Preliminary and Incomplete
Comments Welcome

Policy Uncertainty and Informational Monopolies: The Case of Monetary Policy

by Larry E. Jones and Rodolfo E. Manuelli[1]

This version: June 4, 1997
First version: April 1, 1995

[1] MEDS, Kellogg School, Northwestern University and Department of Economics, University of Wisconsin-Madison, respectively. We would like to thank NSF for providing financial support for this research. Part of this work was done when both authors visited the Bank of Portugal, which provided a stimulating research environment. We also thank seminar participants at several universities for their comments, and John Kennan and Tom Sargent for many useful conversations on these topics.

1. Introduction

There are at least three important reasons why volatile government policy can adverse effects on welfare. First, uncertainty about policy can decrease utility for the standard risk aversion reasons. Ssecond, volatility can result in a confusion on the part of individual agents between real and policy shocks. This creates a costly signal extraction problem for the agents in the economy. Third, even in the absence of any real shocks to the system variable policy can create situations in which agents on opposite sides of trades can be asymmetrically informed about the state of policy. This third effect can operate even when all agents are risk neutral. In this paper, we concentrate on the third aspect of policy volatility, and model the possibility of informational monopolies which arise due to the strategic use of these informational differences.

Although this problem could arise in any setting in which government's actions are uncertain, a natural environment to study these issues is where there is uncertainty about monetary policy. (The high level of monetary policy uncertainty and its relationship to the average level of inflation is well documented, see Barro (1995).) Thus, this research continues an important line of inquiry into the mechanisms through which uncertain monetary policy has real effects begun with the seminal work of Lucas (1972). In that paper, Lucas first showed that asymmetric information about the realization of the money supply can result in monetary shocks affecting output since agents cannot perfectly distinguish real from monetary shocks. Since that initial work on the subject, a variety of papers have studied different environments and different mechanisms through which uncertain monetary policies have real effects (see, for example, Lucas (1987), (1989), Lucas and Woodford (1993), Eden (1994), Benabou and Gertner (1993), and Jovanovic and Ueda (1996)). A common feature in this work is that asymmetric information about monetary policy plays a leading role. This paper contributes to this literature by offering a new perspective on the roles played by two elements: the strategic use of information and the resulting incentives for costly information gathering activities. This is one possible explanation for the observation that the size of the financial intermediary sector is larger in countries with high and variable inflation (cf. Aiyagari and Eckstein (1993)).

The model that we analyze is a simple bargaining model in which buyers and sellers are potentially differentially informed. When a buyer and a seller are matched they bargain over the terms of the transfer of one unit of an indivisible good. If the parties agree to transfer the good, the buyer makes a money payment to the seller. Residual cash balances are used to buy units of a divisible good. We assume that the realization of the money supply follows a commonly known stochastic law and choose units of the divisible good so that its price equals the money supply. For most of the paper we consider a static version of the model to simplify the presentation. We later show that the same results apply to a situation in which individuals face repeated bargaining problems, and have infinite horizons.

In the game that we study, we assume that the seller announces a price and then buyer decides whether or not to purchase the indivisible good. (Following Samuelson (1984) and Myerson

(1985), it can be shown that this is the mechanism that maximizes the welfare of the seller given incentive compatibility and individual rationality on the part of the buyer.) In this way, the basic model mirrors that of Chwe (1995). For this model, it can be shown that a natural neutrality result holds: Proportional changes in the stochastic process determining the money supply are neutral. That is, changes in the money supply process that have the form of a positive constant times the "old" money supply process give rise to the same real allocation. Further, it can be shown that if either both parties are informed or both parties are uninformed, there are no real effects of variability in the money supply. (This result is a special case of that given in Chwe, that if information is symmetric, monetary policy has no real effects.)

This environment produces some interesting results both when the information structure is taken as given, and in the case in which individual agents determine whether they will be informed about the realization of the money supply.

Consider first the case of a buyer who is perfectly informed and a seller who is uninformed. We show that, depending on parameters, it is possible that a "lemons problem" (cf. Akerloff (1970)) in money can arise. That is, since both parties to the agreement share a common, but uncertain, value of the good, 'money,' the seller, when setting his price must take into account the fact that the only time the buyer will accept the offer is when the value of money is low (i.e., the money supply is large). This uncertainty about the value of cash has a non-monotonic effect on prices: for small levels of variability of the money supply, increases in variance decrease prices, while for high levels the opposite holds. When volatility is high enough, this last effect dominates trade causing the probability of exchange to decrease to zero even though it is common knowledge among the parties that the buyer values the good more highly than the seller. This welfare loss (i.e., the probability of trade being less than one) is simply a problem of an informational monopoly induced by the policy uncertainty. We interpret this as a possible explanation of the observation by Heymann and Leijonhufvud (1995) that during the Argentine hyperinflation of the eighties many shop owners refused to trade and posted signs that indicated they were " 'Closed for Lack of Prices' " (page 104).

To see the importance of the informational monopoly we show that even for parameter values in which the probability of trade is one, independent of the variance of the money supply, changes in volatility can have redistributive effects. This is, we believe, a feature common to asymmetries of information induced by policy volatility: Since both parties share an interest in the outcome of the policy variable, the informational problem is of the common value, rather than the independent value form. It is because of this form of informational asymmetry that there can be both welfare losses from private information and common knowledge of gains to trade. (For general overviews of these two cases, see Myerson (1985) and Kennan and Wilson (1993).)

Even though our emphasis is on the real effects of different monetary regimes, this model shares many of the features of other monetary models that rely on asymmetric information and uncertainty in the money supply process. For example, note that since the buyer accepts the offer if the "real price" is low (the realization of the money supply is high), and rejects the offer if the price is high (the realization of the money supply is low), expansionary monetary policy results in higher sales and, in this sense, in high output. Thus, the model reproduces the standard implications of many sticky price models: expansionary monetary policy increases output, and prices are not fully responsive to monetary shocks (in this example they are independent of the realization of the money supply).

This characterization depends critically on our assumption that the buyer is informed and the seller is not. We study the other alternatives and show that when both parties share the same information structure the probability of trade is one; while when buyers are uninformed and sellers are informed high variability of the money supply results in a probability of trade less than one, and in contractionary effects of unforeseen increases in the money supply. This raises the question of the determination of the information structure in this setting.

We next consider the case in which agents choose their information levels. We study a two stage game in which buyers and sellers can choose to learn the realization of the money supply at some cost. In the second stage they are randomly paired. Our previous description of the effects of policy uncertainty on the equilibrium of a given match, suggests that the payoffs to becoming informed are low at both low and high levels of uncertainty. At low levels of variability the expected value of any given amount of money is close to its certainty value. Information is not very valuable. At the other end, high variance results in high prices and low probability of a transaction. In this case, information is not very valuable either. This intuition carries over to the equilibrium in which all agents choose their information structure as long as the parties values for the indivisible good are not too far apart. We show that for both low and high variances all individuals choose to be uninformed. For intermediate values of the variance of the money supply, the unique symmetric equilibrium is in mixed strategies with varying fractions of buyers and sellers being informed. In this region the inefficiency of equilibrium is due to two factors: first, trade occurs with probability less than one; second, real resources are used to acquire socially useless information.

In the case in which buyers' and sellers' valuations are far apart, policy uncertainty has a smaller impact on the probability of trade (which is one over a much larger region). However, the artificially created value of information induces agents to spend resources on acquiring information and, even for high levels of policy variability, the equilibrium is inefficient. Since the good is traded with high probability, information does not lose its value when the variance of the money supply increases. Thus, unlike the case of similar valuations, there is an equilibrium in which a positive fraction of both buyers and sellers are informed for all values of the variance of the money supply exceeding some critical level. In this case the inefficiency is mostly associated with the resources devoted to acquiring information.

In both of these cases, our model implies that the cross sectional distribution of prices for the same good (across stores or matches) is not degenerate because different sellers will be differentially informed. This agrees with the evidence in high inflation countries (see Lach and Tsiddon (1992), Eden (1994) and the studies in Sheshinski and Weiss (1993)). An alternative class of models with this feature is that in which inflation induces search. In those models (see, for example, Benabou (1988), (1992), Benabou and Gertner (1993), Diamond (1993), and Tommasi (1994)) the key effect is that, given that consumers have a positive cost of searching, higher inflation increases “effective” search costs and results in firms having monopoly power. In our model, this dispersion is driven just by informational asymmetries and hence is independent of firms' costs, search costs and the costs of changing prices. Thus, our model provides yet another mechanism through which monetary policy can result in price dispersion.

In addition to its implications on prices, the model makes predictions about the distribution of informed and uninformed agents, and shows that --due to the strategic interaction-- these are not monotonic in our measure of the variability of monetary policy. The model does not support the view that higher variance makes information more valuable and, hence, more people should be informed. The reason for this is that the value of information depends on the policy in place as well as the actions of other agents. In this sense, the fraction of “other” agents that is informed is an important as the variability of the money supply in determining the decision to become informed on the part of any single individual.

The papers that are closest to this one are Casella and Feinstein (1990) and Chwe (1995). Cassella and Feinstein consider a dynamic bargaining situation in which there is no asymmetric information and inflation acts as a tax. In their particular formulation, inflation reduces the effective discount rate and this, in turn, affects the outcome of the bargaining game. The key result in their paper is that producers will charge different prices depending on the age (or wealth since they are perfectly correlated) of the buyers that are in the store. Chwe (1995) considers a model in which buyers and sellers are asymmetrically informed about both the value of money and their private valuations of the indivisible good. He shows that there will be no welfare loss associated with private information about money as long as the value of money is common knowledge or agents share the same priors and information partition. He also shows that when individuals have different priors, monetary uncertainty can increase the sum of expected utilities. Chwe also studies the effect of an exogenously changing fraction of informed traders and shows that the resulting equilibrium displays Phillips curve like features. Our work emphasizes the point that it is the informational monopoly that is important and that its degree of importance depends on how variable monetary policy is.

In section 2 we present the basic model for fixed information structures. Section 3 explores the impact of changes in policy uncertainty upon the equilibrium when information structures are not allowed to change. In sections 4 and 5 we concentrate on the case in which the differences in the valuation of the indivisible good between buyers and sellers is small. In section 4 we study a two stage game. In the first stage individuals choose their information structure, and in the second stage they are randomly matched. We show that an equilibrium exists and we partially characterize it. Section 5 contains most of our comparative statics results. In section 6 we discuss the case of large differences in valuation, while section 7 extends the model to a dynamic framework. Finally, section 8 offers some preliminary conclusions.

2. The Basic Model and Equilibrium with Symmetric Information

In this section we describe a basic bargaining problem between a buyer and a seller with a fixed information structure. We model this as a game with a take it or leave it offer from the seller followed by an accept/reject decision by the buyer. We analyze the equilibrium when the two parties are symmetrically informed/uninformed and show that monetary policy is neutral in this case.

Both the buyer and the seller derive utility from two goods: an indivisible good which gives utility v^b to the buyer (if purchased) and v^s to the seller, and another which we call general consumption. We assume that utility is additive and given by,

$$u^b (x^b,c)= v^b x^b + c,$$

where x^b, restricted to be either 0 or 1, represents consumption of the indivisible good and c is the level of general consumption. We assume that subsequent units of the indivisible good do not yield any utility. We treat the seller symmetrically and assume that his utility function is given by,

$$u^s (x^s,c)= v^s x^s + c.$$

At this point we take the valuations of both buyers and sellers as given, and assume that $v^b > v^s$. This assumption implies that trading the indivisible good is always the ex post efficient outcome.

The buyer has an initial endowment of money m_b . If the seller announces the price p and the buyer purchases one unit of the indivisible good, the buyer's consumption of the general consumption good is $c^b = (m_b-p)/M$. Here we have imposed that the price of the consumption good is equal to the money supply M. This simply follows from our choice of units of the divisible good, and can be rationalized by imposing a cash-in-advance constraint on divisible consumption that opens after the value of M becomes common knowledge. As it will become clear when we discuss a dynamic version of this model (section 7) it is possible to interpret M as the growth rate of the money supply, with last period's money supply normalized to one. In what follows we will usually use the “stock” interpretation of M, although the reader can, without any formal changes, think of M as a growth rate. Let the seller's endowment be m_s. Then, if he sells the indivisible good, his level of general consumption is $(m_s+p)/M$. Without loss of generality assume that $m_b=\lambda M$, while $m_s=(1-\lambda)M$, where $0 \leq \lambda \leq 1$. With these conventions it is possible to write the indirect utility function (with some abuse of notation) for each of the parties given that they know M and the price p has been announced by the seller as,

$$(2.1) \quad u^b (x^b,p,M)= v^b x^b + \lambda - x^b p/M,$$

$$(2.2) \quad u^s (x^b,p,M)= v^s (1-x^b) +1-\lambda + x^b p/M.$$

We restrict the seller’s announced price to be measurable function of his information. We take M to be a random variable, and assume that its distribution is common knowledge.

First, consider the case in which both parties are informed. Here, given a price p announced by the seller, the buyer's problem is to choose whether or not to buy the object. Since, by assumption, he knows the realization of the random variable, M, he must choose the action to maximize his conditional expected utility given M. In this case, he will buy the object as long as $p \le v^b M$. Since the seller knows this and knows the value of M, the equilibrium has the seller charging the price $p(M) = v^b M$. The good is exchanged with probability one and the seller extracts all of the surplus from the buyer. We summarize the equilibrium in the following proposition.

Proposition 2.1. In the case of an informed buyer meeting an informed seller, (I,I), the equilibrium outcome is characterized by,

(i) The indivisible good will be traded with probability one. [q(I,I)=1]
(ii) The price announced by the seller is $p(M)= v^b M$.
(iii) The expected utilities of the two parties are,

$$W^s(I,I) = 1-\lambda + v^b,$$
$$W^b(I,I) = \lambda \;.$$

(iv) Monetary policy is neutral. The allocation and (both ex ante and ex post) utilities of two agents do not depend on the distribution for M.

Note: Throughout, we will denote the 'information state' by a pair, (Y,Z) where Y and Z $\in$ {U,I}, Y denotes the informational status of the buyer and Z is that of the seller. Of course, U means that the agent is uninformed and I means that the agent is informed.

Next, consider the case of an uninformed buyer meeting an uninformed seller. Here, since the seller does not know the value of M, the announced price does not depend on M. Thus, the expected utility of the buyer if the price is p is his unconditional expected utility. If he purchases the good utility is,

$$\lambda + v^b - pE(M^{-1}),$$

while the expected utility of not buying the good is λ. Thus, the rule the buyers uses is: buy if the price is less than or equal to $v^b/E(M^{-1})$. It is clear that --if the seller decides to sell at all-- he will always charge the price $p = v^b/E(M^{-1})$. (Recall that given the information structure the seller is restricted to making price offers that do not depend on M.) Thus,

$$p(M)= v^b/E(M^{-1}). \qquad (2.3)$$

Whether the seller will choose to sell the indivisible good depends on the comparison of its utility in the two cases. It is easy to check that the seller will choose to sell if and only if the price it can get is greater than or equal to $v^s/E(M^{-1})$. Since this is less than p(M) trade will occur with probability one at the monopoly price $p = v^b/E(M^{-1})$. To simplify notation in what follows, define

μ to be $E(M^{-1})$.

We summarize this discussion in the following proposition.

Proposition 2.2. In the case of an uninformed buyer meeting an uninformed seller, (U,U), the equilibrium outcome is characterized by the following,

(i) The indivisible good is traded with probability one. [q(U,U)=1]
(ii) The price announced by the seller is $p(M)= v^b/\mu$.
(iii) The expected utilities of the two parties are,
$$W^s(U,U) = 1-\lambda + v^b,$$
$$W^b(U,U) = \lambda.$$
(iv) Monetary policy is neutral. The allocation and (ex ante, expected) utilities of two agents are the same as if the random variable 1/M was equal to its mean value with probability one. Thus, they do not depend on the distribution for M. Ex post welfare does depend on the realization of M, however.

Note that, for this informational structure, prices depend on the mean of the inverse of the money supply and --as before-- are independent of relative wealth. Payoffs (and income) are independent of monetary policy, and depend just on initial wealth and monopoly power.

The neutrality results presented here (i.e., Proposition 2.1 (iv) and Proposition 2.2 (iv)) are reminiscent Chwe's (1995) common knowledge results. There, it is shown that whenever both parties share the same prior beliefs (which is the only case we consider) and their information partitions are the same, money is neutral.

3. The Case with Asymmetric Information

In this section, we generalize the model presented above to consider cases of asymmetric information. We consider both the case of an uninformed seller facing an informed buyer as well as an informed seller facing an uninformed buyer. We show that in these two cases, money is not neutral in general. The character of the equilibrium depends both on which party is informed and the distribution governing the money supply, M. There are some general properties, however. In general, the probability of trade is strictly less than one (as long as there is 'enough variance') and hence equilibrium is inefficient. The one exception to this is when there is no variability at all in the money supply. In this case, the good is exchanged with probability one and the outcome is efficient (no matter which party is informed). The model does generate a Phillip's curve, a non-trivial, equilibrium relationship between the level of M and the probability of trade of the good, but the direction of this relationship depends on which party is the informed one.

3.1 Uninformed Sellers and Informed Buyers

Consider the case in which an uninformed seller meets an informed buyer. We assume that the information structure is common knowledge: the seller and the buyer both know who is informed and who is uninformed. We study the analogue of the game analyzed in section 2 in which the seller has all the monopoly power. More precisely, we consider a timing of moves in which the seller announces a price, and the buyer can either accept or reject. We do not allow any counteroffers.

This description of the bargaining environment may seem restrictive at first glance. However, following the arguments in Myerson (1985) and Samuelson (1984), it can be shown that this game implements the outcome of the mechanism that maximizes the utility of the seller given incentive compatibility and individual rationality on the part of the buyer when the buyer is informed and the seller is not:

Proposition 3.1.1 Assume that distribution of M has compact support with a density which is everywhere positive and that the buyer is informed and the seller is not. Then, the outcome of the bargaining game is identical to that of the mechanism that maximizes the utility of the seller given incentive compatibility and individual rationality of the buyer.

