respect to transfers, we obtain the following first-order conditions:

$$
\begin{aligned}
T_{LL} &: p_1 p_2 = \lambda_{LH} + \lambda_{HL} + \lambda_{HH} \\
T_{LH} &: p_1(1-p_2) = \delta^1_{HH} - \lambda_{LH} \\
T_{HL} &: (1-p_1)p_2 = \delta^2_{HH} - \lambda_{HL} \\
T_{HH} &: (1-p_1)(1-p_2) = -\lambda_{HH} - \delta^1_{HH} - \delta^2_{HH} + \eta
\end{aligned} \tag{17}
$$

Differentiating with respect to the elements of the $\mathbf{g}^i$ and using the above expressions, we obtain the following first-order conditions:

$$v_1(g^1_{LL}, g^2_{LL}) = c_L; \qquad v_2(g^1_{LL}, g^2_{LL}) = c_L \tag{18}$$

$$v_1(g^1_{LH}, g^2_{HL}) = c_L; \qquad v_2(g^1_{HL}, g^2_{LH}) = c_L \tag{19}$$

$$v_1(g^1_{HL}, g^2_{LH}) = c_H + (c_H - c_L)\frac{\lambda_{HL}}{(1-p_1)p_2} \tag{20}$$

$$v_2(g^1_{LH}, g^2_{HL}) = c_H + (c_H - c_L)\frac{\lambda_{LH}}{p_1(1-p_2)} \tag{21}$$

$$v_1(g^1_{HH}, g^2_{HH}) = c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \frac{\lambda_{HH} + \lambda_{LH}}{(1-p_1)(1-p_2)}\right) \tag{22}$$

$$v_2(g^1_{HH}, g^2_{HH}) = c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{\lambda_{HH} + \lambda_{HL}}{(1-p_1)(1-p_2)}\right) \tag{23}$$

Define

$$\alpha = \frac{\lambda_{HL}}{\lambda_{LH} + \lambda_{HL} + \lambda_{HH}} \quad \text{and} \quad \beta = \frac{\lambda_{HH}}{\lambda_{LH} + \lambda_{HL} + \lambda_{HH}}$$

Obviously, $\alpha, \beta \geq 0$ and $\alpha + \beta \leq 1$. Moreover, the equation (17) shows that the denominator in the above expressions is equal to $p_1 p_2$. Using α and β we can rewrite the first-order conditions in the form given in the statement of the lemma. The expected informational rent $EIR(1)$ is easily computed:

$$
\begin{aligned}
\frac{EIR^1}{c_H - c_L} =& p_1 p_2 \max\left\{g^1_{HL} + g^2_{HH}, g^1_{HH} + g^2_{HL}, g^1_{HH} + g^2_{HH}\right\} \\
&+ p_1(1-p_2)g^1_{HH} + (1-p_1)p_2 g^2_{HH}
\end{aligned}
$$

QED.

Proof of lemma 4:
The following simple properties of concave functions will be useful later.

Property 2.
Assume that $-v_{22}(g_1, g_2) > -v_{11}(g_1, g_2)$ for $g_1 > g_2$. Consider the following system with $t \geq 0$:

$$\begin{cases} v_1(g_1, g_2) = c_1 - t \\ v_2(g_1, g_2) = c_2 \end{cases}$$

If $c_1 < c_2$, then

$$\frac{dg_1}{dt} > \frac{dg_2}{dt}$$

Proof: By Property 1, $\frac{dg_1}{dt} > 0$. Since the right-hand side of the second equation does not change with t, we can totally differentiate it to get:

$$v_{12}(g_1, g_2)\frac{dg_1}{dt} + v_{22}(g_1, g_2)\frac{dg_2}{dt} = 0$$

From $c_1 < c_2$, it follows that $g_1 > g_2$, and then by assumption $-v_{22}(g_1, g_2) > -v_{11}(g_1, g_2)$. By concavity, $v_{11}v_{22} - v_{12}^2 > 0$. Combining the two inequalities, we get: $-v_{22}(g_1, g_2) > v_{12}(g_1, g_2)$. The result follows.

Property 3
Again, assume that $-v_{22}(g_1, g_2) \geq -v_{11}(g_1, g_2)$ for $g_1 \geq g_2$. Consider the following system:

$$\begin{cases} v_1(g_1, g_2) = c_1 - t \\ v_2(g_1, g_2) = c_2 - t \end{cases}$$

If $c_1 < c_2$, then after fully differentiating and solving the system we get:

$$\frac{dq_1}{dt} - \frac{dq_2}{dt} = \frac{1}{\Delta}(-v_{22}(g_1, g_2) + v_{11}(g_1, g_2)) \geq 0$$

where Δ denotes the determinant of the Hessian of v. This inequality establishes the result.

Next, we turn to the proof of the lemma. At first, let's show how the quantities are ordered in the solution to the relaxed program. For this we need the first-order conditions of lemma 3. Consider two pairs of the first-order conditions: the first pair containing both equations in (18), the second pair containing the second equation from (19) and (20). Since $v_1(g_{HL}^1, g_{LH}^2) > v_1(g_{LL}^1, g_{LL}^2)$, Property 1 implies that:

$$q_{HL}^1 < q_{LH}^2 < q_{LL}^1 = q_{LL}^2$$

When, instead, the second pair includes the first equation in (19) and the first equation in (21), we get:

$$q^2_{HL} < q^1_{LH} < q^1_{LL} = q^2_{LL}$$

Similarly, if we choose the first pair to consist of the first equation in (19) and the first equation in (20), while the second pair is chosen to consist of (22) and (23), by applying Property 1 we get:

$$g^1_{LH} > g^1_{HH},\ q^2_{HL} > q^2_{HH}$$

When, instead, as the first pair we use the second equation from (19) and the second equation from (21), we obtain:

$$g^2_{LH} > g^2_{HH},\ g^1_{HL} > g^1_{HH}$$

Since $g^i_{HL} > g^i_{HH}$, the incentive constraint $IC(LL - HH)$ is non-binding, and therefore $\beta = 0$.

From the first-order conditions (20) and (23) it follows that $q^1_{HL}+q^2_{HH}$ decreases in α, while from (21) and (22) it follows that $g^2_{HL} + g^1_{HH}$ increases in α. Therefore if there exists $\alpha \in (0, 1)$ s.t.

$$g^1_{HL} + g^2_{HH} = g^2_{HL} + g^1_{HH}$$

such α is unique, and in this case both $IC(LL - HL)$ and $IC(LL - LH)$ are binding.

Pick $\alpha = 0$, if for such α

$$g^1_{HL} + g^2_{HH} \leq g^2_{HL} + g^1_{HH}$$

When this inequality is strict, the only incentive constraint binding in the state LL is $IC(LL - LH)$. A necessary, but not sufficient condition for this to be true is $p_1 < \frac{p_2}{1+p_2}$.

Similarly, $\alpha = 1$ if at this value of α

$$g^1_{HL} + g^2_{HH} \geq g^2_{HL} + g^1_{HH}$$

If this inequality is strict, the only incentive constraint binding in the state LL is $IC(LL-HL)$. A necessary, but not sufficient condition for this to be true is $p_2 < \frac{p_1}{1+p_1}$.

The solution to the relaxed program is an optimal mechanism, if it satisfies the omitted individual rationality and incentive compatibility constraints. The incentive compatibility constraints in the state HH are not binding, because

$$g^i_{HH} < \min\{g^i_{HL}, g^i_{LH}\} < g^i_{LL}$$

The upwards incentive constraints $IC(HL-LL)$ and $IC(LH-LL)$ are not binding either, because $g^i_{HH} < g^i_{LL}$.

Constraints $IC(LH-HL)$ and $IC(HL-LH)$ are satisfied if

$$g^i_{HH} \geq g^i_{HL} + g^j_{HH} - g^j_{LH}$$

Obviously, $g^j_{HL} < g^i_{LH}$. The first-order conditions (22) and (23) imply that $g^i_{HH} > g^j_{HH}$ iff $p_i < p_j$. Assuming $p_2 \geq p_1$ without loss of generality, we get that $IC(LH-HL)$ holds. It remains to confirm that $IC(HL-LH)$ is satisfied, i.e.

$$g^1_{LH} - g^2_{HL} \geq g^1_{HH} - g^2_{HH} \tag{24}$$

We are going to show that this inequality holds under assumptions $a)$ and $b)$ of the lemma. At first consider the case $\alpha = 0$. To see that (24) is satisfied, apply **Property 2** to the two pairs of the first-order conditions: (8), (10) and (11), (12).

Suppose now that in the solution to the relaxed program $\alpha > 0$. Then Note that $\alpha = 1$ is impossible when $p_2 \geq p_1$. Therefore,

$$g^1_{HL} + g^2_{HH} = g^2_{HL} + g^1_{HH}$$

From the first-order conditions (9), (10), (11), (12) it follows that this equality can be satisfied only if $g^1_{HL} > g^2_{HL}$, and $g^1_{HH} > g^2_{HH}$. Consequently,

$$\alpha < \frac{\frac{p_2}{1-p_2}}{\frac{p_1}{1-p_1} + \frac{p_2}{1-p_2}} \tag{25}$$

Combining this with our assumption that $\frac{p_2}{1-p_2} \leq \frac{p_1}{1-p_1} + 1$, we obtain:

$$\begin{aligned} v_1(g^1_{LH}, g^2_{HL}) - v_1(g^1_{HH}, g^2_{HH}) &= -(c_H - c_L)\left(1 + \frac{p_1}{1-p_1} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}(1-\alpha)\right) \\ < -(c_H - c_L)\left(\frac{p_2}{1-p_2}\alpha + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right) &= v_2(g^1_{LH}, g^2_{HL}) - v_2(g^1_{HH}, g^2_{HH}) \end{aligned}$$

Then we can apply **Property 2** and **Property 3** to the above inequality to show that (24) holds.

Assume now, that the sufficient conditions given in the lemma do not hold, and the solution to the relaxed program is such that $IC(HL-LH)$ fails. Then the optimal mechanism has to be derived by solving the relaxed program with an additional constraint:

$$T_{HL} - c_H q^1_{HL} - c_L q^2_{LH} \geq T_{LH} - c_H q^1_{LH} - c_L q^2_{HL} \tag{26}$$

When this constraint with an associated multiplier $\mu \geq 0$ is added to the Lagrangian in (16), the new system of the first-order conditions differs from the one in lemma 3 in the following two conditions only:

$$v_1(g^1_{LH}, g^2_{HL}) = c_L - (c_H - c_L)\frac{\mu}{p_1(1-p_2)} \tag{27}$$

$$v_2(g^1_{LH}, g^2_{HL}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2}(1-\alpha-\beta) + (c_H - c_L)\frac{\mu}{p_1(1-p_2)}$$

From the first-order conditions it is easy to obtain that $g^1_{HL} > g^1_{HH}$, and therefore $\beta = 0$. Since (26) must be binding,

$$g^1_{LH} + g^2_{HH} = g^1_{HH} + g^2_{HL}$$

If it is also true that

$$g^1_{HL} + g^2_{HH} > g^1_{HH} + g^2_{HL}$$

then $\alpha = 1$ and $g^2_{HL} > g^1_{HL}$. Thus, *a fortiori* $g^1_{LH} > g^1_{HL}$, and constraint $IC(HL{-}LH)$ is not binding, i.e. we are in case 1 of this lemma.