Proof: The proof is contained in Appendix A.

To analyze the model in this case, consider first the buyer's decision. Since he knows the realization of M, given the price p, the optimal decision is to buy the indivisible good if and only if,

$$v^b + \lambda\text{-}p/M > \lambda, \text{ or equivalently, } v^b + (\lambda M\text{-}p)/M > \lambda M/M.$$

Thus, the buyer's optimal decision rule is,

$$(3.1.1) \qquad \begin{cases} \text{buy if} & p \leq Mv^b \\ \text{do not buy if} & p > Mv^b. \end{cases}$$

The seller knows this rule (this is the sense in which it extracts all the surplus) but --in this case-- does not know the realization of M. Since the seller is restricted to announcing prices that are independent of the realization of M (this is where the measurability restriction is binding), the seller chooses p to maximize,

$$(3.1.2) \qquad V^s(p) = \int \left[\chi_{(p<Mv^b)} [1 - \lambda + \frac{p}{M}] + (1 - \chi_{(p<Mv^b)})[v^s + 1 - \lambda] \right] dF_M$$

where F_M is the cdf of the random variable M. Let p* denote the solution to this maximization problem.

With this investment in notation, it is possible to give two types of neutrality results for this model. First, notice that if the distribution of M is a point mass at some value m*, it follows immediately that the seller will announce the price $p=m^*v^b$. Given this, the buyer will always buy the object and trade will occur with probability one. Note that in this case, the outcome is ex post efficient. This is one sense in which without variability in the money supply, there are no inefficiencies in this model.

A second type of neutrality result holds even in cases in which there may be welfare losses from inflation. This is that the equilibrium outcome of the model is identical for any linear transformation of the money supply process. That is, inspection of the decision rule for the buyer above shows that for the random variable αM with $\alpha>0$, the optimal decision rule for the buyer has a reservation price which is α times as high as with the random variable M. Given this feature, it follows immediately from (3.1.2) that the optimal price from the seller's point of view when the money supply is given by αM is to choose a price equal to αp^* where p* is the optimal price under the original distribution of M. It follows from this that the probability of trade as well as the realized levels of welfare are invariant to this type of change in the distribution of M (although equilibrium nominal prices are affected). For this class of experiments money is neutral. Note that adding a constant to the random variable M does not give a neutrality result. We summarize these results in the following proposition:

Proposition 3.1.2:

(i) If the distribution of M places probability one on the value m*, the equilibrium has the seller charging the price $p = m^*v^b$. The indivisible good is exchanged with probability one and the outcome is efficient.

(ii) Both the probability of trade and the realized levels of utility are invariant to positive multiples in the probability distribution of the money supply. The equilibrium price under a money supply rule given by $M' = \alpha M$ with $\alpha>0$, is αp^* where p* is the equilibrium price under the original money supply rule M.

It turns out that it is convenient to describe (3.1.2) in terms of the distribution of the inverse of the money supply. Thus, let F be the cdf of the random variable 1/M, and assume that it has a density f. The random variable M is assumed bounded, $0<m^L\leq M\leq m^U<\infty$. Note that an equivalent description of the optimal decision rule by the buyer is that he buys the indivisible good if and only if $1/M<v^b/p$. Thus, the seller's indirect utility over prices is given by,

$$V^s(p) = 1 - \lambda + p\int_{1/m^U}^{v^b/p} x\,f(x)\,dx + \left(1 - F\left(\frac{v^b}{p}\right)\right)v^s. \qquad (3.1.3)$$

First, note that if the seller charges the price $p=m^L v^b$ the buyer will accept the offer with probability one. Thus, the seller will always choose prices such that $p \geq m^L v^b$. At the other extreme, any price exceeding $m^U v^b$ will result in a zero probability of trade.

In order to characterize equilibrium behavior in the model, we make a special assumption about the distribution, F:

Assumption 1*: The Probability Model for M*. We will assume that the inverse of the money supply has a uniform distribution on [μ-k,μ+ k], where $0 \leq k < \mu$,

$$1/M \sim U\,[\mu-k, \mu+k].$$

Note that given this specification $m^L=1/[\mu+k]$, and $m^U=1/[\mu-k]$.

To simplify notation, it is useful to express v^s as a fraction of v^b. More specifically, define ϵ by:

$$v^s = (1-\epsilon)v^b.$$

Then, ϵ measures how far apart the values of the buyer and seller are. This, in turn, measures the potential gains from trade between the two parties. Thus, if $\epsilon=1$, $v^s=0$, and the spread between v^s and v^b is maximal and the potential gains from trade are v^b while if $\epsilon = 0$, $v^s=v^b$ and there are no gains from trade.

We need one last bit of notation before describing the equilibrium behavior in this environment. Let $k(\mu,\epsilon)$ be defined (for $\epsilon \leq ½$) by

$$k(\mu,\epsilon) = \mu \frac{[1-(1-2\epsilon)^{1/2}]}{[1+(1-2\epsilon)^{1/2}]}.$$

As it turns out, the qualitative behavior of the equilibrium depends on whether k is smaller or larger than $k(\mu,\epsilon)$ as well as the size of ϵ itself.

The Equilibrium of the Bargaining Game

The seller maximizes (3.1.3) given its expectations about the realization of the money supply. Optimal behavior on the part of the buyer is completely summarized by the “purchasing rule” (3.1.1).

We are now ready to describe the equilibrium of the bargaining game.

Proposition 3.1.3. Let $\hat{p}(\mu,k,\epsilon)$ be the solution to the seller's problem (maximization of (3.1.3) subject to bounds constraints) and let $\hat{q}(\mu,k,\epsilon)$ denote the resulting probability of the object being traded. Then,

i) If $\epsilon<1/2$ the equilibrium is characterized by,

a) If $k\leq k(\mu,\epsilon)$, $\hat{p}(\mu,k,\epsilon)=p^L$ and $\hat{q}(\mu,k,\epsilon)=1$,

b) If $k>k(\mu,\epsilon)$, but $k<\mu$, $p=v^b(1-2\epsilon)^{1/2}/(\mu-k)$ and $\hat{q}(\mu,k,\epsilon)=(\mu-k)[(1-2\epsilon)^{-1/2} - 1]/2k <1$. In this case, trade occurs whenever M is large, i.e., whenever $M \geq p/v^b$.

ii) If $\epsilon\geq 1/2$, $\hat{p}(\mu,k,\epsilon)=p^L$ and $\hat{q}(\mu,k,\epsilon)=1$.

Proof:

The seller's objective function is continuous but not differentiable in the interior of $\Re_+$. The interval $[0,p^L]$ is easy to handle. In this region, the good will be sold with probability one, and the function $V^s(p)$ is increasing in p. It follows that the seller will never announce a price less than p^L. Let p^U be the lowest price such that the buyer does not purchase the good for any realization of M. In our setting p^U is given by $v^b/(\mu-k)$. For prices $p\geq p^U$, the function $V^s(p)$ is just $1-\lambda+(1-\epsilon)v^b$, a constant. It is immediate to check that in the interval (p^L,p^U) function is strictly concave if $\epsilon<1/2$, linear if $\epsilon=1/2$, and strictly convex if $\epsilon>1/2$. We are now ready to discuss the various cases.

i) [$\epsilon<1/2$]. In this case the objective function is concave in the relevant interval ($p\geq p^L$). The first order condition for the seller's maximization problem with the additional requirements that $p\geq p^L$ (with Lagrange multiplier γ) is,

$$\int_y^z xf(x)dx - (1/p)^2 f(v^b/p)[(v^b)^2 - v^b v^s] + \gamma = 0,$$

or,

$$\int_y^z xf(x)dx - (v^b/p)^2 f(v^b/p)\epsilon + \gamma = 0,$$

where $z= v^b/p$ and $y=1/m^U$. Using the specific form for f(x) we get,

$$\gamma = (v^b/p)^2 (1/2k)\epsilon-(1/2k)(1/2)[(v^b/p)^2-(\mu-k)^2].$$

Alternatively, this condition is,

$$(*) \qquad 4k\gamma = (v^b/p)^2 (2\epsilon-1)+(\mu-k)^2.$$

Depending on the size of μ-k, it is possible that even when $p=p^L$, the Lagrange multiplier is positive. Define k_L as the lowest value of k such that (*) is satisfied with $p=p^L$ and $\gamma=0$. Thus, k_L must satisfy,

$$(\mu+k_L))^2 (1-2\epsilon)=(\mu-k_L)^2,$$

or,

$$(1-2\epsilon) = [(\mu-k_L)/(\mu+k_L)]^2,$$

or, recalling the definition of $k(\mu,\epsilon)$ given above,

$$k_L = k(\mu,\epsilon).$$

From the definition of $k(\mu,\epsilon)$ it follows that k_L is unique. We now show that for $k>k_L$ the unique p that solves the first order condition assuming $\gamma=0$ is strictly greater than p^L. To see this impose $\gamma=0$ and solve for p. The solution is,

$$p=v^b(1-2\epsilon)^{½}/(\mu-k).$$

To check that $p>p^L$ we need to check that,

$$v^b(1-2\epsilon)^{½}/(\mu-k) > v^b/(\mu+k),$$

which corresponds to,

$$(1-2\epsilon)^{½} > (\mu-k)/(\mu+k), \text{ or equivalently, } k > k(\mu,\epsilon).$$

The candidate solution automatically satisfies $p<p^U=v^b/(\mu-k)$. Finally, note that the probability of trade is given by $q(\mu,k,\epsilon)=Pr(p < Mv^b) = Pr(1/M < v^b/p)$. Using the expression for $p(\mu,k,\epsilon)$ and the form for F shows that

$$q(\mu,k,\epsilon) = (\mu-k)[(1-2\epsilon)^{½} - 1]/2k$$

as desired.

ii) We first consider the case $\epsilon=1/2$. It is easy to check that the derivative of $V^s(p)$ is negative and given by $-(\mu-k)^2/2k$ on (p^L,p^U). It follows that the optimal policy is to charge p^L, which results in a sale with probability one.

If $\epsilon>1/2$ the function $V^s(p)$ is convex on (p^L,p^U). It is clear that the solution must be a corner solution: the optimal price is either p^L or p^U. The respective utilities are,

$$V^s(p^L) = 1- \lambda + v^b\mu/(\mu+k),$$

$$V^s(p^U) = 1- \lambda + v^b (1-\epsilon).$$

For the seller to weakly prefer p^U over p^L it must be the case that,

$$\mu/(\mu+k) \geq (1-\epsilon),$$

or --given the restrictions on ϵ in this case-- that,

$$½ < \epsilon \leq k/(\mu+k).$$

This, however, leads to a contradiction since the right side of the previous inequality is bounded above by ½. It follows that the seller will always choose p^L, and the indivisible good is sold with probability one.

■

This simple model delivers the standard result in many fixed (or predetermined) (see Lucas (1989) and Lucas and Woodford (1993) for examples with explicit micro foundations) price models: an expansionary monetary policy reduces the real cost of goods for sale and it increases output. In our case, the result is extreme due to the indivisibility of the good; for low values of the money supply there is no trade (and output is low), while for high values, output is efficient. The model also suggests an asymmetric interpretation of the reluctance to trade on the part of store owners documented by Heymann and Leijonhufvud (1995). They describe the following scene that took place in December 1989 when the inflation rate reached suddenly increased from a few percentage points per month to over 20% per week in a short period.

> " A customer finds a good inside a shop, with a clearly marked price, and decides to buy it. The shopkeeper refuses; he explains that the posted price has no significance, because he cannot be sure that the wholesaler will not double his own price the next day. When asked what he would do if someone offered to pay double the marked price, the shopkeeper answers that he would not sell anyway, for what if the wholesale price tripled before he replaced the good?" (page 106, footnote 19)

This, it seems to us --although Heymann and Leijonhufvud apparently disagree with this interpretation--, shows the effect of uncertainty about the value of money when deciding the terms of trade.

As Proposition 3.1.3 shows, the qualitative behavior of the equilibrium depends on whether ϵ is less or greater than one. In the remainder of this subsection, as well as in sections 4 and 5, we concentrate on the case $\epsilon<1/2$. The other case --large differences in valuation-- is discussed in section 6 and Appendix D.

Changing the volatility of M

Next, we will discuss the consequences of changing the volatility of the money supply upon the equilibrium when the buyer is informed, but the seller is not. Proposition 3.1.3 gives us a partial, but incomplete accounting for what happens when uncertainty about policy is increased. There, holding μ fixed, as k is increased, the probability of trade decreases toward zero. Notice here however that we are simultaneously changing both the average value of M (it goes to infinity) and its variance (we are holding μ= E(1/M) fixed). The problem is that changes in k holding μ constant **are not** equivalent to increases in the variance of M, holding its mean constant. For this reason, we will be interested in **simultaneously** changing k and μ so that E(M) is fixed. In this way, we can isolate the effect of changing the volatility of policy without changing its average value. Because of this, it is useful to summarize some properties of the family of distributions that we are working with.

Let the mean of M be m*, m* = E(M). Then it follows that,

$$m^* = [\ln(\mu+k)-\ln(\mu-k)]/2k.$$

Thus, we can find for each k, the value of μ, denoted μ(k) that satisfies this expression for a given value of m*. It is easy to check that,

$$\mu(k) = k(1+e^{2m^*k})/(e^{2m^*k}-1).$$

It is convenient to display the cumulative distribution function of M given the parameters (m*,k). This is given by,

$$F(m;m^*,k) = e^{2m^*k}/(e^{2m^*k}-1) - 1/(2mk).$$

It is straightforward to calculate the variance of M given (μ(k),k). It is,

$$\sigma_M^2(k,m^*) = 1/[\mu(k)^2-k^2] - (m^*)^2.$$

There are two properties of this family of distributions that are worth emphasizing. First, for any given mean, m*, $\lim_{k\to\infty}\sigma_M^2(k,m^*)=\infty$, and $\lim_{k\to 0}\sigma_M^2(k,m^*)=0$, and $\sigma_M^2(k,m^*)$ is a monotone increasing function of k. Thus, it is appropriate to index monetary policies by k, with higher k's (holding m* constant) corresponding to higher variance --but constant mean-- economies. Finally, it can be shown that increasing k with μ = μ(k) is a mean preserving spread on the distribution of M.

In section 3.1 we described the critical value k(μ,ϵ), which played a critical role in determining the efficiency of an allocation. This was defined for a given value of the mean of the inverse of the money supply, μ. Since we are interested in changes in the variance of the money supply holding its mean (m*) constant, it is necessary to describe the critical value of k that corresponds to the threshold effect associated with k(μ,ϵ). To this end, let k_1^* be the unique value of k that satisfies,

$$k_1^*=k(\mu(k_1^*),\epsilon).$$

A simple calculation shows that $k_1^*=\ln[(1-2\epsilon)^{-½}]/2m^*$.

As described in Proposition 3.1.3, the qualitative nature of the solution depends on how far apart the valuation of the buyer and the seller are. In particular, if $\epsilon<1/2$, for $k\leq k_1^*$, the probability of trade is one while for $k>k_1^*$ it is strictly less than one.

From now on, to simplify notation, we will ignore the dependence of the equilibrium variables on ϵ and m*. In the conclusion we briefly discuss the impact of changing them .

The Impact of Variability on Prices

From Proposition 3.1.3 it follows that the equilibrium price charged by an uninformed seller when faced with an informed buyer is,

$$(3.1.4) \qquad p(I,U,k) = v^b m^* \max\{(e^{2m^*k}-1)/2m^*ke^{2m^*k},\ (1-2\epsilon)^{½}\ (e^{2m^*k}-1)/2m^*k\}.$$

This is equivalent to,

$$(3.1.5) \qquad p(I,U,k) = v^b m^* \max\ \{\Phi_1(k),\Phi_2(k)\},$$

where,

$$\Phi_1(k) = (e^{2m^*k}-1)/2m^*ke^{2m^*k} \text{ and}$$

$$\Phi_2(k) = (1-2\epsilon)^{½}\ (e^{2m^*k}-1)/2m^*k.$$

Direct calculations show that $\Phi_1(k)$ is continuous, decreasing and differentiable, and $\lim_{k\to 0} \Phi_1(k)=1$, $\lim_{k\to\infty}\Phi_1(k)=0$. $\Phi_2(k)$ is a differentiable and increasing function, and it satisfies $\lim_{k\to 0} \Phi_2(k)=(1-2\epsilon)^{½}$, and $\lim_{k\to\infty}\Phi_2(k)=\infty$. Moreover, the unique value of k that satisfies $\Phi_1(k)=\Phi_2(k)$ is k_1^*. Figure 1 displays candidate Φ_i functions . From (3.1.5), it follows that as the variance of the money supply increases there is a region in which prices decrease. However, once the variance --as indexed by k-- gets past the value k_1^*, additional increases result in higher prices. Thus, in this case, prices are a V-shaped function of the variance of the money supply. To understand this somewhat puzzling response of equilibrium prices to changes in the volatility of the money supply we consider the two regions defined by k_1^* separately. Consider first the region of decreasing prices. In this region --in which trade occurs with probability one-- an increase in the variance of the money supply forces the seller to lower prices to guarantee a sale, so that even in the lowest realization of the money supply the buyer finds advantageous to purchase the indivisible good. Thus, in this region, an increase in variance reduces the seller's monopoly position: uncertainty about the value that buyers put on the object and the desire to sell results in lower prices. Of course, lowering the price reduces consumption of the divisible good on the part of the seller. At some point this decrease in consumption is sufficiently high that the seller decides to charge slightly higher prices and face the possibility of trading with probability less than one. Thus, in this second region --to the right of k^*_1 in Figure 1-- increases in the variance are accompanied by price increases and --as we will show shortly-- reductions in the transaction volume. Thus, we summarize this discussion in the following result.