Thus, $\alpha < 1$ if $IC(HL-LH)$ is binding. When $\alpha \in (0,1)$, $g^1_{HL} = g^1_{LH}$, i.e. pooling occurs. To see that the other 'horizontal' constraint $IC(LH{-}HL)$ still holds, note that *i)* $g^2_{LH} > g^1_{HL} = g^1_{LH}$; *ii)* $q^1_{HH} > q^2_{HH}$.

A corner solution with $\alpha = 0$ is possible, when p_2 and p_1 are significantly different. In this case

$$g^1_{HL} + g^2_{HH} \leq g^1_{HH} + g^2_{HL}$$

Since (24) holds as equality, $g^1_{LH} \geq g^1_{HL}$.

Also, notice that from (20) and (21) $g^1_{HL} > g^2_{HL}$. Combining this with the first inequality in the previous paragraph, we get $q^1_{HH} > q^2_{HH}$. In combination with $g^2_{LH} > g^1_{LH}$ this establishes that the other 'horizontal' constraint $IC(LH-HL)$ is not binding. It is immediate that no upward incentive constraints could bind. **QED.**

Homotopy construction

Define the function $V(t)$ for $t \in [0,1]$ as follows.

$$
\begin{aligned}
V(t) = \max_{\mathbf{h}} & \left(v(h^1_{LL}, h^2_{LL}) - c_L(h^1_{LL} + h^2_{LL})\right) p_1 p_2 \\
& + \left(v(h^1_{LH}, h^2_{HL}) - c_L h^1_{LH} - \left[c_H + \frac{p_2}{1-p_2}(c_H - c_L)((1-\alpha) + t\alpha)\right] h^2_{HL}\right) p_1(1-p_2) \\
& + \left(v(h^1_{HL}, h^2_{LH}) - \left[c_H + (c_H - c_L)\frac{p_1}{1-p_1}(\alpha + (1-\alpha)t)\right] h^1_{HL} - c_L h^2_{LH}\right)(1-p_1)p_2 \\
& + \left(v(h^1_{HH}, h^2_{HH}) - \left[c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \frac{p_1 p_2(1-\alpha)(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^1_{HH}\right. \\
& \left. - \left[c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{p_1 p_2 \alpha(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^2_{HH}\right)(1-p_1)(1-p_2) \qquad (28)
\end{aligned}
$$

For each $t \in [0,1]$ there is a unique quantity vector $\mathbf{q}^i(t)$ solving the corresponding maximization problem.

Substituting $t = 0$ and $t = 1$ in the above expression, we can convince ourselves that $V(0)$ ($V(1)$) equals the payoff from the optimal single-agent (two-agent) mechanism, and $\mathbf{q}^i(0)$=$\mathbf{g}^i$ is the optimal quantity vector in the single-agent mechanism, while $\mathbf{q}^i(1)$ =$\mathbf{q}^i$ is the optimal quantity vector in the two-agent mechanism.

The vector $\mathbf{q}^i(t)$ is characterized by the following set of the first-order conditions:

$$
v_1(q^1_{LL}(t), q^2_{LL}(t)) = v_2(q^1_{LL}(t), q^2_{LL}(t)) = v_1(q^1_{LH}(t), q^2_{HL}(t)) = v_2(q^1_{HL}(t), q^2_{LH}(t)) = c_L \qquad (29)
$$

$$
v_1(q^1_{HL}(t), q^2_{LH}(t)) = c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left[\alpha + (1-\alpha)t\right] \qquad (30)
$$

$$
v_2(q^1_{LH}(t), q^2_{HL}(t)) = c_H + (c_H - c_L)\frac{p_2}{1-p_2}\left[(1-\alpha) + \alpha t\right] \qquad (31)
$$

$$
v_1(q^1_{HH}(t), q^2_{HH}(t)) = c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}(1-\alpha)(1-t)\right) \qquad (32)
$$

$$
v_2(q^1_{HH}(t), q^2_{HH}(t)) = c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha(1-t)\right) \qquad (33)
$$

By the envelope theorem,

$$
\frac{dV(t)}{dt} = (c_H - c_L)p_1 p_2 \left[(q^2_{HH}(t) - q^2_{HL}(t))\alpha + (q^1_{HH}(t) - q^1_{HL}(t))(1-\alpha)\right]
$$

From this expression the result in the text follows.

Proof of Proposition 5:

Here the proof is given for the case when the condition $v_{12}(q_1, q_2) \leq -v_{11}(q_1, q_2)$ for $q_1 \leq q_2$ does not hold, and the solution to the relaxed program fails a 'horizontal' incentive constraint. If the horizontal constraint is non-binding, then the proof is simple because $\mu = 0$ below.

First, solve for the optimal single-agent mechanism. By lemma 4, the solution produces a modified Lagrange multiplier $\alpha \in [0, 1]$ and a Lagrange multiplier $\mu > 0$. Similarly to (28) above, we construct a homotopy between a single-agent and a two-agent problem by defining $V(t)$ for $t \in [0, 1]$ as follows:

$$
\begin{aligned}
V(t) = \max_{\mathbf{h}} & \left(v(h^1_{LL}, h^2_{LL}) - c_L(h^1_{LL} + h^2_{LL})\right) p_1 p_2 \\
+ & \left(v(h^1_{LH}, h^2_{HL}) - \left(c_L - (c_H - c_L)\frac{\mu}{p_1(1-p_2)}(1-t)\right) h^1_{LH}\right. \\
- & \left.\left[c_H + (c_H - c_L)\frac{p_2}{1-p_2}[(1-\alpha(1-t) + \frac{\mu}{p_1(1-p_2)}(1-t)]\right] h^2_{HL}\right) p_1(1-p_2) \\
+ & \left(v(h^1_{HL}, h^2_{LH}) - \left[c_H + (c_H - c_L)\frac{p_1}{1-p_1}(\alpha + (1-\alpha)t)\right] h^1_{HL} - c_L h^2_{LH}\right) (1-p_1)p_2 \\
+ & \left(v(h^1_{HH}, h^2_{HH}) - \left[c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \frac{p_1 p_2(1-\alpha)(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^1_{HH}\right. \\
- & \left.\left[c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{p_1 p_2 \alpha(1-t)}{(1-p_1)(1-p_2)}\right)\right] h^2_{HH}\right) (1-p_1)(1-p_2) \qquad (34)
\end{aligned}
$$

For any $t \in [0, 1]$ let $(q^i_{LL}(t), q^i_{LH}(t), q^i_{HL}(t), q^i_{HH}(t))$, where $i \in \{1, 2\}$, denote the unique solution to this problem.

Observe that the principal's net benefit P_1 from the optimal single-agent mechanism is equal to $V(0) - \mu(c_H - c_L)(q^1_{LH}(0) - q^2_{HL}(0))$, and her benefit from the optimal two-agent mechanism P_2 is equal to $V(1)$. Also note that if $\mu > 0$, then $q^1_{LH}(0) - q^2_{HL}(0) = q^1_{HH}(0) - q^2_{HH}(0)$, i.e. $IC(HL - LH)$ is binding. Using the envelope theorem obtain:

$$
\begin{aligned}
\frac{V'(t)}{c_H - c_L} = & -\mu\left(q^1_{LH}(t) - q^2_{HL}(t)\right) \\
& - p_1 p_2\left(\alpha(q^2_{HL}(t) - q^2_{HH}(t)) + (1-\alpha)(q^1_{HL}(t) - q^1_{HH}(t))\right)
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\frac{P_1 - P_2}{c_H - c_L} = & -\mu\left(q^1_{LH}(0) - q^2_{HL}(0)\right) + \mu\int_0^1 q^1_{LH}(t) - q^2_{HL}(t)dt \\
& + p_1 p_2\left(\int_0^1 \alpha(q^2_{HL}(t) - q^2_{HH}(t)) + (1-\alpha)(q^1_{HL}(t) - q^1_{HH}(t))dt\right)
\end{aligned}
$$

It is easy to establish that $q^1_{LH}(t) - q^2_{HL}(t)$, $q^1_{HL}(t) - q^1_{HH}(t)$ and $q^2_{HL}(t) - q^2_{HH}(t)$ are positive and decreasing in t. From the first-order conditions in the proof of the lemma 4 obtain:

$$\mu \leq (1-p_1)p_2 + \alpha p_1 p_2$$

Using this inequality, it can be established that:

$$q^1_{HH}(0) - q^2_{HH}(0) < q^1_{LH}(1) - q^2_{HL}(1) + \frac{\alpha}{\mu}(q^2_{HL}(1) - q^2_{HH}(1)) + \frac{1-\alpha}{\mu}(q^1_{HL}(1) - q^1_{HH}(1))$$

Since $q^1_{LH}(0) - q^2_{HL}(0) = q^1_{HH}(0) - q^2_{HH}(0)$, we get $P_1 - P_2 > 0$. **QED.**

Asymmetric Supports Case:

Assume that:

$$v(g_1, g_2) = a(g_1 + g_2) - \frac{b}{2}(g_1^2 + g_2^2) + d g_1 g_2$$

where $b > d > 0$. Denote $\Delta_2 = c^2_H - c^2_L$ and $\Delta_1 = c^1_H - c^1_L$, and without loss of generality assume that $\Delta_2 > \Delta_1$.

We will demonstrate that if $\frac{\Delta_2}{\Delta_1}$ is sufficiently large, a two-agent mechanism dominates because of distortions imposed by a horizontal incentive constraint $IC(HL-LH)$ in the single-agent mechanism. First of all, notice that the horizontal incentive constraint $IC(HL-LH)$ in the single-agent mechanism can be written as:

$$\Delta_2 g^2_{HH} \geq \Delta_2 g^2_{HL} + \Delta_1 g^1_{HH} + \Delta_1 g^1_{LH}$$

or:

$$\Delta_2 (g^2_{HL} - g^2_{HH}) \leq \Delta_1 (g^1_{LH} - g^1_{HH}) \tag{35}$$

It is easy to show that the solution to the relaxed single-agent problem, which is given in lemma 3, fails this constraint if

$$d\Delta_2 > b\Delta_1$$

Let's assume that this is the case. Then the optimal single-agent mechanism is derived as in lemma 4. It has the properties that the constraint $IC(HL-HL)$ is binding, i.e. (35) holds as equality. From the first-order condition in the proof of lemma 3 we can solve for the optimal quantities and show that for the quadratic benefit function 35 is equivalent to:

$$\begin{aligned}
&\Delta_2 \left(-\mu(b-d) + b\Delta_2 \frac{p_2}{1-p_2} \left(1 + \frac{p_1}{1-p_1}\right) \alpha \right) \\
&+ d\Delta_1 \Delta_2 \left(1 + \frac{p_1}{1-p_1}\left(1 + (1-\alpha)\frac{p_2}{1-p_2}\right)\right) \\
&= b\Delta_1^2 \frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}\right)(1-\alpha) + d\Delta_1\Delta_2 \left(1 + \frac{p_2}{1-p_2}\left(1 + \alpha \frac{p_1}{1-p_1}\right)\right)
\end{aligned} \tag{36}$$

Also, in the optimal mechanism both $IC(LL-LH)$ and $IC(LL-HL)$ are binding, which implies that:

$$\Delta_2 g^2_{HL} + \Delta g^1_{HH} = \Delta_1 g^1_{HL} + \Delta_2 g^2_{HH}) \tag{37}$$

Using (37), the right-hand side of 36 can be rewritten as:

$$\Delta_1 \left(\mu(b-d) + d\Delta_2 \left(1 + \frac{p_1}{1-p_1} \left(1 + (1-\alpha) \frac{p_2}{1-p_2} \right) \right) + d\Delta_2 \frac{p_2}{1-p_2} \left(1 + \frac{p_1}{1-p_1} \right) \alpha \right)$$

where $\mu \geq 0$ is a Lagrange multiplier associated with the constraint $IC(LL-HH)$. If $\frac{\Delta_2}{\Delta_1}$ is large, $g^2_{HL} - g^2_{HH}$ has to be sufficiently small for the 'horizontal' incentive constraint $IC(HL-LH)$ to be satisfied and, consequently, a two-agent mechanism becomes superior.