Proposition 3.1.4. Let $\epsilon<1/2$. Then the equilibrium price announced by an uninformed seller who meets an informed buyer --p(I,U,k)-- is a V shaped function of k. It decreases for $k \leq k^*_1$, and it increases for $k \geq k^*_1$. Moreover, it equals $v^b m^*$ when k is zero, and it converges to infinity as k goes to infinity.

In the spirit of Benabou and Gertner (1993), it is possible to interpret the ratio of p(I,U,k) and v^s as a measure of markup. As in Benabou and Gertner, the model predicts that changes in the variability of inflation (or real shocks in their setting) can either increase or decrease markups. In their setting, the key element is the size of search costs: if search costs are high, higher uncertainty reduces search and increases prices. If search costs are low the opposite happens. In this model, the emphasis is shifted to the magnitude of the uncertainty, and its interaction with informational monopolies: for low levels of variability markups decrease as the value of information increases, when buyers are endowed with information; for high levels of uncertainty, the value of information is lower due to the inefficiencies in pricing and hence higher variance increases prices --the value of the informational monopoly decreases.

The Effect upon the Probability of Trade

As described in Proposition 3.1.3 the probability of trade depends on the size of the variance of the money supply. As before, we study increases in k holding E(M)=m* constant. From Proposition 3.1.3 it follows that for k in the interval $[0,k^*_1]$ the probability of trade is one. In the region $k>k^*_1$ the probability of trade is given by,

$$q(I,U,k)= \Pr[1/M<v^b/p].$$

Given our assumptions that the distribution of 1/M, is U[μ(k)-k, μ(k)+k], a straightforward calculation shows that the probability of trade is,

$$q(I,U,k)= ½ + \xi(k), \tag{3.1.6}$$

where,

$$\xi(k)= (e^{2m^*k}-1)^{-1}[(1-2\epsilon)^{-½} - (½)(1+e^{2m^*k})]. \tag{3.1.7}$$

It is easy to check that $\xi(k^*_1) = ½$, $\lim_{k\to\infty}\xi(k)=-½$, and that ξ(k) is decreasing in k. We summarize these results in the following proposition.

*Proposition 3.1.*5. Let $\epsilon<1/2$. Denote by q(I,U,k) the probability that the indivisible good is traded when an informed buyer is matched with an uninformed seller. Then, q(I,U,k) =1 for $k \leq k^*_1$, and $0<q(I,U,k)<1$ for $k>k^*_1$. Further, q(I,U,k) is decreasing in k and $\lim_{k\to\infty} q(I,U,k)=0$.

Thus, when the variability of the money supply is large relative to the mean, the probability of trade converges to zero. In this case, the high variance of the money supply decreases the expected real value of money balances obtained by the seller. Thus, the relative valuation of the indivisible good in terms of the consumption good decreases. In this case, the seller is willing to part with the good only if he can get enough consumption in return. To accomplish this, the announced price converges to infinity (see Proposition 3.1.4), but this drives the probability of trade to zero.

The Welfare Effects of Variable Inflation

How do changes in the stochastic process governing the money supply affect welfare? At the individual level there are two possible effects: a redistributive effect that results in changes in the real value of the transfer between buyers and sellers whenever there is a transaction, and an efficiency effect that is captured by a probability of trade less than one. Consider the seller first. In the region $k \leq k^*_1$, expected utility is given by,

$$\text{(3.1.8a)} \qquad W^s(I,U,k) = 1-\lambda+v^b(1+e^{2m^*k})/2e^{2m^*k}.$$

For the region $k > k^*_1$, the appropriate expression is,

$$\text{(3.1.8b)} \qquad W^s(I,U,k) = 1-\lambda+(v^b/2(e^{2m^*k}-1))[(1-2\epsilon)^{-1/2}-(1-2\epsilon)^{1/2})]+ [1-q(I,U,k)](1-\epsilon)v^b,$$

where q(I,U,k) is given by (3.1.6). Similarly, it is possible to calculate the expected utility of the buyer. Direct calculations show that in the region $k \leq k^*_1$, the expected payoff to the buyer is,

$$\text{(3.1.9a)} \qquad W^b(I,U,k) = \lambda + v^b[1-(1+e^{2m^*k})/2e^{2m^*k}],$$

while in the region $k > k^*_1$, expected utility is,

$$\text{(3.1.9b)} \qquad W^b(I,U,k)=\lambda-(v^b/2(e^{2m^*k}-1))[(1-2\epsilon)^{-1/2}-(1-2\epsilon)^{1/2})] + q(I,U,k)v^b.$$

There are two interesting features. First, in the region $k \leq k^*_1$, the utility of the seller decreases as k increases, while the utility of the buyer increases. The reason for this is --as described before-- the incentives that sellers have to lower their prices in order to secure a sale, in a region in which selling strictly dominates. (In this region, the probability of trade q(I,U,k) equals one.) Thus, in this region, increases in the coefficient of variation of the money supply process result in a redistribution of income from sellers to buyers by reducing the former's monopoly power.

In the region $k > k^*_1$ the situation is more complex. Each of the two welfare functions involve the term

$(v^b/2(e^{2m^*k}-1))[(1-2\epsilon)^{-½}-(1-2\epsilon)^{½})]$,

as well as a term containing the probability of trade. The interpretation of this term is simple. It gives the value to the seller of transferring the good --conditional on a sale occurring. This term enters positively in the sellers expected utility and negatively in the buyers utility. In addition, each of the two parties derives utility from consuming the indivisible good. It is easy to check that $(v^b/2(e^{2m^*k}-1))[(1-2\epsilon)^{-½}-(1-2\epsilon)^{½})]$ is decreasing in k. Thus, as k increases the seller's expected surplus --even conditional on selling the good-- decreases. On the other hand, the value of not selling increases, as the probability of trade decreases. The opposite holds for the buyer.

What happens to total welfare? Note that in this world of linear utility functions, welfare is equivalent to expected income measured in units of the divisible good. Let aggregate welfare (or expected income) be given by $W=W^s+W^b$. It is straightforward to calculate that in the region $k \leq k_1^*$ expected income is given by,

(3.1.10a) $$W(I,U,k)= 1+v^b.$$

Note that this is maximal, the outcome here is ex post efficient. In the region $k > k_1^*$, expected income is,

(3.1.10b) $$W(I,U,k)= 1+v^b[q(I,U,k) + (1-\epsilon)(1-q(I,U,k))].$$

It follows that changes in k do not affect aggregate income in the region $k \leq k_1^*$, but they have redistributive consequences. On the other hand, in the region $k > k_1^*$, increases in k decrease the probability of trade which, in our setting, implies inefficient outcomes. Throughout this region, real money balances are equal to one (when deflating by the price of consumption which we make equal to M), thus conventional calculations of the welfare loss associated with inflation --the area under the demand curve-- would show no impact. Finally, the difference between the ex-post efficient level of aggregate income --$1+v^b$-- and the expected level of income as given by (3.1.10b) is a measure of the expected loss from inflation. This difference is $(v^b-v^s)(1-q(I,U,k))$, which is just the "premium" that buyers put on the indivisible good times the probability of not trading. Thus, as k increases and q(I,U,k) converges to 0, the size of the expected welfare (and income) loss goes to v^b-v^s. We summarize the results in the following proposition.

Proposition 3.1.6. Let $\epsilon<1/2$. Let $W^j(I,U,k)$ denote the expected welfare (and expected income) of agent j, j=b,s; and let W(I,U,k) be given by $W^b(I,U,k)+W^s(I,U,k)$. Then,

i) $W^s(I,U,k)$ is decreasing for all values of k.

ii) $W^b(I,U,k)$ is increasing for $k \leq k_1^*$, and decreasing for $k \geq k_1^*$.

iii) W(I,U,k) is constant and equal to its maximal value $1+v^b$ for $k \leq k_1^*$. For $k \geq k_1^*$, W(I,U,k) is given by $1+v^b[q(I,U,k) + (1-\epsilon)(1-q(I,U,k))]$ which is a decreasing function of k which converges to $1+v^b(1-\epsilon)$ as k goes to infinity.

3.2 Asymmetric Information: Informed Sellers and Uninformed Buyers

Finally, we study the bargaining problem between an uninformed buyer and an informed seller, (U,I). In this case the seller knows M and can announce prices which are contingent on M. More precisely, we will analyze perfect Bayesian equilibria of the game in which seller's are first informed about the value of M, second, sellers announce a price and finally, buyers, having seen the price, but not M, decide whether or not to buy the object. A perfect Bayesian equilibrium is then a choice of price by the seller for every outcome of M, p(M), an accept/reject rule for the buyer given the price (but not M), X(p)= 0 or 1, and a set of beliefs about the true state of nature for the buyer given that the seller has offered the price p(M). Of course, X(p) must maximize the buyer's utility given his beliefs and the price rule, p(M), must (for each M) maximize the sellers utility. Finally, the beliefs of the buyer must be credible.

As a first step, it can be shown that the neutrality result, Proposition 3.1.2, has a direct analogue in this case. Specifically, if M=m* with probability one, the good is exchanged with probability one, and prices are proportional to m*. Further, both the realized levels of utility and the probability of trade are invariant to money supply processes that are positive multiples of each other. Prices simply change by the magnitude of the positive multiple.

To analyze the model, first consider the problem of a buyer. Since we will only be interested in pure strategy equilibria, a buyer's strategy can equally well be summarized by an 'acceptance set.' Accordingly, let A be the set of prices for which the buyer will accept and trade will occur. For simplicity, we will assume that the buyer can only choose among A's which contain their supremum. This is a restriction on the strategy space of the buyer. Note further that given any acceptance set, A, the only relevant price from this set is its supremum, sup A, since if the seller wants to sell the object to the buyer given what he knows about M, it is always in his interest to charge the maximal acceptable price. If he doesn't want to sell to the buyer, he can charge any price above sup A, but p^U is a natural candidate. Given this, we can equivalently represent the problem of the buyer as choosing a reservation price, which we will denote by p^r.

To characterize equilibrium in this setting, we will first treat p^r parametrically and impose rationality on the part of the buyer below.

Given a level of p^r, we can characterize the optimal behavior of the seller. If the seller chooses the price p^r, he sells the object and hence receives utility $1-\lambda + p^r/M$, while if he charges any price higher than p^r, he does not sell the object and hence receives utility $1-\lambda + v^s$. Thus, he will charge the price p^r if and only if $M \le \min(m^U, p^r/v^s) \equiv M^*$. If , on the other hand, $M>M^*$, he will charge some price higher than p^r. Without loss of generality, we will assume that he charges the highest possible price, p^U, in this case.

This description of the optimal behavior of the seller given any reservation price rule determines the beliefs of the buyer using Bayes Rule. If the buyer sees the price p^r, the conditional distribution on 1/M given $p=p^r$ is uniform on the interval [1/M*, μ+k], while if the buyer sees the price p^U, the conditional distribution on 1/M given $p=p^U$ is uniform on the interval [μ-k,1/M*]. As is standard in these models, beliefs are not pinned down for any other values of p without making further assumptions.

Given these restrictions on beliefs, we will now derive the restrictions on buyers' behavior that results from optimal decision making on his part. First, the utility received by the buyer if he buys and the price is p^r is given by:

$$U = \lambda + \int_{m^L}^{M*} [v_b - \frac{p^r}{M}] dF_{M|p^r} = \lambda + v_b - \frac{p^r}{2}\left(\frac{1}{M*} + \frac{1}{m^L}\right).$$

On the other hand, if he does not buy, his utility is λ. Thus, the restrictions imposed by rationality by the buyer reduces to two cases. The first is if $p^r/v^s \le m^U$. In this case, optimality of the buyers decision rule can be simplified to:

$$p^r \le m^L v^b (1+\epsilon).$$

It is straightforward to check that these strategies (with the price of the seller equal to p^U whenever M>M*) do in fact constitute an equilibrium as long as $p^r \le \min(m^L v^b (1+\epsilon), m^U v^s)$.

Notice that $\min(m^L v^b (1+\epsilon), m^U v^s) = m^L v^b (1+\epsilon)$ if and only if $k \ge \epsilon\mu$, and $\min(m^L v^b (1+\epsilon), m^U v^s) = m^U v^s$ if and only if $k \le \epsilon\mu$. Thus, if $k \le \epsilon\mu$, there are equilibria of this type with p^r up to $m^U v^s$. The relevance of this is that if $p^r = m^U v^s$, the probability of trade is equal to one and the outcome is efficient.

On the other hand, if $p^r \ge m^U v^s$, note that this implies a probability of trade equal to one given optimal behavior on the seller's part and, optimality on the buyer's part reduces to $p^r \le v^b \mu$. These two (i.e., $p^r \ge m^U v^s$ and $p^r \le v^b \mu$) are mutually consistent if and only if $k \le \epsilon\mu$.

Among these equilibria, we will concentrate on the one that maximizes the utility of the seller. An alternative selection criteria would be to choose the equilibria that maximize the probability of trade. When $k \le \epsilon\mu$ our selected equilibrium is one of many that results in a probability of trade equal to one. When $k > \epsilon\mu$, our selected mechanism is the only one that makes the probability of trade maximal. Thus, there are no conflicts between these two alternative criteria. More precisely, our criterion selects the equilibrium in which p^r is at its maximal level. This is given by $p^r = v^b \mu$ when $k \le \epsilon\mu$ and $p^r = m^L v^b (1+\epsilon)$ when $k > \epsilon\mu$. Since we are interested in studying the limits to efficiency we will only study this is equilibrium.

In this equilibrium, when $k > \epsilon\mu$, trade occurs whenever $m^L \le M \le M^* = m^L(1+\epsilon)/(1-\epsilon)$. Thus, the probability of trade is given by:

$$P\left(m^L \leq M \leq m^L \frac{(1+\epsilon)}{(1-\epsilon)} \right) = P\left(\frac{(1-\epsilon)}{m^L(1+\epsilon)} \leq 1/M \leq 1/m^L \right) = P\left((\mu+k)\frac{(1-\epsilon)}{(1+\epsilon)} \leq 1/M \leq \mu+k \right) = \frac{\epsilon(\mu+k)}{(1+\epsilon)k}.$$

When $k\to\infty$ with $\mu=\mu(k)$, it follows that this probability of trade converges to $2\,\epsilon/(1+\epsilon)$. Note that this is strictly less than one (and strictly positive) whenever $\epsilon < 1$ ($\epsilon > 0$). In the case that $k \leq \epsilon\, \mu$, this maximum probability of trade is given by one. Since μ and k are not independent, it is convenient to describe the two regions in terms of a cutoff level of k. Let k_2^* be the level of k such that $\mu(k)=\epsilon^{-1}k$. It is easy to check that,

$$k_2^* = \ln[(1+\epsilon)/(1-\epsilon)]/2m^*.$$

It also follows that $k \leq \epsilon\,\mu$, if and only if $k \leq k_2^*$; and $k > \epsilon\,\mu$, if and only if $k > k_2^*$. We summarize this discussion in the following proposition:

Proposition 3.2.1. In the case of an uninformed buyer meeting an informed seller, (U,I), the equilibrium outcome is characterized by the following:

(i) If $k \leq k_2^*$, there are a continuum of equilibria indexed by the buyer's reservation price, p^r where $p^r \leq v^b/\mu(k)$. These equilibria have a probability of trade q(U,I,k) equal to one for $m^U v^s \leq p^r \leq v^b/\mu(k)$. The equilibrium welfare of the agents (in the equilibrium that maximizes the welfare of the seller) is given by:

$W^b(U,I,k) = \lambda$, and,
$W^s(U,I,k) = 1-\lambda + v^b$.

(ii) If $k > k_2^*$, there are a continuum of equilibria indexed by the buyer's reservation price, p^r where $p^r \leq m^L v^b (1+\epsilon)$. The indivisible good will be traded with probability $q(U,I,k) = [\epsilon/(1+\epsilon)][2e^{2m^*k}/(e^{2m^*k}-1)] < 1$. Trade occurs when $M \leq m^L (1+\epsilon)/(1-\epsilon)$, i.e., when M is low. The equilibrium welfare of the agents (in the equilibrium that maximizes the welfare of the seller) is given by:

$W^b(U,I,k) = \lambda$, and,
$W^s(U,I,k) = 1-\lambda + v^b [1- \epsilon + \epsilon\, q(U,I,k)]$.

(iii) As $k\to\infty$, the maximum probability of trade converges to $2\,\epsilon/(1+\epsilon)$.

Here, since the seller has a monopoly on market power (by virtue of moving first in the bargaining game), and information, the buyer is pushed to his reservation utility for all values of k. Note that in this informational setting, the Phillip’s curve goes the opposite way from the previous case. That is, here trade occurs when M is small. Finally, note that the equilibrium price in this setting does not depend on M, even though it could (i.e., the seller knows M).

4. Endogenous Information Structures

So far we have taken the information structure of the buyer-seller pairs as given. However, as the discussion in the previous section show there are policies (high k policies) for which the private gains from being informed are potentially very large. A natural extension of the analysis is to allow for the endogenous choice of the information structure. To that effect we study a two stage game. In the first stage, individuals choose whether to become informed or not. This decision is made simultaneously by all players (buyers and sellers). In the second stage --given the information structure-- the buyers and sellers are randomly matched, and they engage in the bargaining problem described in the previous two sections. We assume that in the second stage the information structure is common knowledge; in other words, both parties to a match know whether the other is informed or not. Thus, the equilibrium strategies for the second stage are those described in sections 2 and 3. It is clear that a second stage equilibrium always exists and is unique. In this section, we study the equilibrium of a one period game in information strategies that has as its payoffs the expected utilities that were calculated in the previous section. The notion of equilibrium that we use is Nash equilibrium.

We assume that the cost of becoming informed is given by c (measured in units of the divisible consumption good). This cost of acquiring information can be interpreted broadly as encompassing private time and expenditure costs which can be affected by the government. As an example, secrecy rules associated with the meetings of the FOMC have helped create a sizable industry of Fed watchers who try to estimate --given the fragmentary information available-- the Fed's actual policy. If the Fed plainly announced in unequivocal terms its intentions and policies it is possible to imagine that this industry would greatly shrink. This is an example in which c would correspond to the cost of purchasing information from Fed watchers. Alternatively, many accounts of hyperinflations emphasize that individuals spend a great deal of time trying to determine the real value of alternative transactions. This, in our model, corresponds to finding the actual value of M, since this is all that is required to evaluate a proposed transaction.

From the point of view of the model we need to specify the payoffs from being informed and uninformed. We will consider only symmetric equilibria in pure strategies extended to the natural asymmetric situation when the unique equilibrium is in mixed strategies.

We first, consider the first stage payoffs. They are given by,

Table I: Payoffs in the First Stage (ϵ<1/2)

Seller Buyer	Uninformed (U)	Informed (I)
Uninformed (U)	$1-\lambda + v^b$ λ	$1-\lambda + v^b\Upsilon^s$ - c λ
Informed (I)	$1-\lambda + v^b\Upsilon^s$ $\lambda + v^b\Upsilon^b$ - c	$1-\lambda + v^b$ -c λ-c

The functions Υ^j j= b,s are defined in Appendix B. To make the information acquisition problem non-trivial we assume that the cost of information is less than the value of the gains from trading the indivisible good. Thus, we assume $c<\epsilon v^b$. If $c\geq\epsilon v^b$, it follows that the unique Nash equilibrium is (U,U), independent of the level of volatility of M.

Our main characterization result is:

Theorem 4.1. Let ϵ<1/2. For all values of k a Nash equilibrium exists. If $c\geq\epsilon v^b$, the unique first stage equilibrium strategies are (U,U). (As before we first denote the buyers' strategy.) If $c<\epsilon v^b$, there exists values of k denoted $k_1(c)$ and $k_2(c)$ such that,

i) $k_1(c) < k^*_1 < k_2(c)$, $k_1(c)$ is decreasing in c and converges to 0 as c goes to zero; $k_2(c)$ is increasing in c and converges to ∞ as c goes to zero.

ii) For $k \in [0,k_1(c))$, the unique symmetric Nash equilibrium strategies are (U,U).

iii) For $k = k_1(c)$ there are two pure strategy Nash equilibria. The Nash equilibrium strategies are given by (U,U) and (I,U). In addition, there are infinitely many mixed strategy Nash equilibria. The equilibrium strategies are given by any mixture over {I,U} on the part of the buyer, and the mixture that puts mass one on U on the part of the seller.

iv) For $k \in (k_1(c),k_2(c))$ there are no pure strategy Nash equilibria. There exists a unique mixed strategy equilibrium, and the equilibrium strategies are characterized by the following probabilities that buyers (π^b) and sellers (π^s) are informed (i.e. they choose I),

$$\pi^b(k) = [(c/v^b)+(1-\Upsilon^s(U,I,k))]/[(1-\Upsilon^s(I,U,k)) +(1-\Upsilon^s(U,I,k))].$$
$$\pi^s(k) = [v^b\Upsilon^b(I,U,k)-c]/v^b\Upsilon^b(I,U,k).$$

v) For $k \geq k_2(c)$ there is a unique Nash equilibrium. The equilibrium strategies are (U,U)

Proof: See Appendix C.

In the region in which monetary policy is not too volatile ($k < k_1(c)$), even an informed seller who could extract all the surplus would choose to be uninformed. The "lost" surplus is not too large. Similarly, in this region, the cost of information exceeds the gains to a potential buyer even if he is assured to meet an uninformed seller (the only case in which he is able to extract part of the surplus). It follows that neither buyers nor sellers will choose to become informed. In this region information is "intrinsically" not very valuable; that is, its low value depends on being close to certainty, and not on the players' strategies. At the point $k=k_1(c)$, buyers are indifferent between acquiring information and remaining uniformed. At this point, the gains from being informed (which accrue if they meet an uninformed seller) just offset the cost. Thus there are two pure strategy Nash equilibria and a continuum of mixed strategy equilibria.

For higher values of k --more specifically for $k_1(c)<k<k_2(c)$-- there are no pure strategy equilibria. At the lower end of this region (low volatility) a large fraction of the buyers are informed (how large depends on k_2^*; if $k_2^*>k_1(c)$ large is 100%), since the gains from being informed when meeting an uninformed seller exceed the cost. On the other hand, almost none of the sellers is informed. The reason is that, for k close to $k_1(c)$, sellers do not stand to gain at all from acquiring information: the potential gain is equal to the cost since they are going to meet informed buyers with probability one. In general, as k increases, the fraction of buyers who are informed decreases. The fraction of informed sellers increases up to the point $k=k_1^*$, and then decreases. To understand the increase in the region $k_1(c)<k<k_1^*$, note that in this region all pairs of buyers and sellers, except possibly (this depends on the value of k_2^*) for the case of informed sellers meeting uninformed buyers, trade the indivisible good with probability one. Increases in k in this sub-region redistribute income from uninformed sellers to informed buyers. Since sellers get all the "realized" surplus (i.e. buyers get zero in this region, but again depending on k_2^* trade will not occur with probability one and hence, there will be some lost surplus) whenever they are informed, the incentives that they have to limit the amount of redistribution lie at the heart of their decision to become informed. In this sub-region, the bigger the value of k the larger the amount redistributed when they meet an informed buyer and, hence, the larger the number of informed sellers. In the sub-region defined by $k_1^*< k <k_2(c)$, the gains from being informed are smaller because both when an informed buyer meets an uninformed seller, and an uninformed buyer meets an informed seller, trade will not always occur. In fact, the probability of trade for a pair (I,U) decreases with k, and so do the benefits from becoming informed.

For values of k exceeding $k_2(c)$, the private gains from becoming informed are too small, and the unique Nash equilibrium is (U,U). The reason for this is however quite different from the low k case. In this region there are two cases. First, the potential gain to a buyer from being informed for a given fixed strategy on the part of uninformed sellers is increasing in k. The problem is that uninformed sellers choose increasingly higher prices (we are in the increasing region of the price function in Figure 1) and, thus, reduce the probability of trade. Thus, it is because of the equilibrium response of sellers that buyers find it optimal not to pay for information. In some sense this is an example in which two wrongs produce a right. The first wrong corresponds to a policy that increases uncertainty. The second can be associated with the sellers desire to maintain their monopolist position and the consequent high prices that they charge. In the case of an informed seller meeting an uninformed buyer, it is clear that buyers have no incentive to acquire information (they get no surplus in that case) and, hence, sellers do not choose to become informed.

The distribution of information --in the case of intermediate levels of variability of the money supply-- is not degenerate. Moreover, the region in which some agents actively acquire information increases as the cost of acquiring information decreases.

5. Comparative Statics with Endogenous Information

The previous section established the existence of an equilibrium. It turns out that to characterize the equilibrium it is necessary to know that value of k_2^* relative to the interval $(k_1(c),k_2(c))$. The case in which the probability of trade is highest corresponds to a high k_2^* (see Proposition 3.2.1), and this is the case that we discuss in this section. It turns out that depending on the value of ϵ, k_2^* can lie on either side of k_1^*. For ϵ small, one can show that $k_2^* > k_1^*$. In all cases, $k_2^* < k_2(c)$. In this section we find it useful to make an assumption about the joint values of ϵ and the costs of acquiring information (which, of course, we assume to be less than ϵv^b, so that there is a mixed strategy equilibrium).

Define the set A as follows,

$$A \equiv \{(c/(\epsilon v^b), \epsilon) \in (0,1) \times (0,1/2) \mid (1+\epsilon) \geq (1-\epsilon)(1-2\epsilon)^{-1/2} \text{ and } [(c/\epsilon v^b)(1-\epsilon)+\epsilon(1+\epsilon)] \leq [(c/\epsilon v^b)(1+\epsilon)+1-\epsilon][1-(1-\epsilon)^{1/2}]\}.$$

The first inequality guarantees that $k_2^* > k_1^*$, while the second implies that $\pi^b(k)$ is decreasing in all its range. It is easy to check that the set A is nonempty. In particular, small values of ϵ and relatively large values of $c/(\epsilon v^b)$ are in A. For the set of economies satisfying A, we fully explore the impact upon the equilibrium behavior of changes in the variability of the monetary policy process as measured by k.

The Distribution of Information

How does policy uncertainty affect the equilibrium amount of information? In the context of this model, an appropriate measure is given by the total fraction of the population that is informed. In addition to its impact on the overall fraction of people informed, changes in k affect the identity of those informed. Given Theorem 4.1 it is clear that no agents are informed if $k \in [0,k_1(c)) \cup [k_2(c), \infty)$. Thus, we restrict attention to the set $[k_1(c),k_2(c))$. The following proposition summarizes our results.

Proposition 5.1 Assume that $(c/(\epsilon v^b), \epsilon) \in A$. Then the distribution of information satisfies

i) Aggregate information $(\pi^b(k)+\pi^s(k))/2$:

a) $(\pi^b(k)+\pi^s(k))/2 \in [0,1/2]$ for $k=k_1(c)$,
b) $(\pi^b(k)+\pi^s(k))/2 = ½$ for $k_1(c) \leq k \leq k_1^*$,
c) $(\pi^b(k)+\pi^s(k))/2$ is less than 1/2, for $k_1^*<k<k_2(c)$. Moreover, $(\pi^b(k)+\pi^s(k))/2$ is decreasing in this region.

ii) Individual information:

a) $\pi^b(k_1(c))=1$, $0<\pi^b(k_2(c))<1$, and $\pi^b(k)$ is decreasing in k,
b) $\pi^s(k_1(c)) = \pi^s(k_2(c)) = 0$. The function $\pi^s(k)$ is increasing for $k<k_1^*$ and decreasing for $k>k_1^*$.

Proof:

It follows from the results in Theorem 4.1 once one notes that for $k\leq k_2^*$, $1=\Upsilon^s(U,I,k)$ and an informed seller who meets an uninformed buyer gets all the surplus since they exchange the good with probability one. In this region $\pi^b(k) = c/[v^b(1-\Upsilon^s(I,U,k))]$. Moreover, for $k<k_1^*$, $1=\Upsilon^s(I,U,k) +\Upsilon^b(I,U,k)$. The monotonicity results follow from the properties of the $\Upsilon^j(I,U,k)$ functions derived in Appendix B. For $k \geq k_2^*$, direct calculations show that the second inequality in the definition of the set A implies that $\pi^b(k)$ is decreasing

■

The behavior of the different measures of information is illustrated in Figure 2. First, note that the total fraction of informed agents is a non-monotonic function of our measure of variability --the variance of the money supply. At low levels of k it is zero; it jumps to ½ of the population for moderate k's and it remains constant up to the point in which the equilibrium strategies for the pair (Informed buyer, Uninformed seller) call for trading with probability less than one. At this point --which corresponds to k_1^*-- the fraction of informed individuals decreases. It is zero for large k's.

The distribution of information displays substantial changes as a function of k. For low k's in the range in which it pays to acquire information, the fraction of informed buyers is large (it is 100% at $k_1(c)$) and the fraction of informed sellers is low (it is 0% at $k_1(c)$). In this region of low k's --more precisely in the interval $[k_1(c),k_1^*]$-- the indivisible good is traded with probability one. The buyers have a huge incentive to acquire information because --whenever they are informed-- they can get better deals from the seller (who wants to charge a low price to guarantee a sale). No such a deal is given to uninformed buyers. As k increases, the value of the transfer increases. This makes sellers more willing to invest in information (informed sellers capture all the surplus), and buyers less willing to do so (they only obtain a surplus when they meet an uninformed seller, and the fraction of uninformed sellers is decreasing). This relative behavior continues over the range in which the good is traded with probability one, and the only issue is the allocation of the surplus between the two parties. For larger values of k in the region in which information is acquired -- formally in the interval $[k^*_1, k_2(c))$-- the indivisible good is not traded all the time whenever an informed buyer meets an uninformed seller. Thus, from the point of view of the buyer, the expected returns from being informed decrease and, consequently, the fraction of informed buyers decreases. Since the threat of informed buyers is decreasing, and the gain from selling to an informed buyer is decreasing as well, the sellers' best response is to reduce their demand for information. Thus, the fraction of both sellers and buyers that acquires information declines. In the region $[k^*_2, k_2(c))$, even the uninformed buyer who meets an informed seller fails to trade with probability one.

For high k's the equilibrium is such that no agent finds it advantageous to acquire information. This is probably the consequence of our static model. In a dynamic setting one can think of the money supply at t as given by $M_t = M_0 + \sum_{j=0}^{t} x_j$. In such a setting, the relevant variable is the last piece of information. Note that the variance of the prior distribution of M_t increases with the length of the period in which the agents have chosen not to acquire information. Thus, it is possible that in such a model individuals would acquire information after a certain number of periods. In our static framework being uninformed means being equally uninformed. In a dynamic framework, the "age" of the last piece of information is relevant to determine the "degree" of information.

The Effects on Income and Welfare

What happens to welfare and income as k changes? In this setting it seems natural to describe welfare as given by the function W --as defined in sections 2 and 3-- minus the costs of acquiring information. If the costs of acquiring information are just waste of resources, this measure corresponds to real income as well. In the region in which the equilibrium is in mixed strategies we use the probabilities assigned to each information option to construct the probability of a meeting with information set (i,j). Since we assume a large number of buyers and sellers, these probabilities are also fractions of the population. We then have the following characterization of real income.

Proposition 5.2 Assume that $(c/(\epsilon v^b), \epsilon) \in A$. Let welfare (and real income) be defined by $I(k) = W(k) - c[\pi^b(k)+\pi^s(k)]$. Then,

i)	$I(k)=1+v^b$,	for $k<k_1(c)$,
ii)	$I(k) \in [1+v^b, 1+v^b-c]$,	for $k=k_1(c)$,
iii)	$I(k)=1+v^b-c$,	for $k_1(c)<k\leq k^*_2$,
iv)	$I(k)= 1+v^b-c - a(k)$, $a(k)>0$,	for $k^*_2 \leq k < k_2(c)$
v)	$I(k)=1+v^b$,	for $k>k_2(c)$.

Proof: In case i), the unique equilibrium is (U,U). Thus there are no costs of acquiring information. Since the pair (U,U) trades the indivisible good with probability one, real income is maximized. Case ii) corresponds to case ii) in the statement of Theorem 4.1. If the equilibrium is (U,U), there are no income losses. On the hand, if all buyers choose to be informed, real income is $1+v^b-c$. Since any mixed strategy equilibrium in which the buyers randomize between these two strategies is an equilibrium as well, any value of real income in $[1+v^b,1+v^b-c]$ can be an equilibrium outcome.

To prove iii) it is useful to separately prove the results for the subsets $(k_1(c),k_1^*]$, $[k_1^*, k_2^*]$, and $(k_2^*, k_2(c))$. First, consider the subset $(k_1(c),k_1^*]$. It is necessary to compute $\pi^b(k)+\pi^s(k)$. From Proposition 5.1, it can be verified that $\pi^b(k)+\pi^s(k)=1$. The claim then follows from the observation that in the region $k\leq k_1^*$, and for all pairs of information, the probability of trade q(I,U,k) equals one. Next, we consider the region $[k_1^*, k_2^*]$. Here the situation is more complicated. A fraction $\pi^b(1-\pi^s)$ of all pairs have information set (I,U). Income --ignoring information costs for now-- in this case is given by Proposition 3.3 and equals $1+v^b(1-\epsilon(1-q))$. In the other three cases --which, of course, occur with probability $1-\pi^b(1-\pi^s)$-- income is just $1+v^b$. Thus, average income is,

$$I(k)=1+v^b-c+c(1-\pi^b-\pi^s)-v^b\epsilon(1-q)\,\pi^b(1-\pi^s).$$

We will show that $c(1-\pi^b-\pi^s)=v^b\epsilon(1-q(I,U,k))\,\pi^b(1-\pi^s)$.To do this we calculate $\pi^b+\pi^s$ for this subset. Using the results in Theorem 4.1 and Appendix B, it follows that $1-\Upsilon^s(I,U,k)=\Upsilon^b(I,U,k)\epsilon(1-q)$. Thus, $\pi^b=c/[v^b(1-\Upsilon^s(I,U,k))]$, and $1-\pi^s=c/(v^b\Upsilon^b(I,U,k))$. Using these values it follows that $c(1-\pi^b-\pi^s)=v^b\epsilon(1-q(I,U,k))\,\pi^b(1-\pi^s)$ if and only if $(1/\Upsilon^b(I,U,k))-(1/(1-\Upsilon^s(I,U,k)))=\epsilon(1-q(I,U,k))/[\Upsilon^b(I,U,k)(1-\Upsilon^s(I,U,k))]$. This holds since $1-\Upsilon^s(I,U,k)=\Upsilon^b(I,U,k)\epsilon(1-q(I,U,k))$.

Next consider the region $[k_2^*, k_2(c))$. It is useful to define $a\equiv 1-q(U,I,k) \equiv \epsilon[1-\Upsilon^s(U,I,k)]$. Define $\pi^b(k,a)$ by,

$$\pi^b(k,a) = [(c/v^b) + \epsilon a]/[1-\Upsilon^s(I,U,k) + \epsilon a].$$

Note that $\pi^b(k,0)$ corresponds to the fraction of informed buyers in the region $(k_1(c),k_2^*]$. It is easy to check that, even in the region $[k_2^*,k_2(c))$, $c(1-\pi^b-\pi^s)=v^b\epsilon(1-q(I,U,k))\,\pi^b(1-\pi^s)$. With this result, it is possible to show that,

$$I(k)=1+v^b-c - \epsilon v^b\{(\pi^b(k,a) - \pi^b(k,0))((c/(\epsilon v^b) + (1-\pi^s)(1-q(I,U,k)) + (1-\pi^b(k,a))\pi^s a\}.$$

Since in this region a>0 (the reader can check that a=0 in $(k_1(c), k_2^*]$), and $\pi^b(k,a)$ is increasing in a, the term in set brackets is strictly positive. If we define the function a(k) to equal the expression in set brackets (of course, a is, itself, a function of k) this completes the proof of part iv).