To establish this result, let us consider a mechanism with an additional restriction $g^2_H = g^2_{HL} = g^2_{HH}$. It is easy to show that when this restriction is imposed, optimal single-agent and two-agent mechanisms are identical and the optimal quantities (denoted by $\hat{g}$) satisfy the following three first-order conditions:

$$1) v_1(\hat{g}^1_{HL}, \hat{g}^2_{LH}) = c^1_H + \Delta_1 \frac{p_1}{1-p_1}$$

$$2) v_1(\hat{g}^1_{HH}, \hat{g}^2_{HH}) = c^1_H + \Delta_1 \frac{p_1}{1-p_1}$$

$$3) p_1(1-p_2) \left(v_2(\hat{g}^1_{LH}, \hat{g}^2_H) - c^2_H - c^2_H - \Delta_2 \frac{p_2}{1-p_2} \right) + (1-p_1)(1-p_2) \left(v_2(\hat{g}^1_{HH}, \hat{g}^2_H) - c^2_H - c^2_H - \Delta_2 \frac{p_2}{1-p_2} \right) = 0$$

When the first-order conditions for the low-cost quantities are added to the above three equations, we can solve for the optimal quantities in the pooling mechanism:

Let $W(P)$ denote the principal's profit in this mechanism (where P stands for pooling), while $W(1)$ ($W(2)$) denote the principal's profit in the single-agent (two-agent) mechanism. To establish the superiority of the two-agent mechanism, we need to show that $W(2) - W(P) > W(1) - W(P)$.

At first let us compute $W(2) - W(P)$. Using the first-order conditions for optimal quantities in lemma 1 we obtain:

$$
\begin{aligned}
W(2) - W(P) = p_1(1-p_2) \Bigg(& v(q^1_{LH}, q^2_{HL}) - c^1_L q^1_{LH} - (c^2_H + \Delta_2 \frac{p_2}{1-p_2}) q^2_{HL} \\
& - v(\hat{g}^1_{LH}, \hat{g}^2_H) + c^1_L \hat{g}^1_{LH} + (c^2_H + \Delta_2 \frac{p_2}{1-p_2}) \hat{g}^2_H \Bigg) \\
& + (1-p_1)(1-p_2) \Bigg(v_2(q^1_{HH}, q^2_{HH}) - c^1_H q^1_{HH} - (c^2_H + \Delta_2 \frac{p_2}{1-p_2}) q^2_{HH} \\
& - v_2(\hat{g}^1_{HH}, \hat{g}^2_H) + c^1_H \hat{g}^1_{HH} + (c^2_H + \Delta_2 \frac{p_2}{1-p_2}) \hat{g}^2_H \Bigg) \\
& = p_1(1-p_2) b \int_{\hat{g}^2_H}^{q^2_{HL}} q dq + (1-p_1)(1-p_2) b \int_{\hat{g}^2_H}^{q^2_{HH}} q dq \qquad (38)
\end{aligned}
$$

The last equality is obtained using the first-order conditions and integration by parts. Substituting in for q^2_{HL}, q^2_{HH} and $\hat{g}^2_H$, it is easy to obtain that:

$$
W(2) - W(P) = \frac{b^3}{2(b^2 - d^2)^2} p_1 (1 + \frac{p_1}{1-p_1})(1-p_2) \Delta_1^2
$$

It remains to compute $W(1) - W(P)$. For our purposes it is sufficient to provide an upper bound to it. Performing computations similar to that in (38) we obtain:

$$
\begin{aligned}
W(1) - W(P) &= p_1(1-p_2) b \int_{\hat{g}^2_H}^{g^2_{HL}} q dq + (1-p_1)(1-p_2) b \int_{\hat{g}^2_H}^{g^2_{HH}} q dq \\
&= p_1(1-p_2) \frac{b}{2} \left((g^2_{HL})^2 - (\hat{g}^2_H)^2 \right) - (1-p_1)(1-p_2) \frac{b}{2} \left((g^2_{HH})^2 - (\hat{g}^2_H)^2 \right) \qquad (39)
\end{aligned}
$$

It is easy to show that in the optimal single-agent mechanism:

$$
p_1(1-p_2) \left(g^2_{HL} - \hat{g}^2_H \right) < (1-p_1)(1-p_2) \left(g^2_{HH} - \hat{g}^2_H \right)
$$

Therefore,

$$
W(1) - W(P) \leq p_1(1-p_2) \frac{b}{2} \left(g^2_{HL} - \hat{g}^2_H \right) \left(g^2_{HL} - g^2_{HH} \right) \Big) < p_1(1-p_2) \frac{b}{2} \left(g^2_{HL} - \hat{g}^2_H \right)^2 \qquad (40)
$$

From (36) we obtain that:

$$
\begin{aligned}
g^2_{HL} - g^2_{HH} &= \frac{p_2}{1-p_2} (1 + \frac{p_1}{1-p_1}) \alpha (b+d) \frac{\Delta_2 \Delta_1}{\Delta_2 + \Delta_1} \\
&+ (b+d) \left(1 + \frac{p_1}{1-p_1} (1 + (1-\alpha) \frac{p_2}{1-p_2}) \right) \frac{\Delta_1^2}{\Delta_2 + \Delta_1}
\end{aligned}
$$

Substituting the above expression into (40), we obtain that if Δ_2 is large enough and p_2 is small enough, $W(2) - W(P) > W(1) - W(P)$. It follows that a two-agent mechanism is optimal when costs are distributed sufficiently asymmetrically.

Proof of lemma 7:

Differentiating the first-order conditions (32) and (33) we have the following system of two equations: [4]

$$\begin{cases} v_{11} dq^1_{HH}(t) + v_{12} dq^2_{HH}(t) = -(1-\alpha)\frac{p_1 p_2}{(1-p_1)(1-p_2)} dt \\ v_{12} dq^1_{HH}(t) + v_{22} dq^2_{HH}(t) = -(\alpha+\beta)\frac{p_1 p_2}{(1-p_1)(1-p_2)} dt \end{cases}$$

This system can be solved to give:

$$\begin{cases} \frac{dq^1_{HH}(t)}{dt} = \frac{-(1-\alpha)v_{22} + (\alpha+\beta)v_{12}}{v_{11}v_{22} - v_{12}^2} \frac{p_1 p_2}{(1-p_1)(1-p_2)} \\ \frac{dq^2_{HH}(t)}{dt} = \frac{-(\alpha+\beta)v_{11} + (1-\alpha)v_{12}}{v_{11}v_{22} - v_{12}^2} \frac{p_1 p_2}{(1-p_1)(1-p_2)} \end{cases}$$

Combining the equations in the above system we have:

$$\begin{aligned} &(1-\alpha)\frac{dq^1_{HH}(t)}{dt} + (\alpha+\beta)\frac{dq^2_{HH}(t)}{dt} \\ &= \frac{-(1-\alpha)^2 v_{22} + 2(\alpha+\beta)(1-\alpha)v_{12} - (\alpha+\beta)^2 v_{11}}{v_{11}v_{22} - v_{12}^2} \frac{p_1 p_2}{(1-p_1)(1-p_2)} \\ &> \frac{-(1-\alpha)^2 v_{22} + 2(\alpha+\beta)(1-\alpha)\sqrt{v_{11}v_{22}} - (\alpha+\beta)^2 v_{11}}{v_{11}v_{22} - v_{12}^2} \frac{p_1 p_2}{(1-p_1)(1-p_2)} \\ &= \frac{(\sqrt{-v_{22}}(1-\alpha) - (\alpha+\beta)^2\sqrt{-v_{11}})^2}{v_{11}v_{22} - v_{12}^2} \frac{p_1 p_2}{(1-p_1)(1-p_2)} \geq 0 \end{aligned}$$

The first inequality follows because $v(.,.)$ is concave and its Hessian is negative-definite.

To establish the second part of the lemma, we will only show that $q^1_{HL}(t)$ is increasing. The proof for $q^2_{HL}(t)$ is completely symmetric. Fully differentiating the first-order conditions (29) and (30) we obtain:

$$\begin{cases} v_{11} dq^1_{HL}(t) + v_{12} dq^2_{LH}(t) = \frac{p_1}{1-p_1}(1-\alpha) dt \\ v_{12} dq^1_{HL}(t) + v_{22} dq^2_{LH}(t) = 0 \end{cases}$$

From the above system it easily follows that:

$$\frac{dq^1_{HL}(t)}{dt} = \frac{v_{22}}{v_{11}v_{22} - v_{12}^2} \frac{p_1}{1-p_1}(1-\alpha) < 0$$

[4] The argument of $v(.,.)$ is $(q^1_{HH}(t), q^2_{HH}(t))$; it is omitted for brevity

where all derivatives are evaluated at $(q^1_{HL}(t), q^2_{LH}(t))$. **QED.**

Proof of Proposition 8:

By lemma 6 it is sufficient to show that $IC(LL-HH)$ is binding in the optimal single-agent mechanism. The proof is by contradiction. Suppose otherwise, i.e. $IC(LL-HH)$ is not binding. Then $\beta = 0$ and $q_i^{HH} < q_i^{HL}$ for each $i \in \{1,2\}$, which implies that both $IC(LL-HL)$ and $IC(LL-LH)$ have to be binding. Hence,

$$q^1_{HL} + q^2_{HH} = q^1_{HL} + q^2_{HH}$$

Assuming without loss of generality that $p_1 \leq p_2$, from the first-order conditions in lemma 3 it follows that the above equality can hold only if:

a) $q^1_{HL} > q^2_{HL}$ which is equivalent to:

$$\frac{p_1}{1-p_1}\alpha < \frac{p_2}{1-p_2}(1-\alpha) \tag{41}$$

b)$q^1_{HH} > q^2_{HH}$ which is equivalent to:

$$\frac{p_1}{1-p_1}(1+\frac{p_2}{1-p_2}(1-\alpha)) < \frac{p_2}{1-p_2}\left(1+\frac{p_1}{1-p_1}\alpha\right) \tag{42}$$

Define $\hat{q}^1_{HH}$ implicitly to satisfy:

$$v_1(\hat{q}^1_{HH}, \hat{q}^1_{HH}) = v_1(q^1_{HH}, q^2_{HH}) = c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left(1+\frac{p_2}{1-p_2}(1-\alpha)\right) \tag{43}$$