Finally, note that in the region corresponding to v) the unique Nash equilibrium is (U,U). Thus, the same reasoning as in i) establishes the desired result.

■

Note: As the reader can verify, the proof of this result uses only the assumption that $k_2^* > k_1^*$, and, hence, the assumption that $(c/(\epsilon v^b), \epsilon) \in A$ can be weakened.

To summarize, inflation variability has a negative impact upon income, which is given by the costs of acquiring (essentially useless) information. If these costs are significant so is the welfare loss. However, it is not clear that these costs are not components of measured GDP. To the extent that they capture time that could be used in production they reduce measured income. However, it is possible to broadly interpret these costs as including payments to firms specialized in evaluating economic perspectives and costs associated with the operation of markets that are "created" by variable policies. If the correct interpretation is the latter, then they do not subtract from conventionally measured GDP. This is, of course, another instance in which GDP is not an accurate measure of welfare. To capture the effect of policy variability on conventional GDP in this case, we construct an alternative measure of income which ignores the cost of acquiring information. Let this measure be given by $Y(k)=I(k) + c[\pi^b(k)+\pi^s(k)]$. It is immediate to calculate Y(k) given the previous result. It is given by,

$$Y(k) \begin{cases} 1+v^b, & \text{for } k \leq k_1^* \\ 1+v^b-v^b\epsilon[(1-q(I,U,k))\pi^b(k)(1-\pi^s(k))] & \text{for } k_1^* \leq k \leq k_2^* \\ 1+v^b-v^b\epsilon[(1-q(I,U,k))\pi^b(k)(1-\pi^s(k)) + (1-q(U,I,k))(1-\pi^b(k))\pi^s(k) & \text{for } k_2^* \leq k < k_2(c) \\ 1+v^b & \text{for } k_2(c) \leq k. \end{cases}$$

Note that, using this definition, the variability of the money supply has no effect up to k_1^* and then the appropriate measure of loss of output is the difference in the valuation of the indivisible good between buyers and sellers ($v^b\epsilon$) multiplied by the probability of not trading (1-q(k)), and weighted by the frequency of this pair. In addition, for $k \geq k_2^*$, there is another source of welfare loss since the pair (U,I) also trades with probability less than one.

Figure 3 displays the behavior of both measures of income as a function of k.

The Behavior of Prices

How do prices behave? For small values of k, prices of the indivisible good are unresponsive to monetary shocks and identical in a cross sectional sample of pairs of buyers and sellers. Both buyers and sellers choose to be uninformed when $k < k_1(c)$. As pointed out in section 3, prices decrease with increases in the variance of the money supply. In this region the ratio of the standard deviation of prices and money supply $\sigma_p(k)/\sigma_M(k)$ is zero: prices are unresponsive to money supply shocks.

For values of k in the interval $[k_1(c),k_2(c))$, the equilibrium is in mixed strategies. In this region, each information pair trades at different prices. Thus, the model predicts that the cross-sectional variability of prices is positive. This agrees with the findings by Eden (1994) and the studies collected by Sheshinski and Weiss (1993) in which more variable inflation is associated with more variability of intra period prices. The distribution of prices has the same qualitative characteristics for all values of k in the mixed strategy region. However, the actual prices charged by the informed seller who meets an uninformed buyer (I,U) depend on the location of k relative to k_2^*. To illustrate our point we consider the case $k \geq k_2^*$. The distribution of prices across information pairs (and, given our assumptions, population of traders) is given by,

(I,U) $v^b m^* \max\{(e^{2m^*k}-1)/(2m^*ke^{2m^*k}), (1-2\epsilon)^{1/2} (e^{2m^*k}-1)/2m^*k\}$, with probability $\pi^b(1-\pi^s)$.

(U,U) $v^b m^* (e^{2m^*k}-1)/[m^*k(1+e^{2m^*k})]$ with probability $(1-\pi^b)(1-\pi^s)$.

(U,I) $v^b m^* (e^{2m^*k}-1)/(2m^*ke^{2m^*k})$ with probability $(1-\pi^b)\,\pi^s$.

(I,I) $v^b M$ with probability $\pi^b\,\pi^s$.

Given k and the realization of M, the average price of the indivisible good over all the trading pairs (of course, we assume that there are a large number of buyers and sellers so that probabilities equal fractions of the population) is of the form,

$$\bar{p}(M;k)=\alpha_0 + \alpha_1 M,$$

where $\alpha_1 = v^b\pi^b\pi^s$. Thus, prices in this region are sluggish in the sense that they are less responsive to monetary shocks than what would be predicted by a full information flexible price model ($p=v^bM$). The ratio $\sigma_p(k)/\sigma_M(k)$ is $v^b\pi^b\pi^s$. This measure of relative standard deviation displays an inverted U-shape: it is 0 at both $k=k_1(c)$ and $k=k_2(c)$, it increases up to the point $k=k_1^*$; and it is decreasing in the region $(k_1^*,k_2(c))$.

Given the realization of M, let $\sigma_p^2(M,k)$ be the cross sectional variance of prices (again taken across all pairs of traders). Then simple calculations show that $\partial\sigma_p^2(M,k)/\partial M \gtrless 0$ depending on whether $M \gtrless \hat{M}(k)$. Thus for large values of M (large inflation relative to the average) the model predicts a higher level of price variability, while for small realizations of the money supply it predicts a decrease in variability.

Even though the model has implications about the relationship between variability of the money supply process and the cross sectional variance of prices most of the available evidence (see Eden (1994)) refers to cross sectional variances for a given regime. Thus, the empirical results are not directly applicable to our model.

The Equilibrium Phillips Curve

As pointed out in section 3, the effect of an expansive monetary shock --a high value of M-- is to induce an increase in output (measured as units of the indivisible good traded) in the (I,U) case, and a contraction in the (U,I) case. Thus, potentially, the Phillips curve can bend backwards, and the effect of monetary shocks on output are dependent on the value of k. In particular, for $k \leq k_1^*$, all information pairs trade with probability one; thus the Phillips curve is perfectly flat. In the region $[k_1^*, k_2^*]$ only the pair (I,U) fail to trade for some realizations of the monetary shock, and those are low values of M (see Proposition 3.1.3). Thus, in this region the Phillips curve bends "upward." The region $[k_2^*, k_2(c))$ is more interesting because both the (I,U) pairs and the (U,I) pairs may not trade depending on the realization of M. To determine the general equilibrium effects of monetary shocks fix k. Then, the pair (I,U) trades whenever the realization of M exceeds a threshold which we denote $\bar{M}(I,U,k)$. From Proposition 3.1.3, it follows that,

$$\bar{M}(I,U,k) = (1-2\epsilon)^{1/2}(e^{2m^*k}-1)/(2k).$$

On the other hand, the pair (U,I) trades whenever M falls short of $\bar{M}(U,I,k)$, where $\bar{M}(U,I,k)$ is given by,

$$\bar{M}(U,I,k) = [(1+\epsilon)(e^{2m^*k}-1)]/[2k(1-\epsilon)e^{2m^*k}].$$

It turns out that the shape of the relationship between realizations of M (holding k constant) and a measure of output (say, fraction of pairs that trade the indivisible good) depends on the location of the thresholds $\bar{M}(I,U,k)$ and $\bar{M}(U,I,k)$. Simple calculations show that there are two cases which correspond to different regions in k space. Let $g(k) \equiv e^{2m^*k}$. Then, the two cases are (we ignore the "boundary" given by equalities; the reader can check that the obvious behavior obtains)

Case I: $g(k_2^*) < g(k) < g(k_2^*)g(k_1^*)$.

> In this case $\bar{M}(I,U,k) < \bar{M}(U,I,k)$. Thus, in the region $[m^L, \bar{M}(I,U,k)]$ all information pairs but (I,U) trade; output is "moderately" low. In the region $[\bar{M}(I,U,k), \bar{M}(U,I,k)]$ all information pairs trade, and output is high. Finally, in the region $[\bar{M}(U,I,k), m^H]$ the only pair that fails to trade is (U,I); hence, output is again "moderately" low. Overall, the relationship between M and output resembles an inverted-U.

Case II: $g(k_2^*)g(k_1^*) < g(k)$.

> In this case $\bar{M}(I,U,k) > \bar{M}(U,I,k)$. Thus, in the region $[m^L, \bar{M}(U,I,k)]$ all information pairs but (I,U) trade; output is "moderately" low. In the region $[\bar{M}(U,I,k), \bar{M}(I,U,k)]$ neither the (I,U) or the (U,I) information pairs trade, and output is low. Finally, in the region $[\bar{M}(I,U,k), m^H]$ the only pair that fails to trade is (U,I); hence, output is again "moderately" low. Overall, the relationship between M and output has a U-shape.

It is clear that even a simple model like the one in this paper does not give robust predictions about the shape of the aggregate Phillips curve. The results depend on the amount of variability (relative to the mean) and suggest in all cases a non-linear relationship. This suggests that more empirical work --which would allow for nonlinearities as well as regime (i.e. k) changes-- is necessary to assess the relevance of this model to explain the behavior of aggregate output during high inflation.

6. Large Gains from Trade

So far we have concentrated on the case in which the sellers are, in many cases, unwilling to part with the indivisible good because even if they can extract all the surplus from the buyers the gain is not too large. We now look at the other case (formal statements and proofs are in Appendix D). Formally this corresponds to $v^b > v^s/2$ (or $\epsilon \geq 1/2$). The key differences are that when an informed buyer and an uninformed seller meet, the equilibrium outcome is that they always trade. In order to guarantee this, prices must decline with the variance of the money supply.

As in the small ϵ case, there are incentives to become informed because --even if the indivisible good is traded-- the terms of trade depend upon the information structure. It can be shown that when the money supply is not too variable (k is small) neither buyers or sellers choose to become informed. The reason is simple: the "loss" from being uninformed are low when the money supply is not too variable. As the money supply increases, buyers have a large incentive to become informed. More precisely, there is threshold level of k such that, at that point, all the buyers and none of the sellers are informed. Further increases in k result in the proportion of informed buyers declining and the proportion of informed sellers increasing. Unlike the small ϵ case, even for arbitrarily high values of the variability of the money supply there are private incentives to become informed. More formally, the fraction of the population --both buyers and sellers-- who is informed converges to a positive number as k goes to infinity.

The intuition for this is simple: in this case the *potential* gains from trade are so large that even high variability of the money supply is not enough to eliminate the *equilibrium* gains from trade. Since the distribution of these equilibrium gains depends on the information structure, both sides of the market find it privately advantageous to acquire information to affect the distribution of the gains.

There is another difference between this case and the small ϵ case worth noting. In the case of large differences of valuation, the only situation in which a buyer-seller pair fails to trade is whenever an uninformed buyer meets an informed seller. The "problem" (which occurs only for high values of k) is that the informed seller would like to convey accurately the value of M, only when M is low. However, if it announces a low p, the buyer interprets that as evidence of an extremely low value of M and refuses to purchase the good. Thus, in equilibrium, the good is not traded when M is low. Thus, in this case, the Phillips curve bends backwards, as high unexpected shocks do not affect output but low unexpected money supply realizations are contractionary.

Overall, this case highlights that it is possible for the equilibrium value of information not to diminish as the variability of the money supply increases, and that its value is mainly driven by the returns to the informational monopoly.

7. A Dynamic Version

Even though the analysis has been conducted within the framework of a static model, it is possible to embed the one period problem into a fully specified, general equilibrium dynamic model. In this section we discuss the details of such an extension; and we allow the representative family to have an infinite planning horizon. In order to simplify the presentation, we consider the case in which informed buyers meet uninformed sellers. The extension to the case of endogenous information can be done along the same lines, and the arguments are the same, at some notational cost.

In a multiperiod setting, it seems natural to assume that last period's money supply is known to all parties. Thus, the relevant source of uncertainty is not the level of the money supply, but the level relative to last period's level. One simple way of modeling changes in the money supply process and --at the same time-- preserving the results from the static version, is to consider the growth rate of the money supply as the (potentially unknown) random variable. From a formal point of view this requires reinterpreting the variable M not as the level of the money supply at t, but as its growth rate. With this interpretation, all the results in the static case apply to the dynamic case.

It is useful to maintain symmetry by introducing the fiction of a representative family composed of a pair: a buyer and a seller. At the beginning of the period --and before the money supply is realized-- the pair has wealth given by,

$$(7.1) \quad a_t = m_t + R_t b_t,$$

where m_t is nominal balances at the beginning of period t, b_t is the stock of nominally denominated one period bonds, and R_t is the nominal interest rate. In this section we use lower case letters to denote individual levels of a variable, and capital letters will denote economy wide averages. Each household is endowed with e units of the divisible good, and one unit of labor that can be used to produce one unit of the indivisible good. As in Lucas (1980), it is assumed that households do not consume the goods that they produce: they can sell their stock of divisible consumption, and produce and sell their one unit of indivisible consumption, and buy similar goods in the market. Both goods are perishable.

To preserve the notion of decentralized exchange, we assume that at the beginning of the period the pair splits. The buyer first goes to the financial market where, after the monetary shock is realized, he/she decides on the composition of the family's portfolio of money and bonds. Then the buyer goes to the market for the indivisible good where he/she is randomly paired with a seller from another family. The seller member of the pair stays at the family's location and waits for the arrival of a buyer from another family. There is no uncertainty about whether there will be a match: all sellers are paired with one buyer --and only one buyer-- in each time period. After the potential buyer has arrived, the seller makes a price offer and bargaining occurs according to the specification in section 3. After trading in the indivisible good ends, trading in the divisible good opens. The buyer uses the family's cash balances --including whatever cash the seller obtained from selling the indivisible good-- to purchase divisible consumption. The seller sells the endowment of the divisible good at the market price. Cash obtained from the sale of the endowment of the divisible good increases next period's money balances but cannot be used to purchase this period's divisible consumption.

Consider first the buyer side of the family. The buyer first goes to the financial market and receives a transfer from the government, N_t. Thus, after the transfer, wealth is given by,

$$m_t + N_t + R_t b_t.$$

N_t is common to all families and equal to the change in the money supply; it follows that $N_t = M_t - M_{t-1}$, where M_t is the per family money supply at time t. It is useful to consider the stochastic process for the money supply as given by,

$$(7.2) \quad M_t = \gamma_t M_{t-1}.$$

Since M_{t-1} is observed by everybody at the beginning of period t, the only source of uncertainty is γ_t. In order to match the results from the previous sections, assume that γ_t is an i.i.d. random variable, with distribution function F_M, and mean m*. We also assume that the inverse of the growth rate of the money supply, $1/\gamma_t$, has c.d.f given by F, with an everywhere positive density given by f. Thus, in the dynamic version, the key random variable is the growth rate of the money supply, instead of the level.

The informed buyer chooses a portfolio of money and bonds, satisfying,

$$(7.3) \quad m' + b_{t+1} \leq m_t + N_t + R_t b_t.$$

Next the buyer is matched with an uninformed seller, and has to decide whether to accept or reject the offer to buy the indivisible good. At the same time, the seller of the family --without knowledge of the realization of γ_t, picks a price for the indivisible good that it can produce and sell. We assume that the family's utility from consuming leisure that could be used to produce the indivisible good is v^s, and the utility from consuming other family's indivisible good is v^b.

After both members of the family finish their transactions in the market for indivisible good (which, of course, includes the possibility of no trade), they get together and go to the market for the divisible good. At this point everybody knows the current realization of the money supply. The pair again splits: the buyer takes their cash balances (but not their bonds; this is the standard cash in advance assumption) and decides how much to buy of the indivisible good. The seller sells their endowment at the market price. The proceeds from this sale cannot be used to purchase divisible consumption this period, and are used to increase the stock of money that the family brings into the following period.

We next argue that the buyer's and seller's decision rules in this dynamic version of the model, match those derived in section 3.1. To do this we work backwards from the last decision of period t, the simultaneous sale and purchase of the indivisible good.

Let V(m,Rb, M,B,q) be the value function of a family that has levels of money and bonds given by m and Rb, respectively, when the aggregate levels of money (post transfer) and bonds are M and

B, and the price of divisible consumption in terms of money is q. Let r(m,m',Rb,R'b',M,B,q) be the indirect period utility function for a family that behaves optimally, and starts the period with a portfolio given by (m,b) and ends the period with a portfolio given by (m',b'). Using primes to denote next period variables, Bellman's equation is given by,

$$V(m,Rb, M,B,q) = \max \{r(m,Rb,m',R'b',M,B,q) + \beta E[V(m',R'b', M',B',q') | M,B,q]\},$$

where the maximization is over (m',b') and the other choices described above. Given the stationary nature of the problem, the arguments in Stokey and Lucas (1989) show that the solution to the household's dynamic optimization problem is completely described by the stationary solution to Bellman's equation. Even though general arguments can be used to establish existence and uniqueness of the function V that satisfies the above functional equation, in order to characterize the solution to the household's problem, it is useful to explicitly characterize the solution V. To this end, it is possible to show (we will verify this claim later) that the solution V is of the form,

$$(7.3) \quad V(m_{t+1}, R_{t+1}b_{t+1}, M_{t+1}, B_{t+1}) = \hat{V}_0(M_{t+1}) + v_1[(m_{t+1}+R_{t+1}b_{t+1})/q_{t+1}],$$

where $\hat{V}_0$ is an integrable function, and v_1 is a constant. In addition, we hypothesize that the equilibrium price level is given by,

$$(7.4) \quad q_t = M_t e^{-1}.$$

This hypothesis simply anticipates an implication of the cash in advance constraint: the total value of the divisible good is equal to the money supply.