$\hat{q}^1_{HH}$ is well-defined, and satisfies $q^2_{HH} < \hat{q}^1_{HH} \leq q^1_{HH}$. Next, define $\hat{q}^2_{LH}$ as a solution to:

$$v_2(\hat{q}^1_{HH}, \hat{q}^2_{LH}) = c_L \tag{44}$$

The hypothesis that $IC(LL-HH)$ is not binding will be contradicted if $\hat{q}^1_{HH} > q^1_{HL}$, which in turn is true when:

$$v_1(\hat{q}^1_{HH}, \hat{q}^2_{LH}) < c_H + (c_H - c_L)\frac{p_1}{1-p_1}\alpha$$

Using (43), this inequality can be rewritten as:

$$v_1(\hat{q}^1_{HH}, \hat{q}^2_{LH}) - v_1(\hat{q}^1_{HH}, \hat{q}^1_{HH}) = \int_{\hat{q}^1_{HH}}^{\hat{q}^2_{LH}} v_{12}(q, \hat{q}^1_{HH})dq$$

$$< -(c_H - c_L)\frac{p_1}{1-p_1}\left(1+\frac{p_2}{1-p_2}\right)(1-\alpha) \tag{45}$$

Combining (43) and (44) obtain:

$$v_2(\hat{q}^2_{LH}, \hat{q}^1_{HH}) - v_1(\hat{q}^1_{HH}, \hat{q}^1_{HH}) = \int_{\hat{q}^1_{HH}}^{\hat{q}^2_{LH}} v_{11}(q, \hat{q}^1_{HH})dq$$
$$= -(c_H - c_L)\left(1 + \frac{p_1}{1-p_1} + (1-\alpha)\frac{p_1 p_2}{(1-p_1)(1-p_2)}\right)$$

Therefore, (45) will be satisfied if

$$1 - \delta(p) > \frac{\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}\right)(1-\alpha)}{1 + \frac{p_1}{1-p_1} + (1-\alpha)\frac{p_1 p_2}{(1-p_1)(1-p_2)}}$$

The fraction on the right-hand side of the above inequality is increasing in $1-\alpha$. Since $1 - \alpha \leq 1$, if we choose

$$\delta(p_1, p_2) < \frac{1}{1 + \frac{p_1}{1-p_1} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}} < 1$$

then (45) is always satisfied. **QED.**

Proof of Proposition 9:

Without loss of generality suppose that $p_1 \leq p_2$. We will show that if p_1 is sufficiently small, then $IC(LL-HH)$ is binding in the optimal single-agent mechanism. Assume otherwise, i.e. $q^i_{HH} < q^i_{HL}$ for both $i \in \{1,2\}$. Consider the following sequence of equalities:

$$\begin{aligned}
&v_1'(q^1_{HH}, q^2_{HH}) - v_1'(q^1_{HL}, q^2_{LH}) \\
&= v_1'(q^1_{HH}, q^2_{HH}) - v_1'(q^1_{HL}, q^2_{HH}) + v_1'(q^1_{HL}, q^2_{HH}) - v_1'(q^1_{HL}, q^2_{LH}) \\
&= -\int_{q^1_{HH}}^{q^1_{HL}} v_{11}''(q, q^2_{HH})dq - \int_{q^2_{HH}}^{q^2_{LH}} v_{12}''(q^1_{HL}, q)dq \\
&= (c_H - c_L)\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}\right)(1-\alpha) \qquad (46)
\end{aligned}$$

If we define $\bar{q}_{HL}, \bar{q}_{LH}$ implicitly as solutions to the following system of equations:

$$\begin{cases} v_1(\bar{q}_{LH}, \bar{q}_{HL}) = c_L \\ v_2(\bar{q}_{LH}, \bar{q}_{HL}) = c_H \end{cases}$$

Then $q^i_{HL} \leq \bar{q}_{HL} < \bar{q}_{LH} \leq q^i_{LH}$. Next, define $\underline{q}_{HL}, \underline{q}_{HL}$ implicitly:

$$\begin{cases} v_1(\underline{q}_{LH}, \underline{q}_{HL}) = c_L \\ v_2(\underline{q}_{LH}, \underline{q}_{HL}) = c_H + (c_H - c_L)k \end{cases}$$

where $k \geq \max(1, \frac{p_2}{1-p_2})$. We choose $k \geq 1$ to allow p_2 to be very small. Since by assumption $q^2_{HH} < q^2_{HL}$,

$$-\int_{q^2_{HH}}^{q^2_{LH}} v''_{12}(q^1_{HL}, q)dq > \min_{h \in [\underline{q}_{HL}, \bar{q}_{HL}]} -\int_{\bar{q}_{HL}}^{\bar{q}_{LH}} v''_{12}(h, q)dq = T > 0$$

The minimization is well-defined, because the integral is continuous in h. The minimized value T is positive because the cross-partial derivative is negative everywhere. Notice that by construction T is independent of p_1 (and also p_2). Therefore, if p_1 is sufficiently small, (46) can hold only if $q^1_{HL} < q^1_{HH}$. But in this case $IC(LL-HH)$ is binding, and by lemma 6 a two-agent mechanism is superior. **QED.**

Proof of Proposition 10:

Let's fix $p_2 > 0$ and consider a sequence $p^n_1 \in (p_2, 1)$ s.t. $\lim_{n\to\infty} p^n_1 = 1$. If the single-agent mechanism is superior for all n, then the optimal quantity vectors $\mathbf{g}^{i,n}$ are such, that in the first-order conditions of lemma 3 $\beta_n = 0$. Moreover, for all n

$$g^{1,n}_{HL} + g^{2,n}_{HH} = g^{2,n}_{HL} + g^{2,n}_{HH} \quad \text{and} \quad g^{i,n}_{HL} > g^{i,n}_{HH}$$

As p^n_1 increases to 1, $\frac{p^n_1}{1-p^n_1} \to \infty$. Then there are two possible cases:

Case 1. $\frac{p^n_1}{1-p^n_1}\alpha_n \to \infty$ (or has a subsequence going to infinity). According to the first-order condition (9), $v_1(g^{1,n}_{HL}, g^{2,n}_{LH}) \to \infty$, and therefore $g^{1,n}_{HL} \to 0$.

First-order conditions (11) and (12) imply that

$$v_1(g^{1,n}_{HH}, g^{2,n}_{HH}) \to \infty \quad \text{and} \quad v_2(g^{1,n}_{HH}, g^{2,n}_{HH}) \to \infty$$

Therefore, $g^{1,n}_{HH} \to 0$ and $g^{2,n}_{HH} \to 0$. However, from (10) it follows that $g^{2,n}_{HL}$ remains bounded away from zero.

Consequently, for some n

$$g^{1,n}_{HL} + g^{2,n}_{HH} < g^{2,n}_{HL} + g^{1,n}_{HH}$$

Therefore, $\alpha_n = 0$. But this in turn implies that $q^{2,n}_{HH} > q^{2,n}_{HL}$. Thus, a two-agent mechanism is superior. It is easy to show that for all $p_1 \geq p^n_1$ this remains true.

Case 2. There exists $0 < K < \infty$ s.t. $\forall n$ $\frac{p^n_1}{1-p^n_1}\alpha_n \leq K$. In this case $\alpha_n \to 0$. Notice that $\forall n$ $\frac{p^n_1}{1-p^n_1}\alpha_n > \frac{p^n_2}{1-p^n_2}(1-\alpha_n)$. Otherwise the necessary condition

$$g^{1,n}_{HL} + g^{2,n}_{HH} = g^{2,n}_{HL} + g^{2,n}_{HH}$$

will be violated.

Therefore, $\frac{p_1^n}{1-p_1^n}\alpha_n > \frac{p_2}{2(1-p_2)}$, and there exists a subsequence $\frac{p_1^m}{1-p_1^m}\alpha_m \to M$ s.t. $0 < M < \infty$. Along this subsequence $\alpha_m \to 0$, $g_{HL}^{1,m} \to \bar{g}_{HL}^1 > 0$, and $g_{HL}^{2,m} \to q_{HL}^2$, where q_{HL}^2 is an optimal quantity in the two-agent mechanism.

We also have:

$$\begin{aligned} &v_1(g_{HH}^{1,m}, g_{HH}^{2,m}) \to \infty \\ &v_2(g_{HH}^{1,m}, g_{HH}^{2,m}) = c_H + (c_H - c_L)\frac{p_2}{1-p_2}(1+M) < \infty \end{aligned}$$

From the two above equations it follows that, if $\lim_{q_1 \to 0} v_2(q_1, q_2) = \infty \; \forall q_2 > 0$, i.e. the condition (a) in the statement of the lemma is satisfied, then for large enough m $g_{HH}^{2,m} > \bar{g}_{HL}^2 > 0$, and, thus, a two-agent mechanism is superior for all $p_1 > p_1^m$.

If the condition $a)$ fails but $b)$ holds, then as $v'(g_{HH}^{1,m}, g_{HH}^{2,m}) \to \infty$, it follows that $g_{HH}^{2,m}$ will increase to become larger than $\bar{g}_{HL}^2$. Thus, $IC(LL-HH)$ becomes binding at some point, and according to lemma 6 a two-agent mechanism is superior. **QED.**

Proof of Proposition 11:

At first, let's show that the the constraint $IC(LL-HH)$ is not binding in the solution to the relaxed program, if $\epsilon(p)$ defined in the statement of the lemma is small enough.

Consider the first-order conditions (9) and (10) in lemma 3, which characterizes an optimal single-agent mechanism. Obviously, $\min\{\alpha, 1-\alpha-\beta\} \leq \frac{1}{2}$. Define $\bar{g}_{HH}^2$ implicitly by:

$$v_2(g_{HL}^1, \bar{g}_{HH}^2) = c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}(\alpha+\beta)\right)$$

Consider the following sequence of inequalities:

$$\begin{aligned} &v_1(g_{HL}^1, \bar{g}_{HH}^2) = v_1(g_{HL}^1, g_{LH}^2) - \int_{\bar{g}_{HH}^2}^{g_{LH}^2} v_{12}(g_{HL}^1, g)dg \\ &< v_1(g_{HL}^1, g_{LH}^2) - \epsilon(p)\int_{\bar{g}_{HH}^2}^{g_{LH}^2} v_{22}(g_{HL}^1, g)dg \\ &= v_1(g_{HL}^1, g_{LH}^2) - \epsilon(p)\left(v_2(g_{HL}^1, g_{LH}^2) - v_2(g_{HL}^1, \bar{g}_{HH}^2)\right) \\ &= c_H + (c_H - c_L)\frac{p_1}{1-p_1}\alpha + \epsilon(p)(c_H - c_L)\left(1 + \frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}(\alpha+\beta)\right) \end{aligned}$$

If $\alpha \leq \frac{1}{2}$ and $\epsilon(p)$ is sufficiently small,

$$v_1(g_{HL}^1, \bar{g}_{HH}^2) < c_H + (c_H - c_L)\frac{p_1}{1-p_1}$$

which implies that $g^1_{HL} > g^1_{HH}$.