It follows that the expected value of the continuation utility at time t is,

$$E_t\{V(m_{t+1}, R_{t+1}b_{t+1}, M_{t+1}, B_{t+1}\}) = \int \hat{V}_0(\gamma M_t)dF_M + v_1 e[(m_{t+1}+R_{t+1}b_{t+1})/M_t]\int xf(x)dx,$$

or,

$$(7.5) \quad E_t\{V(m_{t+1}, R_{t+1}b_{t+1}, M_{t+1}, B_{t+1}\}) = V_0(M_t) + v_1 e[(m_{t+1}+R_{t+1}b_{t+1})/M_t]\mu,$$

where $\int \hat{V}_0(\gamma M_t)dF_M = V_0(M_t)$.

There are two key features of this value function: first, it is linear in money balances and bonds; second, the effects of the current period money supply --other than deflating nominal wealth-- enter separably, and do not affect the marginal utility of money.

We are now ready to characterize the solution to Bellman's equation and to determine the optimal decision rules. We do this by solving the period problem backwards. That is, we consider the situation faced by a household who has m" dollars in hand and has to decide the level of consumption of the divisible good. Given our hypothesis about the value function, the continuation utility of such a household is given by,

$$c + \beta\{V_0(M_t) + v_1 e\mu[(m_{t+1}+R_{t+1}b_{t+1})/M_t],$$

with,

$$m_{t+1} = m'' +(M_t e^{-1})e - (M_t e^{-1})c,$$

and,

$$(M_t e^{-1})c \leq m''.$$

It is easy to check that if $\beta v_1 \mu \leq 1$, the optimal decision is not to accumulate cash balances, that is, to set $(M_t e^{-1})c=m''$, and $m_{t+1} = (M_t e^{-1})e$. Thus, every family starts the next period with cash balances equal to the average money supply in the previous period, $m_{t+1} = M_t$. We later give conditions under which,

$$(7.6) \quad \beta v_1 \mu \leq 1,$$

is satisfied.

The indirect utility function before the beginning of the divisible good market period is then,

$$(7.7) \quad m''e/M_t + \beta\{V_0(M_t) + v_1 e\mu[1+(R_{t+1}b_{t+1})/M_t].$$

Let's consider the previous stage. As described before, there are two simultaneous dimensions: the buyer's problem, and the seller's problem. Each has to act independently. In general this would pose serious coordination problems. However, in our setting, separability implies that each member of the family can ignore the other's decision. Consider the informed buyer who meets a seller who has posted a price equal to p. Let χ_s denote a random variable that is one if the seller part of the family produces and sells, and zero otherwise. Let p_s the price that the seller part of the family announces. Let the buyer have beliefs over the actions of the seller's part of the family, and let the expectation using those beliefs be denoted by E. (It is not necessary that the buyer have rational beliefs.). If the buyer accepts the offer of the seller he/she has been matched with the family's payoff is ,

$$v^b + e(m' - p + E[p_s\chi_s])/M_t + E[v^s(1-\chi_s)] + \beta\{V_0(M_t) + v_1 e\mu[1+(R_{t+1}b_{t+1})/M_t]\}.$$

If the buyer rejects the offer, the payoff is,

$$v^b + e(m' + E[p_s\chi_s])/M_t + E[v^s(1-\chi_s)] + \beta\{V_0(M_t) + v_1 e\mu[1+(R_{t+1}b_{t+1})/M_t]\}.$$

Thus, the optimal decision rule is,

$$\begin{cases} \text{buy if} & p \leq M_t e v^b \\ \text{do not buy if} & p > M_t e v^b. \end{cases}$$

This decision rule coincides with (3.1.1). Thus, this establishes that buyers in a dynamic setting behave as buyers in the static framework discussed above. To standardize notation, note that we have implicitly used the following law of motion for the stock of cash,

$$m'' = m' - p + p_s\chi_s, \qquad \text{if the buyer accepts the offer,}$$

$$m'' = m' + p_s\chi_s, \qquad \text{if the buyer rejects the offer.}$$

We now consider the seller's part of the family. The seller knows M_{t-1}, and is paired with an informed buyer. Let χ_b be a random variable that equals one if the buyer member of the family purchases one unit of the indivisible good, and zero otherwise. Similarly, let p_b be the price -- unknown to the seller-- that the buyer side of the family will face. Recall that the seller does not even know m', since this has been chosen by the buyer in the financial market and they have not communicated. The seller has beliefs over all these variables, and the expectation is denoted by E. If the seller announces price P, the expected payoff is,

$$E[v^b\chi_b] + eE[(m'-p_b\chi_b)/\gamma_t]/M_{t-1} + P\int_{1/m^U}^{v^b/(P/M_{t-1})} x f(x)\,dx + (1 - F(\frac{v^b}{(P/M_{t-1})}))v^s +$$

$$\beta E_{t-1}\{V_0(M_t) + v_1 e\mu[1+(R_{t+1}b_{t+1})/M_t]\}.$$

It is clear that the only part of the value function that is relevant for the choice of P is,

$$P\int_{1/m^U}^{v^b/(P/M_{t-1})} x f(x)\,dx + (1 - F(\frac{v^b}{(P/M_{t-1})}))v^s \quad .$$

Maximization of this term with respect to P is equivalent to maximizing (3.1.3), with the solution $p=P/M_{t-1}$. Thus, the uninformed seller's optimal choice is the same as in the static case.

Next consider the buyers choice of portfolio. To simplify notation we will impose rational expectations on the part of the buyer. Although not necessary for the argument, it greatly reduces the number of terms. In particular, since the buyer is informed at this stage, he/she can "calculate" all the equilibrium prices. In addition, the buyer knows that if he/she chooses to buy the indivisible good, the seller part of the pair will announce the same price, and will sell with probability one. This implies that the seller knows that whatever money balance he/she chooses at this stage, the same amount will be available for the purchase of divisible goods. In other words, in equilibrium, trading in the indivisible good does not increase or decrease money balances as either both members of the family are active in this market or both --as well as the rest of the economy--

inactive. The value function is,

$$(7.8) \quad v^b\chi + (em')/M_t + v^s(1-\chi) + \beta\{V_0(M_t) + v_1 e\mu[1+(R_{t+1}b_{t+1})/M_t],$$

where χ is random from the point of view of the seller, with $\chi=1$ if the realization of M_t is such that the indivisible good is traded, and $\chi=0$ if the outcome is no trade. The seller maximizes this value function subject to the budget constraint given by (7.3) which, for convenience, we repeat,

$$(7.3) \quad m' + b_{t+1} \leq m_t + N_t + R_t b_t.$$

It is easy to check that for the seller to be indifferent between money and bonds it must be the case that the nominal interest rate is,

$$(7.9) \quad R_{t+1} = (\beta v_1 \mu)^{-1}.$$

There are two things to note. First, given assumption (7.6) (which still has to be verified), (7.9) implies a positive nominal interest rate. Second, it is useful to consider the sequence of money balances for the representative consumer. By representative we mean that the initial money balances, m_t, equal average money balances, M_{t-1}, and that individual bond holdings, b_t, equal average bond holdings which are zero.

For the representative agent, $m_t + N_t + R_t b_t = M_{t-1} + (\gamma_t-1) M_{t-1} = M_t$. Thus, since the family is indifferent between holding money and bonds, we can choose $m'=M_t$. It also follows that $m''=M_t$. This, in turn, yields our quantity theory equation (7.4) since, in equilibrium, $c=e$, and $eq_t = m''$.

To complete the argument we need to show that our guess about the value function is correct, and that (7.6) is satisfied.

From (7.8) we get that the equilibrium level of utility conditional on information available at time t-1 is given by,

$$E_{t-1}\{v^b\chi + v^s(1-\chi)\}+ E_{t-1}\{[em_t + (\gamma_t-1)M_{t-1}]/(\gamma_t M_{t-1})\} + \beta E_{t-1}\{V_0(\gamma_t M_{t-1}) + v_1 e\mu\},$$

where we have imposed, without loss of generality, $b_{t+1} = 0$. As before, χ is a random variable that takes the value 1 if the realization of the money supply is such that the indivisible good gets produced and traded, and 0 otherwise. Collecting terms and using the fact that $E_{t-1}\{1/(\gamma_t M_{t-1})\}= \mu/M_{t-1}$, we get,

$$E_{t-1}\{v^b\chi + v^s(1-\chi)\}+ E_{t-1}\{(\gamma_t-1)/\gamma_t\} + \beta E_{t-1}\{V_0(\gamma_t M_{t-1}) + v_1 e\mu\} + e\mu m_t/M_{t-1}.$$

For our guess to be verified we need,

$$(7.10a) \qquad v_1 e\mu = e\mu,$$

$$(7.10b) \qquad V_0(M_{t-1}) = E_{t-1}\{v^b\chi + v^s(1-\chi)\} + E_{t-1}\{(\gamma_t-1)/\gamma_t\} + \beta E_{t-1}\{V_0(\gamma_t M_{t-1}) + v_1 e\mu\}.$$

The first condition is satisfied if v_1 equals 1. The existence of a function V_0 follows from the observation that the mapping defined by the right hand side of (7.10b) is a contraction mapping (see Stokey and Lucas (1989), Theorem 3.3).

Since (7.6) requires $\beta v_1\mu = \beta\mu \leq 1$, this imposes some restrictions on the expected value of the inverse of the inflation rate. For the class of distributions that we consider satisfaction of this constraint requires,

$$\beta k(1+e^{2m^*k})/(e^{2m^*k}-1) \leq 1.$$

This condition is more likely to be satisfied for smaller values of the variance of the growth rate of the money supply --as indexed by k-- and for higher values of the mean growth rate of the money supply m*. It is weaker (i.e., implied by) than the requirement that the lowest realization of the growth rate of the money supply is strictly greater than one.

Finally, the dynamic version adds some restrictions to equilibrium nominal interest rates. Interest rates are constant and given by,

$$R = (e^{2m^*k}-1)/[\beta k(1+e^{2m^*k})].$$

The model implies that nominal interest rates increase with mean inflation, even though the elasticity is less than one. Note, however, that the expected real interest rate --the expectation of the properly deflated nominal interest rate-- is constant and equal to β^{-1}. Increases in uncertainty, as measured by increases in k, reduce the level of nominal interest rates. Thus, “high” nominal interest rates are not necessarily a good indicator of the potential welfare losses associated with high inflation variability. In this model the opposite can be true: for a given level of mean inflation, increases in uncertainty can decrease income, welfare and nominal interest rates.

We summarize the results of this section in the following Proposition.

Proposition 7.1. Consider a buyer-seller household with an infinite horizon, in a setting in which the buyer side of the household is informed and the seller uninformed. Then, if (m,k*) and the discount factor β satisfy $\beta k(1+e^{2m^*k})/(e^{2m^*k}-1) \leq 1$,

i) The equilibrium decision rules coincide with those derived for the static case (Proposition 3.1.3)

ii) The equilibrium one period nominal interest rate is $R = (e^{2m^*k}-1)/[\beta k(1+e^{2m^*k})]$, which is a non-linear function of both average inflation, m*, and its variability, k.

7. Conclusion

In this paper we have presented a model in which perfectly anticipated inflation is superneutral: if the variance of the money (or the growth rate of the money supply in the dynamic interpretation) supply is zero, the real equilibrium is independent of the mean of the money supply. Moreover, any changes in the money supply process that do not change its coefficient of variation are neutral as well. On the other hand, it was shown that increases in the variability of the money supply holding its mean constant can have substantial real effects. The key feature is that --even when the decision to become informed is endogenous-- there is ex-post asymmetric information. This asymmetry gives the possessor of information an "informational monopoly." This informational monopoly can result in both distributional and allocative effects.

The nature of these non-neutralities depends in a complicated way on the size of the gains from trade. In the case of large gains from trade, increased policy uncertainty results in information gathering because knowledge of the money supply confers an informational monopoly. In the case in which the potential gains from trade are smaller, the situation is more complex. Changes in the magnitude of uncertainty change the value of information for two reasons. First, there are objective gains from being informed (i.e., knowledge of how much a dollar buys). Second, through the impact on the probability of trade, higher variance reduces the probability of trade and, hence, the value of information.

These distributional effects capture the idea that changes in the variability of the money supply affect the value of the good "knowledge of the actual value of the money supply." Since the equilibrium prices depend on the value of this variable, a more volatile money supply affects prices. More precisely, in the case in which an informed buyer meets an uninformed seller, for small levels of variability of the money supply (a situation in which sellers would like to sell with probability one), an increase in the variance of the money supply makes the "poorest" potential buyer poorer. In this setting, the seller lowers the price in order to induce all the buyers to acquire the good. Efficiency --which in this simple setting corresponds to trading the good-- is unaffected by a higher variance in the money supply, but buyers are better off since they face lower prices. Thus, the informational monopoly partially compensates for the economic monopoly enjoyed by sellers in this model.

In addition to distributional issues, there are allocative effects of changes in the variability of monetary policy. In the setting of this paper there is a loss of efficiency if the indivisible good is not traded. This, indeed, is sometimes the case. Again consider the match of an informed buyer and an uninformed seller. An increase in the variance of the money supply decreases the value (in terms of consumption) of any given monetary quantity. If the variance of the money supply is sufficiently high, the seller "protects" himself by announcing a very high price. Of course, when the realization of the money supply is low, the ex-post real price of the indivisible good is high, and the buyer refuses to purchase it. Thus, in some cases, the good is not traded giving an efficiency loss.

There is a second mechanism though which variable monetary policy affects efficiency. This is through the amount of resources devoted to acquiring information. Because there are private rents associated to being informed, it is in the private interest of economic agents to acquire information up to the point where the gains from being informed equal the private benefits. The costs of acquiring information are social costs. They correspond to real resources. On the other hand, the social benefits of information are zero: Being informed confers an advantage relative to the other party but no overall gain. We showed that --when the valuation of the buyers and the sellers are close-- individual agents will engage in information acquisition for intermediate values of the variance of money supply. The fraction of informed traders is constant up to a given level of the standard deviation of the money supply, and then decreases. For high levels of variability, no agents are informed. On the other hand, when the difference in the valuation of the indivisible good between the two agents is large, there is always a positive fraction of the population engaged in information acquisition. This is in spite of the fact that the good is traded with high probability (and even probability one in an arbitrarily large region). Here, it is basically the monopoly power (or the distributional effects) associated with information that drives private decisions.

In both cases, we obtain the result that for low levels of variability of the money supply the equilibrium is efficient. Thus, this model is consistent with the view that “small” amounts of uncertainty about a policy do not have any aggregate effects. It is important to emphasize that in this model, policy variability is hard to justify except, as a possible second best policy, in the case of a government that wants to redistribute income. Overall, the picture that emerges is that policy uncertainty can be quite costly.

What does the model say about some popular views associated with “well functioning” markets? Overall, it fails to support the view that nominal prices that fully reflect monetary shocks are in some sense associated with better allocations than “sluggish prices.” More specifically, for low variability of the money supply we show that the equilibrium price is constant (independent of the money supply) and the allocation is efficient. In an intermediate range of variability of the money supply prices are an affine (but not linear) function of the realization of the money supply. If we compute the ratio of the standard deviation of prices and the money supply, it is always less than one. Moreover, this ratio is not monotonic in the variance of the money supply: it displays an inverted U shape in a region in which allocations are not efficient.

The model is also consistent with cross-sectional price dispersion. Whenever the buyers' and sellers' valuation of the indivisible good are close, we observe cross-sectional price dispersion only for intermediate values of our measure of policy uncertainty. In the case in which the buyers' and sellers' valuations of the indivisible good are far apart, the model predicts that there will be cross sectional price variability for all values of policy uncertainty greater than a given threshold. We also show that the variance of the cross sectional distribution of prices is not a monotonic function of the realization of the money supply: for small values of M, an increase in M decreases the cross sectional variance, while the result is opposite for large values of M.

Although we emphasize the effect of policy variability on the price of the indivisible good, the model contains two goods, and the price of the divisible good (consumption) is “fully flexible” and --by choice of units-- equal to the realization of the money supply, M. Thus, in addition to its implications about overall level and variability of prices, the model has implications about the relative price of divisible and indivisible goods.

There are three dimensions of the model that we have held constant: the relative valuations of the two agents (within two broad ranges), the cost of acquiring information and the mean money supply. From our discussion, it follows that changes in the mean money supply holding the coefficient of variation constant (i.e. holding k constant) are superneutral. Prices increase proportionally, but the real allocation remains unchanged. As one suspects, a lower cost of acquiring information increases the region in which there is a mixed strategy equilibrium. In terms of the welfare consequences of an increase in the cost of information there are two results. For some values of k an increase in the cost of information eliminates the mixed strategy Nash equilibrium and this results --using aggregate output as a measure-- in a welfare increase. However, an increase in c in the region in which the equilibrium is in mixed strategies reduces aggregate welfare.

Next, consider the impact of reducing the differences in valuation between buyers and sellers. In the case of small differences ($\epsilon<1/2$), a decrease in ϵ (valuations closer to each other) has mixed effects. First it decreases k_1^*, and hence the region in which the probability of trade is one; it also increases q --the probability of trade-- for each value of k. Finally one can show that although it does not affect the low end of the region in which there is a mixed strategy equilibrium, it reduces the high end. In the case where differences are large ($\epsilon \geq ½$), this decrease increases the region in which the good is traded with probability one (it increases k_2^*) and, hence, it is welfare improving. In addition, it increases the region in which the fraction of the population that chooses to be informed is maximal. From this perspective, it decreases welfare.

There are two extensions of the model that are of particular interest. The first is to endogeneize the valuations of buyers and sellers. To do this, it is necessary to explicitly describe a production and a storage technology and to consider a dynamic model. We hope that such an extension will shed light on the production versus storage/speculation decisions that seem to characterize an important trade off that producers face in high uncertainty environments. The second extension is to drop the assumption that the information structure is common knowledge. That is, when the parties are matched they do not know whether the other party is informed. Preliminary results show that for the high variance case the value of information does not decrease as k increases and, hence, that there is a Nash equilibrium in mixed strategies for all (high) values of k.