By a symmetric argument we can establish that if $1-\alpha-\beta \leq \frac{1}{2}$ and $\epsilon(p)$ is small enough, then

$$v_1(\bar{g}^1_{HH}, g^2_{HL}) < c_H + (c_H - c_L)\frac{p_2}{1-p_2}$$

Then $g^2_{HL} > g^2_{HH}$.

In any case, either $g^1_{HL} > g^1_{HH}$ or $g^2_{HL} > g^2_{HH}$, and therefore $IC(LL-HH)$ is not binding, $\beta = 0$, and by lemma 6 the relaxed program gives an optimal single-agent mechanism.

When $IC(LL-HH)$ is not binding,

$$g^1_{HL} + g^2_{HH} = g^2_{HL} + g^1_{HH}$$

Assuming without loss of generality that $p_2 \geq p_1$, the first-order conditions in lemma 3 imply that $g^1_{HL} \geq g^2_{HL}$ and $g^1_{HH} \geq g^2_{HH}$. These two inequalities impose the following constraints on α:

$$\frac{1}{2} - \frac{1}{2}\frac{\frac{p_2}{1-p_2} - \frac{p_1}{1-p_1}}{\frac{p_1 p_2}{(1-p_1)(1-p_2)}} \leq \alpha \leq \frac{\frac{p_2}{1-p_2}}{\frac{p_1}{1-p_1} + \frac{p_2}{1-p_2}} < 1$$

Although the inequality on the right imposes an upper bound on α strictly below 1, the inequality on the left imposes a strictly positive lower bound only if $p_2 < \frac{p_1}{1-p_1}$. The following argument establishes that α is bounded above 0. Let $\tilde{q}^1_{HL}(t)$ and $\tilde{q}^2_{HH}(t)$ be the solutions to the following system:

$$\begin{cases} v_1(\tilde{q}^1_{HL}(t), \tilde{q}^2_{HH}(t)) = c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left[\alpha + (1-\alpha)t\right] \\ v_2(\tilde{q}^1_{HL}(t), \tilde{q}^2_{HH}(t)) = c_H + (c_H - c_L)\left(\frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right) \end{cases}$$

Next, define $\tilde{q}^2_{LH}(t)$ to solve:

$$v_2(\tilde{q}^1_{HL}(t), \tilde{q}^2_{LH}(t)) = c_L$$

It is easy to see that $\tilde{q}^1_{HL}(1) \geq q^1_{HH}(t)\ \forall t \in [0,1]$. Also, by Property 1 established in the appendix $\tilde{q}^1_{HL}(t) > q^1_{HL}(t)$. Now consider the following sequence of inequalities:

$$\begin{aligned} v_1(\tilde{q}^1_{HL}(t), \tilde{q}^2_{LH}(t)) &= v_1(\tilde{q}^1_{HL}(t), \tilde{q}^2_{HH}(t)) + \int_{\tilde{q}^2_{HH}(t)}^{\tilde{q}^2_{LH}(t)} v_{12}(\tilde{q}^1_{HL}(t), q)dq \\ &> v_1(\tilde{q}^1_{HL}(t), \tilde{q}^2_{HH}(t)) + \epsilon(p)\int_{\tilde{q}^2_{HH}(t)}^{\tilde{q}^2_{LH}(t)} v_{22}(\tilde{q}^1_{HL}(t), q)dq \\ &= c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1}\left[\alpha + (1-\alpha)t\right] - \epsilon(p)\left(1 + \frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right)\right) \end{aligned} \tag{47}$$

Define $\hat{t}_1 = \hat{t}_1(t, \epsilon)$ to be such that:

$$\frac{p_1}{1-p_1}\left[\alpha + (1-\alpha)t\right] - \epsilon(p)\left(1 + \frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right) = \frac{p_1}{1-p_1}\left[\alpha + (1-\alpha)\hat{t}_1\right] \tag{48}$$

Then

$$q^1_{HL}(\hat{t}_1(t)) \geq \tilde{q}^1_{HL}(t)$$

Because $q^1_{HL}(t)$ is decreasing in t, we obtain $\forall s < \hat{t}_1$ and $\forall z \in [0,1]$ $q^1_{HL}(s) > q^1_{HL}(\hat{t}_1(1)) >\geq q^1_{HH}(z)$. Since α is bounded from above by a number less than 1, $\hat{t}_1$ gets arbitrarily close to 1 as $\epsilon(p)$ is decreased.

When α is bounded away from 0, we can define $\tilde{q}^2_{HL}(t)$, $\tilde{q}^1_{HH}(t)$ and $\tilde{q}^1_{LH}(t)$ in a similar way, and show that there exists $\hat{t}_2 = \hat{t}_2(t, \epsilon)$ s.t. $q^2_{HL}(\hat{t}_2(t)) \geq \tilde{q}^2_{HL}(t)$. Then $\forall s < \hat{t}_2(1)$ and $\forall z \in [0,1]$, $q^2_{HL}(s) > q^2_{HL}(\hat{t}(1)) > q^2_{HH}(z)$, and $\hat{t}_2(1) \to 1$ as $\epsilon(p) \to 0$.

The next step is to show that if ϵ is small enough, then for all $t < t^*$ where t^* is close to 1:

$$q^i_{HL}(1) \geq q^i_{HH}(t)$$

Define $\tilde{q}^2_H$ as follows:

$$v_2(q^1_{HL}(1), \tilde{q}^2_H) = c_H + (c_H - c_L)\frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha$$

Consider the following sequence of inequalities:

$$\begin{aligned} v_1(q^1_{HL}(1), \tilde{q}^2_H) &= v_1(q^1_{HL}(1), q^2_{LH}(1)) - \int_{\tilde{q}^2_H}^{q^2_{LH}(1)} v_{12}(q^1_{HL}(1), q)dq \\ &< v_1(q^1_{HL}(1), q^2_{LH}(1)) - \epsilon(p)\int_{\tilde{q}^2_H}^{q^2_{LH}(1)} v_{22}(q^1_{HL}(1), q)dq \\ &= c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \epsilon(p)\left(1 + \frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right)\right) \end{aligned} \tag{49}$$

Since α is bounded below 1, for any $\tau_1 \in (0,1)$ we can choose ϵ small enough that the following equality holds:

$$\begin{aligned} &\frac{p_1}{1-p_1} + \epsilon(p)\left(1 + \frac{p_2}{1-p_2} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}\alpha\right) \\ &= \frac{p_1}{1-p_1} + \frac{p_1 p_2}{(1-p_1)(1-p_2)}(1-\alpha)(1-\tau_1) \end{aligned} \tag{50}$$

This implies that $q^1_{HL}(1) > q^1_{HH}(s)$ for all $s \leq \tau_1$. Collecting the results obtained so far, we can conclude that for any t_1^* we can choose $\epsilon(p)$ small enough that $\forall s \leq t_1^*\ q^1_{HL}(s) - q^1_{HH}(s) > \max_{t\in[0,1]}(q^1_{HH}(t) - q^1_{HH}(t))$. Consequently,

$$\int_0^1 q^1_{HL}(t) - q^1_{HH}(t)dt > \int_{1-2t_1^*}^1 q^1_{HL}(t) - q^1_{HH}(t)dt > 0$$

If α is bounded from below above 0, then a symmetrical argument can be used to establish that $\int_0^1 q^2_{HL}(t) - q^2_{HH}(t)dt > 0$, and the proof is complete.

When $p_2 > \frac{p_1}{1-p_1}$, we have to consider the possibility that α becomes arbitrarily small, as ϵ is decreased. Then, if we consider the versions of equations (48) and (50) for the good 2, $\hat{t}_2(t)$ may fail to converge to t, or/and t_2^* will fail to converge to 1. To rule this out notice the following four inequalities which are satisfied by the optimal single-agent mechanism:

$$\frac{p_2}{1-p_2}(1-\alpha)(1-\epsilon) < v'(g^2_{HL}, g^2_{HL})(1-\epsilon) < \frac{p_2}{1-p_2}(1-\alpha) - \epsilon(p)c_L$$

and

$$c_H + (c_H - c_L)\frac{p_2}{1-p_2}(1 + \frac{p_1}{1-p_1}\alpha)(1-\epsilon) < v'(g^2_{HH}, g^2_{HH})(1-\epsilon) <$$
$$c_H + (c_H - c_L)\frac{p_2}{1-p_2}\left(1 + \frac{p_1}{1-p_1}\alpha\right) - \epsilon\left(c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}(1-\alpha)\right)\right)$$

The above inequalities imply that $g^2_{HL} - g^2_{HH} \to 0$ if $\alpha, \epsilon \to 0$.

The following inequality follows from the first-order conditions:

$$c_H + (c_H - c_L)\frac{p_1}{1-p_1}\alpha(1-\epsilon) < v'(g^1_{HL}, g^1_{HL})(1-\epsilon) < c_H + (c_H - c_L)\frac{p_1}{1-p_1}\alpha - \epsilon c_L$$

Since $g^1_{HH} > g^2_{HH}$, we also have:

$$v'(g^1_{HH}, g^1_{HH}) > c_H + (c_H - c_L)\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}(1-\alpha)\right)$$

Thus,

$$v'(g^1_{HL}, g^1_{HL}) - v'(g^1_{HH}, g^1_{HH}) \to (c_H - c_L)\frac{p_1}{1-p_1}\left(1 + \frac{p_2}{1-p_2}\right)$$

But then $g^1_{HL} + g^2_{HH} = g^2_{HL} + g^1_{HH}$ can no longer hold, because $g^1_{HL} - g^1_{HH}$ remains bounded from below. Hence α can not converge to 0, and the proof is complete. **QED.**

Proof of Proposition 12:

At first, we will prove the proposition for $p_1 = p_2 = p$, and then show that the result applies if p_2 lies in some neighborhood $U(p)$ of p.

When $p_1 = p_2 = p$, $g^1_{HH} = g^2_{HH} = g_{HH}$, $g^1_{HL} = g^2_{HL} = g_{HL}$. Substituting $\alpha = \frac{1}{2}$ and $\beta = 0$ in the first-order conditions of lemma 3, we obtain:

$$
\begin{aligned}
v_1(g_{LH}, g_{HL}) &= c_L \\
v_2(g_{LH}, g_{HL}) &= c_H + (c_H - c_L)\frac{1}{2}\frac{p_1}{1-p_1} \\
v_1(g_{HH}, g_{HH}) &= c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + \frac{1}{2}\frac{p_1^2}{(1-p_1)^2}\right)
\end{aligned}
$$

Using the assumption $\frac{v_{12}(q_1,q_2)}{v_{11}(q_1,q_2)} \le 1 - \gamma < 1$ in conjunction with the above equations, it is easy to establish that for p close enough to 1, $g_{HL} > g_{HH}$. Hence, in the relaxed program $IC(LL - HH)$ is not binding, and its solution is a unique optimal single-agent mechanism.