It is clear that this model, as is, is not adequate to conduct quantitative analyses of the effects of variable monetary policies. There are at least two important considerations that we have, by construction, ignored. These cut in oppostie directions in terms of the welfare cost of inflation. First, we have not allowed for agents to meet after they reach a state of common knowledge. Because of this, the analysis probably does not apply to situations of repeat transactions. For this reason, the model as is, will overstate the welfare cost of varaibility in inflation. On the other hand, in contrast to the standard cash goods/credit goods treatment of the impact of inflation, the welfare cost that we find extends to all nominally denominated transactions, and not only to those in which cash is the medium of exchange. This second effect, increases the cost of inflation relative to the one derived in that cash/credit good calculation. Our hope is that by studying a very simple setting we have contributed to identifying new potential channels through which policy variability can affect real allocations. If these new effects are considered important enough we expect that the next generation of models will be more realistic in its description of the economic environment.

BIBLIOGRAPHY

Aiyagari, R., and Z. Eckstein, (1993), "Credit Goods Production in a Two Sector Monetary Growth Model," *Federal Reserve Bank of Minneapolis Working Paper*.

Akerloff, G., (1970), "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 89, pp:488-500

Barro, R., (1995), "Inflation and Economic Growth," *Bank of England Quarterly Bulletin*, pp:166-176, May. (Another version is in NBER Working Paper 5326, October 1995)

Benabou, R., (1988), "Search, Price Setting and Inflation," *Review of Economic Studies*, LV, pp:353-376.

Benabou, R. and R. Gertner, (1993), "Search with Learning from Prices: Does Increased Inflationary Uncertainty Lead to Higher Markup," *Review of Economic Studies*, 60, 69-94, January.

Casella, A. and J. Feinstein, (1990), "Economic Exchange During Hyperinflations," *Journal of Political Economy*, Vol. 98, No. 1, pp:1-27.

Chwe, M., (1995?), "Money Neutrality is Equivalent to Common Knowledge and Game Theoretic Bargaining Foundations of the Phillips Curve," Department of Economics, University of Chicago.

Diamond, P.A., (1993), "Search, Sticky Prices and Inflation," *Review of Economic Studies*, 60, pp:53-68, January.

DeGregorio, J and F. Sturzenegger, (1994), "Credit Markets and the Welfare Costs of Inflation," NBER Working Paper No. 4873, October.

Eden, B., (1994), "Inflation and Price Dispersion," Bank of Israel Research Department Working Paper No. 94.13, August.

Fischer, S., (1991), "Growth, Macroeconomics and Development," *NBER Macroeconomics Annual 1991*, Vol. 6, ed. Olivier Jean Blanchard and Stanley Fischer, pp. 329–64. Cambridge, Mass.: MIT Press/National Bureau of Economic Research.

___________, (1993), "The Role of Macroeconomic Factors in Growth," Working Paper 4565. National Bureau of Economic Research.

Heymann, D and A Leijonhufvud, (1995), *High Inflation*, Clarendon Press, Oxford.

Imrohoroglu, A. and E. Prescott, (1991), "Seignorage as a Tax: A Quantitative Evaluation," *Journal of Money, Credit and Banking*," 23, pp:462-475.

Jovanovic, B. and M. Ueda, (1996), "Contracts and Money,"Working paper, University of Pennsylvania, June.

Kennan, J. and R. Wilson, (1993), "Bargaining with Private Information," *Journal of Economic Literature*, Vol XXI, pp:45-104, March.

Kormendi, R.L and P. G. Meguire, (1985), "Macroeconomic Determinants of Growth: Cross-Country Evidence," *Journal of Monetary Economics*, 16, pp:141-163, September.

Lach, S. and D. Tsiddon, (1992), "The Behavior of Prices and Inflation: An Empirical Analysis of Disaggregated Price Data," *Journal of Political Economy*, Vol. 100, No. 2, pp:349-389, April.

Lucas, R. E. Jr, (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, pp:103-124, April.

______________ (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, pp:103-124, April.

______________ (1987), *Models of Business Cycles*, Basil Blackwell.

______________ (1989), "The Effects of Monetary Shocks When Prices are Set in Advance, Working Paper, November.

Lucas, R. E. Jr, and M. Woodford (1993), "Real Effects of Monetary Shocks in an Economy with Sequential Purchases," NBER Working Paper No. 4250, January.

Myerson, R. and M. R. Satterthwaite, (1983), "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, Vol. 29, No. 2, pp:265-281, April.

Myerson, R., (1985), "Analysis of Two Bargaining Problems with Incomplete Information," in *Game Theoretic Models of Bargaining*, A. Roth, Ed., Cambridge University Press, Cambridge.

Samuelson, W., (1984), "Bargaining Under Asymmetric Information," *Econometrica*, Vol. 52, No. 4, July, 995-1006.

Sheshinski, E., and Y. Weiss, (1993), *Individual and Aggregate Price Dynamics*. MIT Press.

Stokey, N.L. and R.E. Lucas (with E.C. Prescott), (1989), *Recursive Methods in Economic Dynamics*, Harvard University Press.

Tommasi, M , (1994), “The Consequences of Price Instability on Search Markets: Towards Understanding the Effects of Inflation,” *American Economic Review*, Vol 84, No. 5, pp:1385-1396, December.

Appendix A

In this appendix, we present the proof that the equilibrium outcome of the bargaining game presented in section 2.1, where the seller moves first by setting a price (take it or leave it) and the buyer reacts optimally to this offer (and then the game ends) coincides with the outcome of the mechanism that maximizes the seller's utility subject to incentive compatibility and individual rationality on the part of the buyer.

Although not identical, the proof follows the arguments in Myerson (1985) and Samuelson (1984) very closely.

A mechanism is described by a probability of trading the indivisible good as a function of the state, M, q(M) and a state contingent transfer of the divisible good, T(M) from the buyer to the seller. With this notation, the utility of the buyer is given by:

$$U^b (M) = v^b q(M) + (m_b - T(M))/M = v^b q(M) + \lambda - T(M)/M$$

while that of the seller is given by:

$$U^s (M) = v^s (1-q(M)) + (m_s + T(M))/M = v^s - v^s q(M) + 1-\lambda + T(M)/M.$$

The mechanism, (q(M), T(M)) is feasible if:

1. $0 \leq q(M) \leq 1$ for all M.
2. For all M, M', $v^b q(M) + \lambda - T(M)/M \geq v^b q(M') + \lambda - T(M')/M$.
3. For all M, $v^b q(M) + \lambda - T(M)/M \geq v^b 0 + \lambda - 0/M = \lambda$.

Here, 2 invokes the revelation principle to get that truth telling is an equilibrium (i.e., incentive compatibility holds) and 3 captures individual rationality.

Lemma A.1. A mechanism is feasible if and only if:

1. $0 \leq q(M) \leq 1$ for all M.
2. For all M, M', $M v^b q(M) - T(M) \geq M v^b q(M') - T(M')$.
3. For all M, $M v^b q(M) - T(M) \geq 0$.

Proof: The proof follows directly from the definition of feasibility.

Now, let $\pi(M,M') = M v^b q(M') - T(M')$. This is the utility that the buyer will receive if the true state is M and he announces that the state is M'. Then, using incentive compatibility, one obtains:

Lemma A.2. For any feasible mechanism:

1. $v^b (M-M') q(M') \leq \pi(M,M) - \pi(M',M') \leq v^b (M-M') q(M)$.

2. $\pi(M) \equiv \pi(M,M)$ is Lipschitz continuous with Lipschitz constant v^b (and hence differentiable almost everywhere).
3. q is weakly increasing in M.
4. $d\pi(M)/dM = v^b q(M)$ everywhere π is differentiable.

Proof: 1 follows from applying incentive compatibility. 2 follows immediately from 1. 3 follows from 1 with $M \geq M'$. 4 follows from 1 by taking the limit as (M-M') goes to zero.

Lemma A.3: The mechanism is individually rational for the buyer if and only if $\pi(m^L) \geq 0$.

Proof: This follows from Lemma A.1, 2 and the fact that π is increasing (from Lemma A.2, part 4).

Our next goal is to characterize the utility of the seller for any feasible mechanism. Recall that this is given by:

$$U^s = 1 - \lambda + \int_{m^L}^{m^U} \left[v^s - v^s q(m) + \frac{T(m)}{m} \right] f(m)\, dm = 1 - \lambda + v^s + \int_{m^L}^{m^U} \left[\frac{T(m)}{m} - v^s q(m) \right] f(m)\, dm$$

The idea here is to get rid of T(m)/m in this expression by using what we have already shown about the buyers utility. To do this, note that we can write

$$U^b + U^s = 1 + v^s + \int_{m^L}^{m^U} \left[v^b - v^s \right] q(m) f(m)\, dm$$

and,

$$U^b = \int_{m^L}^{m^U} \left[v^b q(m) + \lambda - \frac{T(m)}{m} \right] f(m)\, dm = \lambda + \int_{m^L}^{m^U} \pi(m) \frac{f(m)}{m} dm$$

Using a change of variables and the fact that π is monotone, we can rewrite this is as:

$$U^b = \lambda + \pi(m^L) \int_{m^L}^{m^U} \frac{f(m)}{m} dm + \int_{m^L}^{m^U} v^b R(s) q(s)\, ds$$

where R(s) is defined by:

$$R(s) = \int_{s}^{m_U} \frac{f(m)}{m} dm.$$

Given this, we can write:

$$U^s = v^s + 1 - \lambda - \pi(m^L) \int_{m^L}^{m^U} \frac{f(m)}{m} dm + \int_{m^L}^{m^U} \left[v^b - v^s - v^b \frac{R(m)}{f(m)} \right] f(m)\, q(m)\, dm.$$

In order to find the mechanism that maximizes the utility of the seller, it is clear that $\pi(m^L) = 0$. Thus, it follows that the mechanism that maximizes the seller's utility is that which maximizes

$$U^s = v^s + 1 - \lambda + \int_{m^L}^{m^U} \left[v^b - v^s - v^b \frac{R(m)}{f(m)} \right] f(m)\, q(m)\, dm.$$

where q must lie between 0 and 1 and must be weakly increasing. Since U^s is linear in q it follows that the solution to this problem is to set q equal to either 0 or 1 only. Thus, we have:

Proposition A.1. The mechanism that maximizes the utility of the seller is of the form q(m) = 0 for all m in $[m^L, m']$ and q(m) = 1 for all m in $(m', m^U]$ for some choice of m'. Thus, the outcome of this mechanism is the same as that of the bargaining game analyzed in sections 2.1 and 2.2.

Proof: That the mechanism is as described follows from the argument given above. To see that this coincides with the bargaining game, note that incentive compatibility requires that the transfer T(M) be the same for all m's with the same probability of exchanging the object. Thus, T(M) can take two possible values: T_1 when the probability of trade is one, and T_0 when the probability of trade is zero. We now argue that T_0 must equal zero. To see this recall that if q(M) is ever zero -- the only relevant case as otherwise it follows that T(M) is constant-- it must be zero at $M=m^L$. However, at this point $\pi(m^L) = -T(m^L) = -T_0$. Since we have shown that $\pi(m^L) = 0$, it follows that $T_0=0$. Hence, the outcome of this mechanism is the same as one which transfers T_1 from the buyer to the seller and charges the price, $p = T_1$ anytime the object is traded. If $q(m^L)=1$, it follows that q(M)=1 for all M, and that in this case the object is always traded and the seller receives a payment equal to $T_1=p$.

Appendix B

In this appendix we describe the welfare of buyers and sellers as derived in section 3 in a way that is more useful to the analysis of the Nash equilibrium in section 4. We first consider the payoffs corresponding to the case in which an informed buyer meets an uninformed seller (I,U).

From section 3.1, denote the seller's payoff as, $W^s(I,U,k)=1-\lambda + v^b\Upsilon^s(I,U,k)$. Using (3.1.8a) and (3.1.8b), it follows that,

$$\Upsilon^s(I,U,k) \begin{cases} \Upsilon^s_1(I,U,k), & \text{if } k \leq k^*_1 \\ \Upsilon^s_2(I,U,k) & \text{if } k > k^*_1, \end{cases}$$

where,

$$\Upsilon^s_1(I,U,k) = (1+e^{2m^*k})/2e^{2m^*k},$$

$$\Upsilon^s_2(I,U,k) = (e^{2m^*k}-1)^{-1}\{(1-\epsilon)e^{2m^*k}-(1-2\epsilon)^{½})\}.$$

It is possible to show that,

(i) $\Upsilon^s(I,U,k)$ is a continuous function of k,
(ii) $\Upsilon^s(I,U,k)$ is a decreasing function of k,
(iii) $\lim_{k\to 0} \Upsilon^s(I,U,k) =1$, $\lim_{k\to\infty} \Upsilon^s(I,U,k)= (1-\epsilon)$,
(iv) $\Upsilon^s(I,U,k)$ is a differentiable function of k, except at k^*_1.

Similar expressions hold for the buyer. Denote the buyer's expected utility by $W^b(I,U,k)=\lambda + v^b\Upsilon^b(I,U,k)$ where,

$$\Upsilon^b(I,U,k) \begin{cases} \Upsilon^b_1(I,U,k)=[1-(1+e^{2m^*k})/2e^{2m^*k}] , & \text{if } k \leq k^*_1 \\ \Upsilon^b_2(I,U,k)= (e^{2m^*k}-1)^{-1}[((1-2\epsilon)^{-½}-(1-2\epsilon)^{½})/2 +((1-2\epsilon)^{-½}-1)], & \text{if } k > k^*_1. \end{cases}$$

Direct calculations show that,

(i) $\Upsilon^b(I,U,k)$ is a continuous function of k,
(ii) $\Upsilon^b(I,U,k)$ is increasing for $k \leq k^*_1$ and decreasing for $k > k^*_1$,
(iii) $\lim_{k\to 0} \Upsilon^b(I,U,k)=0$, $\lim_{k\to\infty}\Upsilon^b(I,U,k)=0$,
(iv) $\Upsilon^b(I,U,k)$ is a differentiable function of k, except at $k=k^*_1$.

Consider next the case in which an uninformed buyer meets an informed seller (U,I). From Proposition 3.2.1, and if $W^s(U,I,k)=1-\lambda + v^b\Upsilon^s(U,I,k)$, it follows that,

$$\Upsilon^{s}(U,I,k) \quad \begin{cases} 1 & \text{if } k \le k_2^* \\ \Upsilon_2^{s}(U,I,k) & \text{if } k \ge k_2^*, \end{cases}$$

where,

$$\Upsilon_2^{s}(U,I,k) = 1-\epsilon + 2\epsilon^2 e^{2m^*k}/[(1+\epsilon)(e^{2m^*k}-1)].$$

Note that $\Upsilon^{s}(U,I,k)$ satisfies:

(i) $\Upsilon^{sb}(U,I,k)$ is a continuous function of k,
(ii) $\Upsilon^{s}(U,I,k)$ is decreasing for $k \ge k_2^*$,
(iii) $\lim_{k\to 0} \Upsilon^{s}(U,I,k)=1$, $\lim_{k\to\infty}\Upsilon^{s}(U,I,k)=1-\epsilon + 2\epsilon^2/(1+\epsilon)$,
(iv) $\Upsilon^{s}(U,I,k)$ is a differentiable function of k, except at $k=k_2^*$.

Appendix C

It is convenient to prove first a series of auxiliary results in the form of lemmas.

Lemma C.1 Let $\epsilon<1/2$, and assume $c<\epsilon v^b$. For each c, there is a unique value of k, denoted $k_1(c)$, such that,

i) $v^b\Upsilon^s(I,U,k_1(c)) = v^b-c$.

ii) $v^b\Upsilon^s(I,U,k) > v^b-c$, for $k < k_1(c)$, and $v^b\Upsilon^s(I,U,k) < v^b-c$ for $k > k_1(c)$.

iii) $k_1(c)$ is decreasing in c and converges to 0 as c goes to zero.

Proof: The results trivially follow from the properties of the function $\Upsilon^s(I,U,k)$ as given in Appendix B. There it is shown that $\Upsilon^s(I,U,k)$ is a decreasing function of k and that $\Upsilon^s(I,U,0)=1$. These properties imply our results.

■

Lemma C.2 Let $\epsilon<1/2$ and assume that $c<\epsilon v^b$. For each c, there exist two distinct values of k, denoted $k_2(c)$ and $k_3(c)$ (without loss of generality assume that $k_3<k_2$) such that,

i) $v^b\Upsilon^b(I,U,k_i(c)) = c$.

ii) $v^b\Upsilon^b(I,U,k) < c$, for $k<k_3(c)$ and $k>k_2(c)$, and $v^b\Upsilon^b(I,U,k)>c$ for $k \in (k_3(c),k_2(c))$.

iii) $k_2(c)$ is increasing in c and converges to ∞ as c goes to zero.

Proof: It follows trivially from the properties of the function $\Upsilon^b(I,U,k)$ described in Appendix B. The key properties are that $\Upsilon^b(I,U,k)$ is a continuous function of k, $\Upsilon^b(I,U,0)=0$, $\lim_{k\to\infty}\Upsilon^b(I,U,k)=0$, and that $\Upsilon^b(I,U,k)$ is increasing for $k\leq k_1^*$ and decreasing for $k>k_1^*$. Finally, the assumption that c is small implies $\Upsilon^b(I,U,k_1^*) > c$. The convergence of $k_2(c)$ to zero as c goes to ∞ follows from $k_2(c)>k_1^*$, and from $\lim_{k\to\infty}\Upsilon^b(I,U,k)=0$, and $\Upsilon^b(I,U,k)$ decreasing for $k>k_1^*$. ■

Lemma C.3 Let $\epsilon<1/2$ and assume that $c<\epsilon v^b$. Then, $k_1(c)=k_3(c)<k_1^*<k_2(c)$.

Proof: The two inequalities follow trivially from the previous result. We next prove that $k_1(c)=k_3(c)$. For $k<k_1^*$, the calculations in Appendix B show that $v^b\Upsilon^s(I,U,k)+ v^b\Upsilon^b(I,U,k)=v^b$. It follows that if $k=k_1(c)$ we have $v^b\Upsilon^s(I,U,k_1(c))+ v^b\Upsilon^b(I,U,k_1(c))=v^b$. This equality is just, $-c+v^b\Upsilon^b(I,U,k_1(c),\epsilon)=0$. However, for $k<k_1^*$ this is the definition of $k_3(c)$.

■

These three results show that $k_1(c)$ is the minimal value of k such that the seller is better off being informed. For values of k less than $k_1(c)$ the uninformed seller who meets an informed buyer realizes higher utility than an informed seller. It follows that no seller will purchase information whenever $k<k_1(c)$. Moreover, buyers who are guaranteed to meet an uninformed seller will choose to be informed if $k_1(c) < k < k_2(c)$. If k is outside that interval, a buyer who meets an uninformed seller is better off --in expected utility terms-- by choosing to be uninformed.

Next, we characterize the values of k for which the buyers are better off purchasing information when they know they will meet an uninformed seller. (This is the only case in which they have any hope of extracting part of the surplus of the match.)

Proof of Theorem 4.1:

i) It follows from Lemmas C.1-C.3

ii) For $k \in (0,k_1(c))$, it follows that $v^b\Upsilon^s(I,U,k) > v^b$-c, and $v^b\Upsilon^b(I,U,k) < c$. Inspection of the payoffs in Table 1 show that playing (U,U) is an equilibrium in dominant strategies, as both players are better off not acquiring information.

iii) At $k = k_1(c)$, we have that $v^b\Upsilon^s(I,U,k) = v^b$-c, and $v^b\Upsilon^b(I,U,k) = c$. First, note that in all cases if the buyer is uninformed, the seller's best response is to be uninformed. This is because in the (U,U) case the seller extracts all the surplus and it does not have to pay for the information. In this region (U,U) is an equilibrium as well. If the buyer is informed, the seller is indifferent between being informed or uninformed (this follows from $v^b\Upsilon^s(I,U,k) = v^b$-c). Thus, the seller's best response to the buyer being informed is either U or I. On the other hand, if the seller chooses U, the buyer's best response is either U or I. However if the seller selects I, the buyer's best response is U. It then follows that the outcome (I,U) is another equilibrium outcome. These arguments also show that a mixed strategy equilibrium of the type describe always exist, since buyers are indifferent between I and U, whenever sellers play U.

iv) The region $k \in (k_1(c),k_2(c))$ corresponds to the case $v^b\Upsilon^s(I,U,k) < v^b$-c, and $v^b\Upsilon^b(I,U,k) > c$. Let $\pi^b(k)$ and $\pi^s(k)$ be, respectively, the probability that buyers and sellers play I. Consider first the payoff of the buyers given the strategy of the sellers. They are given by,

If the buyer chooses U, he/she gets λ.
If the buyer choose I, he/she gets $\pi^s(k)(\lambda-c) + (1-\pi^s(k))(\lambda+ v^b\Upsilon^b-c)$.

For the buyer to be indifferent between the two, it must be the case that $-\pi^s(k)c+(1-\pi^s(k))(v^b\Upsilon^b-c)=0$, or $\pi^s(k)=(v^b\Upsilon^b-c)/v^b\Upsilon^b$. Now, consider the sellers' best response. The seller's payoffs are given by,

If the seller chooses U, he/she gets $\pi^b(k)(1-\lambda+v^b\Upsilon^s(I,U))+(1-\pi^b(k))(1-\lambda+v^b)$.
If the seller chooses I, he/she gets $\pi^b(k)(1-\lambda+v^b) + (1-\pi^b(k))(1-\lambda+v^b\Upsilon^s(U,I)) - c$.

Thus, for sellers to be indifferent between the two actions it must be the case that,

$$\pi^b(k) = [(c/v^b)+(1-\Upsilon^s(U,I))]/[(1-\Upsilon^s(I,U)) +(1-\Upsilon^s(U,I))].$$

It is straightforward to check that these probabilities are between zero and one, and hence that these are equilibrium mixed strategies.

v) At the point $k=k_2(c)$, we have shown that $v^b\Upsilon^s(I,U,k) < v^b-c$, and $v^b\Upsilon^b(I,U,k) = c$. This is the region where, if they knew they would meet an uninformed seller, buyers would be indifferent between acquiring information or not. It turns out that the value of information is zero in this case. Thus, the unique equilibrium is (U,U). Finally, for $k>k_2(c)$, the value of information is too small and U is a dominant strategy for both players. Thus, the unique Nash equilibrium is (U,U). ■

Appendix D: Large Differences in Valuation

In this appendix we summarize the results corresponding to the case in which the gains from trade are large. This formally corresponds to setting $\epsilon \geq 1/2$. First, note that in the cases in which both buyers and sellers share the same information ((U,U) or (I,I)) or in the case of an uninformed buyer meeting an informed seller, (U,I), the equilibrium decisions are independent of the value of ϵ.

The one case where it makes a difference is when an informed buyer meets an uninformed seller, (I,U). In this setting, the analog of Propositions 3.1.4-3.1.6 is the following.

Proposition D.1. Let $\epsilon \geq ½$. The equilibrium variables are given by,

i) $p(I,U,k) = v^b m^*(e^{2m^*k}-1)/[2m^*ke^{2m^*k}] = v^b m^* \Phi_1(k)$. Thus, the equilibrium price is decreasing in k and converges to zero as k goes to infinity.

ii) $q(I,U,k) = 1$ for all k.

iii) $W^s(I,U,k) = 1-\lambda+v^b(1+e^{2m^*k})/2e^{2m^*k}$, $W^b(I,U,k) = \lambda+v^b[1-(1+e^{2m^*k})/2e^{2m^*k}]$, and $W(I,U,k) = 1 + v^b$.

Thus, the case in which the buyers and sellers variations are quite different (this is also the case in which the gains from trade are the largest) is special in the sense that there is no inefficiency. More volatile monetary policies (higher k) redistribute income from sellers to buyers. In the limit --when k goes to infinity-- both buyers and sellers split the surplus. Thus, the limiting case resembles the prediction of a bargaining model with no discount factor. It is clear that policy uncertainty can "redistribute bargaining power" even in a setting in which, nominally, one of the parties has all the bargaining power.

What happens when information is endogenous? The analog of Table I in section 4 is given by,

Table D.I: Payoffs in the First Stage ($\epsilon \geq ½$)

Seller Buyer	Uninformed (U)	Informed (I)
Uninformed (U)	$1-\lambda + v^b$ λ	$1-\lambda + v^b \Upsilon^s(U,I) - c$ λ

Informed (I)	$1-\lambda+v^b(1+e^{2m^*k})/2e^{2k}$ $\lambda+v^b[1-(1+e^{2m^*k})/2e^{2m^*k}]$ - c	$1-\lambda + v^b$ -c λ-c

It is clear that the seller would find profitable to purchase information when this increases his expected payoff when he meets an informed buyer. This is equivalent to,

$$1-\lambda+v^b(1+e^{2m^*k})/2e^{2m^*k} \leq 1-\lambda+v^b-c,$$

or,

$$1-(1+e^{2m^*k})/2e^{2m^*k} \geq c/v^b.$$

There is a unique k that makes this expression an equality. Denote such a k by $\hat{k}(c)$. It follows that for $k \geq \hat{k}(c)$, the sellers' dominant strategy is to acquire information. Inspection of Table D.I shows that a buyer would like to acquire information if and only if $k \geq \hat{k}(c)$. Thus, in this case the Nash equilibrium can be simply described as,

Theorem D.1 Let $\epsilon \geq ½$. For all values of k a symmetric Nash equilibrium exists and is unique. If $c/v^b \geq ½$, the Nash equilibrium strategies are (U,U). If $c/v^b<1/2$, the Nash equilibrium strategies are given by,

i) If $k<\hat{k}(c)$, the Nash equilibrium strategies are (U,U).

ii) If $k=\hat{k}(c)$, the Nash equilibrium strategies are (I,U).

iii) If $k>\hat{k}(c)$, the unique symmetric Nash equilibrium is a mixed strategy equilibrium. Let $\pi^b(k)$ and $\pi^s(k)$ be the fraction (or probabilities) of informed buyers and sellers, respectively. The Nash equilibrium strategy is to be informed with probabilities given by,

$$\pi^b(k) = [(c/v^b)+(1-\Upsilon^s(U,I,k))]/[(1-(1+e^{2m^*k})/2e^{2m^*k})+(1-\Upsilon^s(U,I,k))],$$
$$\pi^s(k) = 1-c/[v^b(1-(1+e^{2m^*k})/2e^{2m^*k})]$$

Proof:

i) In this region, not acquiring information is a dominant strategy for both players. The result trivially follows.

ii) In this region it is possible to show that if all buyers play I, then sellers are indifferent between the two actions {I,U}. If less than 100% of the buyers play I, the sellers' best response is U. On the other hand, if no sellers are informed, buyers are indifferent between the two actions {I,U}. If any fraction of the sellers is informed, the buyers' best response is U. It follows that the unique symmetric Nash equilibrium strategies is for all buyers to be informed, and none of the sellers to acquire any information.

iii) To calculate the mixed strategy equilibrium note that

If the buyer plays U, he/she gets λ.
If the buyer plays I, he/she gets $\pi^s(k)(\lambda\text{-}c) + (1\text{-}\pi^s(k))(\lambda + v^b(1\text{-}(1+e^{2m^*k})/2e^{2m^*k})\text{-}c)$.

For the buyer to be indifferent between the two, it must be the case that $-\pi^s(k)c+(1-\pi^s(k))(v^b(1-(1+e^{2m^*k})/2e^{2m^*k})-c)=0$, or $\pi^s(k)=1-c/v^b(1-(1+e^{2m^*k})/2e^{2m^*k})$ which, by definition of $\hat{k}(c)$, is between zero and one. Now we consider the sellers' best response. The payoffs are given by,

If the seller plays U, he/she gets $\pi^b(k)(1-\lambda+v^b(1+e^{2m^*k})/2e^{2m^*k}))+(1-\pi^b(k))(1-\lambda+v^b)$.
If the seller plays I, he/she gets $\pi^b(k)(1-\lambda)+(1-\pi^b(k))(1-\lambda+v^b\Upsilon^s(U,I)) -c$.

For the seller to be indifferent between the two strategies, it must be the case that $\pi^b(k) = [(c/v^b)+(1-\Upsilon^s(U,I,k))]/[(1-(1+e^{2k})/2e^{2k})+(1-\Upsilon^s(U,I,k))]$. This completes the argument.

■

As in the text, the qualitative behavior of the model depends on the value of k_2^*. In the case of large differences in valuation the key comparison is whether k is lower or higher than k_2^*. The following is a partial characterization for the case $\hat{k}(c)<k_2^*$.

Proposition D.2 Let $\epsilon \geq ½$, and assume that $c/v^b<1/2$, and $\hat{k}(c)<k_2^*$. Then the following obtains:

i) For $k\leq k_2^*$ the probability of trade is one, and it is less than one for $k>k_2^*$.
ii) For $k< \hat{k}(c)$, the unique equilibrium is such that all agents are uninformed ($\pi^b(k)=\pi^s(k)=0$).
ii) The fraction of informed agents given by $(\pi^b(k)+\pi^s(k))/2$ is equal to ½ for all $k\geq\hat{k}(c)\geq k_2^*$, and it less than ½ and decreasing for $k>k_2^*$.
iii) The fraction of informed buyers, $\pi^b(k)$, satisfies,

    a) $\pi^b(\hat{k}(c))=1$, $\pi^b(k)$ is strictly decreasing in k for $k>\hat{k}(c)$, and $\lim_{k\to\infty} \pi^b(k)= [2c/v^b (1+\epsilon)+ 2\epsilon(1-\epsilon)]/[(1+\epsilon) + 2\epsilon(1-\epsilon)]\in (0,1)$,
    b) $\pi^s(\hat{k}(c))=0$, $\pi^s(k)$ is strictly increasing in k for $k>\hat{k}(c)$, and $\lim_{k\to\infty} \pi^s(k)= 1-2c/v^b \in (0,1)$.

iv) Welfare (and income) as given by I(k) satisfies,
    a) $I(k) = 1 +v^b$ for $k<\hat{k}(c)$.
    b) $I(k) = 1 + v^b - c$ for $k\geq\hat{k}(c)\geq k_2^*$.

v) Income gross of information costs is constant and equal $1+v^b$ for $k\leq k_2^*$.

Note that in this case, the only situation in which the agents fail to trade is whenever an uninformed buyer meets an informed seller and the variability of the money supply --as measured by k-- exceeds the critical value k_2^*, and the realization of the money supply is “too low.” In such a case sellers do not believe that buyers will purchase the indivisible good if they announce a low price (they fail to communicate that the money supply is low), and, hence, a high price is announced and the good is not traded.

Figure 1

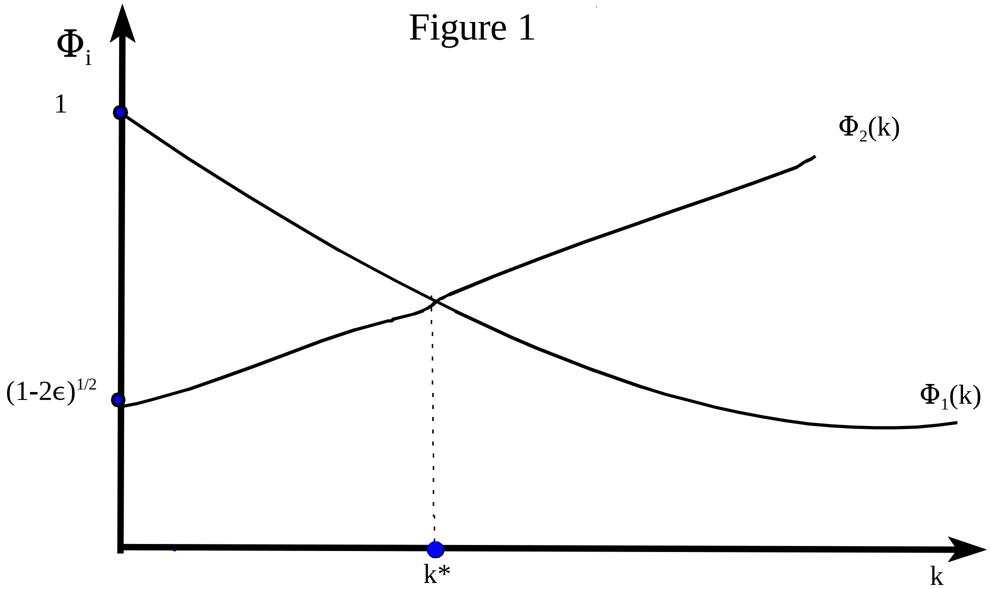

Figure 2

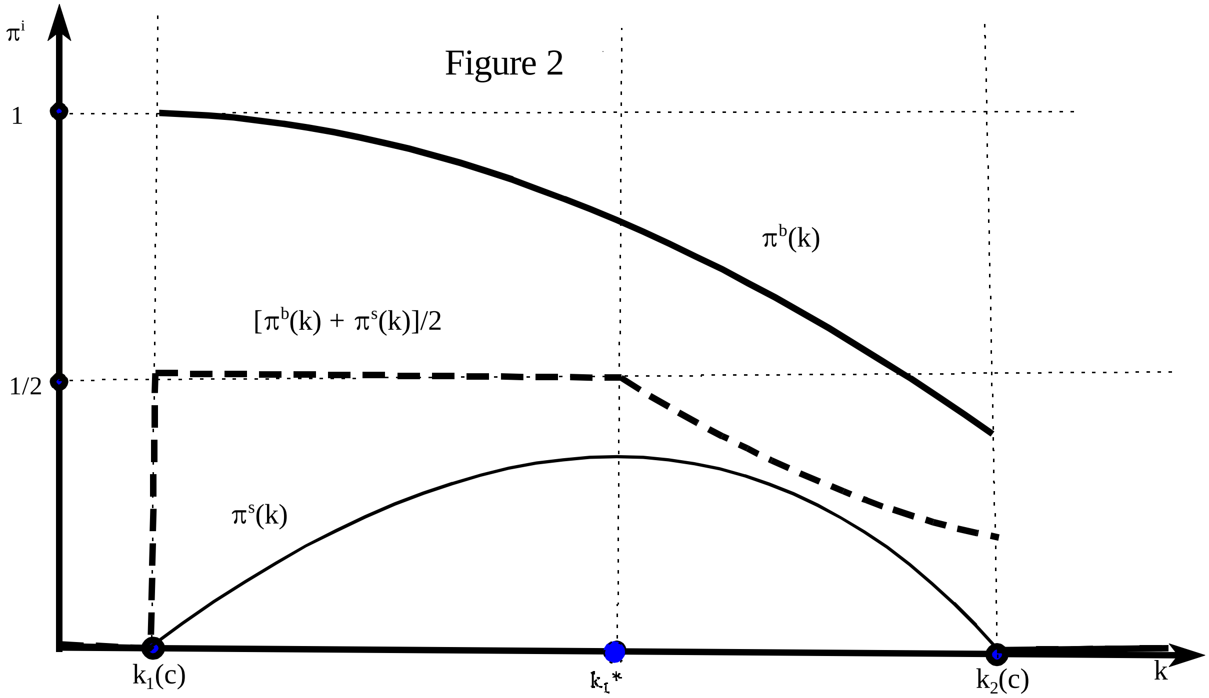

Figure 3