To obtain the desired result, we invoke the homotopy transformation $V(t)$ (defined in equation 28) of the objective function for the single-agent problem ($t = 0$) into the objective function of the two-agent problem ($t = 1$). For each $t \in [0,1]$, $q_{HL}(t)$ and $q_{HH}(t)$ are implicitly defined as follows (the equation defining $q_{LH}(t)$ does not change with t):

$$
\begin{aligned}
v_1(q_{HL}(t), q_{LH}(t)) &= c_H + (c_H - c_L)(1+t)\frac{1}{2}\frac{p_1}{1-p_1} \\
v_1(q_{HH}(t), q_{HH}(t)) &= c_H + (c_H - c_L)\left(\frac{p_1}{1-p_1} + (1-t)\frac{1}{2}\frac{p_1^2}{(1-p_1)^2}\right)
\end{aligned}
\tag{51}
$$

Obviously, $q_{HL}(t)$ is decreasing in t, while $q_{HH}(t)$ is increasing in t, and $q_{HH}(1) > q_{HL}(1)$. A single-agent mechanism is superior if

$$\int_0^1 q_{HL}(t)dt - \int_0^1 q_{HH}(t)dt > 0$$

For any $t \in (0,1)$ define $\tilde{q}^t_{LH}$ implicitly by:

$$v_2(q_{HH}(t), \tilde{q}_{LH}(t)) = c_L$$

Then, $q_{HH}(t) < q_{HL}(1)$, if $v_1(q_{HH}(t), \tilde{q}_{LH}(t)) > c_H + (c_H - c_L)\frac{p}{1-p}$, or equivalently:

$$v_1\left(q_{HH}(t), \tilde{q}_{LH}(t)\right) - v_1\left(q_{HH}(t), q_{HH}(t)\right) > -(c_H - c_L)(1-t)\frac{1}{2}\frac{p^2}{(1-p)^2}$$

Consider the following sequence of inequalities

$$v_1(q_{HH}(t),\tilde{q}_{LH}(t)) - v_1(q_{HH}(t),q_{HH}(t)) = \int_{q_{HH}(t)}^{\tilde{q}_{LH}^t} v_{12}(q_{HH}(t),q)dq$$

$$> (1-\gamma)\int_{q_{HH}(t)}^{\tilde{q}_{LH}^t} v_{11}(q,q_{HH}(t))dq$$

$$= -(1-\gamma)(c_H - c_L)\left(1 + \frac{p}{1-p} + (1-t)\frac{1}{2}\frac{p^2}{(1-p)^2}\right) \tag{52}$$

As p_1 increases to 1, the ratio

$$\frac{(1-t)\frac{1}{2}\frac{p_1^2}{(1-p_1)^2}}{\left(1+\frac{p_1}{1-p_1}+(1-t)\frac{1}{2}\frac{p_1^2}{(1-p_1)^2}\right)}$$

monotonically converges to 1 $\forall t \in (0,1)$, and eventually exceeds $1-\gamma$.

Next, define $t^*(p)$ to be the unique value of the parameter where $q_{HL}(t)$ and $q_{HH}(t)$ are equal to each other and q^* is their value at this point, i.e. $q_{HL}(t^*) = q_{HH}(t^*) = q^*$. Also, let $\hat{t}(p)$ solve $q_{HH}(\hat{t}) = q_{HL}(1)$. Notice that as p is increased towards 1, $\hat{t}(p)$ and $t^*(p)$ become arbitrarily close to 1. Then there exists $\hat{p}$ s.t. for all $p \geq \hat{p}$, $1 - 2\hat{t}(p) \geq \frac{1}{2}$. Obviously, $t^* > \hat{t}$, and we have

$$\int_0^{\frac{1}{2}} (q_{HL}(t) - q_{HH}(t))dt > \int_{t*}^{1} (q^* - q_{HL}(t))dt$$

It remains to show that for p large enough:

$$\int_{\frac{1}{2}}^{t^*} (q^* - q_{HH}(t))dt > \int_{t^*}^{1} (q_{HH}(t) - q^*)dt$$

Differentiating the first-order conditions, we obtain that:

$$\frac{dq_{HH}(t)}{dt} = -\frac{1}{2}\frac{p^2}{(1-p)^2}\frac{1}{v_{11}(q_{HH}(t),q_{HH}(t)) + v_{12}(q_{HH}(t),q_{HH}(t))}$$

If p sufficiently close to 1, $q_{HH}(1) < \rho$. Therefore, by the regularity assumption, $\exists \eta_1, \eta_2, k > 0$ s.t. $\forall t \in [0,1]$:

$$\eta_1 \frac{1}{q_{HH}(t)^k} \leq -v_{11}(q_{HH}(t),q_{HH}(t)) \leq \eta_2 \frac{1}{q_{HH}(t)^k}$$

Then we obtain the following bounds:

$$\frac{1}{2}\eta_1 \frac{p^2}{(1-p)^2} q_{HH}(t)^k \leq \frac{dq_{HH}(t)}{dt} < \eta_2 \frac{p^2}{(1-p)^2} q_{HH}(t)^k$$

Solving the associated differential equations with the boundary condition $q_{HH}(t^*) = q^*$, we obtain that for $t \leq t^*$:

$$q(t) \leq \frac{1}{\left(\left(\frac{1}{q^*}\right)^{k-1} - \frac{p^2}{2(1-p)^2}\eta_1(k-1)(t-t^*)\right)^{\frac{1}{k-1}}}$$

while for all $t \geq t^*$:

$$q(t) \leq \frac{1}{\left(\left(\frac{1}{q^*}\right)^{k-1} - \frac{p^2}{(1-p)^2}\eta_2(k-1)(t-t^*)\right)^{\frac{1}{k-1}}}$$

Using the above inequalities obtain:

$$\int_{\frac{1}{2}}^{t^*} (q^* - q_{HH}(t))dt \geq A = q^*(t^* - \frac{1}{2})$$
$$- \frac{2(1-p)^2}{p^2\eta_1(k-1)}\left[\left(\frac{1}{q^*}\right)^{k-2} - \left(\left(\frac{1}{q^*}\right)^{k-1} - \frac{p^2}{2(1-p)^2}\eta_1(k-1)(t^* - \frac{1}{2})\right)^{\frac{k-2}{k-1}}\right] \quad (53)$$

$$\int_{t^*}^{1} (q_{HH}(t) - q^*)dt \leq B = -q^*(1-t^*)$$
$$+ \frac{(1-p)^2}{p^2\eta_2(k-1)}\left[\left(\frac{1}{q^*}\right)^{k-2} - \left(\left(\frac{1}{q*}\right)^{k-1} - \frac{p^2}{(1-p)^2}\eta_2(k-1)(1-t^*)\right)^{\frac{k-2}{k-1}}\right] \quad (54)$$

The expressions on the right-hand side of (53) and (54) are convex functions of t^* with positive third-order derivatives. Hence, expanding them in a Taylor series to the second-order around 1 gives:

$$\frac{A}{B} \geq \frac{\frac{p^2\eta_1}{2(1-p)^2}(t^* - \frac{1}{2})^2}{\frac{p^2\eta_2}{(1-p)^2}(1-t^*)^2}$$

This ratio increases to infinity as t^* grows to 1, and, consequently, when p is large enough we have:

$$\int_{\frac{1}{2}}^{t^*} (q^* - q_{HH}(t)) > \int_{t^*}^{1} (q_{HH}(t) - q^*)dt$$

This finishes the proof for the case $p_1 = p_2$. By continuity, if p_2 if chosen sufficiently close to p_1, the change in the optimal quantities and the value of the principal's maximized objective will be small enough that a single-agent mechanism remains optimal. **QED.**

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Figure 2: Incentive Structures in the Single-Agent and Two-Agent Problems.

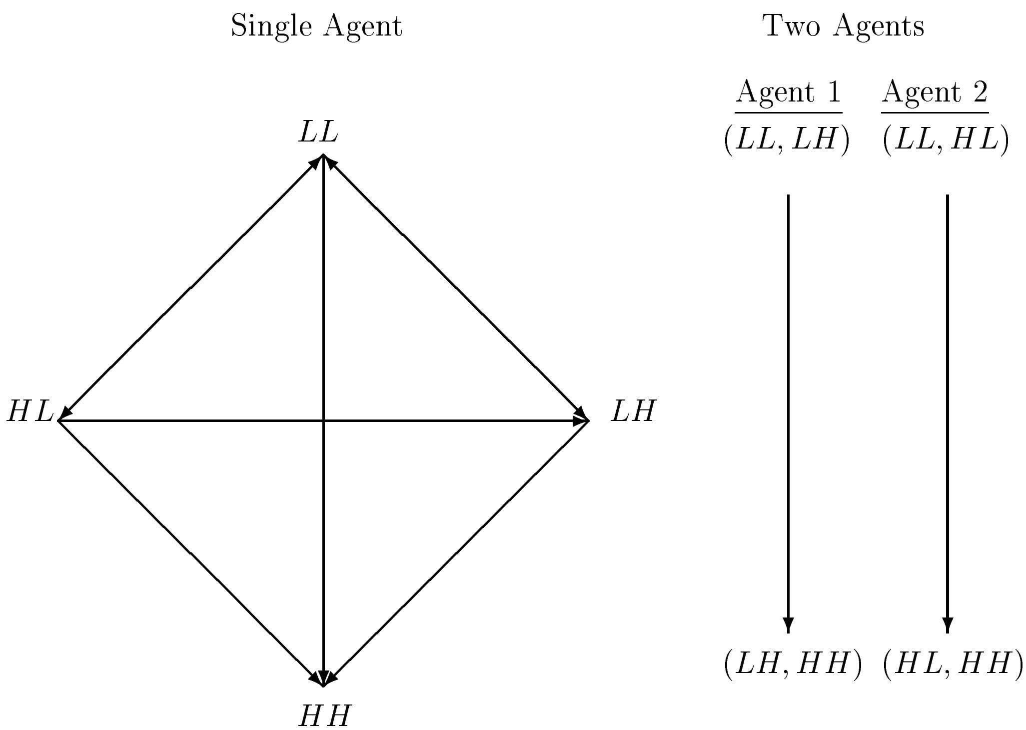

Figure 3: 'Public Good' and 'Extra Distortion' Factors

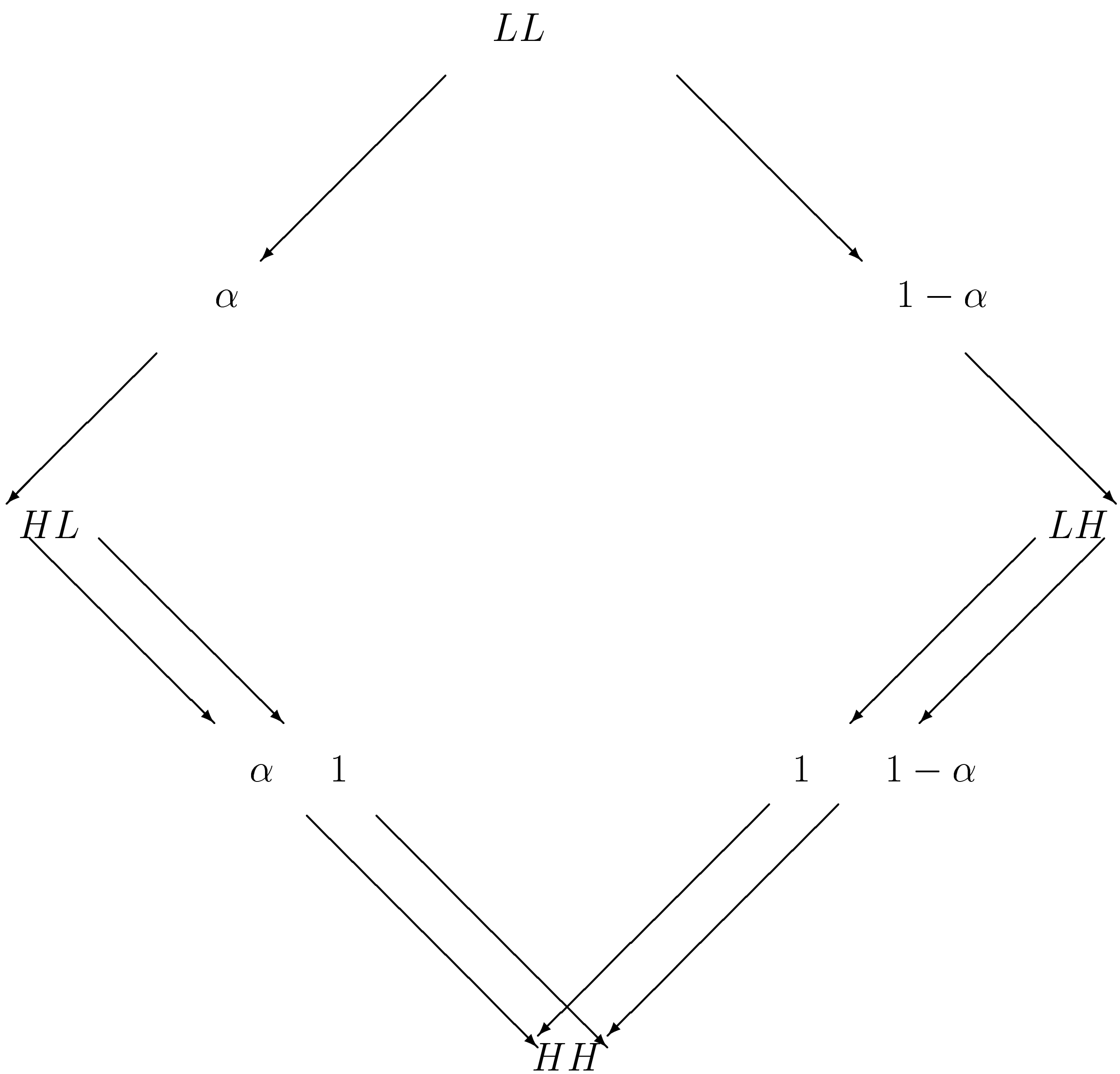

Figure 4: Complementarity effect reinforces the 'public good' factor.

$$\tilde{g}_i(g_j) = argmax_{g_i} v(g_i, g_j) - VC(g_i) - VC(g_j)$$

(VC(.) -virtual cost)

$K_1(-v_{22})$: 'public good' factor

$K_2 v_{12}$: complementarity effect

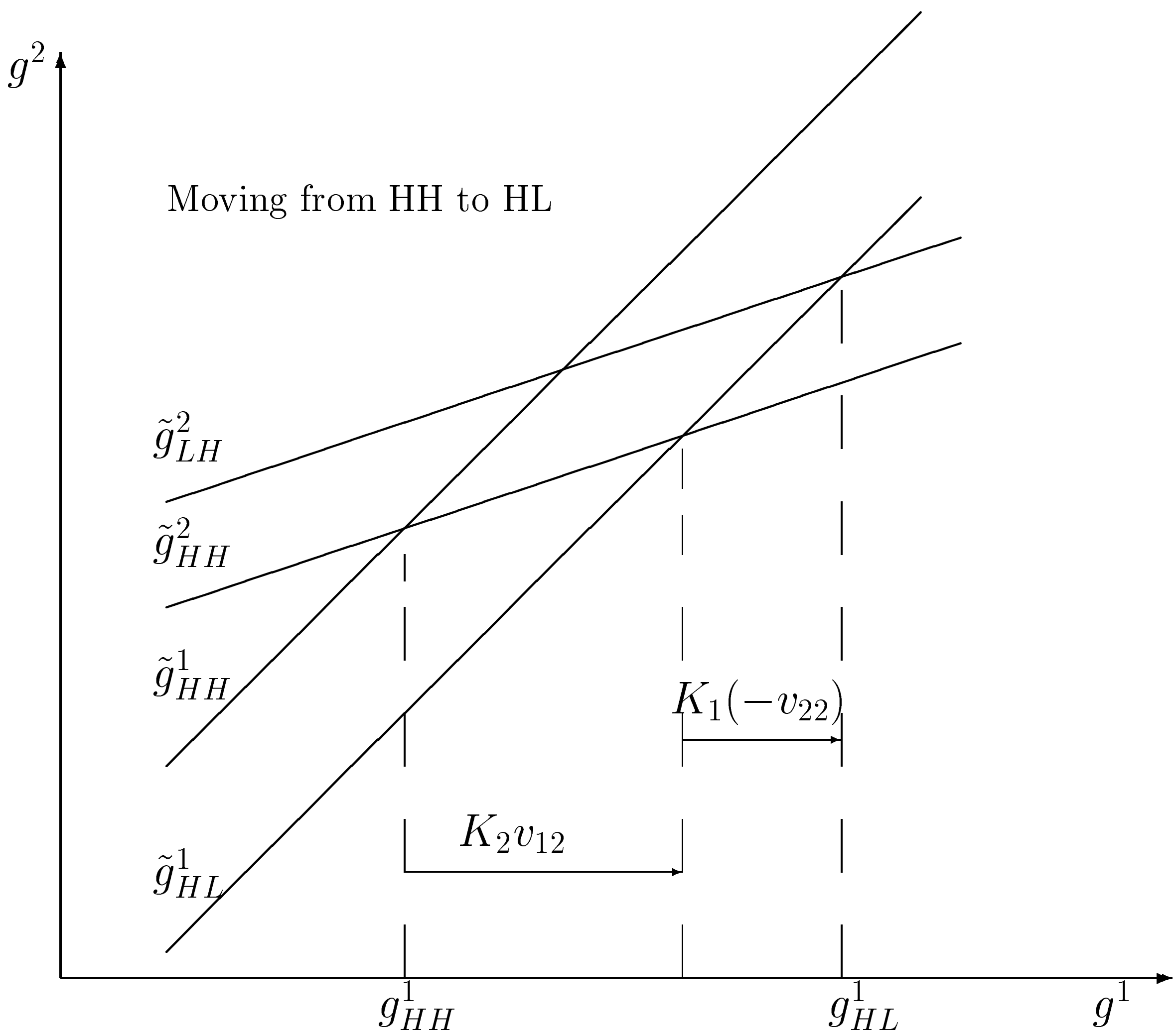

Figure 5: Substitutability effect offsets the 'public good' factor.

$$\tilde{g}_i(g_j) = argmax_{g_i} v(g_i, g_j) - VC(g_i) - VC(g_j)$$

(VC(.) - virtual cost)

$K_1(-v_{22})$: 'public good' factor

$K_2 v_{12}$: substitutability effect

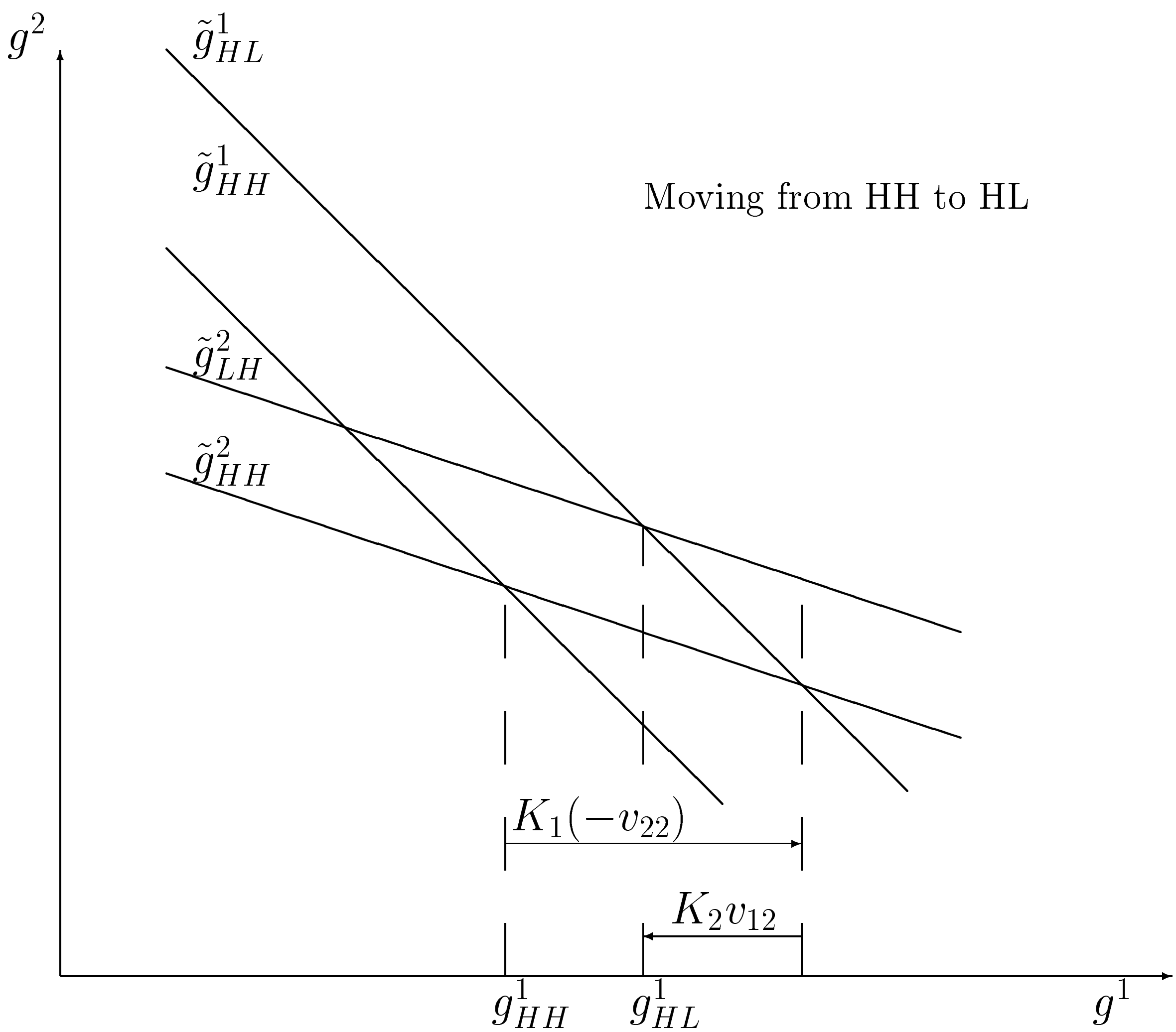

Figure 6: Comparing principal's payoffs under substitutability.

$S_1 > S_2 \Rightarrow$ Single-agent mechanism is superior

$S_1 < S_2 \Rightarrow$ Two-agent mechanism is superior

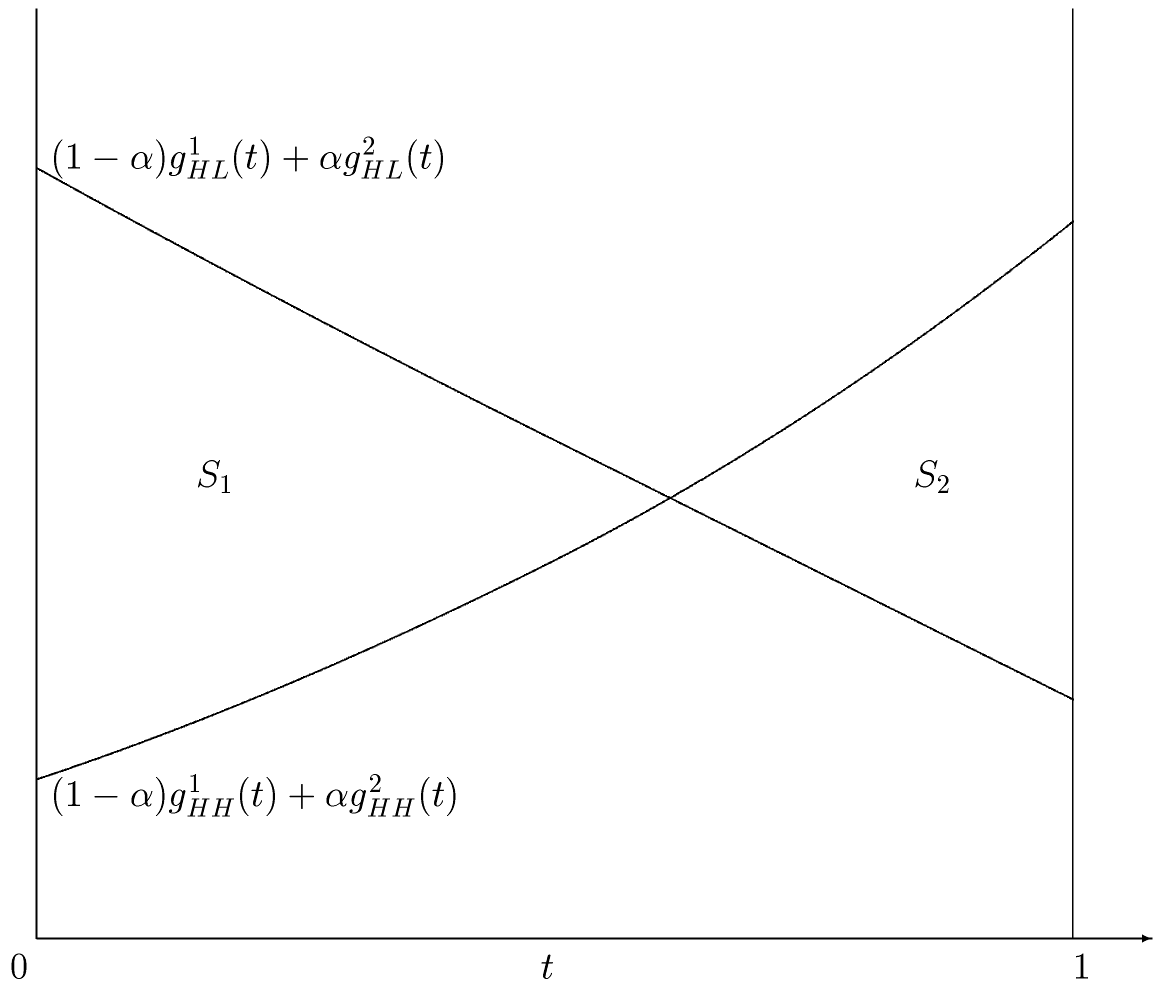

Figure 7: Binding $IC(LL-HH)$: Two-agent mechanism is superior.

$S_1 \subset S_2$

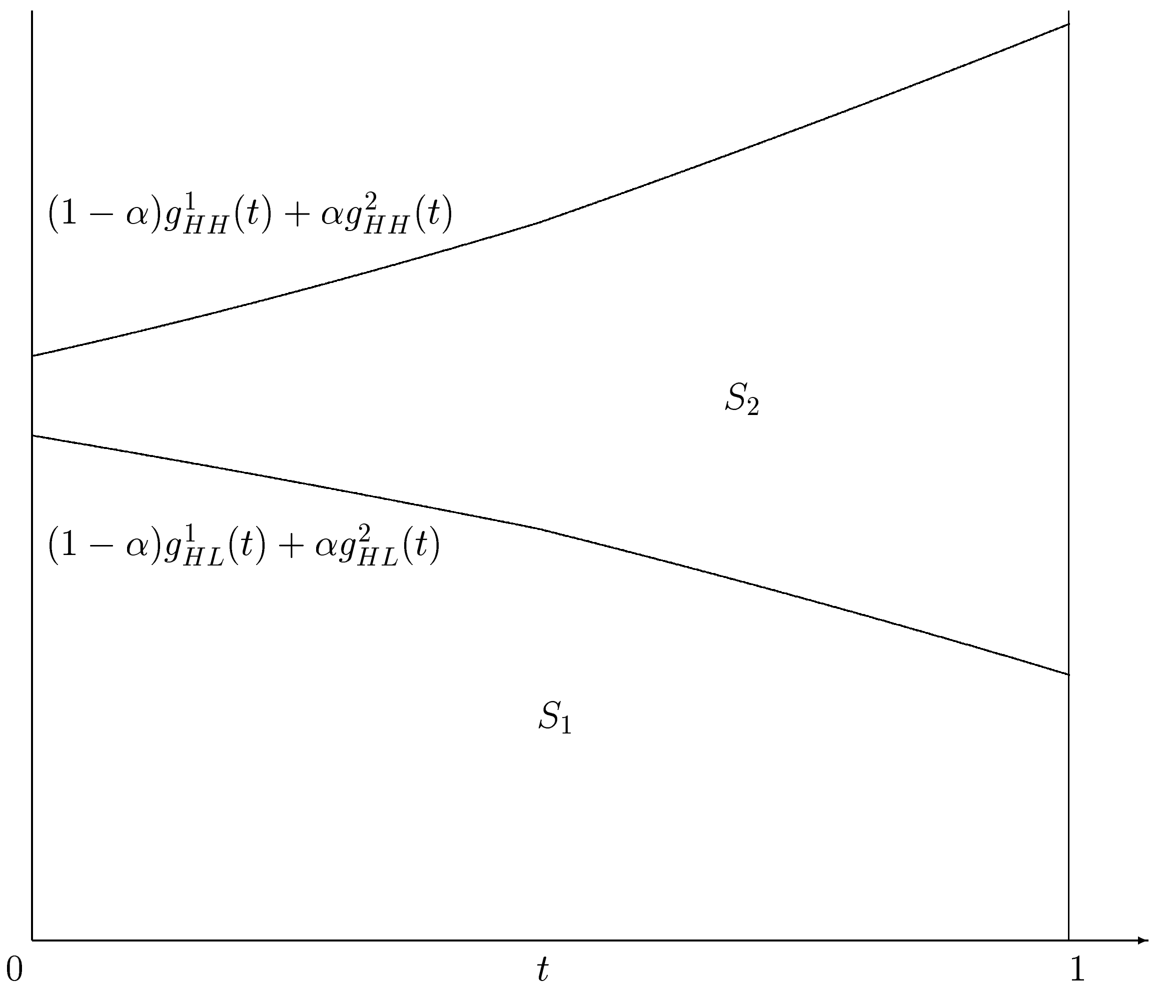

Figure 8: Substitutability: single-agent mechanism is superior, when $\frac{v_{12}}{v_{11}}$ is small.

$$S_1 > S_2$$

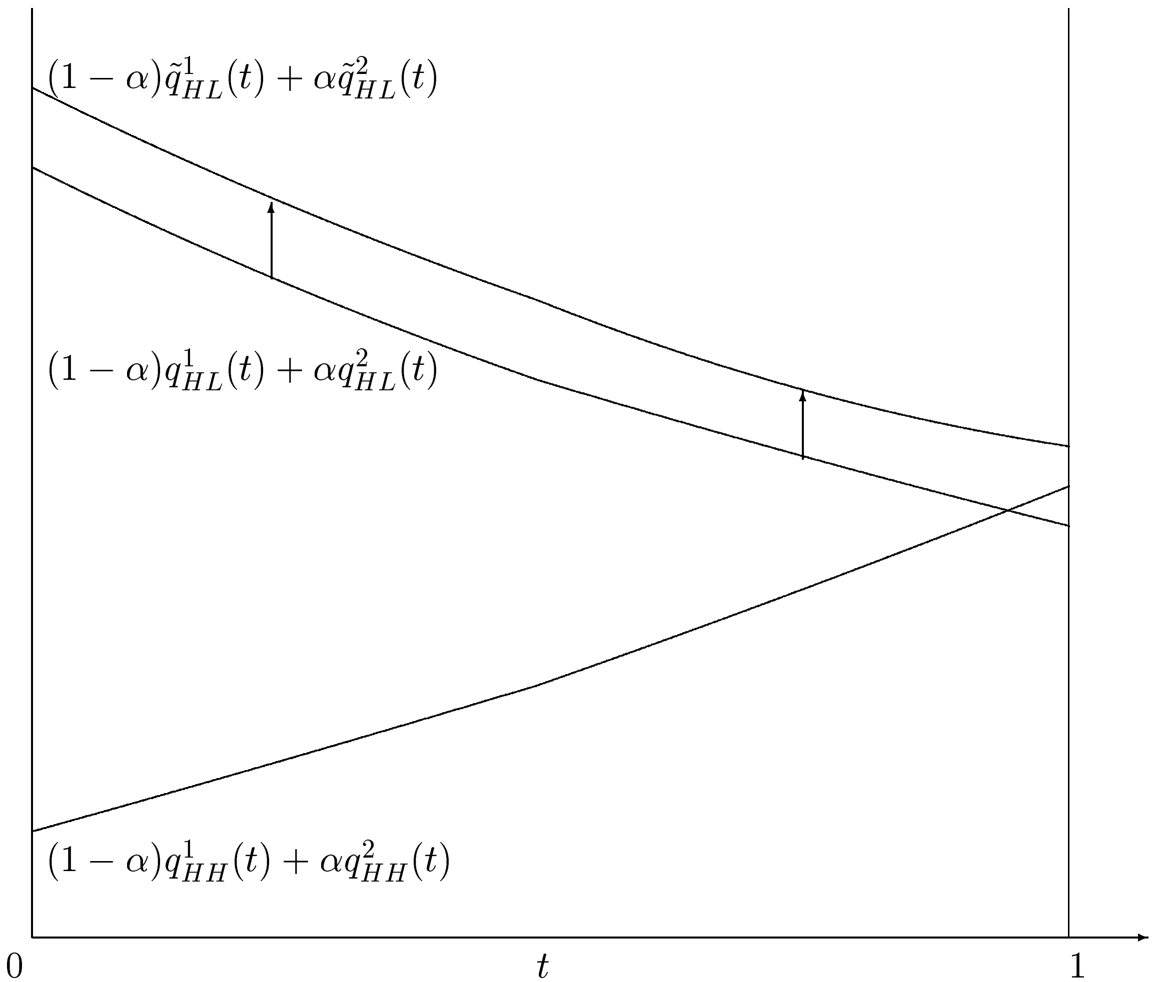

Figure 9: Substitutability: single-agent mechanism is superior, when p_1, p_2 are large.

$$S_1 > S_2$$